**11.11 worksheet**
Math 1101

---

Work in groups of up to three students. Each group can turn in a single write-up. Be sure to include all group member names. The turned in write-up should be written in a readable fashion so that one can figure out what is going on and all steps that have been carried out by only reading the completed document. When you use Mathematica, you may simply write the code that you used and the result in your write-up. It is not necessary for one person to write up the whole thing; I'll accept work written by multiple people in the group.

This should be completed during class on Friday, 4/12. To allow for you to put finishing touches on your work, turn in your work on Friday, 4/19 along with your other discussion problems.

1. The purpose of this question is to figure out how many terms of the Maclaurin series one has to take in order to ensure that the value of the series can be used to compute $\sin(x)$ to a reasonable level of accuracy.

   Let $x$ be any real number. By adding or subtracting multiples of $2\pi$, we can find a number $x_0$ in the interval $[-\pi, \pi]$ so that $\sin(x) = \sin(x_0)$. This means that to compute $\sin(x)$, it suffices to be able to compute $\sin(x_0)$.

   Suppose that for $-\pi \leq x_0 \leq \pi$, we wish to compute $\sin(x_0)$ correct to 8 decimal places. We can do so by finding $k$ so that $T_k(x_0)$ is within distance $10^{-8}$ of $\sin(x_0)$. The following steps lead you to finding $k$.

   (a) Recall that we can represent $\sin(x)$ as follows:

   $$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

   This is an alternating series. What is $b_n$?

   (b) We know that for a convergent alternating series, $R_k$, the difference between the sum and the $k$th partial sum satisfies

   $$|R_k| \leq b_{k+1}.$$

   Using your answer to part (a), find $k$ so that on the interval $[-\pi, \pi]$,

   $$b_{k+1} < 10^{-8}.$$

   (c) If we use the value of $k$ that we found in part (b), it should be the case that for any $x_0$ in the interval $[-\pi, \pi]$, $T_k(x_0)$ should the value of $\sin(x_0)$, correct to at least 8 decimal places. The function `SetPrecision` can be used to compute things to a specified number of decimal places in Mathematica. For example, `SetPrecision[Pi, 10]` will print $\pi$ to 10 decimal places. Note that $T_k(x)$ involves the first $k$ non-zero terms of the series, so the degree of the last term will be $2k+1$.

Using your value of $k$ from part (b) and $x_0 = 1$, check that $T_k(x_0)$ and $\sin(x_0)$ agree to at least 8 decimal places.

(d) Again using the value of $k$ that you found in part (b), compute the value of $\sin(31.5159)$ correct to 8 decimal places.

2. To complete this question, you should follow Example 1 that begins on page 750 of the textbook. In case you don't have your book, you can find the relevant two pages here: `https://ckatkinson.github.io/1102/files/1111.pdf`

   (a) Approximate the function $f(x) = x^{-2}$ by a Taylor polynomial of degree 3 at $a = 2$.

   (b) How accurate is this approximation on the interval $[1.9, 2.1]$?