CPS 430/590.06 **Prof. John Reif**
Design and Analysis of Algorithms **Fall 2013**

Question 1

From largest to smallest, the asymptotic size of the given functions are (with lg taken to represent log_2 and lg^{1000} taken to mean $(lg(n))^{1000}$):

$$lg(n^{1000}) \tag{1}$$
$$(lgn)^n \tag{2}$$
$$n^{lgn} \tag{3}$$
$$n^{lglgn} \tag{4}$$
$$(lgn)^{(lgn)} \tag{5}$$
$$lg^{1000}n \tag{6}$$
$$n^2 \tag{7}$$
$$nlgn \tag{8}$$
$$n \tag{9}$$
$$lgn \tag{10}$$
$$n^{(1/lgn)} \tag{11}$$
$$(1+0.001)^n \tag{12}$$
$$lg_{1000}n \tag{13}$$
$$1 \tag{14}$$

Question 2

The answers to this question rely on the discussion of the master method in chapter 4 of the CLRS textbook.

2.1) $T(n) = 2T(n/3) + 1$, so $a = 2, b = 3, f(n) = 1$). Using the master method, this is case 1: $f(n) = O(n^{\log_3 2-\epsilon})$, so $\mathbf{T(n) = \Theta(n^{\log_3 2})}$.

2.2) $T(n) = 5T(n/4) + n$, so $a = 5, b = 4, f(n) = n$. Again this is case 1 of the master method: $f(n) = O(n^{\log_4 5-\epsilon})$, so $\mathbf{T(n) = \Theta(n^{\log_5 4})}$.

2.3) $T(n) = 8T(n/2) + n^3$, so $a = 8, b = 2, f(n) = n^3$. This is case 2 of the master method: $f(n) = \Theta(n^{\log_2 8})$, so $\mathbf{T(n) = \Theta(n^3 lgn)}$

2.4) $T(n) = T(n^{1/2}) + 1$. This recurrence relation cannot be directly translated into the form of the master method, so we use a change of variables $m = lgn$. $S(m) = S(m/2) + 1$, so $a_m = 1, b_m = 2, f(m) = 1$. This is case 2 of the master method, because $S(m) = \Theta(m^{\log_2 1}) = \Theta(mlgm)$. Changing back to our original variables, $T(n) = T(2^m) = S(m) = \Theta(mlgm) = \mathbf{\Theta(lgn\ lglgn)}$

Question 3

Question 4