Assignment: Module 4

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Contents

Introduction

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# Weigelt Corporation has excess production capacity across three plants and is tasked with producing a
# To solve this problem, we use **linear programming (LP)**, a mathematical optimization technique. The
```

Problem Formulation

1. Formulate a problem that can be solved using R libraries: The problem has been clearly defined with an objective function, decision variables, and constraints. The problem is then translated into a mathematical model, which is solvable using R's optimization libraries.

Decision Variables

Let: - x_{11} = Number of large products produced at Plant 1. - x_{12} = Number of medium products produced at Plant 1. - x_{13} = Number of small products produced at Plant 1. - x_{21} = Number of large products produced at Plant 2. - x_{22} = Number of medium products produced at Plant 2. - x_{23} = Number of small products produced at Plant 2. - x_{31} = Number of large products produced at Plant 3. - x_{32} = Number of medium products produced at Plant 3. - x_{33} = Number of small products produced at Plant 3.

Objective Function

The objective is to maximize profit:

$$\text{Maximize } Z = 420(x_{11} + x_{21} + x_{31}) + 360(x_{12} + x_{22} + x_{32}) + 300(x_{13} + x_{23} + x_{33})$$

Where: - 420 is the profit per large product. - 360 is the profit per medium product. - 300 is the profit per small product.

Constraints

1. **Capacity constraints**:

$$x_{11} + x_{12} + x_{13} \leq 750 \quad \text{(Plant 1)}$$
$$x_{21} + x_{22} + x_{23} \leq 900 \quad \text{(Plant 2)}$$
$$x_{31} + x_{32} + x_{33} \leq 450 \quad \text{(Plant 3)}$$

2. **Storage constraints**:

$$20x_{11} + 15x_{12} + 12x_{13} \leq 13000 \quad \text{(Plant 1 storage)}$$
$$20x_{21} + 15x_{22} + 12x_{23} \leq 12000 \quad \text{(Plant 2 storage)}$$
$$20x_{31} + 15x_{32} + 12x_{33} \leq 5000 \quad \text{(Plant 3 storage)}$$

3. **Sales forecast constraints**:

$$x_{11} + x_{21} + x_{31} \leq 900 \quad \text{(Large products)}$$

$$x_{12} + x_{22} + x_{32} \leq 1200 \quad \text{(Medium products)}$$

$$x_{13} + x_{23} + x_{33} \leq 750 \quad \text{(Small products)}$$

4. **Non-negativity constraint**:

$$x_{ij} \geq 0 \quad \forall i, j$$

2. Represent a problem in a manner that can be solved using R libraries: Using R's `lpSolve` library, the objective function and constraints were represented in matrix form, making it possible to pass the model into the optimization routine.

```r
# load the lpSolve package and
library(lpSolve) # For LPSolve
library(ggplot2) # For Data Visualization
# Coefficients of the objective function
objective <- c(420, 360, 300, 420, 360, 300, 420, 360, 300)

# Constraints matrix
constraints <- matrix(c(
  1, 1, 1, 0, 0, 0, 0, 0, 0,  # Plant 1 capacity
  0, 0, 0, 1, 1, 1, 0, 0, 0,  # Plant 2 capacity
  0, 0, 0, 0, 0, 0, 1, 1, 1,  # Plant 3 capacity
  20, 15, 12, 0, 0, 0, 0, 0, 0,  # Plant 1 storage
  0, 0, 0, 20, 15, 12, 0, 0, 0,  # Plant 2 storage
  0, 0, 0, 0, 0, 0, 20, 15, 12,  # Plant 3 storage
  1, 0, 0, 1, 0, 0, 1, 0, 0,  # Sales for large products
  0, 1, 0, 0, 1, 0, 0, 1, 0,  # Sales for medium products
  0, 0, 1, 0, 0, 1, 0, 0, 1   # Sales for small products
), nrow = 9, byrow = TRUE)

# Right-hand side of the constraints
rhs <- c(750, 900, 450, 13000, 12000, 5000, 900, 1200, 750)

# Direction of the constraints
constraints_dir <- c("<=", "<=", "<=", "<=", "<=", "<=", "<=", "<=", "<=")

# Solve the LP
solution <- lp("max", objective, constraints, constraints_dir, rhs)
```

```r
# Output the solution
print(solution$solution)
```

```
## [1] 350.0000 400.0000   0.0000   0.0000 400.0000 500.0000   0.0000 133.3333
## [9] 250.0000
```

```r
cat("Maximum Profit:", solution$objval, "\n")
```

```
## Maximum Profit: 708000
```

3. Interpret the output from optimization routines: After solving the LP, we interpret the output, such as the optimal production plan and maximum profit, as produced by the `lpSolve` function.

```r
# Optimal solution obtained from lpSolve
large_plant1 <- solution$solution[1]
medium_plant1 <- solution$solution[2]
small_plant1 <- solution$solution[3]
large_plant2 <- solution$solution[4]
medium_plant2 <- solution$solution[5]
small_plant2 <- solution$solution[6]
large_plant3 <- solution$solution[7]
medium_plant3 <- solution$solution[8]
small_plant3 <- solution$solution[9]
max_profit <- solution$objval

# Print the formatted output for Problem  (Weigelt Corporation)
cat("\n=== Output for Problem  (Weigelt Corporation) ===\n")
```

```
##
## === Output for Problem  (Weigelt Corporation) ===
```

```r
cat("\nPlant 1:\n")
```

```
##
## Plant 1:
```

```r
cat("Large products produced: ", large_plant1, "\n")
```

```
## Large products produced:  350
```

```r
cat("Medium products produced: ", medium_plant1, "\n")
```

```
## Medium products produced:  400
```

```r
cat("Small products produced: ", small_plant1, "\n")
```

```
## Small products produced:  0
```

```r
cat("\nPlant 2:\n")
```

```
##
## Plant 2:
```

```r
cat("Large products produced: ", large_plant2, "\n")
```

```
## Large products produced:  0
```

```r
cat("Medium products produced: ", medium_plant2, "\n")
```

```
## Medium products produced:  400
```

```r
cat("Small products produced: ", small_plant2, "\n")
```

```
## Small products produced:  500
```

```r
cat("\nPlant 3:\n")
```

```
##
## Plant 3:
```

```r
cat("Large products produced: ", large_plant3, "\n")
```

```
## Large products produced:  0
```

```r
cat("Medium products produced: ", medium_plant3, "\n")
```

```
## Medium products produced:  133.3333
```

```r
cat("Small products produced: ", small_plant3, "\n")
```

```
## Small products produced:  250
```

```r
# Print the maximum profit
cat("\nMaximum Profit: $", round(max_profit, 2), "\n")
```

```
##
## Maximum Profit: $ 708000
```

```r
# === Output for Problem  (Weigelt Corporation) ===
# Plant 1:
# Large products produced:  350
# Medium products produced:  400
# Small products produced:  0
#
# Plant 2:
# Large products produced:  0
# Medium products produced:  400
# Small products produced:  500
#
# Plant 3:
# Large products produced:  0
# Medium products produced:  133.3333
# Small products produced:  250
#
# Maximum Profit: $ 708000
```

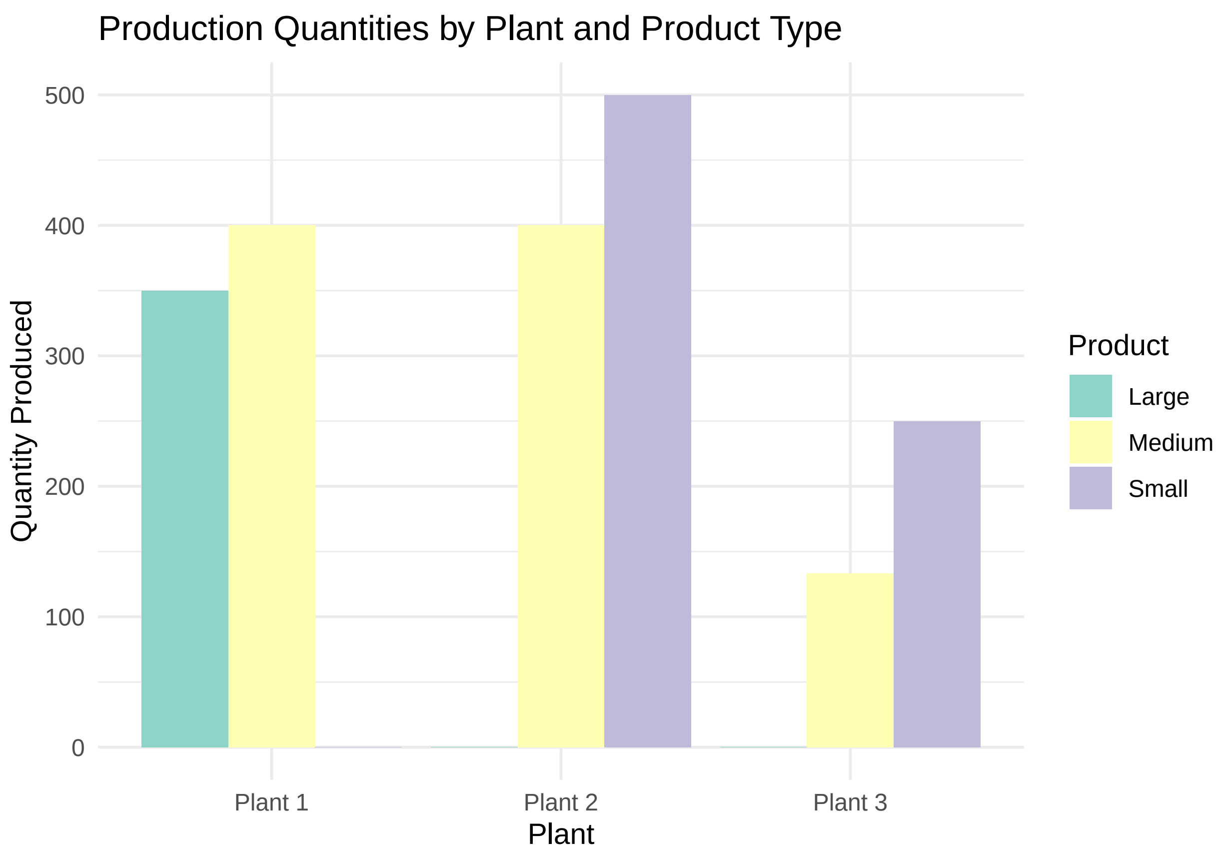
Data Visualization:

 # 4. Interpret the output from a LP solution: The results obtained from R are analyzed and explained, showing how the production of each product size in each plant contributes to maximizing the company's profit, while satisfying all the problem constraints.

```
# Loading the Package.
library(ggplot2) # For Data Visualization.

# Create a data frame with the production values
production_data <- data.frame(
  Plant = rep(c("Plant 1", "Plant 2", "Plant 3"), each = 3),
  Product = factor(rep(c("Large", "Medium", "Small"), times = 3)),
  Quantity = c(large_plant1, medium_plant1, small_plant1,
               large_plant2, medium_plant2, small_plant2,
               large_plant3, medium_plant3, small_plant3)
)

# Plot a grouped bar chart
ggplot(production_data, aes(x = Plant, y = Quantity, fill = Product)) +
  geom_bar(stat = "identity", position = "dodge") +
  labs(title = "Production Quantities by Plant and Product Type",
       x = "Plant", y = "Quantity Produced") +
  theme_minimal() +
  scale_fill_brewer(palette = "Set3")
```



Summary of the Optimal Production Plan for Weigelt Corporation:

```
# The Weigelt Corporation needs to optimize its production of three product sizes (Large, Medium, and Sm

# 1. **Plant 1** will produce 350 large products and 400 medium products. No small products will be pro
# 2. **Plant 2** will focus on medium and small products, producing 400 medium products and 500 small p
# 3. **Plant 3** will primarily produce 133.33 medium products and 250 small products, with no large pr

# The total maximum profit from this production plan is **$708,000**.
```

Key Findings:

```
# 1. **Balanced Production**: The optimal production strategy efficiently allocates large and medium pr
# 2. **Profit Maximization**: The solution maximizes the profit at **$708,000**, utilizing the availabl
# 3. **Specialized Production by Plant**:
#   - **Plant 1** focuses on producing large and medium products.
#   - **Plant 2** specializes in producing medium and small products.
#   - **Plant 3** contributes to producing medium and small products in smaller quantities.
# 4. **No Idle Capacity**: All available production capacity is efficiently utilized across the plants
# 5. **Sales Forecast Alignment**: The production plan is aligned with the sales forecasts, ensuring th
```

Conclusion:

```
# The linear programming solution provides a clear production plan for Weigelt Corporation, helping it

# Additionally, the plan adheres to the sales forecasts, ensuring the company does not overproduce or u
```