

Review sheet for Exam 2

These are **not** sample exam questions, but to be prepared, you should know the material enough to answer them. A useful exercise is to construct an example for as many of the questions as you can.

You may use one side of one-half of a 8.5" \times 11" sheet of paper (46.75in^2) as a cheat sheet. The sheet of paper that you use should be an orientable (hence two-sided) piece of paper.

4.1: Surfaces without boundary

- What is a surface without boundary? Do you know some examples? Can you build them by gluing up polygons?
- What is a triangulation of a surface? When are two surfaces homeomorphic?
- What is the Euler characteristic of a surface? How do you compute it? Does it depend on the triangulation? Is using a triangulation even the most efficient way to do it? Compute some examples.
- What is the connect sum of two surfaces? How is Euler characteristic affected by connect sum?
- What is the genus of a surface? How are genus and Euler characteristic related?

4.2: Surfaces with boundary

- What is a surface with boundary? Do you know some examples?
- What is the Euler characteristic of a surface with boundary? Compute some examples.
- What does it mean for a surface to be orientable?
- How can we determine if two surfaces are homeomorphic using orientation, Euler characteristic, and number of boundary components?
- What does the classification of surfaces say? Given a surface, can you identify it using the classification theorem? Try some examples.

4.3: Genus and Seifert surfaces

- What is the genus of a knot?
- How does Seifert's algorithm work? Try some examples. Compute the Euler characteristic of some Seifert surfaces. Compute their genus (and note that this might not be the genus of the knot; the genus could be less).
- Which knots have genus 0?
- Do you know a family of projections for which Seifert's algorithm always yields a minimum genus surface?
- How does genus behave under composition? Can you use this prove that the unknot is not a composite knot?

6.1: Jones/Bracket polynomials

- What is a Laurent polynomial?
- What is the definition of the bracket polynomial? Can you compute it? Try some examples. Why isn't the bracket polynomial a link invariant?
- How is the writhe of a projection computed?
- What's the definition of the X polynomial? Can you compute it?
- What's the definition of the Jones polynomial?
- How can you use the X polynomial to check for amphicheirality? What does amphicheirality even mean?

6.2: State sum

- What is an A -split? B -split? What is a state of a link projection?
- What is the state sum formulation of the bracket polynomial?
- Know that the number of crossings in a reduced, alternating projection of a link is an invariant of the link. What is this invariant? What does reduced mean?
- How can you use a checkerboard coloring to detect chirality of a knot?