Review sheet for exam 2

The exam covers sections 2.1-2.3, 3.1-3.3, and 4.1-4.5. To study for this exam, I highly recommend doing as many of the practice and discussion problems as you can handle. In addition, take a look at the supplementary exercises at the ends of the chapters. In particular, in chapter 2, think about 1-10, 15-18. In chapter 3, think about 1-8, 11-12. In chapter 4, think about 1-11.

You may use a cheat sheet for this exam. **Your sheet must be written by hand by you**. You may use one side of **half** of a sheet of 8.5in $\times$11in paper. I'll collect your sheets with your exams. They will be returned.

Below, I've listed the topics that the exam will cover.

- 2.1: Matrix operations
  - When and how can you multiply two matrices? How do you do it? How does it correspond to composition of linear transformations?
  - What are the basic properties of matrix multiplication? (See Theorem 2)
  - Also, see the WARNINGS on page 98.
  - What is the transpose of a matrix? What are its properties? (Theorem 3)
- 2.2: Inverse matrices
  - What does it mean to say that a matrix is invertible? What about non-singular?
  - How can you find the inverse of a matrix (see algorithm on page 108)? What if the matrix is 2×2? What is the determinant of a 2×2 matrix?
  - What are some of the basic properties of taking inverses? (See Theorem 6)
  - What are the connections between multiplication by elementary matrices, row operations, and inverse matrices? (See Theorem 7)
- 2.3: Characterizations of invertible matrices
  - How can you tell if a matrix is invertible (see Theorem 8 and the theorem on page 235)?
  - What does it mean for a linear transformation to be invertible?
  - If you know the standard matrix for a linear transformation, how can you find the standard matrix for the inverse transformation?
- 3.1: Introduction to determinants

- What is the definition of a determinant? Can you compute it in practice?
- What is a cofactor of a matrix? Can you compute determinants using cofactors?
- What is the determinant of a triangular matrix? What's a triangular matrix?

- 3.2: Properties of determinants
  - How do row operations change the determinant?
  - If you know the pivots of the echelon form of a matrix and how many row interchanges were used to get there, how can you compute the determinant?
  - What does the determinant have to do with deciding if a matrix is invertible?
  - How does transpose affect determinants? How does determinant behave when taking products of matrices? How is the determinant of a matrix related to the determinant of its inverse?

- 3.3: Cramer's rule, volume, and linear transformations
  - What is the relationship between volume or area and determinants (See Theorems 9 and 10)?

- 4.1: Vector spaces and subspaces
  - What is a vector space? Can you name some examples? What are some sets that aren't vector spaces? See if you can come up with examples that fail for various reasons.
  - Know the basic properties ((1), (2), and (3) on page 191).
  - What is a subspace? Remember that it's easiest to check that something is a vector space by showing that it's a subspace of a known vector space. Play with some examples.
  - What is a linear combination of vectors? What is the span of a set of vectors? Remember that the span of a collection of vectors in a vector space is a subspace.

- 4.2: Null spaces, column spaces, and linear transformations
  - What is the null space of a matrix? If A is $m \times n$, then how does the null space relate to systems of equations? What is it a subspace of? Can you find the null space in examples?

- What is the column space of a matrix? How does it relate to systems of equations? What is it a subspace of? Can you compute it in examples?
- What are the kernel and range of a linear transformation? If you conclude that they're analogous to the null space and column space of a matrix, you're thinking the right thoughts.

- 4.3: Linearly independent sets; bases
  - What is a linearly independent set of vectors in a vector space? How can you detect independence (See Theorem 4)?
  - What is a basis for a vector space? What is the standard basis for $\mathbb{R}^n$? What is the standard basis for $\mathbb{P}^n$?
  - Think about the spanning set theorem (Theorem 5). Keep thinking about it until it is obvious to you.
  - How can you compute bases for the null and column spaces of a matrix?

- 4.4: Coordinate systems
  - Know Theorem 7.
  - How do you compute the coordinates of a vector with respect to a basis? Try some examples. Think about what it means graphically.
  - How does the change-of-coordinates matrix work? Try some examples.
  - What is the coordinate mapping? How is it defined? What are it's properties? Think about this until you're convinced that all finite-dimensional vector spaces really look like $\mathbb{R}^n$ for some n.
  - What is an isomorphism?

- 4.5: The dimension of a vector space
  - What is the dimension of a vector space? What does Theorem 9 say about dimension and collections of vectors?
  - Know the basis theorem (Theorem 12).
  - What is the dimension of the null space of a matrix? What about the column space?