

Homework 3

Due on October 10

Question 1[40 points] Define $[m]$ be $\{0, 1, \dots, m - 1\}$, For each following family of hash functions, determine (1) whether it is universal or not; (2) to chose a function, how many random bits we need.

(I) $H = \{h_{a_1, a_2} : a_1, a_2 \in [m]\}$, where m is a fixed prime and $h_{a_1, a_2}(x_1, x_2) = a_1x_1 + a_2x_2 \bmod m$. Each function has signature $h_{a_1, a_2} : [m]^2 \rightarrow [m]$, it means that it maps a pair of integers in $[m]$ to a single integer in $[m]$.

(II) H is the same, but m is a fixed power of 2.

(III) H is the set of all functions $f : [m] \rightarrow [m - 1]$.

Question 2[30 points] For a set of real numbers $\{x_1, x_2, \dots, x_n\}$,

(1) show that the median is the value of u that minimizes function $\sum_i |x_i - u|$;

(2) show that the mean is the value of u that minimizes function $\sum_i (x_i - u)^2$.

Question 3[30 points] A binary counter(unspecified length) supports two kinds of operations: *increment*(increase its value by one), *reset*(set its value be zero). Prove that, if it starts from counter of value zero, any sequence of n operations take $O(n)$ time.