Homework 7

Math 3302, Fall 2018

Due October 29

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Evaluate the iterated integral over $R = [0, 2] \times [0, 1]$

$$\int\int_R 2xy\, e^{xy^2} dA$$

2. Given that $\int_0^{\pi/2} \frac{dx}{1+\sin^2 x} = \frac{\pi}{2\sqrt{2}}$. Evaluate the double integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{\left(1+\sin^2 x\right)\left(1+\sin^2 y\right)} dxdy$$

3. Find the volume of the solid in the first octant (i.e. $x \geq 0$, $y \geq 0$ and $z \geq 0$), enclosed by the cylinder $z = 9 - y^2$ and the plane $x = 5$.

4. Consider the integral

$$\iint_D 2x^2 e^{xy} dA$$

where D is bounded by $y = x$, $x = 4$ and $y = 0$.

(a) Set up the iterated integrals for both orders of integration, $dxdy$ and $dydx$.

(b) Evaluate either one of your two formulas from part (a).

5. Consider the region R enclosed by $y = x$, $y = -x + 2$, and $y = -\sqrt{1 - (x-1)^2}$. Sketch the region R. Set up the following integrals as one or more iterated integrals, but do not actually compute them:

(a) $\iint_R (x + y)\, dydx$

(b) $\iint_R (x + y)\, dxdy$

6. Evaluate $\iint_D \cos\left(x^2 + y^2\right) dA$, where $D = \left\{(x, y)\,|1 \leq x^2 + y^2 \leq 4\right\}$ is a ring with inner radius 1 and outer radius 2.

7. Consider the area of the shadded region in Figure 1 below, which is enclosed by the circle $x^2 + y^2 = 2$ and the line $x = 1$.

(a) Represent the circle $x^2 + y^2 = 2$ in polar coordinates.

(b) Represent the line $x = 1$ in polar coordinates.

(c) Write down a double integral -in polar coordinates- describing the area of this region

(d) Solve the integral to find the area.

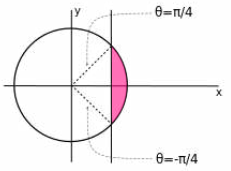

Figure 1: Circle sector