3.11 24, 34, 52

24| Differentiate:

$$\frac{d}{dx}\left(\frac{x^2-2\sqrt{x}}{x}\right) = \frac{d}{dx}\left(x - 2x^{-1/2}\right) = 1 + x^{-3/2}.$$

34| Find equation of TL to

$$y = x^4 + 2x^2 - x \quad \text{at } (1,2).$$

$$y' = 4x^3 + 4x - 1$$

$$y'(1) = 4 + 4 - 1 = 7$$

TL: $y - 2 = 7(x-1)$ $\quad y = 7x - 5$

52| For which x does graph of

$y = x^3 + 3x^2 + x + 3$ have horiz tangent?

Need $y' = 0$.

$$3x^2 + 6x + 1 = 0$$

$$x = \frac{-6 \pm \sqrt{36-12}}{6} = \frac{-6 \pm \sqrt{24}}{6} \leftarrow \text{Thee!}$$

Week 7

3.2 20, 30, 36

20) Find z'.

$$z' = \frac{d}{dw}\left(w^{3/2}(w+ce^{w})\right) = \frac{3}{2}w^{1/2}(w+ce^{w}) + w^{3/2}(1+ce^{w})$$

30) Find f' and f'':

$$f(x) = \frac{x}{3+e^{x}}$$

$$f'(x) = \frac{3+e^{x} - xe^{x}}{(3+e^{x})^2}$$

$$f''(x) = \frac{(e^{x} - e^{x} - xe^{x})(3+e^{x})^2 - (3+e^{x} - xe^{x})\left(e^{x}(3+e^{x}) + e^{x}(3+e^{x})\right)}{(3+e^{x})^{\cancel{2}4}}$$

$$= \frac{-xe^{x}(3+e^{x})^2 - 2(3+e^{x} - xe^{x})\left(e^{x}(3+e^{x})\right)}{(3+e^{x})^4}$$

36) a) Find TL to $y = \frac{x}{1+x^2}$ at $(3, 0.3)$.

$$y' = \frac{1+x^2 - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

At $x=3$, $y'(3) = \frac{1-9}{(1+9)^2} = \frac{-8}{100} = -\frac{2}{25}$

TL: $y - 0.3 = -\frac{2}{25}(x-3)$.

(b) See attached

Might as well check my work:

In[3]:=
```
f[x_] := x / (1 + x^2);
f'[x]
```

Out[3]= $-\frac{2\,x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$

In[4]:=
```
Simplify[%]
```

Out[4]= $\frac{1-x^2}{\left(1+x^2\right)^2}$

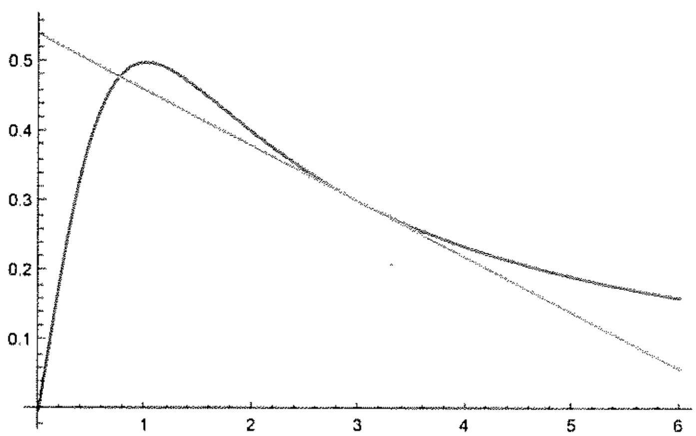
Ok, here's an equation for the tangent line. I've solved for y so that it is a functional description:

In[7]:=
```
y[x_] := -2 / 25 (x - 3) + 0.3;
```

Finally, a plot:

In[8]:=
```
Plot[{f[x], y[x]}, {x, 0, 6}]
```

Out[8]=



3.3 | 10, 24, 34

10 | Differentiate:

$$\frac{d}{dx}\left(\frac{1+\sin x}{x+\cos x}\right) = \frac{\cos x\,(x+\cos x) - (1+\sin x)(1-\sin x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x + \cos^2 x - (1-\sin^2 x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x}{(x+\cos x)^2}$$

24 | Find eq. of TL to $y = \frac{1}{\sin x + \cos x}$ at $(0,1)$

$$y' = \frac{0(\sin x + \cos x) - (\cos x - \sin x)}{(\sin x + \cos x)^2} = \frac{\sin x - \cos x}{(\sin x + \cos x)^2}$$

$$y'(0) = \frac{-1}{(1)^2} = -1.$$

TL: $y - 1 = -x$

$y = -x + 1$

34 | Find the points on $y = \frac{\cos x}{2+\sin x}$ at which TL is horizontal.

$$y' = \frac{-\sin x(2+\sin x) - \cos x(\cos x)}{(2+\sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2}$$

$y' = 0$ iff $\dfrac{-2\sin x - 1}{(2+\sin x)^2} = 0$

$$-2\sin x - 1 = 0$$

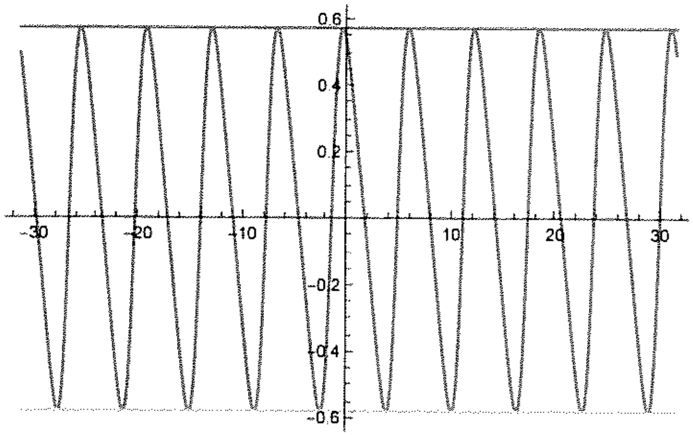
$$\sin x = -1/2$$

$$x = \pi + \frac{\pi}{6} + 2k\pi \quad \text{or} \quad \frac{-\pi}{6} + 2k\pi$$

$$x = \frac{7\pi}{6} + 2k\pi \quad \text{or} \quad \frac{-\pi}{6} + 2k\pi$$

```
f[x_] := Cos[x] / (2 + Sin[x])
tl[x_, a_] := f[a] + f'[a] (x - a) ;
tls1 = Table[tl[x, -Pi / 6 + 2 k Pi], {k, -5, 5}];
tls2 = Table[tl[x, 7 Pi / 6 + 2 k Pi], {k, -5, 5}];
Plot[{f[x], tls1, tls2}, {x, -10 Pi, 10 Pi}]
```

Out[223]=



(Extra work was done with a bunch of tangent lines that all overlap...)