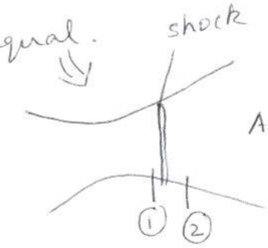
Nonisentropic Flow

Shocks (Sec 3.7)

Shocks are adiabatic processes that cause discontinuous changes in the properties of the fluid passing through them.

* The flow passing through a shock undergoes an irreversible adiabatic process.
* Shocks can occur only in supersonic flows.
* Shocks are so thin that they can be considered to have zero thickness. Thus, even if a shock occurs in a variable area passage, the flow areas just before the shock and just after the shock are equal.



$A_2 = A_1$

I Law: $(h_{o2} - h_{o1}) = \cancel{q}^{\,0} - \cancel{w}^{\,0}$

$$\Rightarrow h_{o2} = h_{o1} \Rightarrow \boxed{T_{o2} = T_{o1}}$$

II Law: $\Delta s = (s_2 - s_1) = (s_{o2} - s_{o1}) = \underbrace{c_p \ln \frac{T_{o2}}{T_{o1}}}_{0} - R \ln \frac{p_{o2}}{p_{o1}}$

$$\Rightarrow \Delta s = -R \ln \frac{p_{o2}}{p_{o1}} \Rightarrow \frac{p_{o2}}{p_{o1}} = e^{-\left(\frac{\Delta s}{R}\right)} < 1$$

The entropy change due to irreversibility causes a decline in stagnation pressure.

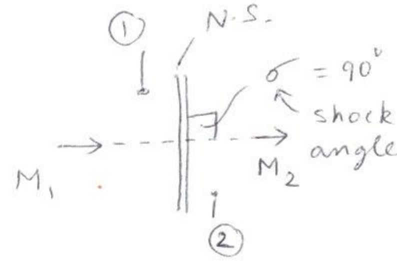
$T_{o2} = T_{o1}$ [energy of the flow does not change]

$p_{o2} < p_{o1}$ ["availability" of energy decreases]

Shocks degrade the capacity of a flow to do useful work.

Normal shock (N.S.)

The plane of the shock is normal (at 90°) to the incoming flow direction.



Summary: $M_1 > 1$, $M_2 < 1$

$T_{0_2} = T_{01}$

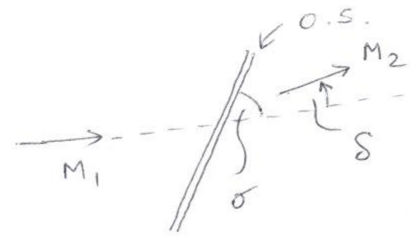
$p_{0_2} < p_{01}$

$T_2 > T_1$, $\underbrace{p_2 > p_1}_{\text{compression process}}$, $u_2 < u_1$

No flow deflection ($\delta = 0$); M_2 aligned with M_1

Oblique shock (O.S.)

The plane of the shock is inclined at angle σ to the incoming flow direction.



Summary: $M_1 > 1$, $M_2 \gtrless 1$

$T_{0_2} = T_{01}$

$p_{0_2} < p_{01}$

$T_2 > T_1$, $p_2 > p_1$, $u_2 < u_1$

$$\sin^{-1}\left(\frac{1}{M_1}\right) \le \sigma \le \frac{\pi}{2}$$

Flow is deflected by angle δ:

$$\tan(\sigma - \delta) = \frac{2\left(1 + \frac{\gamma-1}{2} M_1^2 \sin^2\sigma\right)}{(\gamma+1) M_1^2 \sin\sigma \cos\sigma} \qquad (3.40)$$

Weakest o.s. is a Mach wave, which has $\sigma = \sin^{-1}\left(\frac{1}{M_1}\right)$ at Mach number M_1.

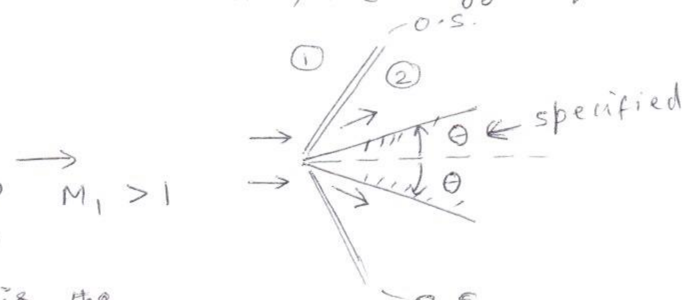
Strongest o.s. is a normal shock; $\sigma = \frac{\pi}{2}$ rad $= 90^\circ$

At any M_1, an o.s. can have σ only between $\sin^{-1}\left(\frac{1}{M_1}\right)$ and $\frac{\pi}{2}$

$$\underbrace{\sin^{-1}\left(\frac{1}{M_1}\right)}_{\text{usually designated } \mu(M_1)} \leq \sigma \leq \frac{\pi}{2} \qquad \left[\, \mu(M_1) \leq \sigma \leq \frac{\pi}{2} \,\right]$$

Ex: At $M_1 = 2$, $\mu(M_1) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$. Thus an o.s. occurring at $M_1 = 2$ can have σ only between 30° and 90°. Shock angles less than 30° are not possible at $M_1 = 2$.

The actual σ is determined by the "trigger" for the shock formation.



The o.s. provides the ~~flow~~ mechanism to deflect the flow by angle θ; that is, the shock deflection angle is $\delta = \theta$

Figures (3.10), (3.11) and (3.12) are charts from which property changes across shocks (both normal and oblique) can be found. [$\gamma = 1.4$]

Fig 3.10 is the graphical equivalent of the $M_1 - \sigma - \delta$ relation of Eq (3.40). For a given M_1 and δ, there are two different σ angles that are possible ($\underset{\text{strong}}{\sigma_{str}}$, $\underset{\text{weak}}{\sigma_{wk}}$) $\sigma_{str} > \sigma_{wk}$

Ex: $M_1 = 2$, $\delta = 15^\circ \Rightarrow \sigma_{str} = 79^\circ$ and $\sigma_{wk} = 45^\circ$

The strong shock solution causes larger changes in flow properties than the weak shock solution $\left[\left(\frac{T_2}{T_1}\right)_{str} > \left(\frac{T_2}{T_1}\right)_{wk}, \right.$
$\left. \left(\frac{p_2}{p_1}\right)_{str} > \left(\frac{p_2}{p_1}\right)_{wk}, \left(\frac{p_{o2}}{p_{o1}}\right)_{str} < \left(\frac{p_{o2}}{p_{o1}}\right)_{wk} \right]$

Fig 3.11 gives stagnation pressure ratios for specified M_1 and δ. Again, there are strong and weak solutions.

Ex: $M_1 = 2$, $\delta = 15^\circ \Rightarrow \left(\frac{p_{t2}}{p_{t1}}\right)_{str} = 0.75, \left(\frac{p_{t2}}{p_{t1}}\right)_{wk} = 0.96$

Fig 3.12 gives the Mach number after the shock (M_2). Again, there are strong and weak solutions.

Ex: $M_1 = 2$, $\delta = 15^\circ \Rightarrow M_{2,str} = 0.68$, $M_{2,wk} = 1.44$

$M_2 > 1$ after a weak shock (mostly)

$M_2 < 1$ after a strong shock (always)

In practice, weak shocks form. So we work with weak solutions.

Even though Fig 3.11 gives the stagnation pressure ratio, the static pressure ratio can be readily found from the results of Fig 3.11 and Fig 3.12

$$p_2 = \frac{p_{o2}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}}, \quad p_1 = \frac{p_{o1}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{p_2}{p_1} = \frac{p_{o2}}{p_{o1}}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Ex: $M_1 = 2$, $\delta = 15^\circ$

weak solution: $\frac{p_{o2}}{p_{o1}} = 0.96$, $M_2 = 1.44$, $\sigma = 45^\circ$

$$\frac{p_2}{p_1} = (0.96)\left[\frac{1+(0.2)(2)^2}{1+(0.2)(1.44)^2}\right]^{3.5} = 2.23$$

similarly, $T_2 = \frac{T_{o2}}{1+\frac{\gamma-1}{2}M_2^2}$, $T_1 = \frac{T_{o1}}{1+\frac{\gamma-1}{2}M_1^2}$

$$\frac{T_2}{T_1} = \cancelto{1}{\frac{T_{o2}}{T_{o1}}} \cdot \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

Ex: $M_1 = 2$, $\delta = 15^\circ$

weak solution: $\frac{T_2}{T_1} = \frac{1+(0.2)(2)^2}{1+(0.2)(1.44)^2} = 1.27$

Use also can be made of online ~~shock~~ gas dynamic calculators such as the "Compressible Aerodynamics Calculator."