4.9 | 12, 40, 48

12) Find antiderivative.

$$g(x) = \frac{5 - 4x^3 + 2x^6}{x^5} = 5x^{-6} - 4x^{-3} + 2$$

$$G(x) = -x^{-5} + 2x^{-2} + 2x + C$$

40) Find f if $f''(t) = \frac{3}{\sqrt{t}}$, $f(4) = 20$, $f'(4) = 7$.

$f''(t) = 3t^{-1/2}$, so $f'(t) = 6t^{1/2} + C$

$7 = 6 \cdot 4^{1/2} + C$, so

$-5 = C$

$f(t) = \frac{12}{3} t^{3/2} - 5t + D$

$= 4t^{3/2} - 5t + D$

$f(4) = 20 = 32 - 20 + D$

$D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$

48) Find a function f s.t. $f'(x) = x^3$ and $x + y = 0$ is tangent to f.

For $y = -x$ to be tangent to $f'(x)$, need that a point satisfying $f(x) = y$ to also satisfy $y = -x$.

$f(x) = \frac{x^4}{4} + C$, so need $-x = \frac{x^4}{4} + C$ Also, we need that $f'(x) = -1$ at some point. $\therefore$ ~~at~~ $x = -1$ is where line is tangent. Hence.

$1 = \frac{1}{4} + C$, so $C = 3/4$. $f(x) = \frac{x^4 + 3}{4}$