week 3)

1.2) 4, 20, 22

4) Row reduce and identify pivot positions + columns.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & -3 & -12 & -18 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim \underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}}_{\text{pivot columns}}$$

◯ = pivot positions.

20) Find h, K so that system has
(a) no sol'n, (b) a unique sol'n, (c) many sol'ns.

$$\begin{aligned} x_1 - 3x_2 &= 1 \\ 2x_1 + hx_2 &= K \end{aligned} \sim \begin{bmatrix} 1 & -3 & 1 \\ 2 & h & K \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 1 \\ 0 & 6+h & K-2 \end{bmatrix}.$$

↓

(a) System has no sol'n if
$6+h=0$ and $k-2\neq 0$.
For example $h=-6$, $k=3$.

(b) unique solution if $6+h\neq 0$.
For example $h=4$.

(c) many sol'ns if $6+h=0=k-2$, so
for $h=-6$ and $k=2$.

22 (a) True (Thm 1)
(b) False (Thm 2)
(c) False (see Thm 1)
(d) True.
(e) False. It might be inconsistent.

1.3 | 14, 16, 24

14 | Is $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ a linear combo of columns of $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}$?

Let's see: If so, then system correspondy to aug matrix has sol'n:

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$ Yep. This is consistent,

so $\vec{b}$ is a l.c. of the columns of A.

16 | $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$.

For what h is $\vec{y} \in \text{Span}\{\vec{v}_1, \vec{v}_2\}$?

We're looking for all h s.t. system corresponding to the following is consistent!

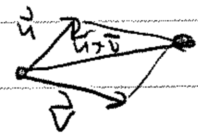
$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 0 & 2h-5+9 \end{bmatrix}.$$

This is consistent only when $2h+4=0$

$$\underline{\underline{h=2}}$$

24 (a) False!



(b) True.

(c) True

(d) False. $\vec{u}$ results...

(e) False. Why would the book suggest this!?

1.4 | 16, 18, 26

16 Let

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Show $A\vec{x} = \vec{b}$ is not consistent for all $\vec{b}$.
Describe set of $\vec{b}$ for which it is consistent.

Plan: I'll row reduce augmented matrix and see what happens:

$$\begin{bmatrix} 1 & -2 & -1 & b_1 \\ -2 & 2 & 0 & b_2 \\ 4 & -1 & 3 & b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -2 & -2 & 2b_1+b_2 \\ 0 & 7 & 7 & b_3-4b_1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & 1 & 1 & \frac{-2b_1-b_2}{2} \\ 0 & 0 & 0 & c \end{bmatrix} \quad \text{where}$$

$$c = -7\left(\frac{-2b_1-b_2}{2}\right) + b_3 - 4b_1$$

$$= \frac{14b_1}{2} + \frac{7b_2}{2} + b_3 - 4b_1 = 3b_1 + \frac{7}{2}b_2 + b_3.$$

System is consistent only when $c = 0$.

$\downarrow$

When $c=0$, $b_1 = \frac{-7}{6} b_2 - \frac{1}{3} b_3$, so the set of $\vec{b}$ for which system is consistent is

$$\mathcal{B} = \left\{ \begin{bmatrix} -7/6\, b_2 - 1/3\, b_3 \\ b_2 \\ b_3 \end{bmatrix} \middle| \; b_i \in \mathbb{R} \right\}.$$

Note that $\mathcal{B} = \text{Span}\left\{ \begin{bmatrix} -7/6 & -1/3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

18) Can every vector in $\mathbb{R}^4$ be written as linear combo of columns of the matrix B, displayed below?

If so, then thm says B has 4 pivot positions. Let's see!

$$B = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 0 & 1 & 3 & -11 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

No!

26) $\vec{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$.

Note that $2\vec{u} - 3\vec{v} - \vec{w} = \vec{0}$. $^{(*)}$ Use this fact to find x_1, x_2 satisfying

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}.$$

By $(*)$, $2\vec{u} - 3\vec{v} = \vec{w}$.

The matrix equation is of the form

$$\begin{bmatrix} \vec{u} & \vec{v} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{w},$$ so

take $x_1 = 2$, $x_2 = -3$.

1.5 6, 18, 24, 38

6 + 18 Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix}$. Find solution set to $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 5 \\ 13 \\ -8 \end{bmatrix}$. Since solutions have form $\vec{p} + \vec{v}_h$, we can solve both simultaneously by working out the non-homogeneous version.

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -3 & 3 & 3 \\ 0 & 3 & -3 & -3 \end{bmatrix}$$

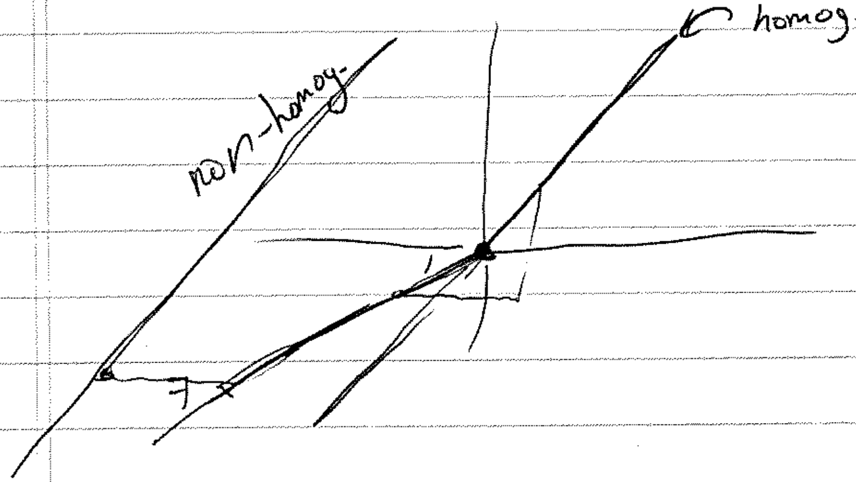
$$\sim \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$x_1 - x_3 = 7$
$x_2 - x_3 = -1$, so

Sol'n set is (to non-homg) $\left\{ \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \;\middle|\; x_3 \in \mathbb{R} \right\}$.

For the homogeneous system, sol'n set is $\mathrm{Span}\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$.

Geometrically:



24 (a) False! $\vec{0}$ is always a sol'n.

(b) False! $\left\{ x+y-z+w=0 \right.$ has a solution $\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$.

(c) ~~True!~~ → ?

(d) False! Need $\vec{b}=\vec{0}$.

(e) True! See theorem.

38) If $A\vec{w}=0$, then $A(c\vec{w}) = cA\vec{w}$
$= c\vec{0} = \vec{0}$.