

Homework 1

Math 3302, Fall 2018

Due August 31

1. Suppose the graph of a function $f(x)$ is given. Write equations for the graphs that are obtained from the graph of $f(x)$ as follows:

- (a) Shifts the graph 3 units upward
- (b) Shifts the graph 2 units to the left
- (c) Reflects the graph across the x -axis
- (d) Stretches the graph vertically by a factor of 4

For example, if the prompt were “Reflects the graph across the y axis” the answer would be $y = f(-x)$

2. Differentiation

- (a) Using appropriate rules, compute the derivative $\frac{dy}{dx}$ if

$$y = e^x \frac{x}{x+1}$$

- (b) Use implicit differentiation to find $\frac{dy}{dx}$ if

$$x = \tan(y).$$

Hint: draw a triangle to help you write the results in terms of x .

3. Compute the following indefinite integrals:

- (a) $\int 2x\sqrt{1+x^2}dx$
- (b) $\int xe^x dx$

4. A particle traces out the trajectory given by the parametric curve

$$x = t^3 - t$$

$$y = \sqrt{3}(t^2 - 1)$$

- (a) Sketch the graphs of the points of the functions $x(t)$ and $y(t)$
- (b) Using the results from part (a), sketch the trajectory of the particle in the (x, y) plane.
- (c) What is the local distance travelled by the particle during the interval $t \in [-1, 1]$?

5. Consider the polar curve

$$r = \sin(2\theta).$$

- (a) Sketch the graph of this function for $\theta \in [0, 2\pi]$
- (b) What is the area enclosed by the graph over the interval $\theta \in [0, \frac{\pi}{2}]$

6. Find an equation of the sphere if one of its diameters has end points $(2, -3, 5)$ and $(0, -1, 3)$.

7. Consider the vector $\vec{v} = 3\vec{i} - 2\vec{j} + 18\vec{k}$.

(a) Find the unit vector \vec{u} in the same direction as \vec{v} .

(b) Find the vector \vec{w} in the opposite direction as \vec{v} that has magnitude 6.

8. Let $\vec{a} = -2\vec{i} + 7\vec{j} - \vec{k}$ and $\vec{b} = 5\vec{i} + 3\vec{j}$. Compute the following:

(a) $\vec{a} + \vec{b}$ (d) $\|\vec{a}\|$

(b) $2\vec{a} + 7\vec{b}$ (e) $\|\vec{a} + 3\vec{b}\|$

(c) $6\vec{a} - 4\vec{b}$ (f) $\|2\vec{a}\|$

9. Determine whether the given vectors are perpendicular, parallel or neither:

(a) $\vec{u} = 8\vec{i} + 12\vec{j} - 4\vec{k}$, $\vec{v} = -6\vec{i} - 9\vec{j} + 3\vec{k}$

(b) $\vec{u} = \langle -1, 1, 2 \rangle$, $\vec{v} = \langle 1, -2, -1 \rangle$

(c) $\vec{u} = \langle a, b, c \rangle$, $\vec{v} = \langle 0, c, -b \rangle$,

10. If $\vec{r} = \langle x, y, z \rangle$, $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, show that the vector equation

$$(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$$

is the equation of a sphere. What are the coordinates of the center of this sphere? What is the radius of the sphere?