5.1: 4, 12, 20

4a: Estimate the are under the graph of f(x) = Sqrt[x] from 0 to 4 using four rectangles and right endpoints. Sketch the graph and the rectangles. Is it an over or underestimate?

```
rsumL[f_, a_, b_, n_] := N[Module[{dx = (b - a) / n}, Sum[f[a + i dx] dx, {i, 0, n - 1}]]]
rsumR[f_, a_, b_, n_] := N[Module[{dx = (b - a) / n}, Sum[f[a + i dx] dx, {i, 1, n}]]]
rsumM[f_, a_, b_, n_] :=
 N[Module[{dx = (b - a) / n}, Sum[f[a + dx / 2 + i dx] dx, {i, 0, n - 1}]]]
rectsR[f_, a_, b_, n_] := Module[{dx = (b - a) / n},
  Show[DiscretePlot[f[x], {x, a, b, dx}, ExtentSize → Left], Plot[f[x], {x, a, b}]]]
rectsL[f_, a_, b_, n_] := Module[{dx = (b - a) / n},
  Show[DiscretePlot[f[x], {x, a, b, dx}, ExtentSize → Right], Plot[f[x], {x, a, b}]]]
```

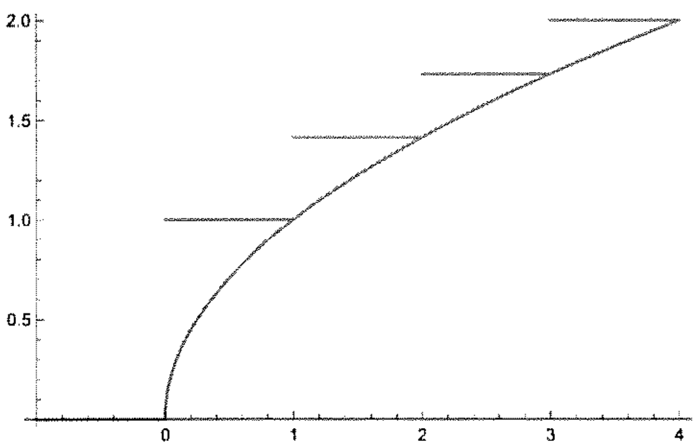
```
Clear[f]
```

```
f[x_] := Sqrt[x]
rsumR[f, 0, 4, 4]
```

```
6.14626
```

Above is the approximation. Below are the rectangles:

```
rectsR[f, 0, 4, 4]
```

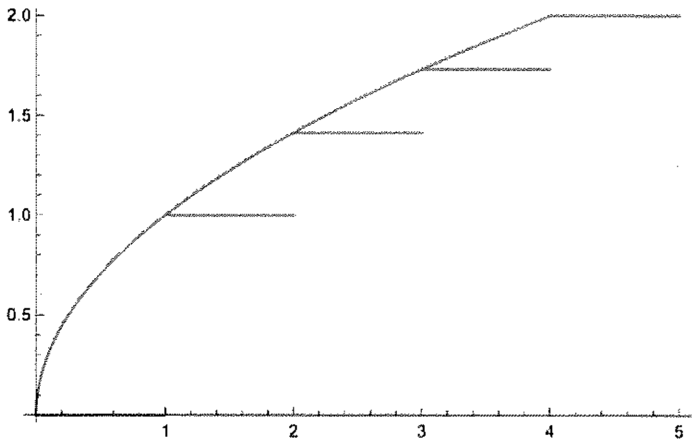


This appears to be an overestimate.

b) Repeat the above with left-hand endpoints.

```
rsumL[f, 0, 4, 4]
rectsL[f, 0, 4, 4]
```

```
4.14626
```



The approximation is as above. The graphs shows that this is an underestimate (ignore the rectangle over the interval [4,5]).

12. Speedometer readings for a motorcycle at 12 second intervals are as follows:

```
v[0] = 30;
v[12] = 28;
v[24] = 25;
v[36] = 22;
v[48] = 24;
v[60] = 27;
```

a) Estimate distance traveled by using velocities at the beginning of the time intervals. This is a left-hand sum:

```
rsumL[v, 0, 60, 5]
```

```
1548.
```

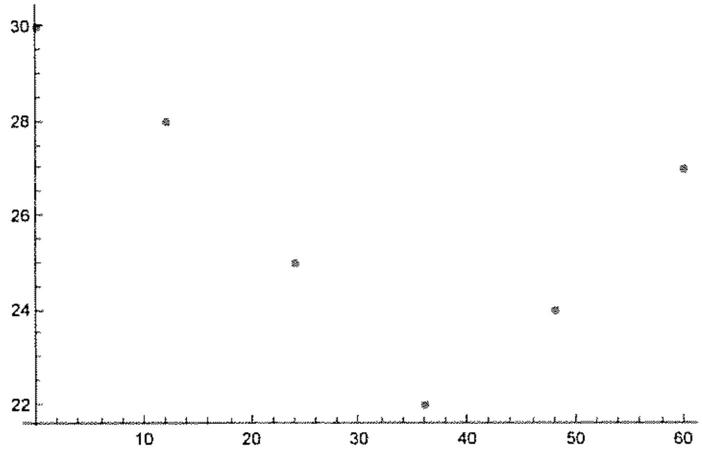
The motorcycle traveled around 1548ft.

b) Estimate using the end of the intervals. This is a right-hand sum:

```
rsumR[v, 0, 60, 5]
```

```
1512.
```

```
ListPlot[Table[{t, v[t]}, {t, 0, 60, 12}]]
```



It appears that the left-sums are an over estimate and the right-sums are an underestimate.

5.1|20

Find a region whose area is equal to

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5+\frac{2i}{n}\right)^{10}.$$

right sum?

Δx

$\Delta x = \frac{2}{n}$ and $a=5$, so $\frac{2}{n} = \frac{b-5}{n}$

$2 = b-5$

$7 = b$.

$f(x)=x^{10}$, so this limit is the area under $f(x)=x^{10}$ over $[5,7]$.

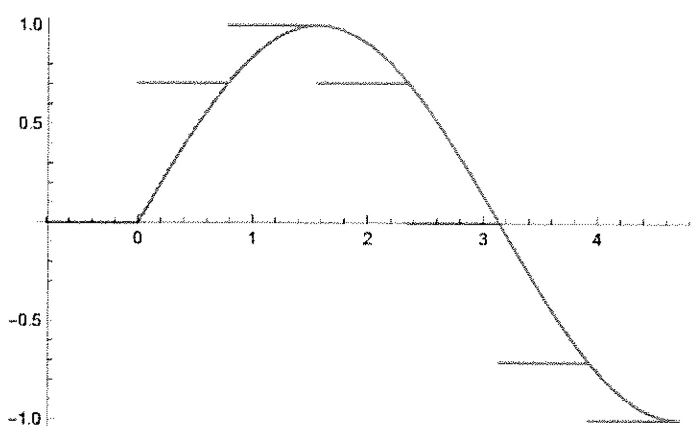
5.2: 4, 16, 40

```
rsumL[f_, a_, b_, n_] := N[Module[{dx = (b - a) / n}, Sum[f[a + i dx] dx, {i, 0, n - 1}]]]
rsumR[f_, a_, b_, n_] := N[Module[{dx = (b - a) / n}, Sum[f[a + i dx] dx, {i, 1, n}]]]
rsumM[f_, a_, b_, n_] :=
 N[Module[{dx = (b - a) / n}, Sum[f[a + dx / 2 + i dx] dx, {i, 0, n - 1}]]]
rectsL[f_, a_, b_, n_] := Module[{dx = (b - a) / n}, Show[
   DiscretePlot[f[x], {x, a, b, dx}, ExtentSize → Right], Plot[f[x], {x, a, b}]]]
rectsR[f_, a_, b_, n_] := Module[{dx = (b - a) / n},
  Show[DiscretePlot[f[x], {x, a, b, dx}, ExtentSize → Left], Plot[f[x], {x, a, b}]]]
```

4a: Find the Riemann sum for f(x) = sin(x) for x in [0,3pi/2] with six terms taking the right-hand endpoints to be the sample points. Explain what this means with a sketch.

```
f[x_] := Sin[x]
rsumR[f, 0, 3 Pi / 2, 6]
rectsR[f, 0, 3 Pi / 2, 6]
```

```
0.55536
```



This means that 0.55536 is an approximation to the net or signed area under the curve.

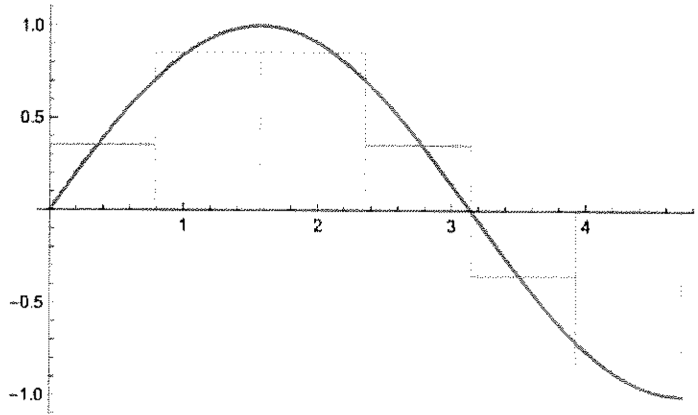
4b: Repeat the previous part with midpoints. Below is some code to plot a midpoint sum that I found on stackexchange.

```
rsumM[f, 0, 3 Pi / 2, 6]
```

```
1.02617
```

In[15]:=
```
With[{a = 0, b = 3 Pi / 2, n = 6}, rectangles = Table[{Opacity[0.05],
    EdgeForm[Gray], Rectangle[{a + i (b - a) / n, 0}, {a + (i + 1) (b - a) / n,
      Mean[{f[a + i (b - a) / n], f[a + (i + 1) (b - a) / n]}]}]}, {i, 0, n - 1, 1}];
  Show[Plot[f[x], {x, a, b}, PlotStyle → Thick, AxesOrigin → {0, 0}],
   Graphics@rectangles]]
```

Out[15]=



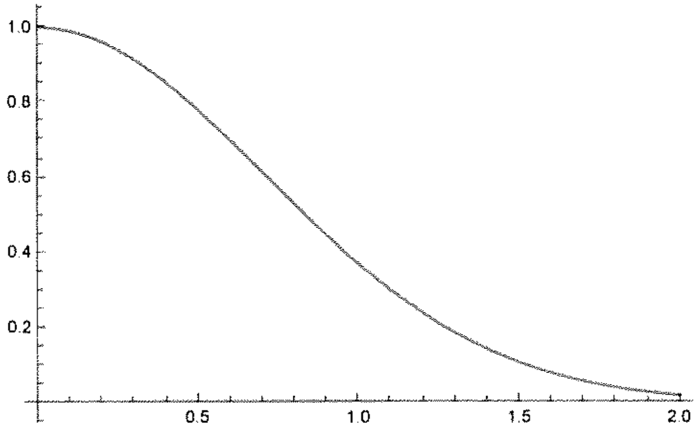
16: Make a table of values of left and right Riemann sums L_n and R_n for the integral of
e^(-x^2) from 0 to 2 with n = 5, 10, 50, 100. Below, the first column is n, the second is L_n and the third is R_n.

In[21]:=
```
g[x_] := E^(-x^2)
l[n_] := rsumL[g, 0, 2, n]
r[n_] := rsumR[g, 0, 2, n]
row[n_] := {n, l[n], r[n]}
Column[{row[5], row[10], row[50], row[100]}]
```

Out[25]=
```
{5, 1.07747, 0.684794}
{10, 0.980007, 0.78367}
{50, 0.901705, 0.862438}
{100, 0.891896, 0.872262}
```

Since the function is decreasing on the interval, L_n is always an upper bound on the integral and R_n is always a lower bound. This means that the integral lies between 0.87226 and 0.891896.

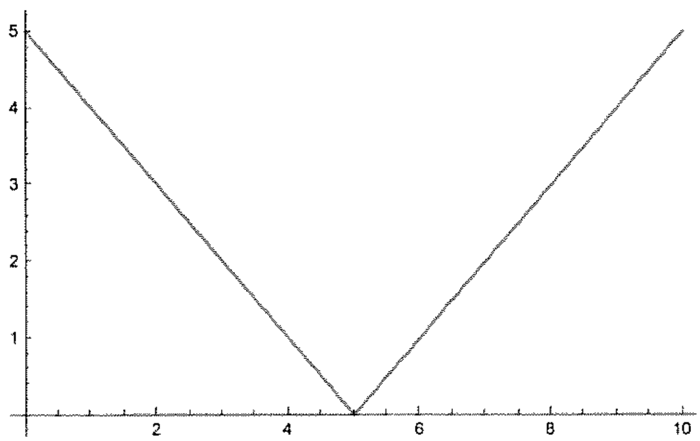
```
Plot[g[x], {x, 0, 2}]
```



Since the function is not decreasing on all of [-1,2], we can't make a similar statement for the integral over that interval.

40: Evaluate the integral from 0 to 10 of |x-5| by computing areas.

```
Plot[Abs[x - 5], {x, 0, 10}]
```



This integral is the sum of the areas of the two triangles shown. Each of these triangles has area 5*5/2, so the integral has value 25.

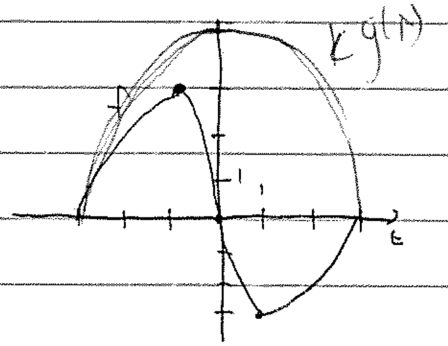
```
Integrate[Abs[x - 5], {x, 0, 10}]
```

25

Yep.

5.3a) | 4, 16, 22

4.



$g(x) = \int_{-3}^{x} f(t)\,dt$.

a) $g(-3) = g(3) = 0$.

b) ~~$g(-2) \approx 1$~~
$g(-1) \approx 3.5$
$g(0) \approx 5.3$

d) g is maximized at $x=0$ since $g'(x)<0$ for $x>0$.

c) g is increasing on $(-3, 0)$ since $g'(x) = f(x) > 0$ on that interval.

e) Sketch g.

(f) $g' = f$, so we've already drawn it.

16 Find y':

$$y' = \frac{d}{dx}\int_{1}^{\cos x} (1+v^2)^{10}\,dv = (1+\cos^2 x)^{10}\cdot(-\sin x).$$

22|

$$\int_0^1 \left(1+\frac{1}{2}u^4-\frac{2}{5}u^9\right)du = \left(u+\frac{u^5}{10}-\frac{u^{10}}{25}\right)\Big|_0^1$$

$$= 1+\frac{1}{10}-\frac{1}{25} = \frac{50+5-2}{50} = \frac{53}{50}.$$

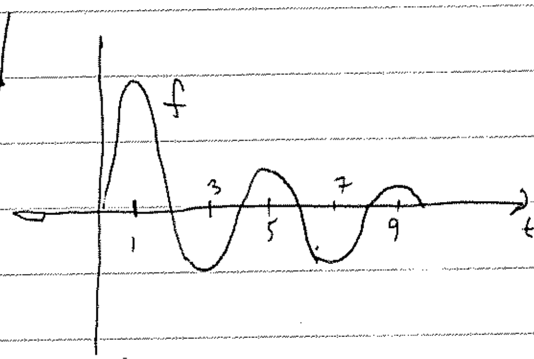
5.3b: 54, 64, 66

54

$$\frac{d}{dx}\left(\int_{\tan x}^{x^2}\frac{dt}{\sqrt{2+t^4}}\right)=\frac{d}{dx}\left(\int_{37}^{x^2}\frac{dt}{\sqrt{2+t^4}}-\int_{37}^{\tan x}\frac{dt}{\sqrt{2+t^4}}\right)$$

$$=\frac{2x}{\sqrt{2+x^8}}-\frac{\sec^2 x}{\sqrt{2+\tan^4 x}}.$$

64



$$g(x)=\int_0^x f(t)\,dt$$

$$g'(x)=f(x)$$

g'	+	−	+	−	+
	2	4	6	8	10

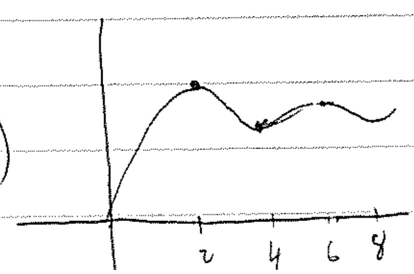
(a) max: 2, 6
min 4, 8

(b) $x=2$.

(c) g concave down when f is decreasing:

$(1,3)\cup(5,7)\cup(9,10)$.

d)



66 Compute

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{i \cdot \frac{1}{n}} \cdot \frac{1}{n} = \int_0^1 \sqrt{x}\,dx$$

$$= \frac{2}{3} x^{3/2} \Big|_0^1 = 2/3.$$