Mass ratio, Mole ratio relationship

$$r = \frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{\dot{n}_{ox}\,\bar{M}_{ox}}{\dot{n}_f\,\bar{M}_f}$$

$\dot{m}$ - mass flow rate (kg/s), $\dot{n}$ - mole flow rate (kmol/s)

$$\frac{\dot{n}_{ox}}{\dot{n}_f} = \frac{\dot{m}_{ox}}{\dot{m}_f} \cdot \frac{\bar{M}_f}{\bar{M}_{ox}} = r\,\frac{\bar{M}_f}{\bar{M}_{ox}}$$

Ex: A $H_2 - O_2$ LPRE has $r = 4$. Write the reaction model.

$$\frac{\dot{n}_{ox}}{\dot{n}_f} = r\,\frac{\bar{M}_f}{\bar{M}_{ox}}$$

. Here, fuel is H_2 and oxidizer is O_2.

$$\frac{\dot{n}_{O_2}}{\dot{n}_{H_2}} = r\,\frac{\bar{M}_{H_2}}{\bar{M}_{O_2}} = 4\,\frac{2}{32} = 0.25$$

If $\dot{n}_{H_2} = 1$ kmol/s, $\dot{n}_{O_2} = (0.25)(1) = 0.25$ kmol/s

The reaction model is

$$H_2 + 0.25\,O_2 \longrightarrow \text{Products}$$

↓ ASIDE

Stoichiometry: Reaction in which fuel and oxidizer are combined in the correct proportion to completely react with each other, and forms the complete reaction product.

For the $H_2 - O_2$ reaction, the stoichiometric reaction is

$$H_2 + 0.5\,O_2 \longrightarrow H_2O$$ ← complete reaction product

$$r_{stoic} = \frac{\dot{m}_{O_2}}{\dot{m}_{H_2}} = \frac{(0.5)(32)}{(2)} = 8$$

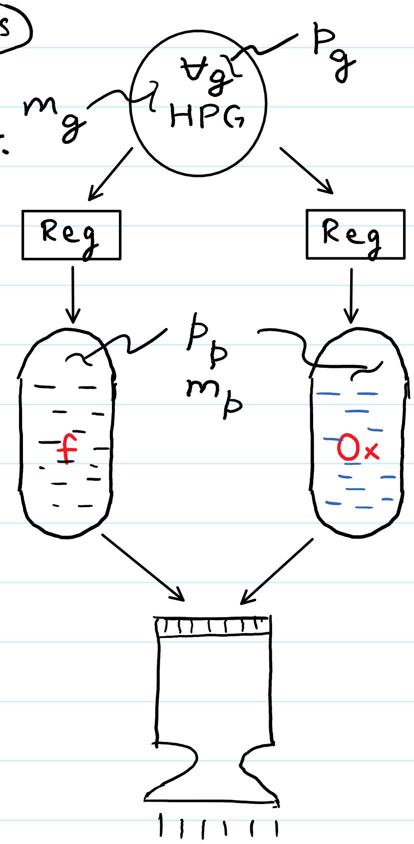
The reaction, $H_2 + 0.25\,O_2 \longrightarrow$ Products, has $r = 4$ and is fuel-rich; that is, the rocker is using a fuel-rich reactant mixture.

Use the equilibrium software to get the temperature and composition of the product mixture. Use the software to reproduce the results of Table 12.2.
In an adiabatic reaction process, enthalpy does not change. If the combustor velocity is small, the pressure is approximately constant.

Across a rocket combustor, both the enthalpy and the pressure are constant.

LPRE Propellant Feed Systems

① Gas Pressure Feed System (inert gas)



p – pressure of high pressure gas (HPG):

p_g – pressure in the gas tank,
initially (@ $t=0$) p_{g0}, $p_g = p_g(t)$

p_p – pressure in the propellant tank

m – mass of the HPG:

m_g – mass in the gas tank,
initially m_{g0}, $m_g = m_g(t)$

m_p – mass in the propellant tank,
initially zero, $m_p = m_p(t)$

The HPG is used to expel the propellants from their tanks to the thrust chamber at the desired pressure, set by the regulator.

Assume adiabatic expansion of the HPG, and apply the I law to find p_g at the instant a volume V_p of the propellant (fuel and oxidizer together) has been delivered to the thrust chamber. The propellant is delivered to the thrust chamber at pressure p_p.

Initial HPG energy $= m_{g0} c_v T_{g0}$ [T_{g0} is the initial HPG temperature]

"Final" HPG energy $= m_g c_v T_g + m_p c_v T_p$

Work done by HPG $= p_p V_p$

Heat $= 0$ (adiabatic)

T_g – temperature of HPG in gas tank.

T_p – temperature of HPG in propellant tank.

$$\Delta U = Q - W$$

$$(m_g c_v T_g + m_p c_v T_p) - m_{g0} c_v T_{g0} = 0 - p_p V_p$$

The ideal gas relation, $pV = mRT$, yields $mT = \frac{pV}{R}$.

Using this in the I law,

$$\left(c_v \frac{p_g V_g}{R} + c_v \frac{p_p V_p}{R}\right) - c_v \frac{p_{g0} V_g}{R} = 0 - p_p V_p$$

$$\Rightarrow c_v \frac{p_g V_g}{R} = c_v \frac{p_{g0} V_g}{R} \underbrace{- c_v \frac{p_p V_p}{R} - p_p V_p}_{-\frac{c_p}{R} p_p V_p}$$

(or) $$p_g = p_{g0} - \gamma \frac{V_p}{V_g} p_p$$

when the pressure of the HPG in the gas tank (p_g) becomes p_p, no more delivery is possible. By setting $p_g = p_p$ in the above equation, we can find the minimum required initial pressure of the HPG in the gas tank (p_{g0}) to deliver volume V_p of propellant to the thrust chamber at pressure p_p.

$$p_p = p_{g0} - \gamma \frac{V_p}{V_g} p_p$$ [the volume of gas tank V_g is given]

or,

$$V_g = \frac{\gamma p_p V_p}{p_{g0} - p_p}$$ [the initial pressure of the HPG in the gas tank is given]

Ex:

Consider the pressurized delivery of $0.28\ m^3$ of monopropellant into the thrust chamber of a rocket engine at 10 MPa. The HPG is initially at a pressure of 40 MPa in the gas tank. Find the minimum volume of the gas tank with

a) helium ($\gamma = 1.67$, $\bar{M} = 4$) as the HPG, and

b) nitrogen ($\gamma = 1.29$, $\bar{M} = 28$) as the HPG.

Also, for each case, find the mass of the HPG. The initial temperature of the HPG (T_{g0}) is 300 K.

a) $p_{g0} = 40$ MPa, $p_p = 10$ MPa, $V_p = 0.28\ m^3$, $\gamma = 1.67$,

$$R = \frac{\bar{R}}{\bar{M}_{He}} = \frac{8314.3}{4} = 2078.6\ \frac{J}{kg\text{-}K}$$

$$V_g = \frac{\gamma p_p V_p}{p_{g0} - p_p} = \frac{(1.67)(10)(0.28)}{(40 - 10)} = 0.156\ m^3$$

$$V_g = \frac{\gamma p_p V_p}{p_{go} - p_p} = \frac{(1.67)(10)(0.28)}{(40-10)} = 0.156\ m^3$$

$$\rho_{go} = \frac{p_{go}}{R_{He} T_{go}} = \frac{(40)(10^6)}{(2078.6)(300)} = 64.15\ \frac{kg}{m^3}$$

$$m_{go} = \rho_{go} V_g = (64.15)(0.156) = 10\ kg$$

b) $V_g = 0.120\ m^3$, $m_{go} = 54\ kg$ (verify)

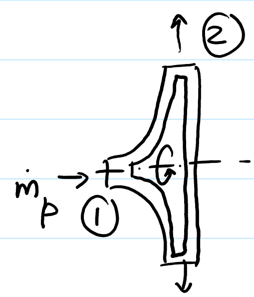
(2) Pump Feed System

↓ASIDE

Useful relation : $T\,ds = dh - \frac{dp}{\rho}$ (A)

↑

Turbopump



I Law : $\dot{m}_p h_{o1} - \dot{W}_{pump} = \dot{m}_p h_{o2}$

(or) $(h_{o2} - h_{o1}) = -\frac{\dot{W}_p}{\dot{m}_p} = -w_p$

(or) $\Delta h_o = -w_p$ (B)

An ideal pump is isentropic. Applying the relation (A) with stagnation conditions,

$$T_o\, \cancel{ds_o}^{\,0} = dh_o - \frac{dp_o}{\rho_o}$$

$$\Rightarrow dh_o = \frac{dp_o}{\rho_o} = \frac{dp_o}{\rho}$$ [For liquids ρ is Constant]

Integrating, $\Delta h_{os} = \dfrac{\Delta p_0}{\rho}$ (C)
(denotes isentropic process)

(B) and (C) $\Rightarrow$ $-w_{ps} = \Delta h_{os} = \dfrac{\Delta p_0}{\rho}$

Isentropic operation does not extend to real pumps. The work required to generate the same pressure rise (Δp_0) is

$$w_p \overset{\text{def}}{=} \frac{w_{ps}}{\eta_p} = -\frac{(\Delta p_0/\rho)}{\eta_p},$$

where η_p is the pump efficiency.

Pump power $\mathcal{P}_p = \dot{m}_p w_p = -\dot{m}_p\left[\dfrac{(\Delta p_0/\rho)}{\eta_p}\right]$

$$|\mathcal{P}_p| = \dot{m}_p\left[\frac{(\Delta p_0/\rho)}{\eta_p}\right]$$

Ex: Find the total pump power required to supply a kerosene-LOX LPRE thrust chamber at a total $\dot{m}$ of 92 kg/s. $r = 2.5$. The pressure rise for the kerosene pump is 3.8 MPa, and that for the LOX pump is 4.6 MPa. The efficiency of each pump is 65%. $SG_{ker} = 0.8$, $SG_{LOX} = 1.14$

$$\dot{m} = (\dot{m}_{ker} + \dot{m}_{LOX}) = \dot{m}_{ker}(1+r)$$

$$\dot{m}_{ker}(1+2.5) = 92$$

$$\Rightarrow \dot{m}_{ker} = \frac{92}{3.5} = 26.3\ \frac{kg}{s},\quad \dot{m}_{LOX} = r\,\dot{m}_{ker} = (2.5)(26.3) = 65.7\ \frac{kg}{s}$$

see terms inside the curly brackets of Eqn 13.14

$$\mathcal{P}_p = \left[\frac{\dot m(\Delta p_0)}{\eta_p \rho}\right]_{ker} + \left[\frac{\dot m(\Delta p_0)}{\eta_p \rho}\right]_{Lox}$$

$$\boxed{\begin{aligned} \rho &= (SG)\,\rho_w \\ \rho_{ker} &= (0.8)(1000) \\ &= 800\ kg/m^3 \\ \rho_{Lox} &= 1.14(1000) \\ &= 1140\ kg/m^3 \end{aligned}}$$

$$= \left[\frac{(26.3)(3.8)(10^6)}{(0.65)(800)}\right] + \left[\frac{(65.7)(4.6)(10^6)}{(0.65)(1140)}\right]$$

$$= 4.08(10^5) + 1.92(10^5) = 6(10^5)\ W = 0.6\ MW$$

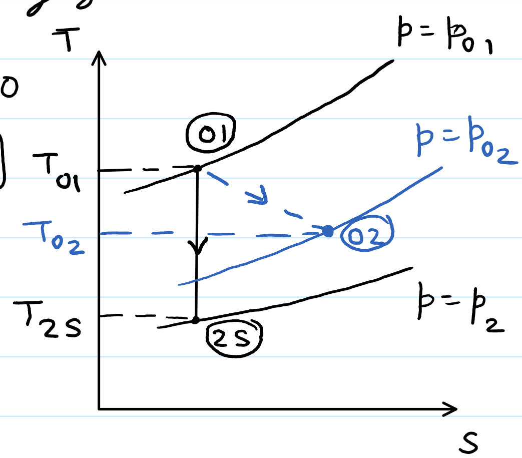
Turbine

I Law: $\dot m_T (h_{0_2} - h_{0_1}) = -\mathcal{P}_T$

$$\Rightarrow (h_{0_2} - h_{0_1}) = -\frac{\mathcal{P}_T}{\dot m_T} = -w_T$$

(or) $w_T = (h_{0_1} - h_{0_2}) = c_p(T_{0_1} - T_{0_2})$

Ideal turbine is isentropic. Since the kinetic energy of the turbine exhaust is wasted, the turbine is designed to make it as small as possible, ideally zero. Thus



$$w_{Ts} = (h_{0_1} - h_{2s}) \qquad \left[h_{0_2} = h_2 + \cancel{\frac{u_2^2}{2}}^{\,0}\right]$$

$$= c_p(T_{0_1} - T_{2s})$$

$$\eta_{Ts} = \frac{w_T}{w_{Ts}} = \frac{\cancel{c_p}(T_{0_1} - T_{0_2})}{\cancel{c_p}(T_{0_1} - T_{2s})}$$

$$\cancel{T_{01}}\left(1 - \frac{T_{02}}{T_{01}}\right) \qquad \left(1 - \frac{T_{02}}{T_{01}}\right) \qquad 1 - \frac{T_{02}}{T_{01}}$$

$$= \frac{\cancel{T_{01}}\left(1 - \frac{T_{02}}{T_{01}}\right)}{\cancel{T_{01}}\left(1 - \frac{T_{2s}}{T_{01}}\right)} = \frac{\left(1 - \frac{T_{02}}{T_{01}}\right)}{1 - \left(\frac{p_{2s}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1 - \frac{T_{02}}{T_{01}}}{1 - \left(\frac{p_2}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}}$$

Turbine Equations:

$$P_T = \dot{m}_T w_T, \quad w_T = c_p\left(T_{01} - T_{02}\right), \quad \eta_{TS} = \frac{1 - \frac{T_{02}}{T_{01}}}{1 - \left(\frac{p_2}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}}$$

{See equation above Eqn (13.13)

The turbine wheel is turned by the applied pressure ratio (p_{01}/p_{02}) across it. The turbine power is used to turn the fuel and oxidizer pumps. $P_T = P_p$.

Ex: A turbine, powered by a gas generator, is used to drive the pumps of the previous example. The turbine entry stagnation conditions are 2 MPa and 800 K. The turbine discharge static pressure is 0.14 MPa. Calculate the fraction of the total propellant flow to be diverted to the gas generator. $\bar{M} = 34$, $\gamma = 1.3$. $\eta_{ts} = 70\%$. The turbine-pump shaft has mechanical efficiency (η_m) of 90%.

$$R = \frac{\bar{R}}{\bar{M}} = \frac{8314.3}{34} = 244.5 \ \frac{J}{kg-K}$$

$$c_p = \frac{\gamma R}{\gamma - 1} = 1060 \ \frac{J}{kg-K}$$

$$P_T = \dot{m}_T w_T = \dot{m}_T \eta_{ts} w_{Ts}$$

$$\frac{T_{02}}{} \qquad 1 - \frac{T_{02}}{}$$

$$\eta_{TS} = 0.7 = \frac{1 - \frac{T_{02}}{T_{01}}}{1 - \left(\frac{p_2}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1 - \frac{T_{02}}{T_{01}}}{1 - \left(\frac{0.14}{2}\right)^{\frac{1.3-1}{1.3}}}$$

$$\Rightarrow \frac{T_{02}}{T_{01}} = 0.6790$$

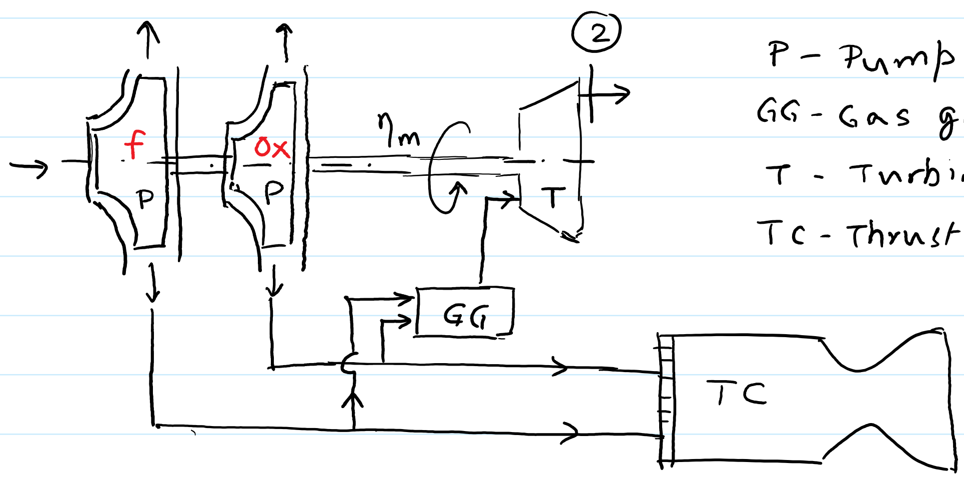
$$T_{02} = 0.6790\, T_{01} = 0.6790\,(800) = 543.2 \text{ K}$$

$$\mathcal{P}_T = \dot{m}_T\,(1060)(800 - 543.2) = 272208\, \dot{m}_T$$

$$\eta_m \mathcal{P}_T = \mathcal{P}_p$$

$$(0.90)(272208\, \dot{m}_T) = 6\,(10^5) \Rightarrow \dot{m}_T = 2.45 \frac{kg}{s}$$

$$\text{Fraction} = \frac{\dot{m}_T}{\dot{m}} = \frac{2.45}{92} = 0.0266, \text{ or } 2.66\,\%$$



P – Pump
GG – Gas generator
T – Turbine
TC – Thrust chamber