

## AE 414, Space Propulsion

### Thermodynamic properties

$p$  - pressure ( $\text{N/m}^2$ , or Pa) [ $1 \text{ Pa} \stackrel{\text{def}}{=} 1 \frac{\text{N}}{\text{m}^2}$ ]

$T$  - temperature (absolute) (K).

$\rho$  - density ( $\text{kg/m}^3$ )

$\mathcal{V}$  - volume ( $\text{m}^3$ )

$m$  - mass (kg)

$v \stackrel{\text{def}}{=} \frac{\mathcal{V}}{m}$  (specific volume) ( $\text{m}^3/\text{kg}$ )

$$v = \frac{1}{\rho}$$

$E$  - internal energy (J)

$e$  - specific internal energy

$$e = \frac{E}{m} \text{ (J/kg)} ; e = e(T, v)$$

$H$  - enthalpy;  $H = E + p\mathcal{V}$

$h$  - specific enthalpy;  $h = h(T, p)$

$$h = \frac{H}{m} \text{ (J/kg)}$$

$$h = \frac{H}{m} = \frac{E}{m} + p \frac{\mathcal{V}}{m} = e + \frac{p}{\rho}$$

$S$  - Entropy (J/K)

$$s = \frac{S}{m} \text{ (specific entropy) (J/kg-K)}$$

$$c_v = \left( \frac{\partial e}{\partial T} \right)_v \quad \text{constant volume specific heat} \left( \frac{\text{J}}{\text{kg} \cdot \text{K}} \right)$$

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p \quad \text{constant pressure specific heat} \left( \frac{\text{J}}{\text{kg} \cdot \text{K}} \right)$$

### I AND II LAWS OF THERMODYNAMICS

$$\text{I Law: } dE = \delta Q - \delta W$$

← heat ← work

$$\text{II Law: } dS = \left( \frac{\delta Q}{T} \right)_{\text{rev}} \quad (2.9)$$

Heat is considered positive if added to system.  
Work is considered positive if done by system.

It can be shown that  $TdS = dE + p dV$  }

$$H = E + pV \Rightarrow dE = dH - p dV - V dp$$

$$TdS = dH - V dp$$

Dividing by mass  $m$ ,  $T ds = dh - v dp$  ← very important relationship

(or)  $T ds = dh - \frac{dp}{\rho}$  ←

If a fluid is in motion, the energy is made up of  $E$ , the internal energy;  $\frac{mu^2}{2}$ , the kinetic energy; and  $mgz$ , the potential energy. The potential energy is negligible in the devices we study in this course. Thus the total energy is  $\left( E + \frac{mu^2}{2} \right)$  [see (2.6)]

## Perfect (Ideal) Gas

$$p v = R T \quad (2.13)$$

specific volume ( $\text{m}^3/\text{kg}$ )  
pressure ( $\frac{\text{N}}{\text{m}^2}$  (or) Pa) thermodynamic (absolute) temperature (K)  
specific gas constant ( $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ )

$$\rho = \frac{1}{v} \Rightarrow p = \rho R T \quad (\text{another form of the thermal equation of state})$$

density ( $\text{kg}/\text{m}^3$ )

A gmol of a substance represents an amount of substance with mass equal to its molar mass ( $\bar{M}$ ) {molecular weight, obsolete}.

$$1 \text{ gmol } \text{H}_2 \equiv 2 \text{ gm } \text{H}_2 \quad \left\{ \bar{M}_{\text{H}_2} = 2 \frac{\text{g}}{\text{gmol}} \right\}$$

$$1 \text{ kgmol } \text{H}_2 \equiv 2000 \text{ gm } \text{H}_2 = 2 \text{ kg } \text{H}_2 \quad \left\{ \bar{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kgmol}} \right\}$$

gmol is simply written as mol

kgmol is simply written as kmol

$$\bar{M}_{\text{H}_2} = 2 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{O}_2} = 32 \frac{\text{kg}}{\text{kmol}}, \quad \bar{M}_{\text{N}_2} = 28 \frac{\text{kg}}{\text{kmol}}, \quad \bar{M}_{\text{CO}_2} = 44 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{M}_{\text{H}_2\text{O}} = 18 \frac{\text{kg}}{\text{kmol}}$$

$$\bar{R} - \text{universal gas constant}, \quad \bar{R} = 8314.3 \frac{\text{J}}{\text{kmol}\cdot\text{K}}$$

$$R = \frac{\bar{R}}{\bar{M}} = \frac{\frac{\text{J}}{\text{kmol}\cdot\text{K}}}{\frac{\text{kg}}{\text{kmol}}} \Rightarrow \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

$\bar{R}$  same for all gases

$R$  different for different gases

$$R_{H_2} = \frac{\bar{R}}{\bar{M}_{H_2}} = \frac{8314.3}{2} = 4157.2 \frac{J}{kg-K}$$

$$R_{O_2} = \frac{\bar{R}}{\bar{M}_{O_2}} = \frac{8314.3}{32} = 259.8 \frac{J}{kg-K}$$

Similarly,  $R_{N_2} = 296.9 \frac{J}{kg-K}$ ,  $R_{CO_2} = 189 \frac{J}{kg-K}$ ,

$$R_{H_2O} = 461.9 \frac{J}{kg-K}$$

see page 34

$e = e(T)$ , does not depend on specific volume

$h = e + \frac{p}{\rho} = e + RT \Rightarrow h = h(T)$ , does not depend on pressure.

$$\left. \begin{aligned} c_v &= \left( \frac{\partial e}{\partial T} \right)_v = \frac{de}{dT} \\ c_p &= \left( \frac{\partial h}{\partial T} \right)_p = \frac{dh}{dT} \end{aligned} \right\} (2.15)$$

$\gamma$  - specific heat ratio

$$\gamma \stackrel{\text{def}}{=} \frac{c_p}{c_v} \quad (\text{see page 35})$$

$c_p$ ,  $c_v$  and  $\gamma$  vary with  $T$

$$\frac{dh}{dT} = \frac{de}{dT} + R \Rightarrow c_p = c_v + R$$

$$\boxed{\begin{aligned} c_p &= \frac{\gamma R}{\gamma - 1} \\ c_v &= \frac{R}{\gamma - 1} \end{aligned}}$$

Consider  $Tds = dh - \frac{dp}{\rho}$  for a perfect gas

$$dh = c_p dT \text{ (from 2.15), } \rho = \frac{P}{RT} \text{ (from 2.13)}$$

$$Tds = c_p dT - \frac{RT}{P} dp$$

$$\Rightarrow ds = c_p \frac{dT}{T} - R \frac{dp}{P} \quad (2.17)$$

Oftentimes,  $c_p$  can be assumed to be constant. Making this assumption, and integrating (2.17) from state ① to state ②,

$$(s_2 - s_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$\Delta s \leftarrow$  entropy change from ① to ②.

Isentropic process is one with no entropy change;

$$ds = dS = 0.$$

(2.9)  $\Rightarrow$  a reversible, adiabatic ( $Q=0$ ) process is isentropic.

The above  $(s_2 - s_1)$  expression yields

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \text{ for an isentropic process}$$

$$\Rightarrow \underbrace{\frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^{\frac{c_p}{R}}}_{\text{isentropic relation for an ideal gas with constant } c_p \text{ or constant } \gamma} = \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \text{ since } c_p = \frac{\gamma R}{\gamma-1}$$

isentropic relation for an ideal gas with constant  $c_p$  or constant  $\gamma$ .