Homework 5

Math 3302, Fall 2018

Due October 10

For each problem, you must show your work (as applicable) to receive credit - if we cannot determine how you performed any step then it will be marked incorrect. While you may use electronic devices to check your work, you should be able to do all of these problems without electronic assistance, since all exams will not allow electronic devices.

1. Find the linear approximation $L(x, y, z)$ to the function

$$f(x, y, z) = 1 - yz\sin(\pi xz)$$

at the point $(1, -1, 2)$. Use this to approximate $f(1.1, -0.9, 1.5)$.

2. Suppose you need to know an equation of the tangent plane to a surface $S(x, y)$ at the point $P(-1, 3, 4)$. However, you do not know the equation for S, but you do know that the two curves

$$\vec{r}_1(t) = \left\langle 2 - 3t,\, 1 + 2t^2,\, 5 - 2t + t^2 \right\rangle$$
$$\vec{r}_2(s) = \left\langle -5 + s^2,\, s^3 - 5,\, 2s^2 - 2s \right\rangle$$

both lie on S and both pass through P. Use this information to find an equation of the tangent plane at P.

3. Consider the functions

$$z = f(x, y), \qquad x = x(s, t), \qquad y = y(s),$$
$$s = s(p, q), \qquad t = t(q),$$

and assume that each function is differentiable.

(a) Draw a tree diagram for the dependencies of each variable on one another.

(b) Which variable(s)s are dependent? Which variables are intermediate? Which variables are independent?

(c) Write out the chain rule for $\frac{\partial z}{\partial p}$.

(d) Write out the chain rule for $\frac{\partial z}{\partial q}$.

4. Use the chain rule to compute $\frac{\partial w}{\partial \theta}$, when $r = 3$ and $\theta = \frac{\pi}{3}$, where

$$w = f(x, y, z) = 2xyz - xy + yz^2,\ x = r\cos\theta, \quad y = r\sin\theta,\ \textit{and } z = r^2\theta$$

5. The function $w = f(x, y, z)$ is implicitly defined by the equation

$$wxy^2 + 2w^2 z = xye^{wxyz}.$$

Use implicit differentiation to find the functions $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$.

6. Find the directional derivative of $f(x, y, z) = xe^z + ye^x + ze^y$ at the point $(0, 0, 0)$ in the direction of the vector $\vec{v} = \langle -2, 6, -3 \rangle$.

7. Suppose that over a certain region of space the electrical potential V is given by the function $V(x, y, z) = 5y^2 - 3yz + xyz$.

(a) Find the rate of change of the potential at the point $P(5, 3, 4)$ in the direction of the vector $\vec{v} = \langle -2, 1, 2 \rangle$.

(b) In which direction(s) does V change the most rapidly at P?