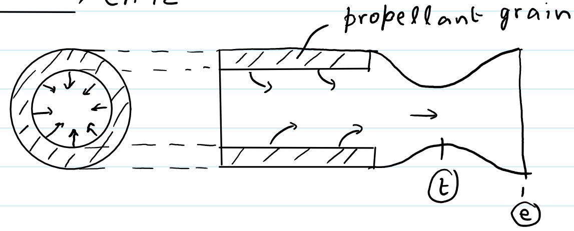
Solid Rocket Motors, Ch 12



Solid propellants

Homogeneous

fuel and oxidizer in the same molecule

Ex: double-base propellant (nitroglycerin – nitrocellulose)

Heterogeneous

Fuel and oxidizer grossly mixed

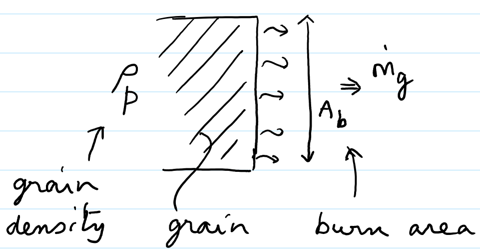
Ex: composite

(plastic-like binder: HTPB

oxidizing crystals: AP, AN, NP, KP, KN

Metal additives: Al, Mg, B

Solid propellants are observed to regress at a uniform rate during their burn (regression rate r)



$$\dot{m}_g = \rho_p A_b r$$

The regression rate is related to pressure as follows:

$$r = a p_o^n \qquad (12.25)$$

n – pressure exponent (independent of temperature)

a – depends on initial temperature of the grain (T_p):

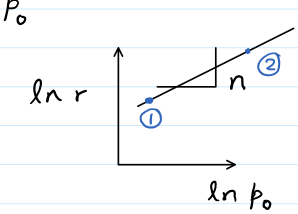
$$a = \frac{A}{T_1 - T_p} \qquad (12.26)$$

{ A, T_1 are constants }

$$(12.25) \Rightarrow \ln r = \ln a + n \ln p_o$$

$$\ln r_2 = \ln a + n \ln p_{o2}$$
$$\ln r_1 = \ln a + n \ln p_{o1}$$

$$(\ln r_2 - \ln r_1) = n(\ln p_{o2} - \ln p_{o1})$$



$$\Rightarrow \quad n = \frac{\ln(r_2/r_1)}{\ln(p_{02}/p_{01})}$$

Quasi-steady model

$$\dot{m}_g = \dot{m}_t$$

$$\dot{m}_g = \rho_p A_b r = \rho_p A_b a p_0^n$$

$$\dot{m}_t = \frac{p_0}{\sqrt{RT_0}} \sqrt{\gamma} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} A_t \quad \{M_t = 1\}$$

Equating $\dot{m}_g$ to $\dot{m}_t$ yields the steady state operating pressure of the rocket motor

$$p_0 = \left[\frac{A_b}{A_t} \frac{a \rho_p}{\sqrt{\frac{\gamma}{RT_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}\right]^{\frac{1}{1-n}} \quad \text{see (12.29)}$$

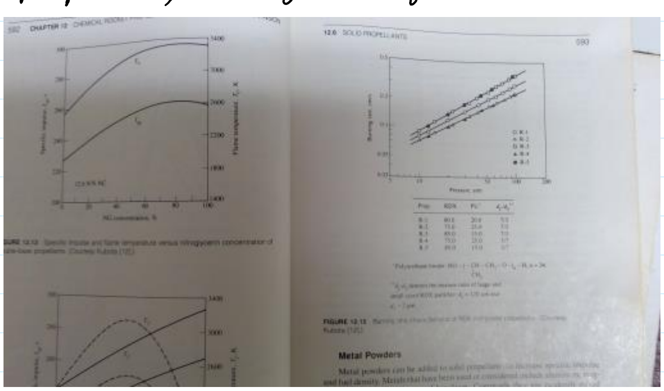
$$\frac{1}{c^*} \quad (\text{Eqn } 11.9)$$

$$p_0 = \left(\frac{A_b}{A_t} a \rho_p c^*\right)^{\frac{1}{1-n}}$$

Ex: Find $\left(\frac{A_b}{A_t}\right)$ required to reach a steady state operating pressure of 14 MPa in a SRM with the following characteristics:

$\rho_p = 1710\ kg/m^3$, $\gamma = 1.27$, $T_0 = 2220\ K$, $\bar{M} = 23$,

R-4 propellant, see Fig 12.15 for burn data



At $p_{01} = 10$ atm, $r_1 = 0.07$ cm/s

At $p_{02} = 100$ atm, $r_2 = 0.22$ cm/s

At $p_{02} = 100$ atm, $r_2 = 0.22$ cm/s

$$n = \frac{\ln\left(\frac{r_2}{r_1}\right)}{\ln\left(\frac{p_{02}}{p_{01}}\right)} = \frac{\ln\left(\frac{0.22}{0.07}\right)}{\ln\left(\frac{100}{10}\right)} = 0.4973$$

$$a = \frac{r_1}{p_{01}^{\,n}} = \frac{(0.07)(10^{-2})}{[10\,(101325)]^{0.4973}} = 7.2186(10^{-7}) \left\{\frac{m/s}{Pa^{0.4973}}\right\}$$

$p_0 = 14$ MPa, $\rho_p = 1710\ kg/m^3$, $\gamma = 1.27$, $R = \frac{\bar{R}}{\bar{M}} = 361.5 \frac{J}{kg\text{-}K}$

$$c^* = \sqrt{\frac{1}{\gamma} R T_0 \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}} = 1353.7\ m/s$$

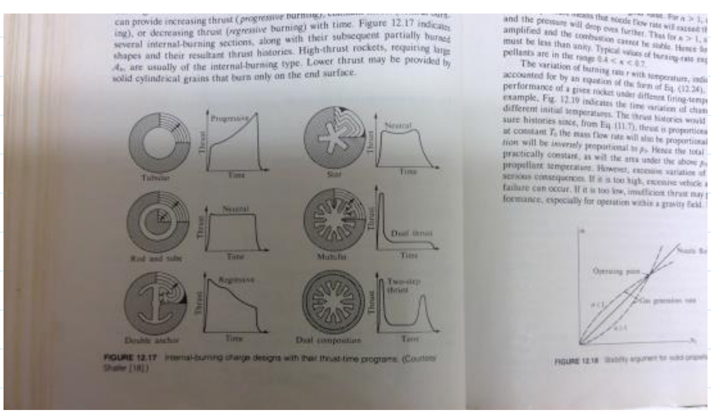
$$p_0 = \left(\frac{A_b}{A_t}\, a \rho_p c^*\right)^{\frac{1}{1-n}}$$

$$\Rightarrow \frac{A_b}{A_t} = \frac{p_0^{\,1-n}}{a \rho_p c^*} = 2341$$

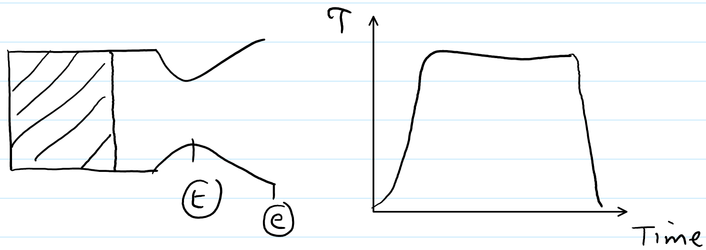
$$\Big\{\, r = a p_0^{\,n} = \underset{\uparrow\ \frac{m/s}{Pa^{0.4973}}}{7.2186\,(10^{-7})}\ \underbrace{[14\,(10^6)]}_{Pa}^{\,0.4973}$$

$$= 0.002584\ m/s = 0.258\ cm/s \,\Big\}$$

See Fig 12.17 for schematics of several internal burning grains and associated thrust histories



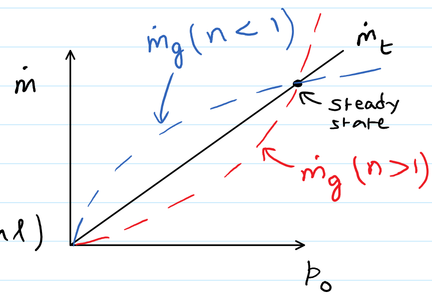
End burning grains burn neutrally.



Burning stability

$n > 1$: explosive behavior
$n < 1$: stable behavior

$0.4 < n < 0.7$ (typical)



The sensitivity of burning rate to initial propellant temperature is defined as

$$\Pi_r \overset{def}{=} \frac{1}{r}\left(\frac{\partial r}{\partial T_p}\right)_{p_o} = \left[\frac{\partial}{\partial T_p}(\ln r)\right]_{p_o} \qquad (12.30)$$

$$r = a p_o^n, \qquad \ln r = \ln a + n \ln p_o$$

$$\Pi_r = \left[\frac{\partial(\ln r)}{\partial T_p}\right]_{p_o} = \frac{d(\ln a)}{dT_p} + 0$$

Since $a = \dfrac{A}{T_1 - T_p}$, $\ln a = \ln A - \ln(T_1 - T_p)$

$$\frac{d(\ln a)}{dT_p} = +\frac{1}{T_1 - T_p}, \text{(or)} \ \frac{1}{a}\frac{da}{dT_p} = \frac{1}{T_1 - T_p}$$

$$\Pi_r = \frac{1}{T_1 - T_p} \qquad (12.31)$$

$0.002\ K^{-1} < \Pi_r < 0.008\ K^{-1}$ (typical Π_r range)

The sensitivity of p_o to initial grain temperature is defined as

$$\Pi_p = \frac{1}{p_o}\left(\frac{\partial p_o}{\partial T_p}\right)_{\frac{A_b}{A_t},\,T_o} = \left[\frac{\partial}{\partial T_p}(\ln p_o)\right]_{\frac{A_b}{A_t},\,T_o}$$

From $p_o = \left(\frac{A_b}{A_t}\, a\, \rho_p\, c^*\right)^{\frac{1}{1-n}}$, and observing only 'a' is initial temperature dependent,

$$\left[\frac{\partial(\ln p_o)}{\partial T_p}\right]_{\frac{A_b}{A_t},\,T_o} = \frac{1}{1-n}\,\frac{d(\ln a)}{dT_p} = \frac{1}{1-n}\,\Pi_r \quad (12.32)$$

$$\Pi_p = \frac{1}{1-n}\,\Pi_r = \frac{1}{(1-n)}\,\frac{1}{(T_1 - T_p)}$$

To lessen the sensitivity of p_o to initial grain temperature, the propellant should have high T_1 and low n. Fig 12.19 shows the impact of the initial grain temperature on p_o.

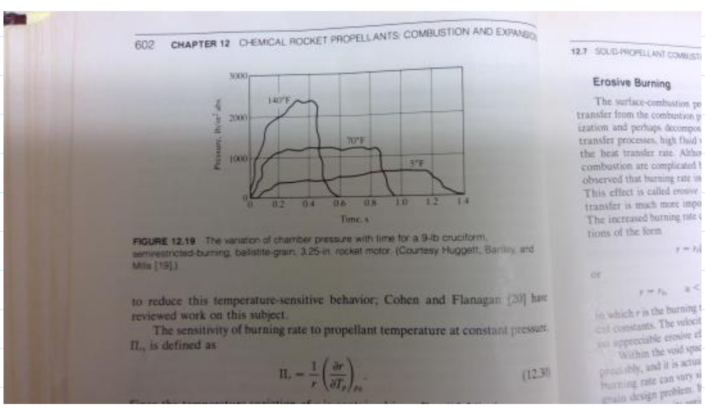

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FIGURE 12.19 The variation of chamber pressure with time for a 9-lb cruciform, semirestricted-burning, ballistite-grain, 3.25-in. rocket motor. (Courtesy Huggett, Bartley, and Mills [19].)

to reduce this temperature-sensitive behavior; Cohen and Flanagan [20] have reviewed work on this subject.

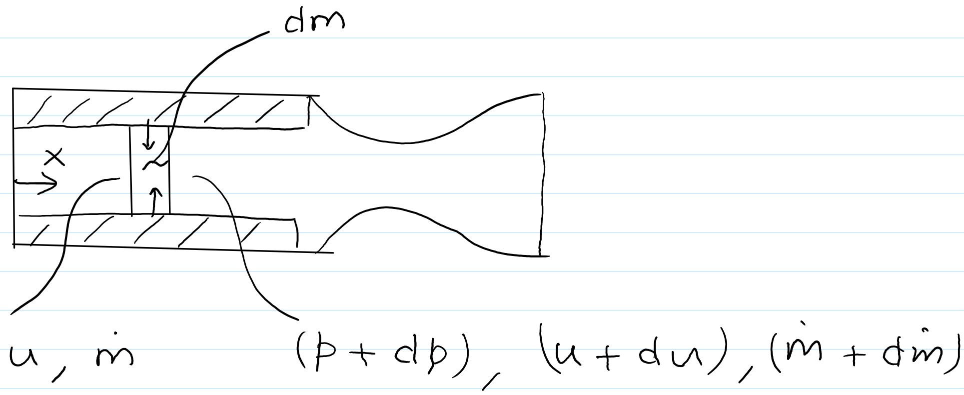
The sensitivity of burning rate to propellant temperature at constant pressure, Π_r, is defined as

$$\Pi_r = \frac{1}{r}\left(\frac{\partial r}{\partial T_p}\right)_{p_0} \quad (12.30)$$

12.7 SOLID-PROPELLANT COMBUSTION

Erosive Burning

Axial pressure variation



x – Momentum :

$$\cancel{\dot{m}u} + (d\dot{m})(0) + \cancel{pA} - (\cancel{p}+dp)A = (\dot{m}+d\dot{m})(u+du)$$

$$= \cancel{\dot{m}u} + u\,d\dot{m} + \dot{m}\,du + \underbrace{\cancel{(d\dot{m})(du)}}_{\text{2nd order}}$$

neglect

$$\Rightarrow -A\,dp = u\,d\dot{m} + \dot{m}\,du = d(\dot{m}u)$$

$$-\int_{p=p_1}^{p} A\,dp = \int_{\dot{m}u=0}^{(\dot{m}u)_x} d(\dot{m}u)$$

$$\Rightarrow A(p_1 - p) = (\dot{m}u)_x$$

$$\Rightarrow p = p_1 - \frac{(\dot{m}u)_x}{A} \qquad (12.35)$$

Since $u = \dfrac{\dot{m}}{\rho A}$,

$$p = p_1 - \frac{1}{\rho}\left(\frac{\dot{m}}{A}\right)^2 \qquad (12.36)$$

If c is the circumference of the grain, the mass flow rate at any station x is

$$\dot{m} = \rho_p (cx) r \qquad \{ A_b = cx \}$$

$$p = p_1 - \frac{RT}{p} \left(\frac{\rho_p c x r}{A} \right)^2$$

$$\Rightarrow p = \frac{p_1}{2} \left[1 + \sqrt{1 - 4RT \left(\frac{\rho_p c r x}{p_1 A} \right)^2} \right] \qquad (12.37)$$