5.2 | 8, 12, 18

8 | Find char. poly + eigenvalues.

$$A = \begin{bmatrix} -4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$P_A(x) = \begin{vmatrix} -4-x & 3 \\ 2 & 1-x \end{vmatrix} = -(4+x)(1-x) - 6 = -(4-3x-x^2) - 6$$
$$= x^2 + 3x - 10 = (x+5)(x-2).$$

$\lambda = 2, -5$ are eigenvalues.

12 | Find char poly.

$$P_A(x) = \left|\begin{bmatrix} -1-x & 0 & 2 \\ 3 & 1-x & 0 \\ 0 & 1 & 2-x \end{bmatrix}\right| = -1-x \begin{vmatrix} 1-x & 0 \\ 1 & 2-x \end{vmatrix} - 3 \begin{vmatrix} 0 & 2 \\ 1 & 2-x \end{vmatrix}$$

$$= (-1-x)(1-x)(2-x) - 3(-2)$$
$$= -(1-x^2)(2-x) + 6$$
$$= -(2-2x^2-x+x^3) + 6$$
$$= -x^3 + 2x^2 + x + 4.$$

18 | Find h so that E_4 is 2-dimensional

$$A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$A - 4I = \begin{bmatrix} 0 & 2 & 3 & 3 \\ 0 & -2 & h & 3 \\ 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 0 & 2 & 3 & 3 \\ 0 & 0 & 3+h & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For E_4 to be 2-dimensional, we need $A-4I$ to have 2 free variables. Hence we need

$\underline{h=-3}$

5.3 | 4, 14, 31+32

4 |

$$\begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}^k = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} (-3)^k & 0 \\ 0 & -2^k \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(-3)^k & -2(-2)^k \\ 2(-3)^k & -1(-1)^k \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3(-3)^k + 4(-2)^k & 6(-3)^k - 6(-2)^k \\ -2(-3)^k + 2(-1)^k & 4(-3)^k - 3(-1)^k \end{bmatrix}$$

14 | Diagonalize:

$A = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ $\lambda = 2, 3$ are eigen values.

$\lambda = 2$:

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad x_1 = -x_2, \; x_3 = 0$$

$$E_2 = \text{Span}\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

$\underline{\lambda = 3}$:

$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -2x_3$

x_2 unconstrained.

$$E_3 = \text{Span}\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

So A is similar to $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ via

$$P = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

3.] Construct a 2×2 matrix that is invertible, but not diagonalizable.

How about $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$?

$P_A(x) = (1-x)^2$, so $\lambda = 1$ is only eigenvalue.

$A - 1 \cdot I = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, so $E_1 = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

Hence A is not diagonal~~ta~~ble.

32) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is diagonal(izable), but is not invertible.