Homework 10

Math 3302, Fall 2018

Due November 19

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Find the gradient vector field of $f(x,y,z) = \sqrt{x^2+y^2+z} - y\ln(z+2x)$.

2. Consider the vector field $\vec{F}(x,y,z) = \langle x^2, y^4, z^6 \rangle$

(a) Compute the line integral of $\vec{F}(x,y,z)$ over the path $\vec{r}: x = t^3$, $y = t$, $z = t^2$, $0 \le t \le 1$.

(b) Find $g(x,y,z)$ so that $\vec{F} = \nabla g$.

3. Determine if the following line integrals $\int_C f(x,y)ds$ are positive, negative or 0 either by graphical analysis or by direct computation.

(a) $f(x,y) = \frac{y}{x^2+y^2}$; C is the top half of the unit circle, starting at $(-1,0)$ and moving clockwise.

(b) $f(x,y) = \frac{y}{x^2+y^2}$; C is the bottom half of the unit circle, starting at $(1,0)$ and moving clockwise.

4. Determine if the following line integrals $\int_C \mathbf{F}\cdot d\mathbf{r}$ are positive, negative or 0 either by graphical analysis or by direct computation.

(a) $\vec{F}(x,y) = \langle x, -y\rangle$; C is the top half of the unit circle, starting at $(1,0)$ and moving counterclockwise.

(b) $f(x,y) = \left\langle x, -\frac{1}{\sqrt{x}} \right\rangle$; C is the part of the parabola $y = x^2$starting at $(1,1)$ and ending at $(2,4)$.

5. Find the work done by the force field $\vec{F}(x,y) = \langle xe^y,\ xy^2 \rangle$ on a particle that moves along the curve $y = 1 - x^2$ from $(1,0)$ to $(0,1)$.

6. A charge q located at $(0,0)$ creates an electric field at (x,y) given by

$$\vec{F} = \frac{K\left(x\vec{i} + y\vec{j}\right)}{(x^2+y^2)^{3/2}},$$

where K is a constant. Calculate the work required to move a charge along the path C_1, the top half of $x^2 + y^2 = 4$.