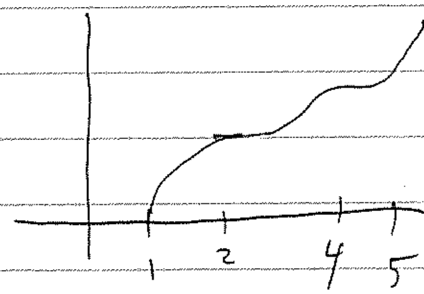
4.1| 10, 36, 56

10| Sketch f, continuous on $[1,5]$ s.t. f has no local max or min but 2, 4 are crit. pts.



36) Find critical numbers.

$$h(p) = \frac{p-1}{p^2+4}$$

$$h'(p) = \frac{+p^2+4 + 2p(p-1)}{(p^2+4)^2} = 0$$

$$-p^2+2p+4=0$$

$$p = 1 \pm \sqrt{5}$$

56) Find absolute extrema

$f(t) = \sqrt[3]{t}\,(8-t)$, $[0,8]$.

$$f'(t) = \tfrac{1}{3}t^{-2/3}(8-t) - \sqrt[3]{t} = \frac{8-t}{3t^{2/3}} - \sqrt[3]{t} = \frac{8-t-3t}{3t^{2/3}}$$

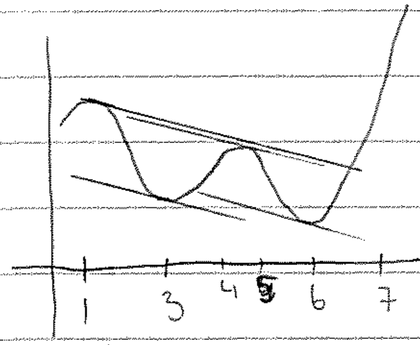
$$= \frac{8-4t}{3t^{2/3}} = 0 \text{ iff } t=2.$$

$f(0)=0$, $f(8)=0$, $f(2) = \sqrt[3]{2}\cdot 6 \leftarrow$ MAX

MIN

4.2 | 8, 12

8 | Estimate the values of c that satisfy the conclusion of MVT on [1,7]



It appears that these values of c are about 3.1, 5, 5.9.

12 | Verify that $f(x) = x^3 + x - 1$ satisfies the hypotheses of MVT. Find all c satisfying the conclusion. (interval is [0,2].

· f is continuous on [0,2] and diff'ble on (0,2) since it's a polynomial.

· $\frac{f(2) - f(0)}{2-0} = \frac{9-(-1)}{2} = 5.$

$f'(x) = 3x^2 + 1 = 5$

$3x^2 = 4$

$x = \pm 2/\sqrt{3}$

4.3 (a) / 8, 10

8) we're given the graph of f'.

a) On what intervals is f increasing?

$(2,4)$, $(6,9)$ since $f' > 0$ on these intervals.

b) f has ~~local~~ a ~~maximum~~ local maximum at $x=4$ since f' switches from $+$ to $-$ there.

f has a local min at $x=2$, $x=6$ since f' switches from $-$ to $+$ there.

c) f is concave up on $(1,3)$, $(5,7)$ and $(8,9)$ since f' is increasing on those intervals.

f is concave down on $(0,1)$, $(3,5)$, $(7,8)$ since f' is decreasing on these intervals.

d) f has inflection points at $x=1$, $x=3$, $x=5$, $x=7$, $x=8$

since f'' changes sign at these points

10) Let $f(x) = 4x^3 + 3x^2 - 6x + 1$

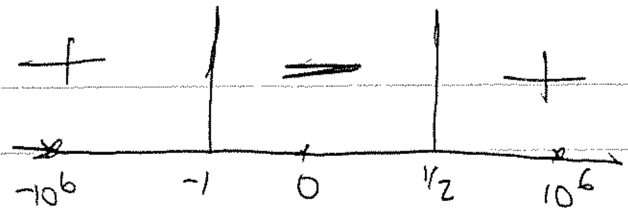
a) find intervals of increase / decrease.

$$f'(x) = 12x^2 + 6x - 6 = 0$$

$$2x^2 + x - 1 = 0$$

$$(2x-1)(x+1) = 0$$

$$x = -1, \tfrac{1}{2}.$$



f is increasing on $(-\infty, -1) \cup (1/2, \infty)$.

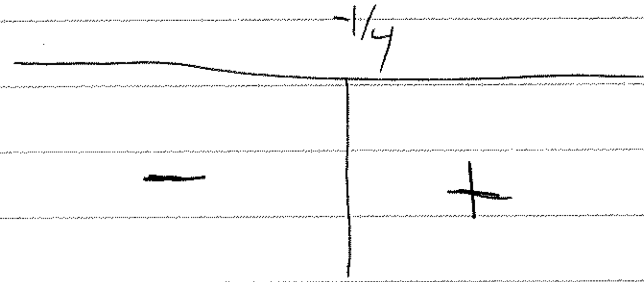
f is decreasing on $(-1, 1/2)$.

(b) $f(-1) = -4 + 3 + 6 + 1 = 6$ is local max value

$f(1/2) = \frac{4}{8} + \frac{3}{4} - 3 + 1 = -3/4$ is local min.

(c) $f''(x) = 24x + 6 = 0$

$x = -1/4$.



f is concave down on $(-\infty, -1/4)$,

concave up on $(-1/4, \infty)$.

$(-1/4, f(-1/4))$ is an inflection point.

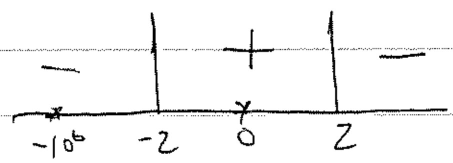
4.3 (b): 20, 30, 46

20| Find local extrema using both 1st and 2nd der tests.

$$f(x) = \frac{x}{x^2+4}$$

$$f'(x) = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = 0 \quad \text{iff} \quad 4-x^2=0$$

$$x = \pm 2.$$



$f(-2) = \frac{-2}{8} = -\frac{1}{4}$ is local min value (at $x=-2$)

$f(2) = \frac{1}{4}$ is local max value (at $x=2$)

$$f''(x) = \frac{-2x(x^2+4)^2 - 2(4-x^2)(x^2+4)\cdot 2x}{(x^2+4)^4}$$

$$= \frac{-2x(x^4+8x^2+16) - 4x(16-x^4)}{(x^2+4)^4}$$

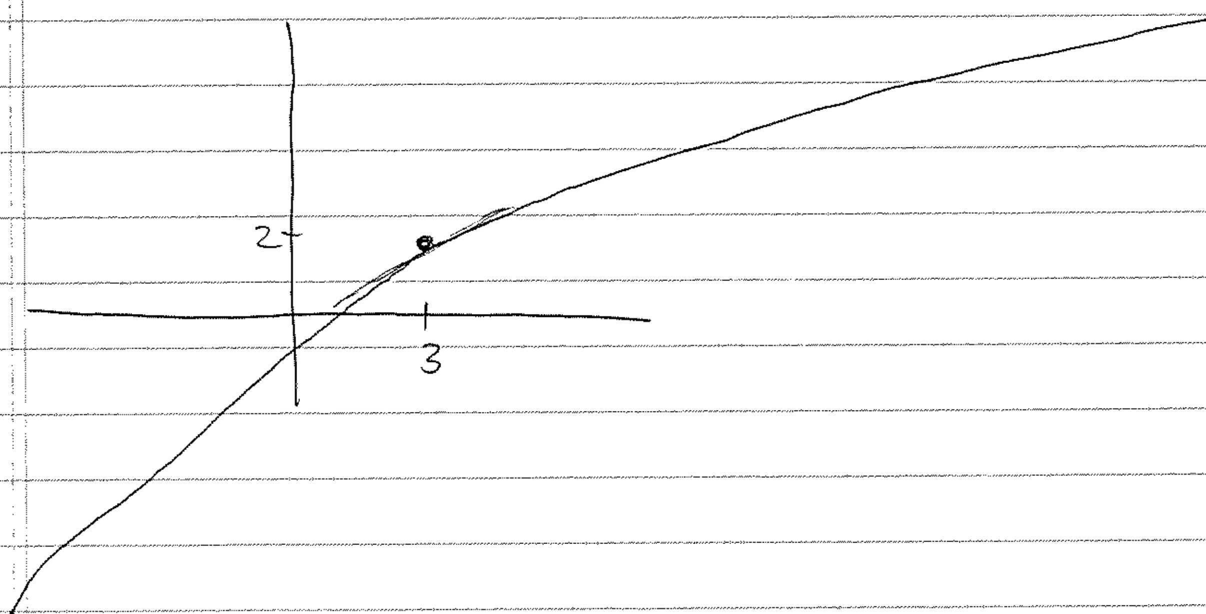
$$= \frac{2x^5 - 16x^3 - 96x}{(x^2+4)^4}$$

$f''(-2) = \frac{-64+128+192}{(x^2+4)^4} > 0$ f'' is odd, so $f''(2) < 0$.

This reconfirms conclusion of 1st der test (which I prefer!).

30) Suppose $f(3)=2$, $f'(3)=1/2$, $f'(x)>0$ for all x and $f''(x)<0$ for all x.

(a) Sketch a graph of f.



(b) $f(x)=0$ has only one solution since f is always increasing. (Otherwise, we'd contradict Rolle's thm)

(c) Is it possible that $f'(2)=1/3$?

No. $2<3$, so since $f''(x)<0$,

$f'(2)>f'(3)$, but $\frac{1}{3}<1/2$.

46 $f(x) = \dfrac{x^2}{(x-2)^2}$

(a) Vertical asymptote at $x=2$
Horiz at $y=1$ since $\lim\limits_{x\to\pm\infty} f(x) = 1$.

(b) $f'(x) = \dfrac{2x(x-2)^2 - 2x^2(x-2)}{(x-2)^4} = \dfrac{2x(x-2) - 2x^2}{(x-2)^3}$

$= \dfrac{-4x}{(x-2)^3} = 0$ iff $x=0$.
(2 is also cp.

f': $-$ | 0 | $+$ | 2 | $-$

dec: $(-\infty, 0)$ inc: $(0, \infty)$

(c) 0 is a local max.

(d) $f''(x) = \dfrac{-4(x-2)^4 + 16x(x-2)^3}{(x-2)^8}$

$= \dfrac{-4(x-2) + 16x}{(x-2)^5} = \dfrac{12x+8}{(x-2)^5} = 0$ iff
$x = \dfrac{-8}{12} = \dfrac{-2}{3}$.

concave up on $(-2/3, \infty)$, concave down on $(-\infty, -2/3)$.

Inflection pt at $\left(-\frac{2}{3}, f\left(-\frac{2}{3}\right)\right) = \left(-\frac{2}{3}, \frac{1}{16}\right)$

e