Experimental Bump Test and Model Validation

Charles Keer and Marielle Lenehan*

Experiment 2: Flexible Link

TA: Stephen Brutch

Section: 09DB

25 October 2023

Embry-Riddle Aeronautical University

1 Aerospace Boulevard

Daytona Beach, FL 32114

*Marielle helped conduct this experiment.

Table of Contents

Table of Figures

1. Procedure

1.1 Finding Stiffness

A bend test was performed on the flexible link while a series of strain gauges and accelerometers were attached to the base. By bending and releasing the flexible link, the link oscillated until it comes to rest. The data acquisition system (DAQ) recorded the angle of the flexible link over time. The natural frequency of the system can be found using this method.

The DAQ system was set up in accordance with the accompanying lab manual. Opening the Simulink and MATLAB model, the model was built and ran. Once data started recording on the computer, the flexible link was bent back and released. This was repeated because the data on the first run was not adequate.

1.2 Model Validation

To validate the model of the flexible link, the stiffness of the link was first found, and the corresponding state space matrices were generated using MATLAB. These state-space values were then translated to the Simulink model to match what our link's stiffness was. The model was then built and ran. The servo motor actuated and a predicted versus actual response of the system was plotted for both the servo angle and angle of the flexible link.

2. Results

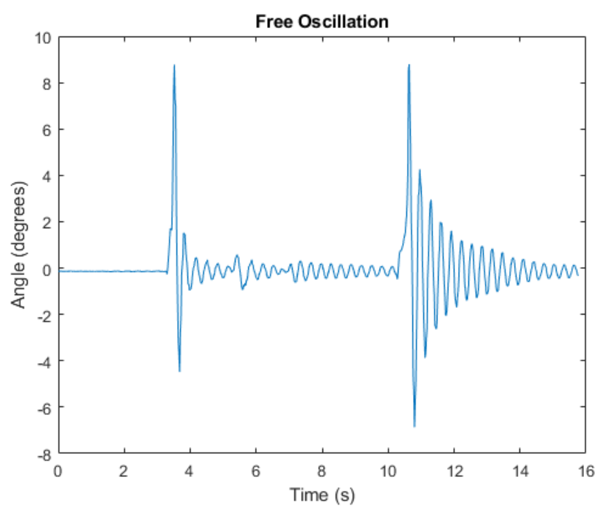

Figure 1. Free oscillation. Note only the data after 10 seconds was used.

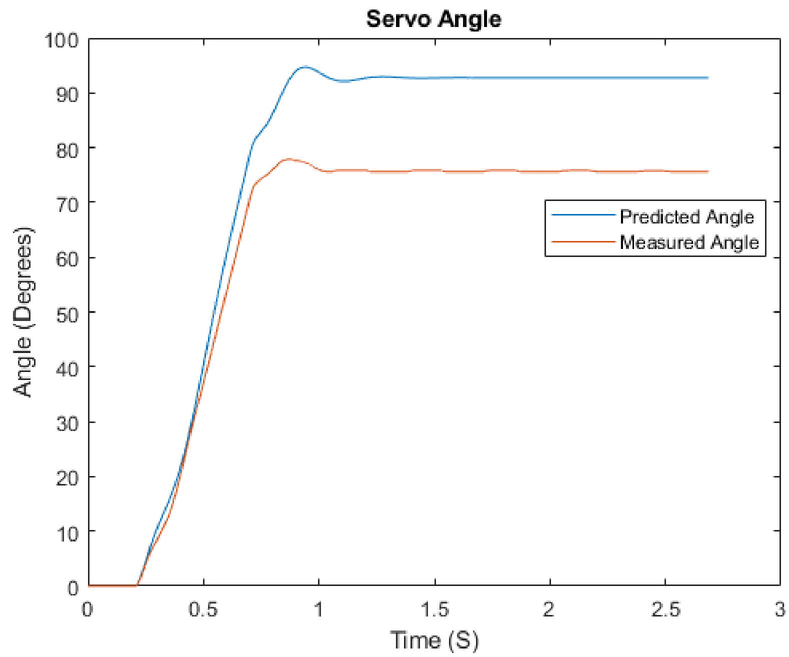

Figure 2. Servo angle in model validation experiment.

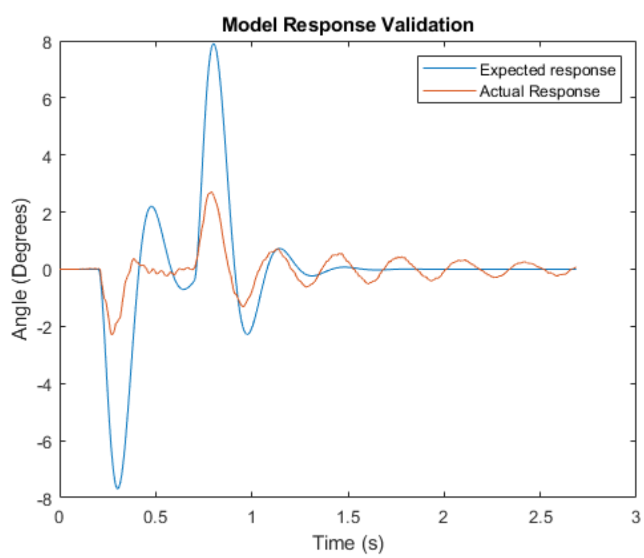

Figure 3. Model validation experiment angle response.

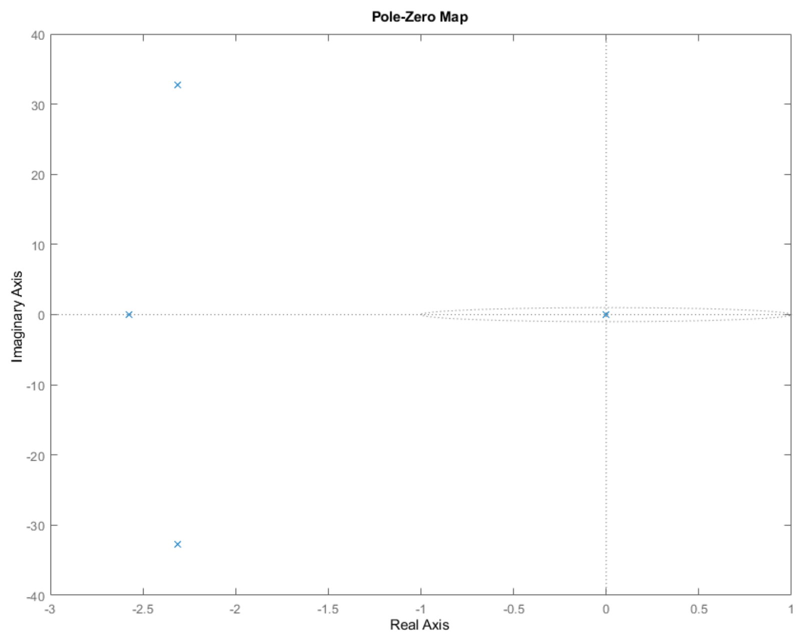

Figure 4. Pole and Zero map of the system.

Table 1. Results from experiment.

Description	Symbol	Value	Unit
Finding Stiffness			
Natural Frequency	ω_n	19.666	Rad/s
Stiffness	K_s	1.4712	N*m/rad
Model Validation			
State-Space Matrix	A	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 705.9 & -7.2 & 0 \\ 0 & -1095.5 & 7.2 & 0 \end{bmatrix}$	
State-Space Matrix	B	$\begin{bmatrix} 0 \\ 0 \\ 479.8 \\ -479.8 \end{bmatrix}$	
State-Space Matrix	C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
State-Space Matrix	D	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	
Open-Loop Poles	OL	$\begin{bmatrix} 0 \\ -2.31 + 32.79i \\ -2.31 - 32.79i \\ -2.57 \end{bmatrix}$	

3. Analysis

The natural frequency of the system was found by analyzing the oscillation time of the free-response oscillation as shown in Figure 1.

$$T_{OSC} = \frac{t_{n+1} - t_1}{n} \tag{1}$$

Using Equation 1, the period of oscillation was found. For this experiment N=1, so t2 was used as the second oscillation point.

$$\omega_d = \frac{2\pi}{T_{OSC}} \tag{2}$$

The damped natural frequency can then be found using Equation 2. To determine the undamped natural frequency, Equation 3 will be used:

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \tag{3}$$

Equations 4 and 5 will then be used to determine the damping ratio and subsidence ratio which are needed to determine the natural frequency:

$$\delta = \frac{1}{n} \ln \frac{O_1}{O_n} \tag{4}$$

$$\zeta = \frac{1}{\sqrt{1+(\frac{2\pi}{\delta})^2}} \tag{5}$$

Finally, Equation 3 can be used to find the natural frequency. It was found that the damped natural frequency was 19.635 while the natural frequency was 19.66, yielding less than a 1% difference.

To calculate the stiffness of the flexible link, the link was assumed to be a “rod” to make calculations easier. The polar moment of inertia for a rod is:

$$J = \frac{m_l L^2{}_l}{3} \tag{6}$$

Where the mass and length of the rod are described in Table 2:

Table 2. Flexible Link Properties

Property	Value
M	0.065 Kg
L	0.419 m

Finally the stiffness, K, can be found using these physical values and Equation 7:

$$K = J{\omega_n}^2 \tag{7}$$

Discrepancies in the model are fairly large for this experiment. One of the biggest reasons was the failure of one of the MATLAB scripts to generate the proper gains for the model once the flexible link's stiffness was found. It output a default set of gains for a stiffness of 1 as opposed to what our link was modeled as. Another reason is slack in the gear train as well as friction in the system. This model can only take so much in as a second order system. The degree of the system would need to increase to get a closer match to the experimental results.

4. Conclusions

This experiment showed how a flexible link could be modeled using strain gauges and accelerometers. The model was then tested against the system. While the model matched to some degree, the gain was not properly set in the experiment, leading to a more imperfect result than was expected. If repeated, the updated gains should yield a more accurate result.

5. References

Quanser Student Workbook Experiment 02 – Quanser