

Homework 2

Due on September 19

Please keep in mind that a solution usually has three parts: description of your algorithm, proof of correctness, complexity analysis.

Question 1[20 points] If all vertices of a binary tree have 0 or 2 children, it is a full binary tree. Define F_n as number of full binary trees with n vertices. (1) Give the value of F_5 by showing all the full binary trees with 5 vertices. (2) Give a recurrence relation for F_n and show that F_n is $\Omega(2^n)$ by induction.

Question 2[20 points] Given an array of size n with duplicated elements, design an $O(n \log n)$ time algorithm to remove all duplicates.

Question 3[20 points] Given an array of size n (no duplicates) which is sorted, design an $O(\log n)$ time divide-and-conquer algorithm to decide whether there exists index j such that $A[j] = j$.

Question 4[20 points] Prove that any array of integers of size n can be sorted in $O(n + (\text{MAX} - \text{MIN}))$ time, where MAX(MIN) is the maximum(minimum) element of the array.

Question 5[20 points] Given two lists of size n_1 and n_2 which are sorted, design an $O(\log n_1 + \log n_2)$ time algorithm to find the i th smallest element of the union list.