

## Homework 2

Math 3302, Fall 2018

Due September 12

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. For the vectors

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$$

$$\vec{b} = 4\vec{i} + 5\vec{j} + 6\vec{k}$$

$$\vec{c} = 7\vec{i} + 8\vec{j} + 9\vec{k}$$

(a) What is  $\vec{a} \cdot \vec{b}$ ?

(b) What is  $\vec{b} \times \vec{c}$ ?

(c) What is  $\vec{a} \cdot (\vec{b} \times \vec{c})$ ?

(d) What is  $\vec{a} \times (\vec{b} \times \vec{c})$ ?

2. Say whether each of the following statements is true or false, and why.

(a) If  $\vec{a} \times \vec{b} = \vec{0}$  and  $\vec{b} \times \vec{c} = \vec{0}$ , then  $\vec{a} \times \vec{c} = \vec{0}$ .

(b) If  $\vec{a} \cdot \vec{b} = 0$  and  $\vec{b} \cdot \vec{c} = 0$ , then  $\vec{a} \cdot \vec{c} = 0$ .

(c) If any two of  $\vec{a}, \vec{b}, \vec{c}$  are parallel, then  $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

(d) If any two of  $\vec{a}, \vec{b}, \vec{c}$  are perpendicular, then  $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{0}$

3. Show that  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$  for all vectors  $\vec{a}, \vec{b}$ , and  $\vec{c}$  in  $V_3$ . *Hint: write  $\vec{a} = \langle a_1, a_2, a_3 \rangle$ ,  $\vec{b} = \langle b_1, b_2, b_3 \rangle$  and  $\vec{c} = \langle c_1, c_2, c_3 \rangle$  where the components may be arbitrary real numbers, perform the relevant calculations on each side, and simplify to show they are equal.*

4. If  $\vec{a} \cdot \vec{b} = 1$  and  $\vec{a} \times \vec{b} = \langle 1, -1, -1 \rangle$ , find the angle  $\theta$  between  $\vec{a}$  and  $\vec{b}$ . *Hint: you may assume that  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ .*

5. Consider a line  $L_1$  through the points  $(2, -9, 5)$  and  $(1, -6, 7)$ , and a line  $L_2$  through the points  $(-3, -2, -1)$  and  $(9, 1, -1)$ .

(a) Write the vector equations for these lines.

(b) Determine whether  $L_1$  and  $L_2$  are parallel, skew or intersecting. If they intersect, find the point of intersection.

6. Consider 5 points:  $P(1, 1, 1)$ ,  $Q(3, 5, 7)$ ,  $R(1, 2, -2)$ ,  $S(2, 1, 2)$ , and  $T(2, 2, 3)$ .

(a) Find the equation of the line through  $P$  and  $Q$ .

(b) Find the equation for the plane containing  $R$ ,  $S$ , and  $T$ .

(c) Construct a line through  $P$ , that is perpendicular to the plane in (b).

(d) Construct a plane containing the line in (a), that is perpendicular to the plane in (b).

7. Consider the line that passes through the point  $(6, -5, 2)$  and that is parallel to the vector  $\langle -1, 1, 3 \rangle$ .

(a) Find symmetric equations for this line.

(b) Find the point at which this line passes through the  $xy$ -plane.

(c) Find the point at which this line passes through the  $yz$ -plane.

(d) Find the point at which this line passes through the  $xz$ -plane.