Review sheet for exam 1

The exam covers sections 1.1-1.5 and 1.7-1.9. To study for this exam, I highly recommend doing as many of the practice and discussion problems as you can handle. In addition, take a look at the chapter 1 supplementary exercises on pages $88 - 90$. All of $1 - 22$ seem like good problems to think about.

You may use a cheat sheet for this exam. You may use one side of a piece of paper such that the side you use has area no more that 46.75 square inches[1]. This is the equivalent to using one side of one–half of a standard sheet of 8.5in $\times$11in sheet of paper. The only thing that may be written on the other side of the sheet is your name. Your sheet must be handwritten by you. No photocopies. I'll collect your sheets with your exams. They will be returned.

Below, I've listed the topics that the exam will cover. I've phrased many as questions.

- 1.1: Systems of linear equations
  - What is a system of linear equations? What is a solution? What is the solution set? When are two systems equivalent?
  - What's the difference between the coefficient matrix of a system and an augmented matrix for the system?
  - Can you solve some systems? Can you do so using the elementary row operations? What does it mean for augmented matrices to be row equivalent?
- 1.2: Row reduction and echelon form
  - What is echelon form? What is reduced echelon form? Why isn't echelon form unique? Is reduced echelon form unique?
  - How do you use row reduction and reduced echelon form to solve a system? You should know how to systematically do this, but I won't ask you about the details of the formal algorithm for doing so.
  - What is a pivot position? Pivot column?
  - What are leading variables? Free variables? How do you use these to describe solution sets?
  - You should know what Theorem 2 means. We've seen that it's important.
- 1.3: Vector equations

[1] If you try to use a Möbius band to cheat the system, note that it only has one side, so you should make your Möbius band out of a quarter sheet of standard letter paper. Also, you'll have no other side on which to write your name.

- What are vectors in $\mathbb{R}^n$? How does vector addition and scalar multiplication work? What do these operations do geometrically?
- What is a linear combination of vectors? What is the span of a set of vectors? What does it mean to say that a set of vectors spans $\mathbb{R}^m$?
- How are vector equations and linear systems related? What does this have to do with span and linear combinations? You should be able to translate seamlessly between these interpretations to answer questions about vectors and linear systems.

- 1.4: The matrix equation
  - What does a matrix equation of the form $A\mathbf{x} = \mathbf{b}$ mean? Be able to translate seamlessly between systems of linear equations, vector equations, and matrix equations (this is essentially Theorem 3).
  - You should have a good feeling for why Theorem 4 is true. A useful exercise would be to prove the theorem. Doing so will likely cement the connections in your head.
  - Know how to multiply $A\mathbf{x}$.

- 1.5: Solution sets of linear systems
  - What is a homogeneous linear system? What is a trivial/nontrivial solution? How can you tell whether or not a homogeneous system has a nontrivial solution?
  - You should be able to describe the solution sets to nonhomogeneous systems. Can you identify the $\mathbf{p}$ and $\mathbf{v_h}$ parts? What does this even mean? Geometrically, what does this mean? (See Theorem 6 to see what I'm talking about here)

- 1.7: Linear independence
  - What is the definition of linear independence/dependence? Work through some examples. Note the connection with homogeneous systems discussed on page 57.
  - Build up some intuition for linear independence (Theorems 7, 8, 9).

- 1.8: Linear transformations
  - What is a transformation? What makes a transformation a linear transformation? What's a matrix transformation? Is it linear?
  - The properties discussed on page 66 are good to know.

  - Play around with some examples to gain intuition for how linear transformations behave.

- 1.9: The matrix of a linear transformation
  - Is every linear transformation represented by a matrix? See Theorem 10. How do you find such a matrix?
  - What does it mean for a transformation to be one-to-one? Onto? How can you tell if a linear transformation is one-to-one or onto? (Theorems 11 and 12).
  - Intuitively speaking, what do one-to-one and onto mean?