Math 3302: Practice Exam 1

Sept. 21, 2018

Name: ________________________

On my honor, I have neither given nor received aid on this exam

Signature: ________________________

The following will be the instructions on your in-class exam (although the number of questions may vary):

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- No calculators or other electronic devices are permitted without the explicit consent of the instructor. Please turn your cell phone off and keep it out of sight at all times
- This exam is open book and open notes.
- Circle or otherwise indicate your final answers.
- This test has x problems and is worth xxx points. Please make sure that you have all of the pages.

#	Score
1	
2	
3	
4	
5	
6	
Total	

1. For the vectors

$$\begin{aligned}\mathbf{a} &= 2\vec{\imath} + 2\vec{\jmath} + 1\vec{k}\\ \mathbf{b} &= 4\vec{\imath} + 1\vec{\jmath} + 1\vec{k}\\ \mathbf{c} &= 5\vec{\imath} + 3\vec{\jmath} + 4\vec{k}\\ \mathbf{d} &= 3\vec{\imath} + 1\vec{\jmath} + 4\vec{k}\end{aligned}$$

compute each of the following, or state why the indicated product does not exist:

(a) $\mathbf{a} \cdot \mathbf{b}$

(b) $\mathbf{c} \times \mathbf{d}$

(c) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(d) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

(e) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

2. Let $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ each be vectors. Indicate whether each of the following products is a vector, scalar, or not defined.

(a) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(b) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

(c) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

(d) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$

(e) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

(f) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

3. For the vectors

$$\begin{aligned}\mathbf{a} &= 1\vec{\imath} + 2\vec{\jmath} + 3\vec{k}\\ \mathbf{b} &= 4\vec{\imath} + 5\vec{\jmath} + 6\vec{k}\end{aligned}$$

Show that $(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = 2\,(\mathbf{a} \times \mathbf{b})$.

4. Consider the lines

$$\begin{aligned}\mathbf{r}_1(t) &= \langle 3, 5, 7\rangle + t\,\langle 1, 2, 3\rangle\\ \mathbf{r}_2(s) &= \langle -2, -1, 0\rangle + s\,\langle 3, 2, 1\rangle\,.\end{aligned}$$

(a) Find the point of intersection of these lines.

(b) Find the equation of the plane that contains these lines.

5. Write an equation for the plane containig the points $(-7, 2, 1)$, $(9, 0, -2)$ and $(-5, -1, 2)$. Is this plane parallel, perpendicular or neither, to the plane

$$2x - 3y + z = 5?$$

6. Consider the two planes $x - z = y + 4$ and $2x + y = z$.

(a) Find the vectors that are perpendicular to these two planes, let's call these vectors $\vec{u}$ and $\vec{v}$.

(b) Parallel planes have parallel $\vec{u}$ and $\vec{v}$. Show that these two planes are not parallel.

(c) Since the planes are not parallel then they intersect in a line, let's call this line $\mathcal{L}$. Find any point on $\mathcal{L}$.

(d) Since $\mathcal{L}$ is parallel to both planes, it is perpendicular to both $\vec{u}$ and $\vec{v}$. Find a vector perpendicular to $\vec{u}$ and $\vec{v}$, that is therefore parallel to $\mathcal{L}$.

(e) Using your results from parts (c) and (d), write the line of intersection for the two planes.

7. Consider the curve $\vec{r}(t) = \sin(3t)\,\vec{i} + t^2\vec{j} + \cos(3t)\,\vec{k}$. Find

(a) $\frac{d}{dr}\vec{r}(t)$

(b) $\vec{T}(t)$

(c) $\frac{d}{dt}\left[\vec{T}(t)\cdot\vec{T}(t)\right]$

8. If $r(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + 3t\vec{k}$ find $\vec{T}, \vec{N}$and $\vec{B}$ when $t = \frac{3\pi}{2}$.

9. Consider the function $f(x,y) = \frac{1}{1+x^2+y^2}$. Find the equations for the following level surfaces for f ans sketch them.

(a) $f(x,y) = \frac{1}{5}$

(b) $f(x,y) = \frac{1}{10}$

10. Describe the level surfaces $f(x,y,z) = k$ for the function $f(x,y,z) = 1 - x^2 - \frac{y^2}{2} - \frac{z^2}{3}$ and the values $k = -1$, $k = 1$, and $k = -2$.

11. What is the arclength of the curve $\vec{r}(t) = \langle e^t\sin(2t), 0, e^t\cos(2t)\rangle$ from $t = 1$ to $t = 2$? Provide an exact expression by performing all derivatives/integrals by hand; you do not need to simplify your answer.

12. Determine whether the curve $\vec{r}(t) = \langle 2t\cos(2t), 2t\sin(2t), t\rangle$ intersects the surface $4z^2 = x^2 + y^2$. If so, where do they intersect?

13. An ostrich runs along a mountain path with coordinates given by $\vec{r}(t) = \left\langle e^t, e^{-t}, \sqrt{2}t\right\rangle$. What is the total distance travelled by the ostrich from $t = 0$ to $t = 4$?

14. Find the curvature of $\vec{r}(t) = \left\langle t, t^2, t^3\right\rangle$ at the point $(1,1,1)$.

15. Find the domain and range of the indicated functions. Be precise; the range can be contained in $\mathbb{R}$, but it does not necessarily cover all of $\mathbb{R}$.

(a) $f(x,y) = 3x^2 - y^2 + 5$

(b) $f(x,y) = 2\ln\left(1 - x^2 - y^2\right) - \frac{1}{x}$

(c) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 4}$

(d) $f(x, y, z) = 2\cos x - 3\sin(y + z)$

16. Find the first partial derivatives of the functions:

(a) $f(x, t) = e^{-t} \cos \pi x$

(b) $f(x, y) = \sin^2(xy^2)$

(c) $f(x, y) = x^y$

17. Let $f(x, y, z) = xy^4 z^3$.Verify that $f_{xyz} = f_{xzy} = f_{zyx}$.

18. Demostrate that the functions $f(x, y) = 5xy$ and $f(x, y) = \arctan(y/x)$ both solve the Laplace equation $f_{xx} + f_{yy} = 0$.

19. In a study of frost penetration it was found that the temperature T at time t (measured in days) at a depth x (measured in feet) can be modeled by the function

$$T(x, t) = T_0 + T_1 e^{-\lambda x} \sin(\omega t - \lambda x),$$

where $\omega = 2\pi/365$ and λ is a positive constant.

(a) Find $T_x(x, t)$. What is its physical significance?

(b) FInd $T_t(x, t)$. What is its physical significance?

(c) Show that T satisfies the heat equation $T_t = \kappa T_{xx}$ where κ is a constant. Write κ in terms of ω and λ.

20. If the following equation implicitely defines z as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$,

$$x^2 yz + 2xz^3 - 3yz = 2xy.$$