Homework 6

Math 3302, Fall 2018

Due October 17

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Consider the surface $x^2 + \frac{y^2}{9} + \frac{z^2}{9} = 1$.

(a) Find the equation of the tangent plane to this surface at the point $\left(\frac{1}{3}, 2, 2\right)$.

(b) Find a point at which the tangent plane to this surface is horizontal. Are there any other such points?

(c) Find a point at which the tangent plane to this surface is vertical. Are there any other such points?

2. Find the absolute maximum and minimum of $f(x,y) = x^2 + xy + y^2$ on the disk $\left\{(x,y) \mid x^2 + y^2 \leq 9\right\}$.

3. Find the absolute maximum and minimum values of the function $f(x,y) = x^2y$ over the region

$$R = \{(x,y) \mid x \geq 0,\ y \geq 0,\ y \leq 1 - x\}.$$

4. Without using Lagrange multipliers, find the shortest distance from the point $(-1,1,2)$ to the plane $2x + y - z = 4$. Solve the plane equation for one variable in terms of the others; plug into the distance formula from the point, and determine the minimum of the resulting function.

5. Use Lagrange multipliers to find the shortest distance from the point $(-1,1,2)$ to the plane $2x+y-z=4$. Note: if you did both problems correctly, your answer should match problem 4 .

6. The plane $4x + y - z = -2$ intersects the cone $x^2 = y^2 + z^2$ in an ellipse. Write the system of equations that you would solve in order to use Lagrange multipliers to find the points on the ellipse having maximum and minimum x values. Compute all relevant derivatives and write your answer only in terms of the variables (x,y,z) and any additional variables that you introduce. *Do not solve the problem, only write the system of equations that would need to be solved.*

7. A smarthphone is built out of a number of components which work together to give the overall "perceived quality." Suppose x and y represent the manufacturing costs allocated to two important components (say, CPU and screen), in units of hundreds of dollars, such that the total cost

$$C(x,y) = x + y.$$

Suppose further that the perceived quality Q of the phone can be expressed as

$$Q(x,y) = \ln(1+x) + \ln(1+2y).$$

What is the maximum perceived quality that can be attained for \$100 in manufacturing costs, i.e. subject to the constraint

$$C(x,y) = x + y = 1?$$