

10.6 Cylinders and quadric surfaces

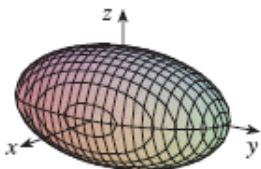
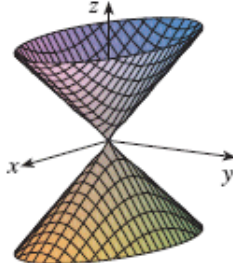
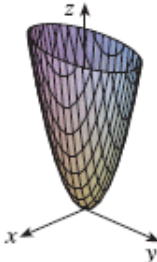
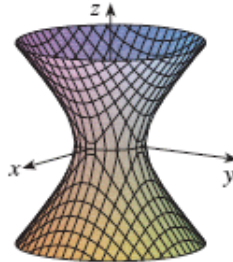
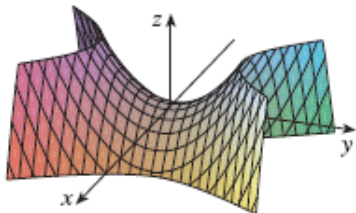
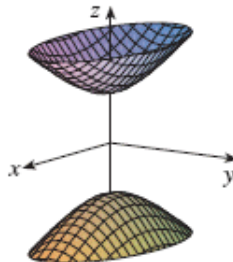
Things to look for:

- are all terms squared?
- do all squared terms have the same sign?
- if they have different signs, which is the odd-dim-out?
- what sign (if any) does the constant have?

Tips when drawing:

- Consider traces of the 3D surface on a given plane by setting the corresponding variable to a constant
 - xy -plane ($z = 0$)
 - planes parallel to xy -plane ($z = \text{const}$)
- Sketch the 2D traces for a number of values of the other variable, for example $z = 0, 1, 2, -1, -2$,
- Draw dotted lines at first, and then fill in lines in the foreground.

Graphs of quadric surfaces

Surface	Equation	Surface	Equation
<p>Ellipsoid</p> 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ <p>All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.</p>	<p>Cone</p> 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ <p>Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.</p>
<p>Elliptic Paraboloid</p> 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ <p>Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.</p>	<p>Hyperboloid of One Sheet</p> 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ <p>Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.</p>
<p>Hyperbolic Paraboloid</p> 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ <p>Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.</p>	<p>Hyperboloid of Two Sheets</p> 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ <p>Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.</p>