Review sheet for Exam 1

These are **not** sample exam questions, but to be prepared, you should know the material enough to answer them. A useful exercise is to construct an example for as many of the questions as you can.

- 1.1: Introduction
  - What is a mathematical knot? When are two knots of the same knot type?
  - What is a projection or diagram of a knot? What is a crossing in a diagram?
  - Can you prove that knots with one or two crossing diagrams are trivial?
  - What is the unknot recognition problem?
- 1.2: Composition
  - How do you compose two knots? Is the composition unique? Can you construct an example showing that it's (potentially) not unique?
  - What are the factor knots of a composite knot?
  - What happens if you compose a knot with the unknot?
  - What is a prime knot?
- 1.3: Reidemeister moves
  - What is an isotopy? What's the difference between an ambient and planar isotopy?
  - What are the three types of Reidemeister moves (knowing this means that you can draw the six diagrams)?
  - Can you actually perform some Reidemeister moves on an example diagram?
  - What is the point of the Reidemeister moves? What does Reidemeister's theorem say?
- 1.4: Links and the linking number
  - What is a link? Can you sketch a few examples?
  - When is a link splittable?
  - What is a knot or link invariant?
  - What is the definition of the linking number of two components of a link? Can you compute it for some examples? Why is it an invariant? What happens if you change the orientation on a component?

- 1.5: Tricolorability
  - What does it mean to say that a knot or link is tricolorable?
  - Try tricoloring a bunch of knots. For example, which of the knots up through 5 crossings are tricolorable? 6 crossings?
  - Why is tricolorability an invariant?
- 1.6: Stick number
  - What is the stick number of a knot? What is the stick number of the unknot? What is the minimal stick number of a nontrivial knot?
- 2.2: Dowker notation
  - Can you write down a DT sequence for an alternating knot? How about a non-alternating knot? Is your answer unique? How can you get different representations of the same knot?
  - Can you draw a knot by looking at its DT sequence?
  - Can you use a DT sequence to detect composite diagrams?
- 2.3: Tangles
  - What is a tangle?
  - What is an integral tangle? Can you draw them? What is a rational tangle? Can you draw them?
  - Can you draw a tangle that is not rational?
  - How can you tell if two rational tangles are equivalent? Try some examples.
  - Do you know how to multiply and add tangles to get algebraic tangles? Try drawing some.
  - How do you get a knot or link from a rational or algebraic tangle? Given two knots or links presented as rational knots, how can you tell if they are the same or not?
- 2.4: Knots and planar graphs
  - Why are knot and link diagrams checkerboard colorable?
  - How do you generate the Tait graph from a knot or link diagram?
  - Given a Tait graph, can you reconstruct the knot or link that it encodes?
- 3.1: Unknotting number

- What is the definition of the unknotting number? Try to compute it for some examples.
- Why is the unknotting number always finite?

- 3.2: Bridge number
  - What is an overpass? A maximal overpass? What is the bridge number of a knot?
  - What do you know about knots with bridge number 1?
  - What do you know about knots with bridge number 2?
  - Can you draw an example of a knot with bridge number 5? Why is this a difficult task?

- 3.3: Crossing number
  - What is the definition of the crossing number of a knot? Why is it an invariant?
  - Can you compute the crossing number of the knot 5_2 (without using the fact that it has a "5" in its name)?
  - Can you do the previous point without appealing to the Kauffman-Murasugi-Thistlethwaite theorem?
  - What do you know about the crossing number of a composite knot?