Homework 6

Due on November 30

Question 1[25 points] Counting heads: Given integer n and k, along with $p_1 ... p_n \in [0, 1]$, you want to determine the probability of obtaining exactly k heads when n biased coins are tossed independently at random, where p_i is the probability that the ith coin comes up heads. Give an $O(n^2)$ algorithm for this task. Assume you can multiply and add two numbers in $[0, 1]$ in $O(1)$ time.

Question 2[25 points] Show that if $P = NP$ then the RSA cryptosystem can be broken in polynomial time.

Question 3[25 points] A $kite$ is a graph on an even number of vertices, say $2n$, in which n of the vertices form a clique and the remaining n vertices are connected in a "tail" that consists of a path joined to one of the vertices of the clique. Given a graph and a goal g, the $KITE$ problem asks for a subgraph which is a kite and which contains $2g$ nodes. Prove that $KITE$ is NP-complete.

Question 4[25 points] In the MULTIWAY CUT problem, the input is an undirected graph $G = (V, E)$ and a set of terminal nodes $s_1, s_2, ..., s_k \in V$. The goal is to find the minimum set of edges in E whose removal leaves all terminals in different components.
(a) Show that this problem can be solved exactly in polynomial time when $k = 2$.
(b) Give an approximation algorithm with ratio at most 2 for the case $k = 3$.
(c) Design a local search algorithm for multiway cut.