Homework 11

Math 3302, Fall 2018

Due November 30

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Consider the vector field $\vec{F} = \langle 1 + y^2 - 2xy,\ 2xy + 1 - x^2 \rangle$.

(a) Show that $\vec{F}$ is a conservative vector field.

(b) Find a scalar field f such that $\vec{F} = \nabla f$.

(c) Determine a path C_1 from the point $(0,0)$ to the point $(1,1)$. Compute

$$\int_{C_1} \vec{F} \cdot \mathrm{d}\vec{r}.$$

(d) Determine a different path C_2 from the point $(0,0)$ to the point $(1,1)$. Again, compute

$$\int_{C_2} \vec{F} \cdot \mathrm{d}\vec{r}.$$

(e) Explain your results from parts (c) and (d).

2. Find a function f such that $\vec{F} = \nabla f$ for $\vec{F}(x,y) = \langle (1+xy)\, e^{xy}, x^2 e^{xy} \rangle$, use it to evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the unit circle centered at the origin, oriented counterclockwise starting at $(0,1)$.

3. Set up the two-dimensional integral that you would solve in order to use Green's theorem to evaluate

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

where $\vec{F} = \left\langle ye^{x^3},\ x^2 e^y \right\rangle$ and C is given by the curves $\vec{r}_1(t) = \langle t,\ -t^2 \rangle$ for $-1 \le t \le 1$, followed by $\vec{r}_2(t) = \langle 2 - t,\ -1 \rangle$ for $1 \le t \le 3$.

4. Use Green's theorem to determine the area enclosed by the curve $\vec{r}(t) = \langle t - t^2, t - t^4 \rangle$ for $0 \le t \le 1$. *Hint: see example 3 (and formula 5) in the reading.*

5. Use the curle to determine whether the vector field $\vec{F}(x,y,z) = \left\langle \frac{2y}{z},\ \frac{2x}{z} + z\sin(yz),\ -\frac{2xy}{z^2} + y\sin(yz) \right\rangle$ is conservative.

6. Show that any vector field of the form

$$\vec{F}(x,y,z) = f(x)\vec{i} + g(y)\vec{j} + h(z)\vec{k}$$

is irrotational.

7. Show that any vector field of the form

$$\vec{F}(x,y,z) = f(y,z)\vec{i} + g(x,z)\vec{j} + h(x,y)\vec{k}$$

is incompressible.

8. Find an equation of the tangent plane to the parametric surface

$$\vec{r}(u,v) = \left\langle 2uv,\ u^2 v,\ \frac{u}{v} \right\rangle$$

at the point $(-4,-4,-2)$.