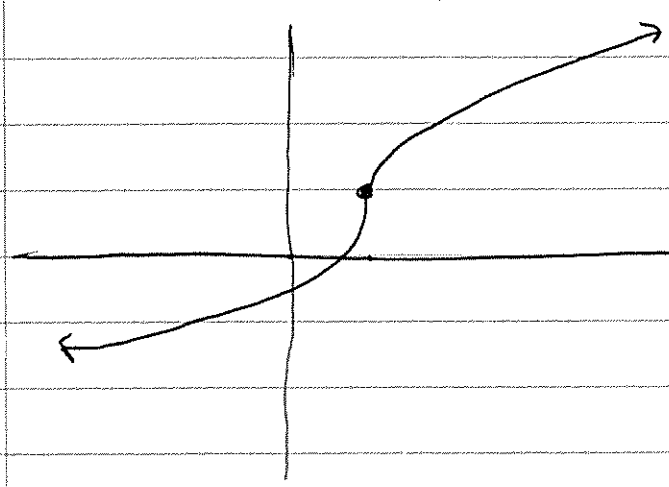
Week 3

1.3] 20, 34, 50

20] Graph by hand.

$y = 1 + \sqrt[3]{x-1}$ ($\sqrt[3]{x}$ shifted right and up by 1



34] Find all functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, (d) $g \circ g$ and their domains.

$f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{1-x}$.

(a) $(f \circ g)(x) = \sqrt{\sqrt[3]{1-x}}$

Need $\sqrt[3]{1-x} \geq 0$

$1 - x \geq 0$

$1 \geq x$

Domain: $(-\infty, 1]$

(b) $(g \circ f)(x) = \sqrt[3]{1-\sqrt{x}}$. Domain $[0, \infty)$.

(c) $(f \circ f)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. Domain $[0, \infty)$

(d) $(g \circ g)(x) = \sqrt[3]{1 - \sqrt[3]{1-x}}$ Domain is $(-\infty, \infty)$.

50]

x	1	2	3	4	5	6
f(x)	3	1	4	2	2	5
g(x)	6	3	2	1	2	3

(a) $f(g(1)) = 5$

(b) $g(f(1)) = 2$

(c) $f(f(1)) = 4$

(d) $g(g(1)) = 3$

(e) $(g \circ f)(3) = 1$

(f) $(f \circ g)(6) = 4$

1.5 | 18, 19, 26

18) Find exponential function $f(x) = c \cdot a^x$ passing through $(0,2)$ and $(2, 2/9)$

$f(0) = 2$, so $2 = c$

$f(2) = \frac{2}{9}$, so $\frac{2}{9} = 2 \cdot a^2$

$\pm\sqrt{\frac{1}{9}} = a$

$a = \pm 1/3 \quad (a > 0)$

$f(x) = 2\,(1/3)^x$.

19) If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h}\right).$$

$$\frac{f(x+h) - f(x)}{h} = \frac{5^{x+h} - 5^x}{h} = 5^x \left(\frac{5^h - 1}{h}\right).$$

2b) Bacterial culture starts with 500 bacteria and doubles every half hour.

$f(x) = ca^x$

$500 = ca^0$, so $c = 500$

$f(0.5) = 1000$, so

$1000 = 500\sqrt{a}$

$4 = a$

$f(x) = 500 \cdot 4^x$.

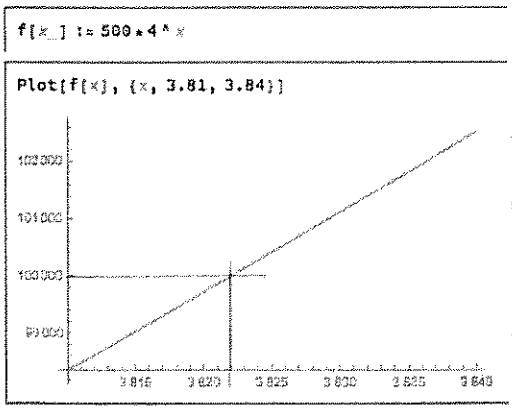
(a) After 3 hours, there are

$f(3) = 500 \cdot 4^3 = 500 \cdot 64 = 32000$ bacteria

(b) well, duh. $f(t)$ bacteria.

(c) $f(2/3) = 500(4)^{2/3} = 1259.92$ bacteria.

(d)



It looks like $f(3.822) \approx 100{,}000$

1.6 26, 38

26) Find formula for inverse:

$$y = \frac{e^x}{1+2e^x}$$

$$x = \frac{e^y}{1+2e^y}$$

$$x + 2xe^y = e^y$$

$$2xe^y - e^y = -x$$

$$e^y = \frac{-x}{2x-1}$$

$$y = \ln\left(\frac{-x}{2x-1}\right) \leftarrow$$

38) Write as a single logarithm:

$$\ln(a+b) + \ln(a-b) - 2\ln c = \ln\left(\frac{a^2-b^2}{c^2}\right)$$