MATH 4221 - Topology 3-3:50 TR, Room: Sci 3665

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Office Hours: M 2:05-3:05pm, T 1-2pm, W 9-10am, T 1:30-2:30pm, F 9-10am, or by appointment

Textbook: “Topology of Surfaces”, by L. Christine Kinsey.

Prerequisites: Math 2202.

Main themes: For a detailed list of the sections that I plan to cover, see the last page of this document.

Topology is the study of the properties of sets and spaces that are preserved by continuous deformation. For example, in topology, a question mark “?” and the lower-case letter “i” are considered to be the same (note that this can actually be font-dependent!). A topologist would consider the letter “O” to be a different topological space from the question mark and lower-case “i”.

Topology is a major and important branch of modern mathematics. It is the language for describing large-scale phenomena of many types of sets. In addition to being interesting in its own right, topology has connections in pure mathematics to real and complex analysis, differential equations, differential and algebraic geometry, and more. The ideas of topology also are increasingly being applied outside of pure mathematics. In biology, topology is being used to understand knotting of DNA. Topology is important for various ideas in physics such as condensed matter physics and topological quantum field theories. In trying to develop quantum computers, researchers grapple with topological problems. In the field of data analysis, people are using topology to understand the large–scale structure of high–dimensional data sets.

The plan for this course is to first develop the basic notions needed to describe topological spaces. We will focus on subsets of $\mathbb{R}^n$. Frequently first courses in topology are very abstract and deal with very pathological examples. We will do our best to avoid this approach due to the limited time that we have together. We'll be skipping over most of the abstract notions in chapter 3 in order to focus on more concrete examples.

We will use our basic notions to fully understand the classification of surfaces. Examples of surfaces are the two–dimensional surfaces of a coffee cup, a basketball, or a doughnut. How can we determine which of these surfaces are topologically the same? What is the set of all possible surfaces? We'll fully answer these questions in this course.

We will also see that many topological spaces can be described in a completely combinatorial way (via cell complexes). The combinatorics of these descriptions give rise to an algebraic structure on topological spaces (homology). This algebraic structure can be described using objects that are very similar to vectors spaces and linear transformations between vector spaces (represented as matrices). The properties of these algebraic objects are very powerful tools for distinguishing topological spaces.

Details of the class:

- **Course webpage:** All information related to the course will be posted on the course webpage. You should bookmark this link:
  http://ckatkinson.github.io/4221/.
  Visit this page regularly as all announcements and assignments will be posted there.
- **Homework:** Homework sets will be due approximately every week.
  Collaboration on homework assignments is permitted and encouraged. If you choose to collaborate with other students, you must write the names of the students with which you collaborated on your submitted homework. All submitted work must be in your own words. What this means in practice is that you should work together to figure out solutions, but should individually write up your solutions. Do not submit solutions that are copied from other students or sources. If you do, I will consider it an act of academic dishonesty and will act accordingly.
- **Exams:** There will be three exams during the semester. The exams will take place during weeks 5, 11, and 16. Since our class meetings are temporally very short, these exams will be take-home exams. For the exams, no collaboration whatsoever is allowed. You may use the course textbook and your notes from class. No other resources are allowed. The only human you may interact with for help on the exam is Prof. Atkinson. Detailed guidelines will be distributed with the exam. There will be no final exam during finals week.
- **Grading:** The university's policy for grades can be found at: http://policy.umn.edu/Policies/Education/Education/GRADINGTRANSCRIPTS.html
  I grade homework assignments and exams with the above guidelines in mind using the following numerical scheme. Your overall score will be rounded to the nearest integer. I reserve the right to change the grading scale at any point, but will not increase the requirements for any letter grades.

Letter	Percentage
A	95-100
A-	90-95
B+	86-89
B	83-86
B-	80-83
C+	76-69
C	73-76
C-	70-73
D+	65-69
D	60-64
F	< 60

  If you are taking the course S-N, then you need 70% to earn an S.
  The components of the course will be combined to calculate your grade as follows:

Homework	55%
Exams	45%

  Although I will not be posting grades online, feel free to ask at any point about where you stand in the course.
- **Extra Credit:** There will be no extra credit.

Univeristy policies: See http://policy.umn.edu/education for the official university policies on education. I will adhere to these policies.

Late work and missed exams: I will only accept late work under exceptional circumstances. Please talk to me as soon as possible if you miss a deadline.

Makeup exams will only be given in the case of legitimate absences as defined by the official university policy: http://policy.umn.edu/Policies/Education/Education/MAKEUPWORK.html. Legitimate absenses must be supported by appropriate documents unless otherwise specified by university policy.

If you have a scheduling conflict and will miss an exam for a documented reason, let me know as far in advance as possible so that we can make arrangements for you to take the exam at another time.

Disability Accommodations:

The University of Minnesota views disability as an important aspect of diversity, and is committed to providing equitable access to learning opportunities for all students. The Disability Resource Center (DRC) is the campus office that collaborates with students who have disabilities to provide and/or arrange reasonable accommodations.

- If you have, or think you have, a disability in any area such as, mental health, attention, learning, chronic health, sensory, or physical, please contact the DRC office on your campus (UM Morris 320.589.6178) to arrange a confidential discussion regarding equitable access and reasonable accommodations.
- Students with short-term disabilities, such as a broken arm, should be able to work with instructors to remove classroom barriers. In situations where additional assistance is needed, students should contact the DRC as noted above.
- If you are registered with the DRC and have a disability accommodation letter dated for this semester or this year, please contact your instructor early in the semester to review how the accommodations will be applied in the course.
- If you are registered with the DRC and have questions or concerns about your accommodations please contact the Coordinator of the Disability Resource Center.

  Additional information is available on the DRC website: http://www.morris.umn.edu/academicsuccess/disability/, or e-mail hoekstra@morris.umn.edu

Here is a link to more policy statements about syllabi: www.policy.umn.edu/Policies/Education/Education/SYLLABUSREQUIREMENTS_APPA.html

Student Learning Outcomes This course is designed to partially satisfy the following *UMM Student Learning Outcomes*: 1a, 1b, 1c, 2a, 2b, 2d, 2g, 4b, 4c

See http://www.morris.umn.edu/committees/Curriculum/Learning_Outcomes_Approved.pdf

The table below displays the sections that I would like to cover. The timing indicated in the "Week" column is highly conjectural. We will go as slow or fast as necessary to keep everyone on board.

Week	Section	Topic
1	Chapter 1	Introduction
	Chapter 2	Point-set topology in $\mathbb{R}^n$
2	2.1	Open and closed sets
	2.2	Relative neighborhoods
3	2.3	Continuity
	2.4	Compact sets
4	2.5	Connected sets
	2.6	Applications
	Chapter 4	Surfaces
5	4.1	Examples of complexes
	4.2	Cell complexes
6	4.3	Surfaces
7	4.4	Triangulations
	4.5	Classification of surfaces
	4.6	Surfaces with boundary
	Chapter 5	The euler characteristic
8	5.1	Topological invariants
	5.2	Graphs and trees
9	5.3	The euler characteristic and the sphere
	5.4	The euler characteristic and surfaces
10	5.5	Map-coloring problems
	5.6	Graphs revisted
	Chapter 6	Homology
11	6.1	The algebra of chains
	6.2	Simplicial complexes
12	6.3	Homology
	6.4	More computations
13	6.5	Betti numbers and the euler characteristic
		Selected topics and applications