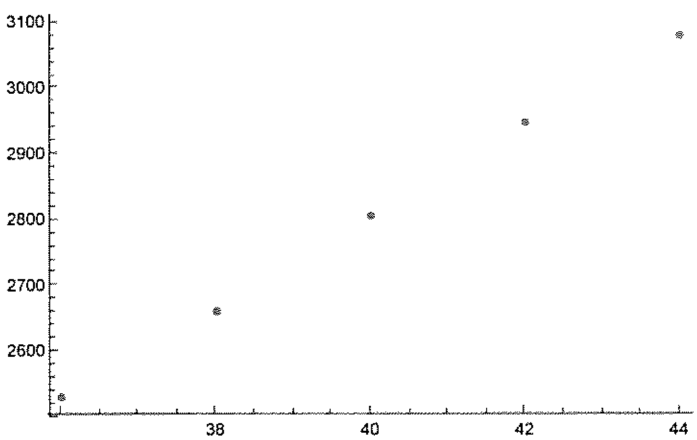
2.1 Discussion problems (2, 6)

2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

The data are as in the data structure shown below. This data structure is called an association. It's analogous to a dictionary in python or a map in c++ (you may ask, "why is CKA using this crazy stuff?" The reason is that I like to learn stuff from the course too! Look at how my secant slope function works to see how this thing works). In each entry of the list, the first entry is the time t in minutes. The second entry is the number of heartbeats recorded until that point.

```
heartbeats = <|
   36 -> 2530,
   38 -> 2661,
   40 -> 2806,
   42 -> 2948,
   44 -> 3080
   |>;
```

```
ListPlot[heartbeats]
```



In parts a through d, we're asked to compute the slope of the secant line over various intervals. I'll define a function that does it for us.

```
hb = heartbeats;
```

```
secantInterval[a_, b_] := N[(hb[b] - hb[a]) / (b - a)];
```

a)

```
secantInterval[36, 42]
```

```
69.6667
```

b)

```
secantInterval[38, 42]
```

```
71.75
```

c)

```
secantInterval[40, 42]
```

```
71.
```

d)

```
secantInterval[42, 44]
```

```
66.
```

It seems that at 42 minutes, the patient's bpm (beats-per-minute) is somewhere between 66 and 71 bpm.

6. If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height in meters t seconds later is given by

```
y[t_] := 10 t - 1.86 t^2;
```

a) Find the average velocity over a few intervals. Parts (i) - (v) correspond the the entries of the list below.

```
ave[{a_, b_}] := N[(y[b] - y[a]) / (b - a) ];
```

```
Map[ave, {{1, 2}, {1, 1.5}, {1, 1.1}, {1, 1.01}, {1, 1.001}}]
```

```
{4.42, 5.35, 6.094, 6.2614, 6.27814}
```

(b) It appears that the instantaneous velocity when t=1 is about 6.27814m/s. The actual instantaneous velocity is equal to the result of the following computation (but this doesn't need to be obvious to you at this point):

```
y'[1]
```

```
6.28
```

2.2 | 6, 16, 18, 26

6) See graph in book.

a) 4 b) 4 c) 4 d) not defined e) 1 f) -1 g) DNE

h) 1 i) 2 j) not def. k) 3 l) DNE

Whoops! out of order.

26)

$$\lim_{x \to -3^-} \frac{x+2}{x+3} = \pm\infty$$ since at -3 it has the form $\frac{-1}{0}$.

If $a < -3$, then the numerator and denominator are both negative, so limit is ∞.

18) Guess limit ~~#~~

$$\lim_{x \to -1} \frac{x^2-2x}{x^2-x-2}$$ by guessing:

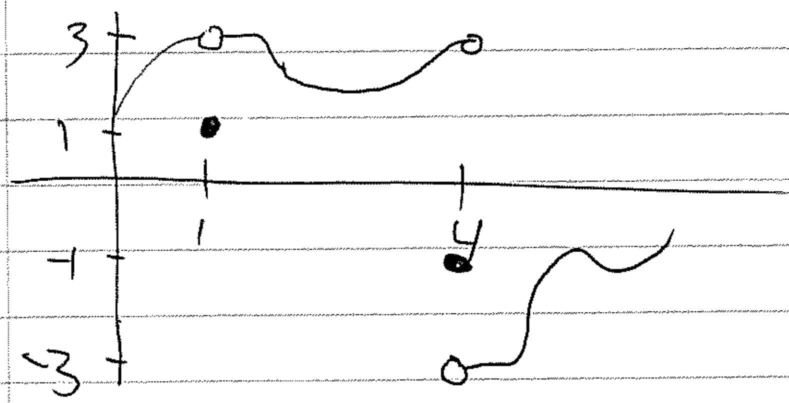
```
f[x_] := (x^2 - 2 x) / (x^2 - x - 2)
Map[f, {0, -0.5, -0.9, -0.95, -0.99, -0.999}]
{0, -1., 9., 19., 99., 999.}

Map[f, {-2, -1.5, -1.1, -1.01, -1.001}]
{2, 3., 11., 101., 1001.}
```

It appears that the limit doesn't exist. I'd guess that it goes to $-\infty$ from the left and $+\infty$ from the right.

16) Sketch a graph of a function f satisfying

$\lim_{x\to 1} f(x) = 3$, $\lim_{x\to 4^-} f(x) = 3$, $\lim_{x\to 4^+} f(x) = -3$

$f(1) = 1$ $\quad$ $f(4) = -1$.



2.3 | 20, 26, 36

20)

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{8+12h+6h^2+h^3-8}{h}$$

$$= \lim_{h\to 0} 12+6h+h^2 = 12.$$

26) $$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0} \frac{t^2+t-t}{t(t^2+t)}$$

$$= \lim_{t\to 0} \frac{1}{t+1} = 1$$

36) If $2x \le g(x) \le x^4-x^2+2$ for all x,

evaluate $\lim_{x\to 1} g(x)$.

Note that $\lim_{x\to 1} 2x = 2$ and

$$\lim_{x\to 1} x^4-x^2+2 = 2.$$

By the squeeze thm, $\lim_{x\to 1} g(x) = 2$.