For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Find the length of the curve $\left\langle \sqrt{1+t^3}, \sqrt{1+t^3}, 0 \right\rangle$ between $t = 0$ and $t = 2$.

2. Let C be the circle of radius 3 centered at the point $(2, 5)$ in the plane.

(a) Give the arclength parametrization $\vec{r}(s)$ of this curve, starting at $(2, 8)$.

(b) Verify that the curvature is constant.

3. Suppose you start at the point $\left(\frac{4\pi}{2}, 3, 0\right)$ and move 5 units along the curve $\langle 4t, 3\sin t, 3\cos t \rangle$ in the negative t direction. Where are you now? *Hint: start by reparameterizing with respect to arclength.*

4. Find the domain and range of the function $f(x, y, z) = \sqrt{x} + \sqrt{16 - x^2 - y^2 - z^2}$.

5. Find a pair of x, y with $0 < x < 1$, $0 < y < 1$ such that $f(x, y) \geq 8$, or show that no values x and y can exist, for each of the following functions of two variables:

(a) $f(x, y) = \frac{1}{x} + \frac{1}{y}$

(b) $f(x, y) = \frac{1}{x - y + 1}$

6. Let $f(x, y, z) = \ln\left(x^2 + \sqrt{y^2 + 2z}\right)$. Find the functions f_x, f_y, and f_z.

7. The *wind-chill index* is a measure of how cold it feels in windy weather, and it is modeled by the function

$$W(T, v) = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16},$$

where T is the temperature (in °C) and v is the wind speed (in km/h). When $T = -5$°C and $v = 25$ km/h, by how much would you expect W to change if:

(a) the actual temperature increases by 1°C? Hint: compute $\frac{\partial W}{\partial T}$.

(b) the wind speed decreases by 1 km/h?