Homework 1

Knot Theory - Atkinson

Due at in class on Friday, January 26.

Here are some guidelines that apply to all homework assignments. I won't include this section on future assignments, but you should still follow the guidelines.

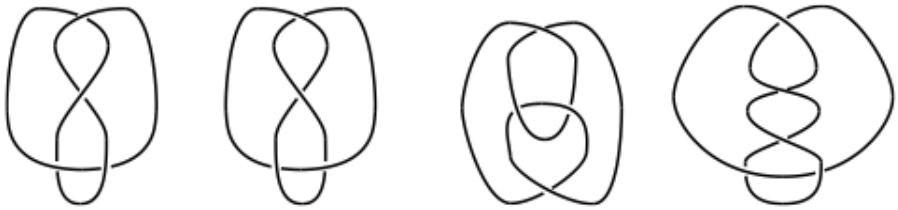
- I will be grading your homework for two things.
  (1) **Mathematical correctness and completeness:** The solutions must be complete, starting from the statement of the problem and ending with the solution or answer. Show all of your steps and include all necessary calculations, diagrams, and graphs. If a calculation uses *Mathematica* or some other computer assistance, include the relevant code and results.
  (2) **Exposition and clarity of communication:** Your solutions should be neatly written up in an organized fashion. Use sentences to explain what you are doing to make it easy for the reader to follow. It will be helpful to explain what your plan of solving the problem is at the beginning for more complicated problems.

  I won't be picky about spelling or grammar, but you should do your best. If English is not your first language, don't worry. You can use this as an opportunity to practice written English.

  In order to more easily write up your solutions neatly, I recommend first figuring out the problem on scratch paper. It is easier to organize your ideas if you have already worked them out.
- I encourage you to work with others in order to figure out how to do the homework problems. The document that you hand in, however, should be entirely your own. No two students' homeworks should be identical in any major way aside from perhaps solution technique. **Include the names of your collaborators.**
- Pages must be stapled or held together with a paper clip. Please do not turn in pages torn from a spiral notebook with jagged, messy edges.

The assignment begins on the next page.

(1) Show that the following projections are actually projections of the same knot. Do so by drawing sequences of pictures that describe the deformations to get from one to another.



(2) Exercise 1.2 on page 4. Prove that every knot with two crossings is trivial.

(3) Exercise 1.8 on page 10. Be sure to also show how to see that the knot in question is composite.

(4) Exercise 1.10 on page 15. You need to show the slow step-by-step process of using only one Reidemeister move at a time.

(5) Sketch a diagram of an unknot having at least 10 crossings. I'll give you two bonus points if your knot is so complicated that I have to use computer assistance to decide if your answer is actually an unknot or not (Yes, Haken's algorithm is not really implementable, but I have my ways...).