Linear Programming Model Assignment-2

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Contents

Problem 1: Backpack Production

Step 1: Define Decision Variables

Let:

- x_1 = Number of Collegiate backpacks produced per week.
- x_2 = Number of Mini backpacks produced per week.

Step 2: Write the Objective Function

The objective is to maximize profit. The profit function is based on the number of Collegiate and Mini backpacks produced and their respective unit profits.

$$\text{Maximize } Z = 32x_1 + 24x_2$$

Where:

- $32x_1$ is the profit from producing x_1 Collegiates.
- $24x_2$ is the profit from producing x_2 Minis.

Step 3: Identify Constraints

1. **Material constraint**: The total nylon used by both models must not exceed 5000 square feet.

$$3x_1 + 2x_2 \leq 5000$$

2. **Labor constraint**: The total labor used by both models must not exceed the available labor hours (1400 hours).

$$0.75x_1 + 0.67x_2 \leq 1400$$

3. **Sales constraint**: The number of backpacks produced cannot exceed the sales forecast.

$$x_1 \leq 1000 \quad \text{and} \quad x_2 \leq 1200$$

4. **Non-negativity constraint**: The number of backpacks produced must be non-negative.

$$x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0$$

Step 4: Full LP Formulation

$$\text{Maximize } Z = 32x_1 + 24x_2$$

Subject to:

$$3x_1 + 2x_2 \leq 5000 \quad \text{(material constraint)}$$

$$0.75x_1 + 0.67x_2 \leq 1400 \quad \text{(labor constraint)}$$

$$x_1 \leq 1000 \quad \text{(sales constraint for Collegiate)}$$

$$x_2 \leq 1200 \quad \text{(sales constraint for Mini)}$$

$$x_1, x_2 \geq 0 \quad \text{(non-negativity constraint)}$$

#Problem 1: Backpack Production for Back Savers

```
# Load the lpSolve package
library(lpSolve)

# Coefficients of the objective function
objective <- c(32, 24)

# Constraints matrix
constraints <- matrix(c(3, 2,     # Material constraint
                        0.75, 0.67,  # Labor constraint
                        1, 0,     # Sales forecast for Collegiate
                        0, 1),    # Sales forecast for Mini
                      nrow = 4, byrow = TRUE)

# Right-hand side of the constraints
rhs <- c(5000, 1400, 1000, 1200)

# Direction of the constraints
constraints_dir <- c("<=", "<=", "<=", "<=")

# Solve the linear programming problem
solution <- lp("max", objective, constraints, constraints_dir, rhs)

# Output the results for Problem 1 in a formatted manner
print("\n=== Output for Problem 1 (Backpack Production) ===\n")
```

```
## [1] "\n=== Output for Problem 1 (Backpack Production) ===\n"
```

```
print("\nCollegiate Backpack:\n")
```

```
## [1] "\nCollegiate Backpack:\n"
```

```
print("Material per unit: 3 sq.ft\n")
```

```
## [1] "Material per unit: 3 sq.ft\n"
```

```
print("Max sales: 1000 units\n")
```

```
## [1] "Max sales: 1000 units\n"
```

```
print("Labor per unit: 45 min\n")
```

```
## [1] "Labor per unit: 45 min\n"
```

```r
print("Profit per unit: $32\n")
```

```
## [1] "Profit per unit: $32\n"
```

```r
print("\nMini Backpack:\n")
```

```
## [1] "\nMini Backpack:\n"
```

```r
print("Material per unit: 2 sq.ft\n")
```

```
## [1] "Material per unit: 2 sq.ft\n"
```

```r
print("Max sales: 1200 units\n")
```

```
## [1] "Max sales: 1200 units\n"
```

```r
print("Labor per unit: 40 min\n")
```

```
## [1] "Labor per unit: 40 min\n"
```

```r
print("Profit per unit: $24\n")
```

```
## [1] "Profit per unit: $24\n"
```

```r
print("\nAvailability:\n")
```

```
## [1] "\nAvailability:\n"
```

```r
print("Total material available: 5000 sq.ft\n")
```

```
## [1] "Total material available: 5000 sq.ft\n"
```

```r
print("Total labor available: 35 workers × 40 hours = 1400 hours\n")
```

```
## [1] "Total labor available: 35 workers × 40 hours = 1400 hours\n"
```

```r
# Print the optimal solution (number of backpacks to produce)
# Install and load the glue package
# install.packages("glue")
library(glue)

# Using glue for string interpolation
cat(glue("Optimal number of Collegiate backpacks to produce: {solution$solution[1]}\n"))
```

```
## Optimal number of Collegiate backpacks to produce: 1000
```

```r
cat(glue("Optimal number of Mini backpacks to produce: {solution$solution[2]}\n"))
```

```
## Optimal number of Mini backpacks to produce: 970.149253731343
```

```r
# Print the maximum profit
cat("Maximum profit: $", solution$objval, "\n")
```

```
## Maximum profit: $ 55283.58
```

```r
# === Output for Problem 1 (Backpack Production) ===
#
# Collegiate Backpack:
# Material per unit: 3 sq.ft
# Max sales: 1000 units
# Labor per unit: 45 min
# Profit per unit: $32
#
# Mini Backpack:
# Material per unit: 2 sq.ft
# Max sales: 1200 units
# Labor per unit: 40 min
# Profit per unit: $24
#
# Availability:
# Total material available: 5000 sq.ft
# Total labor available: 35 workers × 40 hours = 1400 hours
#
# Optimal Solution:
# Optimal number of Collegiate backpacks to produce: 1000
# Optimal number of Mini backpacks to produce: 970.1493
# Maximum profit: $ 55283.58
```

Summary of Key Insights for Problem 1:

1. **Optimal Production Mix**: The solution recommends producing 1000 Collegiate and 970 Mini backpacks to maximize profit, staying within material, labor, and sales constraints.
2. **Resource Utilization**: The total material usage is 4940.3 sq.ft out of 5000, and labor usage is 1399.99 hours out of 1400, indicating nearly perfect utilization of resources.
3. **Profit Maximization**: The production plan results in a maximum profit of $55,283.58, fully utilizing the available resources while meeting demand.
4. **Sales Forecast Alignment**: The plan ensures that sales forecasts are met for both products, with full production of 1000 Collegiate units and nearly full production of 970 Mini units (out of 1200 available).

Problem 2: Weigelt Corporation Production

a. Define the Decision Variables

Let: - x_{11}: Number of large products produced at Plant 1. - x_{12}: Number of medium products produced at Plant 1. - x_{13}: Number of small products produced at Plant 1. - x_{21}: Number of large products produced at Plant 2. - x_{22}: Number of medium products produced at Plant 2. - x_{23}: Number of small products produced at Plant 2. - x_{31}: Number of large products produced at Plant 3. - x_{32}: Number of medium products produced at Plant 3. - x_{33}: Number of small products produced at Plant 3.

These variables represent the number of large, medium, and small products produced at each of the three plants.

---

b. Formulate a Linear Programming Model

Objective: Maximize the total profit from producing large, medium, and small products across the three plants.

Objective Function:

Maximize:

$$Z = 420(x_{11} + x_{21} + x_{31}) + 360(x_{12} + x_{22} + x_{32}) + 300(x_{13} + x_{23} + x_{33})$$

Where: - 420 is the profit per large product. - 360 is the profit per medium product. - 300 is the profit per small product.

Constraints:

1. **Capacity constraints**:

$$x_{11} + x_{12} + x_{13} \leq 750 \quad \text{(Plant 1)}$$

$$x_{21} + x_{22} + x_{23} \leq 900 \quad \text{(Plant 2)}$$

$$x_{31} + x_{32} + x_{33} \leq 450 \quad \text{(Plant 3)}$$

2. **Storage constraints**:

$$20x_{11} + 15x_{12} + 12x_{13} \leq 13000 \quad \text{(Plant 1 storage)}$$

$$20x_{21} + 15x_{22} + 12x_{23} \leq 12000 \quad \text{(Plant 2 storage)}$$

$$20x_{31} + 15x_{32} + 12x_{33} \leq 5000 \quad \text{(Plant 3 storage)}$$

3. **Sales forecast constraints**:

$$x_{11} + x_{21} + x_{31} \leq 900 \quad \text{(Large products)}$$

$$x_{12} + x_{22} + x_{32} \leq 1200 \quad \text{(Medium products)}$$

$$x_{13} + x_{23} + x_{33} \leq 750 \quad \text{(Small products)}$$

4. **Non-negativity**:

$$x_{ij} \geq 0 \quad \text{for all } i, j$$

```r
# Coefficients of the objective function
objective2 <- c(420, 360, 300, 420, 360, 300, 420, 360, 300)

# Constraints matrix
constraints2 <- matrix(c(1, 1, 1, 0, 0, 0, 0, 0, 0,  # Plant 1 capacity
                         0, 0, 0, 1, 1, 1, 0, 0, 0,  # Plant 2 capacity
                         0, 0, 0, 0, 0, 0, 1, 1, 1,  # Plant 3 capacity
                         20, 15, 12, 0, 0, 0, 0, 0, 0,  # Plant 1 storage
                         0, 0, 0, 20, 15, 12, 0, 0, 0,  # Plant 2 storage
                         0, 0, 0, 0, 0, 0, 20, 15, 12,  # Plant 3 storage
                         1, 0, 0, 1, 0, 0, 1, 0, 0,  # Sales constraint for Large
                         0, 1, 0, 0, 1, 0, 0, 1, 0,  # Sales constraint for Medium
                         0, 0, 1, 0, 0, 1, 0, 0, 1),  # Sales constraint for Small
                       nrow = 9, byrow = TRUE)

# Right-hand side of the constraints
rhs2 <- c(750, 900, 450, 13000, 12000, 5000, 900, 1200, 750)

# Direction of the constraints
constraints_dir2 <- c("<=", "<=", "<=", "<=", "<=", "<=", "<=", "<=", "<=")

# Solve the LP problem
solution2 <- lp("max", objective2, constraints2, constraints_dir2, rhs2)

# Store the solution values
large_plant1 <- solution2$solution[1]
medium_plant1 <- solution2$solution[2]
small_plant1 <- solution2$solution[3]
large_plant2 <- solution2$solution[4]
medium_plant2 <- solution2$solution[5]
small_plant2 <- solution2$solution[6]
large_plant3 <- solution2$solution[7]
medium_plant3 <- solution2$solution[8]
small_plant3 <- solution2$solution[9]
max_profit2 <- solution2$objval
```

```r
# Print the solution for Plant 1, 2, and 3
print("Optimal Production Plan for Weigelt Corporation:\n")
```

```
## [1] "Optimal Production Plan for Weigelt Corporation:\n"
```

```r
# Print the solution for all plants
cat("Plant 1 - Large:", large_plant1, "Medium:", medium_plant1, "Small:", small_plant1, "\n")
```

```
## Plant 1 - Large: 350 Medium: 400 Small: 0
```

```r
cat("Plant 2 - Large:", large_plant2, "Medium:", medium_plant2, "Small:", small_plant2, "\n")
```

```
## Plant 2 - Large: 0 Medium: 400 Small: 500
```

```r
cat("Plant 3 - Large:", large_plant3, "Medium:", medium_plant3, "Small:", small_plant3, "\n")
```

```
## Plant 3 - Large: 0 Medium: 133.3333 Small: 250
```

```r
cat("\nMaximum Profit: $", round(max_profit2, 2), "\n")
```

```
##
## Maximum Profit: $ 708000
```

```r
# === Output for Problem 2 (Weigelt Corporation) ===
# "Optimal Production Plan for Weigelt Corporation:\n"
# Plant 1 - Large: 350 Medium: 400 Small: 0
# Plant 2 - Large: 0 Medium: 400 Small: 500
# Plant 3 - Large: 0 Medium: 133.3333 Small: 250
# Maximum Profit: $ 708000
```

Summary of Key Insights for Problem 2:

1. **Balanced Production Strategy**: The solution focuses on Medium and Small products, which yield higher profit margins, optimizing the production mix across all three plants.
2. **Efficient Capacity Utilization**: Plants 1 and 2 are fully utilized, while Plant 3 operates at about 85% capacity, ensuring minimal idle resources.
3. **Profit Maximization**: The production plan achieves a maximum profit of $708,000, leveraging the optimal distribution of products across plants to maximize returns.
4. **Sales Forecast Alignment**: The production plan meets forecasted demand, producing 350 Large, 933.33 Medium, and 750 Small products across the three plants.
5. **Resource Optimization**: By focusing on higher-margin products, the company maximizes profitability while ensuring that capacity and storage are efficiently utilized without exceeding limits.

Conclusion:

1. **Back Savers**: The optimal production of 1000 Collegiate and 970 Mini backpacks yields a maximum profit of $55,283.58 by fully utilizing material and labor resources.
2. **Weigelt Corporation**: The production of 350 Large, 933.33 Medium, and 750 Small products across the three plants results in a maximum profit of $708,000, with efficient use of plant capacity and storage space.