CS 260 – Homework #7

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Problem 1 (5.20 from text)

A *finite automaton* consists of a set of states, which we shall take to be the integers $1..n$ and a table *transitions*[*state*, *input*] giving a *next state* for each *state* and each *input* character. For our purposes, we shall assume that the input is always either 0 or 1. Further, certain of the states are designated *accepting states*. For our purposes, we shall assume that all and only the even numbered states are accepting. Two states p and q are *equivalent* if either they are the same state, or (i) they are both accepting or both nonaccepting, (ii) on input 0 they transfer to equivalent states, and (iii) on input 1 they transfer to equivalent states. Intuitively, equivalent states behave the same on all sequences of inputs; either both or neither lead to accepting states. Write a program using the MFSET operations that computes the sets of equivalent states of a given finite automaton.

Solution

```
Function equivalent(p,q,transitions):
    if p == q:
        return true
    else:
        if (p%2 == q%2) || ((p%2 > 0) && (q%2 > 0))
            if (transitions[p,0] == transitions[q,0]) &&
            (transitions[p,1] == transitions[q,1])
                return true
```

```
        else
            return false

Function equivalent_states(states, transitions):
    eq_states = Initial(states)
    for i in states:
        for j in states:
            if (equivalent(i, j):
                eq_states.Merge(i,j)
    return eq_states
```

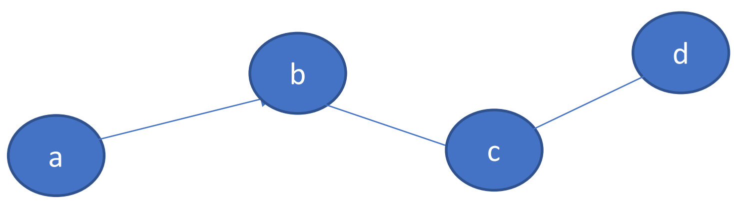
Problem 2

Consider an undirected graph $G = (V,E)$ with $n = |V|$ and $m = |E|$. The degree of a vertex is the number of edges incident on that vertex. Let d_i be the degree of vertex v_i, Show that:

$$\text{SUM}[1..n](\ d_i\ ) = 2m$$

Solution

Consider the following graph:



The nodes have the following degrees: a = 1, b=2, c=2, d=1;
In this instance, n = 4 because it is the number of vertices, and m = 3 because it is the number of edges.

The left-hand side of the equation is just sum of all the degrees, therefore…

1+2+2+1 = 2*3

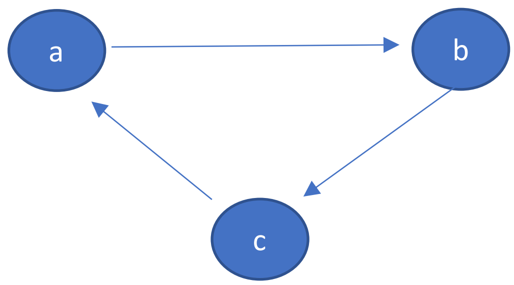
6 = 6

Problem 3

In a directed graph, we can talk about in-degree and out-degree, the number of edges, respectively, arriving and leaving a given vertex. Show that the sum of the in-degrees of a graph is equal to the sum of the out-degrees.

Solution

Consider the following graph:

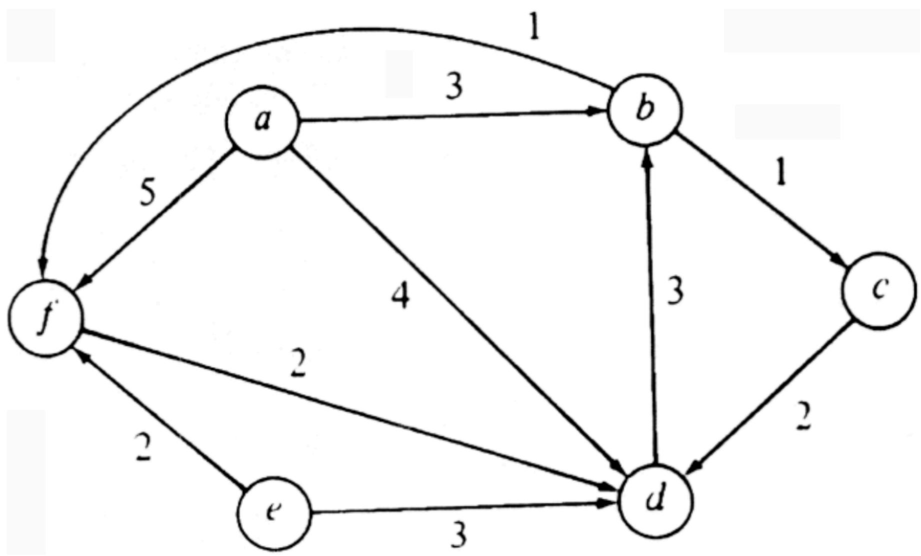


As you can see, every directed edge is coming from one node, into another. Essentially if you were keeping track of the edges, every directed edge would add +1 to both the sum of the in-degrees and out-degrees, which makes both sums equal. For the given graph, this can be seen as each node has each 1 edge directed in and out.

Problem 4 (6.1 from text)

Represent the digraph of Fig. 6.38:

a. By an adjacency matrix give arc costs
b. By a linked adjacency list with arc costs indicated



Solution

a.

	a	b	c	d	e	f
a	0	3	0	4	0	5
b		0	1	3	0	1
c			0	2	0	0
d				0	3	2
e					0	2
f						0

b.