ECE 6560 Final Project

Chris Chow

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1 Introduction

In many medical imaging tasks we receive data in the form of 2D slices of a 3D volume. In order to reconstruct the volume, we need to align the images so that they accurately reflect the topology of the 3D structure. In this project we attempt to parametrize these distortions in terms of a rotation, scaling, and translation of the 2D images from some baseline pose. Very poor results are achieved and it is likely that this is a problem with the implementation.

2 Mathematical Formulation

The original goal of this project was to perform alignment on synthetic blood vessel data. One would expect traditional feature-point matching alignment techniques to perform poorly on such a dataset, where vessels can branch and appear from nowhere. We express an energy that rewards the image overlap being great while penalizing the sum of the images. We follow the formulation in [1].

$$E_{\text{align}} = \sum \frac{\iint_\Omega (I_i - I_j)^2 dA}{\iint_\Omega (I_i + I_j^2 dA} \tag{2.1}$$

The sum is taken over sequential images. Image distortions can be parametrized in $T[p]$.

$$T[p] = \begin{bmatrix} 1 & 0 & \mathrm{t}_x \\ 0 & 1 & \mathrm{t}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{h} & 0 & 0 \\ 0 & \mathrm{h} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \text{-}\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, p = \begin{bmatrix} \mathrm{t}_x \\ \mathrm{t}_y \\ \mathrm{h} \\ \theta \end{bmatrix} \tag{2.2}$$

We could add regularization on the L_2 norm of p to make this scheme perform a bit better in the presence of noise, but that was not tested experimentally.

3 GRADIENT DESCENT

Taking the gradient of the energy functional with respect to the transformation parameter for image i p_i, applying the quotient rule, yields

$$\nabla_{pi} E_{\text{align}} = \sum \frac{\iint_\Omega 2(I_i - I_j)\nabla_{pi} I_i dA}{\iint_\Omega (I_i + I_j)^2 dA} + \frac{2\iint_\Omega (I_i - I_j)^2 dA \iint_\Omega (I_i + I_j)\nabla_{pi} I_i dA}{\left(\iint_\Omega (I_i + I_j)^2 dA\right)^2} \tag{3.1}$$

Since I_i is a function of transformed coordinates acted on by $T[p]$, we express its derivative with the chain rule

$$\nabla_p I_i = \nabla_{xy} * \nabla_p T[p]\vec{x}$$

(3.2)

In homogeneous coordinates

$$= \begin{bmatrix} \frac{\partial I_i}{\partial x} & \frac{\partial I_i}{\partial y} & 0 \end{bmatrix} \nabla T[p] \begin{bmatrix} \text{x} \\ \text{y} \\ 1 \end{bmatrix} \tag{3.3}$$

Since p has a finite number of elements, we can divide up the evaluation of $\nabla T[p]$ into four parts - one for each component of the transform. We use the definition of the transform given in equation 2.2 and take its partial derivative with respect to each element of p.

$$\frac{\partial T[p]}{\partial t_x} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \text{h} & 0 & 0 \\ 0 & \text{h} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \text{-}\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

$$\frac{\partial T[p]}{\partial t_y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \text{h} & 0 & 0 \\ 0 & \text{h} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \text{-}\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.5}$$

$$\frac{\partial T[p]}{\partial h} = \begin{bmatrix} 1 & 0 & \text{t}_x \\ 0 & 1 & \text{t}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \text{-}\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.6}$$

$$\frac{\partial T[p]}{\partial \theta} = \begin{bmatrix} 1 & 0 & \text{t}_x \\ 0 & 1 & \text{t}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{h} & 0 & 0 \\ 0 & \text{h} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{-}\sin(\theta) & \cos(\theta) & 0 \\ \text{-}\cos(\theta) & \text{-}\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

This gives a gradient descent update of $p_{t+1} = -\alpha \nabla_p E_{\text{align}}$.

4 Discretization

The only part of the scheme that needs to be discretized is the calculation of the image gradient. There isn't any stability condition on the update equation because there isn't any relation between the time step and the spatial resolution. However, choosing an appropriately small α will keep the solution from wandering out from a local solution. We select a Sobel kernel to calculate the gradient of the image since it seems to be the most widely used.

5 Experimental Results

We evaluate this scheme on the toy example of an image of an apple. After obtaining poor results on actual synthetic data, we reasoned that the lack of smooth gradients in the sparse blood vessel imagery was causing the scheme to not converge to a useful solution.

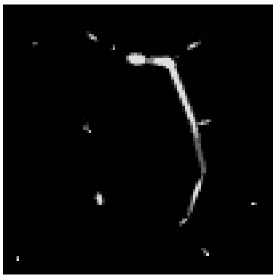

Figure 5.1: Sample synthetic blood vessel imagery

Figure 5.2: Grayscale apple

However, running the scheme on slightly rotated versions of the apple does not lead to convergence. Instead, we get smaller and more rotated versions of the apple until the image vanishes and the cost function saturates.

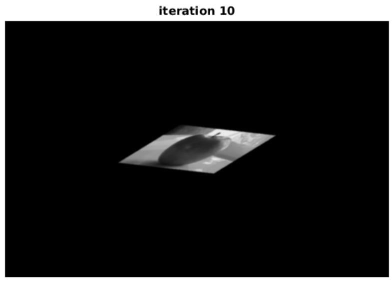

Figure 5.3: Shrinking apple

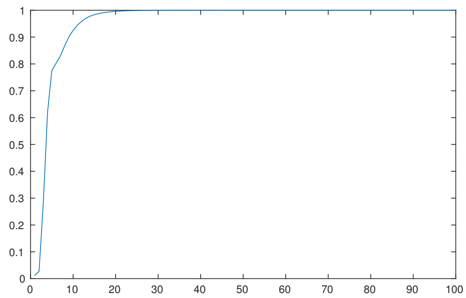

Figure 5.4: Saturated cost function

6 DISCUSSION

In [1] the authors mention that the scheme is very sensitive to local minima. It is possible that the learning parameter for the gradient is too large and this drives the system to divergence. It is also possible that the analytical calculation of the gradients is wrong, or that the implementation of the transform step in MATLAB somehow does not agree with the way that the gradient of the cost was calculated.

Future work could include the addition of a regularization term that would prevent the solution from wandering around in the presence of noise. It is unknown how well this algorithm performs in the presence of significant noise, as it does not seem to take advantage of any topological or geometric properties of the images to find solutions. A more sophisticated cost function could capture some of that information between images and use it to find better solutions.

Another limitation is the low-dimensional parametrization of the image transformation. The variety of things that could happen in a real-world imaging situation is much larger than just translation, scaling, and rotation. A more complicated transformation model might even include arbitrary image-image homographies or even just more terms in the approximation $T[p]$. This problem is interesting because the selection of the parameters of T is in itself a discretization. There is clearly some sort of tradeoff between the fidelity of the transform model and the speed of convergence or even the possibility of convergence, but that is beyond the scope of this project to investigate.

References

[1] Andy Tsai, Anthony Yezzi Jr, William Wells, Clare Tempany, Dewey Tucker, Ayres Fan, W Eric Grimson, and Alan Willsky. A shape-based approach to the segmentation of medical imagery using level sets. *Medical Imaging, IEEE Transactions on*, 22(2):137–154, 2003.