5.1 | 8, 14, 24

8| Is $\lambda = 1$ an eigenvalue of $A = \begin{bmatrix} 4 & -2 & 3 \\ 0 & -1 & 3 \\ -1 & 2 & -2 \end{bmatrix}$?

$$A - \lambda I = \begin{bmatrix} 3 & -2 & 3 \\ 0 & -2 & 3 \\ -1 & 2 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 4 & -6 \\ 0 & -2 & 3 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & -2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$-2x_2 + 3x_3 = 0$

$3x_3 = 2x_2$,

so $\vec{v} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$ is a $\lambda = 1$ eigenvector.

14| Find basis for E_3.

$$A = \begin{bmatrix} 4 & 0 & -1 \\ 3 & 0 & 3 \\ 2 & -2 & 5 \end{bmatrix}. \qquad A - 3I = \begin{bmatrix} 1 & 0 & -1 \\ 3 & -3 & 3 \\ 2 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}. \qquad \begin{aligned} x_1 &= x_3 \\ x_2 &= 2x_3 \end{aligned}$$

$$E_3 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}.$$

24| $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ has only one eigenvalue (with mult 2).