Week 5

2.5 | 20, 32, 42

20 | Why is

$$f(x) = \begin{cases} \dfrac{2x^2-5x-3}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$$

discontinuous at $a=3$?

$f(3)=6$. What about the limit?

$$\lim_{x\to 3} f(x) = \lim_{x \to 3} \frac{2x^2-5x-3}{x-3} = \lim_{x\to 3} \frac{(x-3)(2x+1)}{x-3}$$

$$= \lim_{x\to 3} 2x+1 = 7$$

$7 \neq 6$, so $\lim_{x\to 3} f(x) \neq f(3)$.

32 | Compute:

$$\lim_{x\to\pi} \sin(x+\sin x) = \sin(\pi + \sin(\pi)) = \sin(\pi+1).$$

$\downarrow$

42] Let

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x<2 \\ ax^2-bx+3 & 2 \le x < 3 \\ 2x-a+b & x \ge 3. \end{cases}$$

Find a, b so that f is continuous.

The limits must agree at the "seams".

$$\lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2} x+2 = 4.$$

$$\lim_{x\to 2^+} ax^2-bx+3 = 4a-2b+3, \text{ so}$$

$$\underline{4a-2b+3=4}$$

$$\lim_{x\to 3^-} ax^2-bx+3 = 9a-3b+3.$$

$$\lim_{x\to 3^+} 2x-a+b = 6-a+b, \text{ so}$$

$$\underline{9a-3b+3=6-a+b.}$$

$\downarrow$

Solve system:

$$\begin{cases} 4a-2b=1 \\ 10a-4b=3 \end{cases}$$

$$2a=1$$

$$\boxed{a=1/2.}$$

$$2-2b=1$$

$$1=2b$$

$$\boxed{b=1/2.}$$

2.6| 12, 18, 26

12) See attached sheet.

18| Find limit:

$$\lim_{y\to\infty} \frac{2-3y^2}{5y^2+4y} = \lim_{y\to\infty} \frac{\cancel{y^2}}{\cancel{y^2}} \cdot \frac{2/y^2-3}{5+4/y}$$

$$= \frac{-3}{5}.$$

26| $$\lim_{x\to-\infty} x+\sqrt{x^2+2x} = \lim_{x\to-\infty} \frac{\left(x+\sqrt{x^2+2x}\right)\left(x-\sqrt{x^2+2x}\right)}{x-\sqrt{x^2+2x}}$$

$$= \lim_{x\to\infty} \frac{x^2-x^2-2x}{x-\sqrt{x^2+2x}}$$

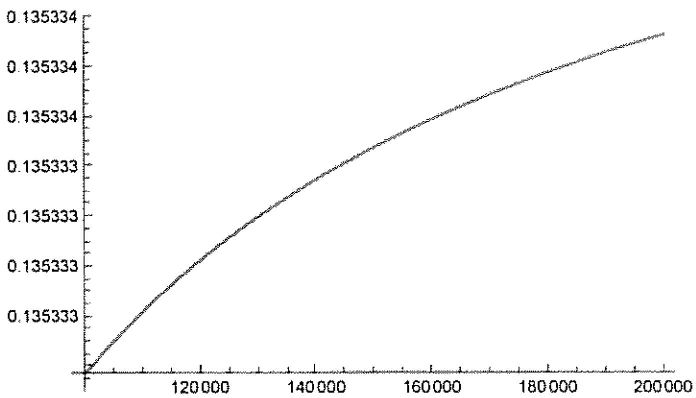
$$= \lim_{x\to-\infty} \frac{-2x}{x-\sqrt{x^2+2x}} = \lim_{x\to-\infty} \frac{-2x}{x-|x|\sqrt{1+2/x}}$$

$$= \lim_{x\to-\infty} \frac{-2x}{x-(-x)\sqrt{1+2/x}} = \lim_{x\to\infty} \frac{-2}{1+\sqrt{1+2/x}} = -1$$

since $x<0$, $|x|=-x$

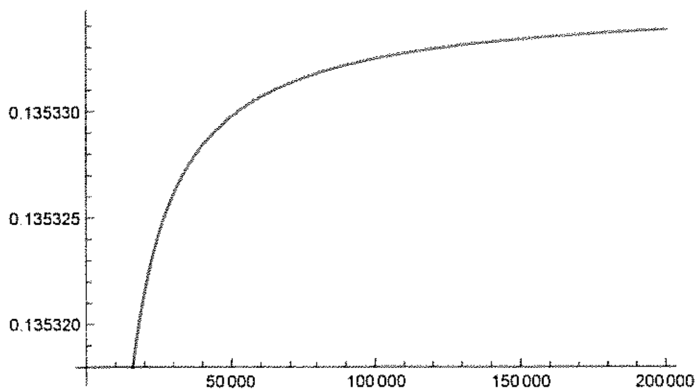
2.6, 12: Consider the function defined below. Use its graph to estimate the limit as x-> infinity correct to two decimal places. My idea is to simply plot the graph far enough out so that all of the y values have the same first two digits.

```
f[x_] := (1 - 2 / x) ^ x
Plot[f[x], {x, 100 000, 200 000}]
```



Looking at this graph, we can see that the values are always near 0.135. Plotting on a bigger range makes it look like this should be the limit.

```
Plot[f[x], {x, 100, 200 000}]
```



For the second part, we are supposed to look at a table of values to estimate the limit correct to four decimal places.

```
Column[Table[{x, N[f[x]]}, {x, 100 000, 110 000, 1000}]]
{100 000, 0.135333}
{101 000, 0.135333}
{102 000, 0.135333}
{103 000, 0.135333}
{104 000, 0.135333}
{105 000, 0.135333}
{106 000, 0.135333}
{107 000, 0.135333}
{108 000, 0.135333}
{109 000, 0.135333}
{110 000, 0.135333}
```

It appears that the limit is approaching 0.1353.

2.7 | 6, 10, 18

6| Find an equation to the tangent line to $y = 2x^3 - 5x$ at $(-1, 3)$.

Let: $f(x) = 2x^3 - 5x$. Then

$$f'(-1) = \lim_{h \to 0} \frac{2(-1+h)^3 - 5(-1+h) - (2(-1)^3 - 5(-1))}{h}$$

$$= \lim_{h \to 0} \frac{2(-1 + 3h - 3h^2 + h^3) + 5 - 5h + 2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{6h - 6h^2 + 2h^3 - 5h}{h}$$

$$= \lim_{h \to 0} 1 - 6h + 2h^2 = 1$$

Eq: $y - 3 = 1 \cdot (x - (-1))$.

10| (a) Find slope of tangent to $y = 1/\sqrt{x}$ at $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h} = \lim_{h \to 0} \frac{1}{h} \left(\frac{\sqrt{a} - \sqrt{a+h}}{\sqrt{a}\sqrt{a+h}} \right)$$

↓

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{a - (a+h)}{\sqrt{a}\sqrt{a+h}\left(\sqrt{a} + \sqrt{a+h}\right)} \right)$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{a}\sqrt{a+h}\left(\sqrt{a} + \sqrt{a+h}\right)} = \frac{-1}{\sqrt{a}\sqrt{a}\,(2\sqrt{a})}$$

$$= \frac{-1}{2a^{3/2}}.$$

(b) Find TL at $a=1$ and $a=4$

$f'(1) = \frac{-1}{2}$, so TL is

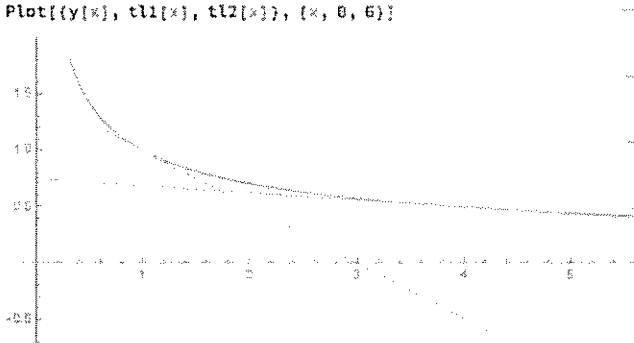
$$y - 1 = \frac{-1}{2}(x-1).$$

$f'(4) = \frac{-1}{2 \cdot 4^{3/2}} = \frac{-1}{16}$, so TL is

$$y - \frac{1}{2} = \frac{-1}{16}(x-4).$$

(c)

```
y[x_] := 1 / Sqrt[x];
tl1[x_] := 1 - 1 / 2 (x - 1);
tl2[x_] := 1 / 2 - 1 / 16 (x - 4);
Plot[{y[x], tl1[x], tl2[x]}, {x, 0, 6}]
```



18 (a) Find an eq. of the tangent line to the graph of $y=g(x)$ at $x=5$ if $g(5)=-3$, $g'(5)=4$.

$$y-(-3)=4(x-5)$$

(b) If the TL to $y=f(x)$ at $(4,3)$ passes through $(0,2)$, find $f(4)$ and $f'(4)$.

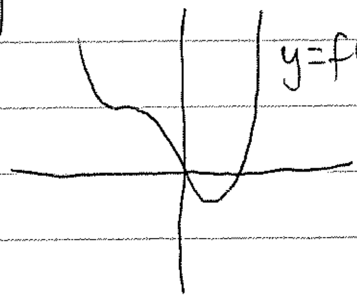
$f(4)=3$.

$$f'(4)=\frac{2-3}{0-4}=\frac{1}{4}.$$

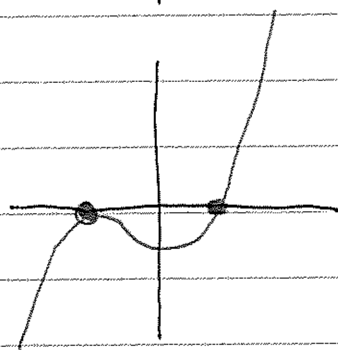
week 6

2.8| 10, 24, 26

10|



y=f(x). Sketch $f'(x)$



24| Find derivative. talk about domains.

$f(x)=x+\sqrt{x}$. This has domain $[0,\infty)$.

$$f'(x)=\lim_{h\to 0}\frac{x+h+\sqrt{x+h}-x-\sqrt{x}}{h} = \cancel{\lim_{h\to 0}\frac{h+\sqrt{x+h}-\sqrt{x}}{h}}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h}+\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0} 1+\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0} 1+\frac{1}{\sqrt{x+h}+\sqrt{x}} = 1+\frac{1}{2\sqrt{x}}.$$ This has domain $(0,\infty)$.

26 $f(x) = \frac{3+x}{1-3x}$ has domain $\{x \in \mathbb{R} \mid x \neq 1/3\}$

$$f'(x) = \lim_{h \to 0} \left(\frac{3+(x+h)}{1-3(x+h)} - \frac{3+x}{1-3x} \right) \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{(3+(x+h))(1-3x) - (3+x)(1-3(x+h))}{h(1-3(x+h))(1-3x)}$$

$$= \lim_{h \to 0} \frac{3 + x + h - 9x - 3x(x+h) - (3 - 9x - 9h + x - 3x^2 - 3xh)}{h(1-3(x+h))(1-3x)}$$

$$= \lim_{h \to 0} \frac{10\cancel{h}}{\cancel{h}(1-3(x+h))(1-3x)} = \frac{10}{(1-3x)^2}.$$

$f'(x)$ has the same domain as $f(x)$.