Homework 1

Due on September 10

Question 1 [30 points] 1, n, n^2, lgn, $(lgn)^n$, $nlgn$, $(lgn)^{lgn}$, n^{lgn}, n^{lglgn}, $n^{1/lgn}$, $log_{1000}n$, $lg^{1000}n$, $lg(n^{1000})$, $(1+0.001)^n$. Sort these functions asymptotically from largest to smallest.

Question 2 [25 points] Solve recurrence relations with Θ bound.
(1) $T(n) = 2T(n/3) + 1$
(2) $T(n) = 5T(n/4) + n$
(3) $T(n) = 8T(n/2) + n^3$
(4) $T(n) = T(n^{1/2}) + 1$

Question 3 [20 points] Prove that solution to recurrence relation $T(n) = 2T(n/2) + O(nlogn)$ is $O(nlog^2n)$.

Question 4 [25 points] The Towers of Hanoi is a mathematical game or puzzle. It consists of three rods, and a number of disks of different sizes which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape. The objective of the puzzle is to move the entire stack to another rod, obeying the following rules: (a) Only one disk may be moved at a time. (b) Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod. (c) No disk may be placed on top of a smaller disk.Give an algorithm to solve the problem for n disks. Analyze how many moves it requires, as a function of n, by setting up a recurrence and solving it.