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CS 383-002
Homework #3

1. Theory

1. Linear regression using least squares estimate (LSE).

$$X = \begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix}$$

Column 1 contains the sole feature, column 2 contains the value to be predicted

std of feature = 4.2282
mean of feature = -0.9

Standardized data with bias feature:

$$X = \begin{bmatrix} 1 & -0.2602 \\ 1 & -0.9697 \\ 1 & -0.4967 \\ 1 & 0.2129 \\ 1 & -1.6792 \\ 1 & -0.2602 \\ 1 & 0.4494 \\ 1 & 1.3954 \\ 1 & -0.0237 \\ 1 & 1.6319 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

Calculate coefficients using: $\theta = (X^T X)^{-1}(X^T Y)$

$X^T X = \begin{bmatrix} 10 & -0.0001 \\ -0.0001 & 9.0002 \end{bmatrix}$

$X^T Y = \begin{bmatrix} 14 \\ -15.7040 \end{bmatrix}$

$\theta = (X^T X)^{-1}(X^T Y) = \begin{bmatrix} 10 & -0.0001 \\ -0.0001 & 9.0002 \end{bmatrix}^{-1} \begin{bmatrix} 14 \\ -15.7040 \end{bmatrix}$

$\theta = \begin{bmatrix} 1.4 \\ -1.7449 \end{bmatrix}$

2. Function $g(x) = (x-1)^4,\ where\ x\ is\ a\ single\ value$

a. Gradient with respect to x:

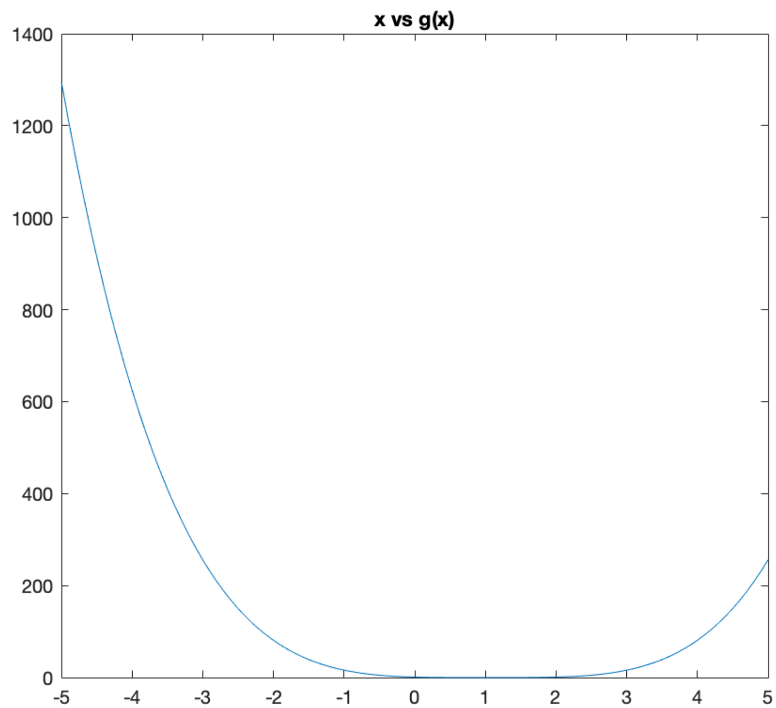
$$\frac{dg}{dx} = 4 * (x-1)^{4-1} * (1) = 4(x-1)^3$$

b. Finding global minimum:

$$0 = 4(x-1)^3$$
$$0 = 4(x^3 - 3x^2 + 3x - 1)$$
$$0 = 4x^3 - 12x^2 + 12x - 4$$
$$4 = 4x^3 - 12x^2 + 12x$$
$$4 = 4x(x^2 - 3x + 3)$$

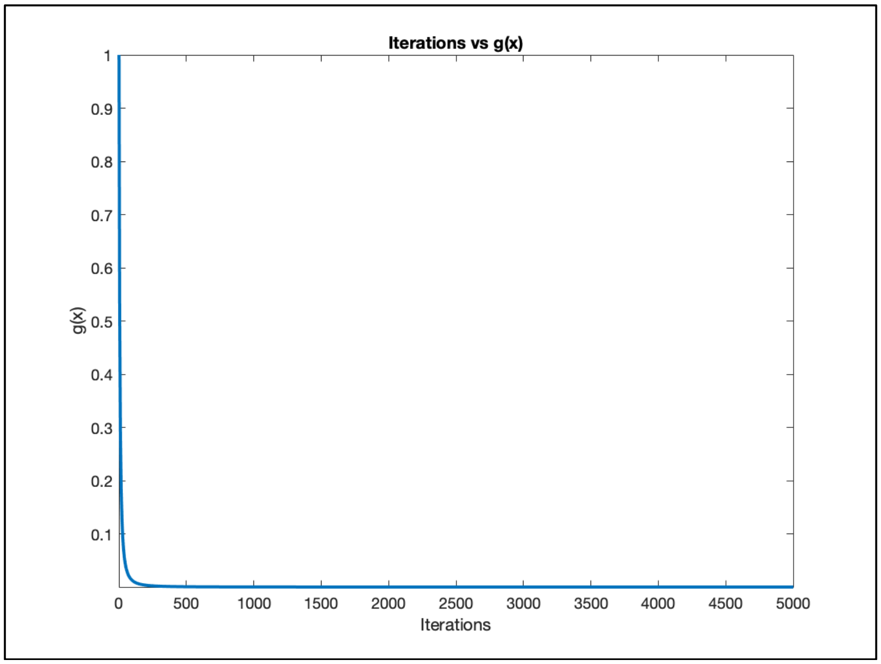
$$4 = 4x => x = 1\ is\ the\ global\ minimum$$

c. Plot x vs g(x) using MATLAB
x = -5 to 5 (in steps of 0.1)



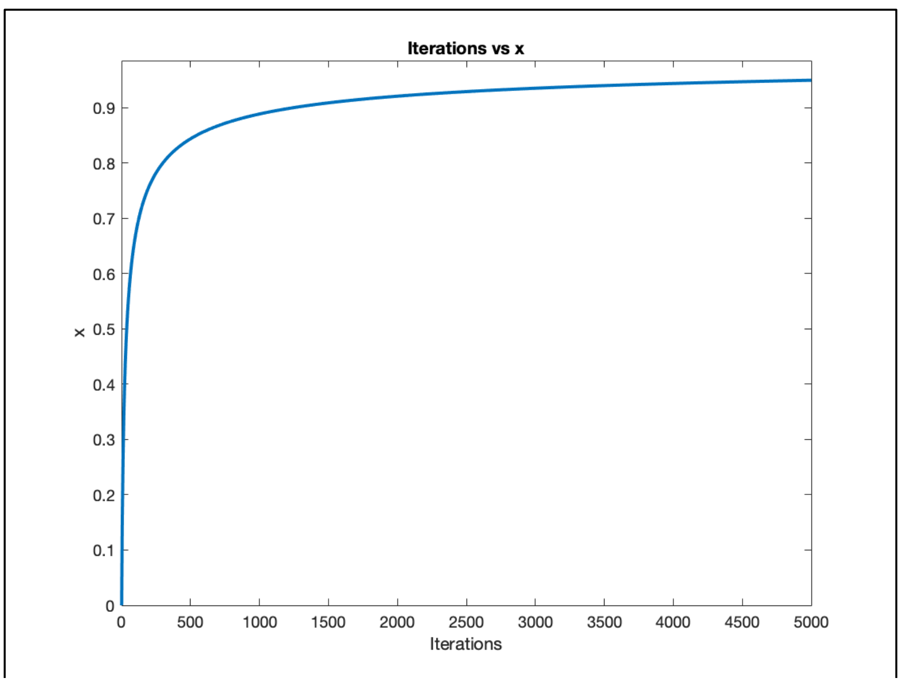
2. Gradient Descent

Fixed Learning Rate



Iterations vs g(x)

Note: X axis adjusted

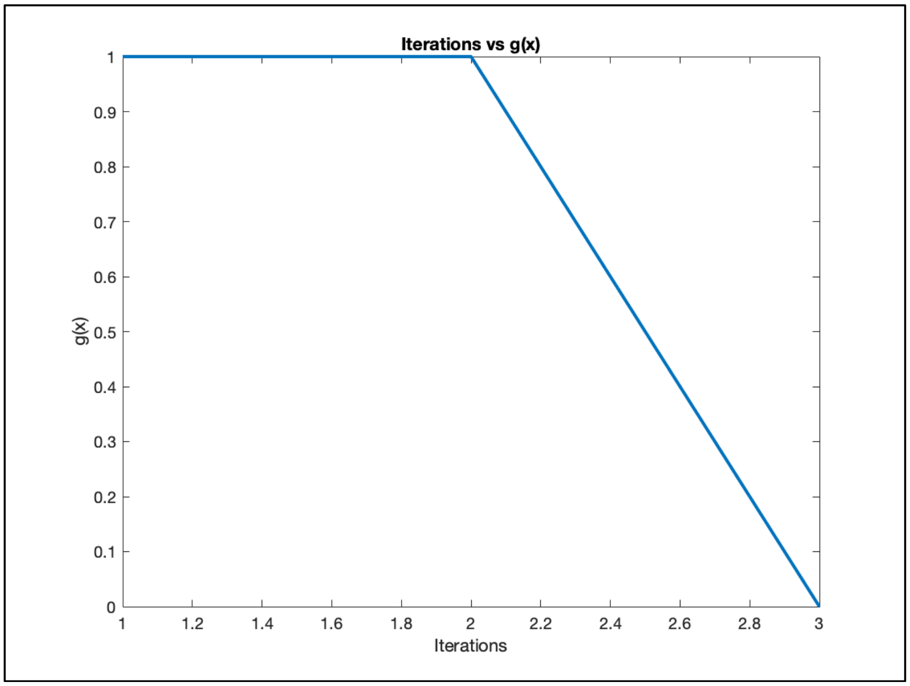


Iterations vs x

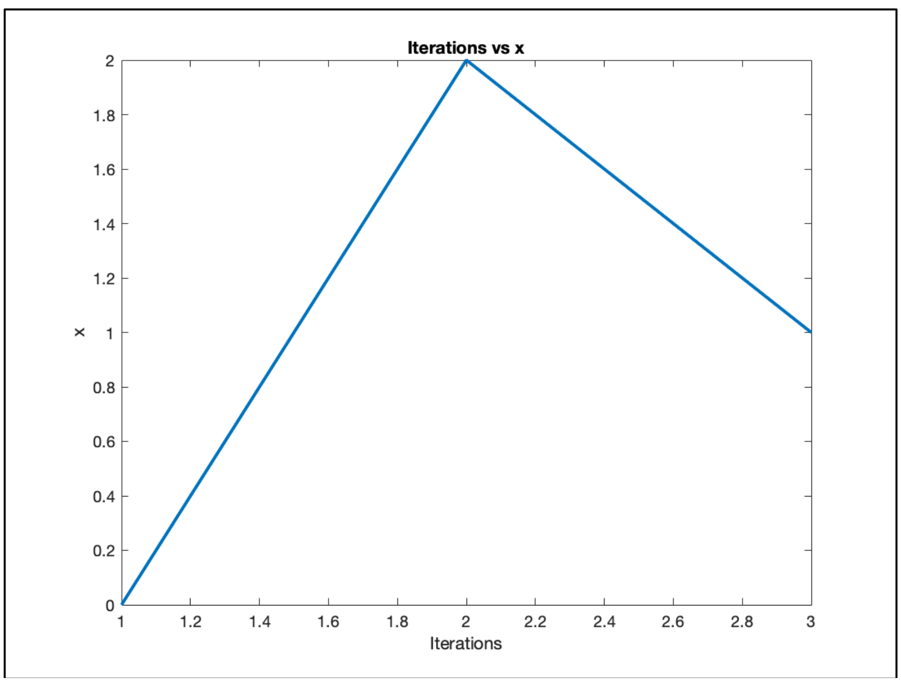
Note: X axis adjusted

Chosen value of n (learning rate) was 0.23

Adaptive Learning Rate



Iterations vs g(x)



Iterations vs x

3. Closed Form Linear Regression

Model: $y = 5404.3105 + 890.5098x_1 - 315.4464x_2$

RMSE	813.2411

4. S-Folds Cross-Validation

S	Average RMSE	Std of RMSE
3	655.9816	45.5404
5	634.3463	23.6618
20	627.0605	12.6290
N (= 44)	623.4051	0