

Week 2

1.11, 12, 14, 28

12] Solve the system:

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned} \rightsquigarrow \begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

The third row implies $0=5$! This system is inconsistent, so has no solutions.

14] Solve:

$$\begin{aligned} 2x_1 - 6x_3 &= -8 \\ x_2 + 2x_3 &= 3 \\ 3x_1 + 6x_2 - 2x_3 &= -4 \end{aligned} \rightsquigarrow \begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{bmatrix}$$

so $x_3 = 2$, $x_2 + 4 = 3$, so $x_2 = -1$, and

$x_1 - 6 = -4$, so $x_1 = 2$. solution is $(2, -1, 2)$.

28] Construct three different augmented matrices for linear systems whose sol. set is $\lambda_1=3, \lambda_2=-2, \lambda_3=-1$.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$