fractional residuals