

3.9 | 12, 20, 24

12 | Snowball melts so that its surface area decreases at  $1 \text{ cm}^2/\text{min}$ . At what rate is the diameter decreasing when diam is  $10 \text{ cm}$ ?

$$A = 4\pi r^2 \text{ and } D = 2r, \text{ so } A = 4\pi \left(\frac{D}{2}\right)^2 = \pi D^2$$

Know  $\frac{dA}{dt} = -1$ . Want  $\left.\frac{dD}{dt}\right|_{D=10}$ .

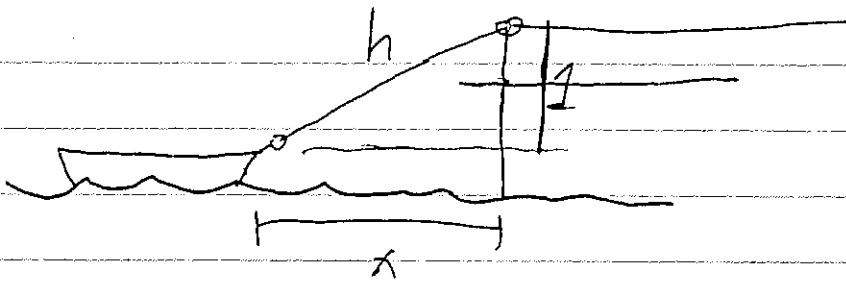
$$\frac{dA}{dt} = 2\pi D \cdot \frac{dD}{dt}, \text{ so}$$

$$\frac{dD}{dt} = \frac{1}{2\pi D} \frac{dA}{dt} \quad \text{when } D=10,$$

$$\frac{dD}{dt} = \frac{1}{20\pi} (-1) \text{ , so diameter is decreasing}$$

$$\text{by } -\frac{1}{20\pi} \text{ cm/min}$$

20 | A boat is pulled into a dock by a rope passing through a pulley  $1 \text{ m}$  above bow of boat. If rope is pulled at  $1 \text{ m/s}$ , how fast is boat approaching dock when it's  $8 \text{ m}$  from dock?



$$h^2 = x^2 + 1. \quad \text{Given } \frac{dh}{dt} = -1, \text{ want } \left. \frac{dx}{dt} \right|_{x=8}$$

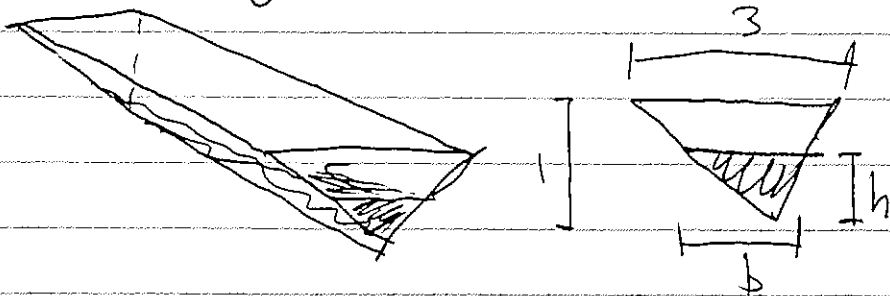
$$2h \frac{dh}{dt} = 2x \frac{dx}{dt}, \text{ so } \frac{dx}{dt} = \frac{h}{x} \frac{dh}{dt}.$$

$$\text{When } x=8, h=\sqrt{65}, \text{ so}$$

$$\frac{dx}{dt} = -\frac{\sqrt{65}}{8} \text{ m/s.}$$

24 | 10 ft long trough w/ cross sections isosceles triangle, with base 3ft and height 1ft.

Trough filled at  $12 \text{ ft}^3/\text{min}$ , how fast is water rising when depth is 6 inches?



$$\frac{h}{b} = \frac{1}{3}, \text{ so } b = 3h$$

$$V = 10 \cdot \frac{1}{2}bh = 15h^2$$

Know  $\frac{dV}{dt} = 12$ , want  $\frac{dh}{dt} \Big|_{h=1/2}$ .

$$\frac{dV}{dt} = 30h \frac{dh}{dt}, \text{ so } \frac{dh}{dt} = \frac{1}{30h} \frac{dV}{dt}.$$

when  $h = 1/2$ ,

$$\frac{dh}{dt} = \frac{1}{15} \cdot 12 = \frac{12}{15} = \frac{4}{5} \text{ ft/min.}$$