

# Review sheet for Final Exam

The best way to study for the exam is to learn how to do all of the assigned practice problems. If you run out of problems, learn to do all of the problems in the book! In addition to the practice problems assigned over the course of the semester, working through the problems in the review sections of the book would make for a useful start. Below are the relevant review problems to look at:

- Chapter 12 review: Think through the “concept check” and “true–false quiz”. Also try 1-7, 11, 15-27, 38.
- Chapter 13 review: Think through the “concept check” and “true–false quiz”. Also try 1-2, 6.
- Chapter 14 review: Think through the “concept check” and “true–false quiz”. Also try 1-8, 13-17, 19-22, 25-29, 31, 35-37, 42-48, 51-56, 59-64
- Chapter 15 review: Think through the “concept check” and “true–false quiz”. Also try 1-34, 39-40, 45-49
- Chapter 16 review: Think through the “concept check” 1-8 and “true–false quiz” 4, 5, 6. Also try 1-9, 11-17.

## 12.1: Coordinates on 3–space

- Be familiar with the Cartesian coordinate system on  $\mathbb{R}^3$ .
- Know how to measure distance in  $\mathbb{R}^3$ .
- Know how to find the equation for a sphere.

## 12.2: Vectors

- Know what a vector is. Understand equivalence of vectors.
- Know how to add vectors and multiply them by scalars, both geometrically and algebraically in terms of components.
- Be able to compute the length of vectors.
- Know what a unit vector is.

## 12.3: Dot product

- Know how to compute the dot product of two vectors in terms of their components.
- Know the algebraic behavior of the dot product.
- Know the theorem relating the dot product of two vectors, their lengths, and the angle between them.
- Be able to determine whether or not two vectors are orthogonal.
- We didn't talk about direction angles or direction cosines. Don't worry about these things for the exam.
- Know how to compute the scalar and vector projections of one vector onto another.

## 12.4: Cross product

- Know how to find the cross product of two vectors, both geometrically and in terms of components.
- Know the theorem relating the length of the cross product of two vectors, the length of each, and the angle between them.
- The length of the cross product of two vectors is equal to the area of the parallelogram that they span.
- Know how the cross product behaves algebraically.

## 12.5: Lines and planes

- Know how to find equations for a line. What data are necessary?
- Be able to parameterize a line segment.
- Know how to find the various equations for planes. Be able to do so given various types of initial data.

## 12.6: Cylinders and quadratic surface

- Know how to find the traces of surfaces defined in terms of equations.

## **13.1: Vector functions and space curves**

- Understand what is meant by a vector valued function.
- Know how these functions correspond to space curves.

## **13.2: Derivatives and integrals of vector functions.**

- Know how to compute derivatives of vector valued functions.
- Know how to interpret derivatives as tangent vectors and how to find equations for tangent lines.

## **14.1: Functions of several variables**

- Be able to use level curves and traces to understand the graphs of functions of several variables.

## **14.3: Partial derivatives**

- Know the definition of partial derivatives and how to compute them.
- Understand the geometric meaning of partial derivatives.
- Know how the various notation for partial derivatives works.
- Know Clairaut's theorem.

## **14.4: Tangent planes**

- Know how to find the tangent plane at a point to a graph of a function of two variables.

## **14.5: The chain rule**

- Know the chain rule.

## 14.6: Directional derivatives and the gradient

- Know the definition of a directional derivative.
- If a function is differentiable, know how to compute the directional derivative using theorem 3.
- Understand the geometric meaning of directional derivatives.
- Know the definition of the gradient vector and understand its geometric properties.
- Be able to compute the tangent plane to a level surface of a function of three variables.

## 14.7: Maximum and minimum values

- Know how to find the critical points of a function of two variables.
- Know the second derivative test and be able to use it.
- Know how to find absolute maxima and minima. You may need to use Lagrange multipliers to do this.

## 14.8: Lagrange multipliers

- Know how to use the technique of Lagrange multipliers.

## 15.1: Double integrals over rectangles

- Understand the concepts behind setting up double integrals over rectangles.

## 15.2: Iterated integrals

- Know how to compute iterated integrals.
- Know Fubini's theorem and how to use it.

### 15.3: Double integrals over more general regions

- Know how to compute an integral over a non-rectangle. How is this derived from integrating over a rectangle?
- Be able to set up double integrals over type I and II regions.
- Be able to change order of integration.
- Know the basic properties of double integrals and how to use them to split up integrals over more complicated regions.

### 15.4: Double integrals in polar coordinates

- Know how to convert to and from polar coordinates.
- Be able to compute double integrals by changing to polar coordinates.

### 15.6: Triple integrals

- Understand the concepts behind setting up triple integrals.
- Know Fubini's theorem for triple integrals and how to use it to integrate over various types of regions.
- Be able to change order of integration.

### 15.7: Triple integrals in cylindrical coordinates

- Understand how cylindrical coordinates work and how to convert between Cartesian and cylindrical coordinates.
- Know how to convert integrals to cylindrical coordinates.

### 15.8: Triple integrals in spherical coordinates

- Understand how spherical coordinates work and how to convert between Cartesian and spherical coordinates.
- Know how to convert integrals to spherical coordinates.

## 15.9: Change of variables

- Understand how transformations of the plane work.
- Know the definition of the Jacobian in 2 and 3 dimensions.
- Know the change of variables formula for double and triple integrals and how to use it.
- In general, be able to evaluate an integral by making an appropriate change of variables (polar, cylindrical, spherical, other).

## 16.1: Vector Fields

- Know what a vector field is.
- Know what a gradient field is.

## 16.2: Line integrals

- Know the definition of the line integral of a scalar function and of a vector field. Be able to compute these.
- Know how to take a line integral with respect to  $dx$  or  $dy$ .

## 16.3: The Fundamental Theorem for Line Integrals

- Know the statement of the theorem and how to use it.
- Understand the connection between conservative vector fields and path independent line integrals.
- Know how to check if a vector field is conservative.
- Given a conservative vector field, be able to find a potential function.

## 16.4: Green's Theorem

- Know the statement of Green's Theorem.
- Be able to use Green's Theorem to evaluate line integrals or to evaluate double integrals.

- Know how to use Green's Theorem to compute area.
- Understand how to use Green's Theorem on regions with holes.