Homework 9

Math 3302, Fall 2018

Due November 14

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Evaluate the triple integral

$$\iiint_E 12xz \, \mathrm{d}V,$$

where E is the solid bounded by the parabolic cylinders $x = z^2$ and $z = x^2$, and the planes $y = 0$ and $y = x + z$.

2. Express the integral $\iiint_E f(x, y, z) \, \mathrm{d}V$ in two of the six different ways $\mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$ and $\mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y$, where E is the solid bounded by the surfaces

$$x^2 + z^2 = 16, \quad y = -3, \quad y = 3.$$

3. Convert the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} xz \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x$$

to cylindrical coordinates. *Do not evaluate the integral, just set it up.*

4. Use cylindrical coordinates to find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 9$ and the sphere $x^2 + y^2 + z^2 = 25$.

5. Prove that dV in spherical coordinates is $\rho^2 \sin\phi d\rho d\theta d\phi$ using the Jacobian matrix, *Hint: compute the absolute value of the determinant (recall cross products) of the matrix*

$$\begin{pmatrix} x_\rho & x_\phi & x_\theta \\ y_\rho & y_\phi & y_\theta \\ z_\rho & z_\phi & z_\theta \end{pmatrix}.$$

6. Use spherical coords and a triple integral to find the volume of a sphere of radius R.

7. Sketch the solid described by the given inequalities in spherical coordinates:

$$2 \leq \rho \leq 3, \qquad 0 \leq \phi \leq \pi/4, \qquad 0 \leq \theta \leq 2\pi.$$

8. Find the amount (mass) of ice-cream in the ice-cream cone formed by a sphere of radius 7 cm centered at the origin and a cone opening upwards from the origin with top radius of 3 cm if the density of ice-cream is given by $\sigma(x, y, z) = z$ g/cm^3.