Homework 8

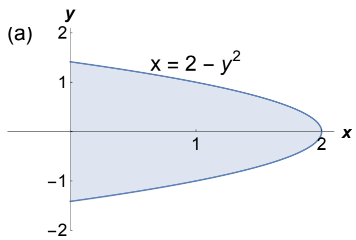
Math 3302, Fall 2018

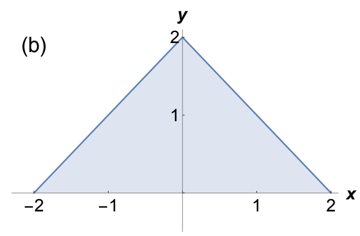
Due November 7

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. A lamina occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

2. Find the centroids of the following figures





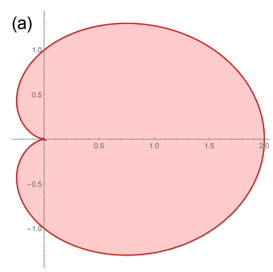
3. Electric charge is distributed over the disk $x^2 + y^2 \leq 1$ so that the charge density at (x, y) is $\sigma(x, y) = \sqrt{x^2 + y^2}$ (measured in Coulumbs per square meter). Find the total charge on the disk.

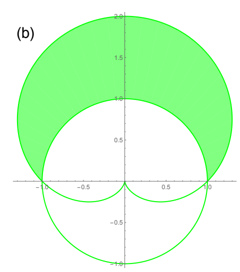
4. The following formula is used for finding the area of a polar region described by the polar curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$:

$$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 \, d\theta$$

(a) Find the area inside the cardiod $r = 1 + \cos\theta$, $0 \leq \theta \leq 2\pi$.

(b) Find the area inside the cardiod $r = 1 + \sin\theta$, $0 \leq \theta \leq 2\pi$, and outside the unit circle $r = 1$.





5. At a certain restaurant, customers must wait an average of 10 minutes for a table. From the time they are seated until they have finished their meal requires an additional 30 minutes, on average. What is the probability that a customer will spend less than an hour at the restaurant, assuming that waiting for a table and completing the meal are independent events? *Hint: waiting times are often modeled by exponential probability densities. Indeed, if X and Y are the random variables for waiting for a table and completing the meal, respectively, then their probability density functions are respectively*

$$p_1(X) = \begin{cases} 0 & if\ X < 0 \\ \frac{1}{10} e^{-X/10} & if\ X \geq 0 \end{cases} \qquad p_2(Y) = \begin{cases} 0 & if\ Y < 0 \\ \frac{1}{30} e^{-Y/30} & if\ Y \geq 0 \end{cases}$$

Since the events are independent, the joint probability for the two events is

$$p(X, Y) = p_1(X) p_2(Y) = \begin{cases} 0 & if\ X < 0\ or\ Y < 0 \\ \frac{1}{300} e^{-X/10} e^{-Y/30} & if\ X, Y \geq 0 \end{cases}$$

The goal is to determine the probability that the combined time X + Y is under 60 minutes.