4.5 | 8, 14, 24

8 | Find a basis for

$$H = \{(a,b,c,d) \mid a - 3b + c = 0\}$$

write $a = 3b - c$, so every element of H has the form

$$(3b-c, b, c, d) = b(3,1,0,0) + c(-1,0,1,0) + d(0,0,0,1).$$

$B = \{(3,1,0,0), (-1,0,1,0), (0,0,0,1)\}$ is a basis. (The vectors clearly span and are independent).

14 | $\dim(\mathrm{Nul}\, A) = \#$ free variables $= 3$

$\dim(\mathrm{Col}(A)) = \#$ pivots $= 4$.

24 | Let $B = \{1, 1-t, 2-4t+t^2\}$ be a basis for $\mathbb{P}_2$. Find

$[p(t)]_B$ where $p(t) = 5 + 5t - 2t^2$.

Write $5+5t-2t^2 = c_1 \cdot 1 + c_2(1-t) + c_3(2-4t+t^2)$

Then $c_1 + c_2 + 2c_3 = 5$

$-c_2 - 4c_3 = 5$

$c_3 = -2$, so

$$[p(t)]_B = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}.$$

4.6 | 4, 16, 24

4]

$$A = \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix} \\ \begin{matrix} \bar{v}_1 & \bar{v}_2 & \bar{v}_3 & v_4 & \bar{v}_5 & \bar{v}_6 \end{matrix} \end{array} \sim B = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 1 & -13 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

rank$(A) = 5$, null $(A) = 1$

Basis for Col$(A) = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_5, \vec{v}_6\}$

Basis for Row$(A) = \{p_1, p_2, p_3, p_4, p_5\}$

$$B \sim \begin{bmatrix} 1 & 0 & -2 & -1 & 4 & 0 \\ 0 & 1 & 0 & 1 & -3 & -2 \\ 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$x_1 = 2x_3 + x_4 - 4x_5$

$x_2 = -x_4 + 3x_5 + 2x_6$

$x_3 = -x_4 + x_6$

$x_5 = x_6 = 0$

So $x_1 = -2x_4 + x_4 = -x_4$

$x_2 = -x_4$

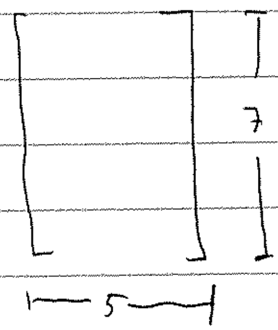
$x_3 = -x_4$

$x_5 = x_6 = 0.$

Nul(A) has basis $\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

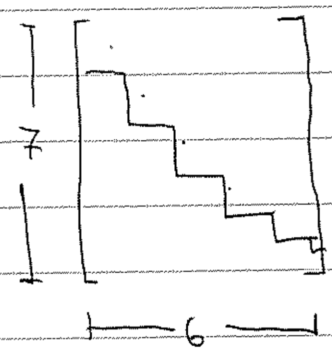
16| If A is 7×5, what is the smallest possible dimension of $Nul(A)$?

$A =$



0! A can have 5 pivots, so can have rank $= 5$.

24| Is it possible for a non-homog system of seven equations in six unknowns to have a unique sol'n for <u>some</u> rHS of constants? For every rHS?



To have a unique solution, we need the matrix of coeffs to have $Nul(A) = 0$. This is possible if $rank(A) = 6$. It cannot have a unique sol'n for all rHS's since $rank(A) \le 6$, so there will always be a row of zeros in ech form.

4.7 2, 6, 14

2) Let $B = \{\vec{b}_1, \vec{b}_2\}$, $C = \{\vec{c}_1, \vec{c}_2\}$ be bases for a vector space V. Suppose

$$\vec{b}_1 = -2c_1 + 4c_2 \text{ and } \vec{b}_2 = 3\vec{c}_1 - 6\vec{c}_2.$$

(a) Find $\underset{C \leftarrow B}{P}$.

$$\underset{C \leftarrow B}{P} = \left[[\vec{b}_1]_C \ [\vec{b}_2]_C \right] = \begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix}.$$

(b) Find $[X]_C$ for $\vec{x} = 2\vec{b}_1 + 3\vec{b}_2$.

$$[\vec{x}]_C = \underset{C \leftarrow B}{P} [\vec{x}]_B = \begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}.$$

6) Let $D = \{\vec{d}_1, \vec{d}_2, \vec{d}_3\}$, $F = \{\vec{f}_1, \vec{f}_2, \vec{f}_3\}$ be bases for v.s. V. Suppose

$$\vec{f}_1 = 2\vec{d}_1 - \vec{d}_2 + \vec{d}_3 \text{ and } \vec{f}_2 = 3\vec{d}_2 + \vec{d}_3 \text{ and}$$
$$\vec{f}_3 = -3\vec{d}_1 + 2\vec{d}_3.$$

(a) $$\underset{D \leftarrow F}{P} = \left[[f_i]_D \right] = \begin{bmatrix} 2 & 0 & -3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}.$$

(b) $$[\vec{f}_1 - 2\vec{f}_2 + 2\vec{f}_3]_D = \underset{D \leftarrow F}{P} [\vec{f}_1 - 2\vec{f}_2 + 2\vec{f}_3]_D = \begin{bmatrix} 2 & 0 & -3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -7 \\ 3 \end{bmatrix}.$$

14) In $\mathbb{P}_2$, find $\underset{\mathcal{E} \leftarrow \mathcal{B}}{P}$ where

$$\mathcal{B} = \{1-3t^2,\ 2+t-5t^2,\ 1+2t\}$$ and

$$\mathcal{E} = \{1, t, t^2\}.$$

$$\underset{\mathcal{E} \leftarrow \mathcal{B}}{P} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{bmatrix}.$$

Write t^2 as a linear combo of elements of $\mathcal{B}$.

$$[t^2]_{\mathcal{E}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad \text{Want } \underset{\mathcal{B} \leftarrow \mathcal{E}}{P} = \left(\underset{\mathcal{E} \leftarrow \mathcal{B}}{P}\right)^{-1}.$$

$$\underset{\mathcal{B} \leftarrow \mathcal{E}}{P} = \begin{bmatrix} 10 & -5 & 3 \\ -6 & 3 & -2 \\ 3 & -1 & 1 \end{bmatrix}, \text{ so}$$

$$[t^2]_{\mathcal{B}} = \begin{bmatrix} 10 & -5 & 3 \\ -6 & 3 & -2 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad \text{and}$$

$$t^2 = 3(1-3t^2) - 2(2+t-5t^2) + 1(1+2t).$$