Example

Reconsider the earlier three-stage rocket example:

$$I_{sp_1} = I_{sp_2} = I_{sp_3} = 370 \text{ s}$$

$$\epsilon_1 = 0.05, \quad \epsilon_2 = 0.0698, \quad \epsilon_3 = 0.1892$$

$$\frac{M_L}{M_{01}} = \frac{68{,}000}{2{,}500{,}000} = 0.0272$$

With $\lambda_1 = 0.3021$, $\lambda_2 = 0.4464$, $\lambda_3 = 0.6126$,

$$\Delta u_{b,imp} = 11023 \text{ m/s}.$$

Let us optimize the vehicle to maximize $\left(\frac{M_L}{M_{01}}\right)$

for $\Delta u_{b,imp} = 11023$ m/s

$$\left\{\frac{1}{\epsilon_1}\left[1 - \frac{1}{\alpha g_e I_{sp_1}}\right]\right\}^{\frac{g_e I_{sp_1}}{\Delta u_{b,imp}}} \otimes \left\{\frac{1}{\epsilon_2}\left[1 - \frac{1}{\alpha g_e I_{sp_2}}\right]\right\}^{\frac{g_e I_{sp_2}}{\Delta u_{b,imp}}} \otimes$$

$$\left\{\frac{1}{\epsilon_3}\left[1 - \frac{1}{\alpha g_e I_{sp_3}}\right]\right\}^{\frac{g_e I_{sp_3}}{\Delta u_{b,imp}}} = e \quad (E)$$

Use MATLAB to solve Equation (E) for αg_e

Then, get λ_i from Eqn. (D):

$$\lambda_i = \frac{\epsilon_i}{\alpha g_e I_{sp_i}(1-\epsilon_i) - 1}$$

$$\frac{M_L}{M_{01}} = \frac{\lambda_1}{1+\lambda_1} \cdot \frac{\lambda_2}{1+\lambda_2} \cdot \frac{\lambda_3}{1+\lambda_3} \quad [\text{from } (10.35)]$$

MATLAB solution:

$\alpha g_e = 0.003554$

$\lambda_1 = 0.2005, \quad \lambda_2 = 0.3125, \quad \lambda_3 = 2.8524$

$\left(\frac{M_L}{M_{01}}\right)_{opt} = 0.0294$

$\left(\Delta u_{b,imp1} = 5688.1 \text{ m/s}, \quad \Delta u_{b,imp2} = 4477.2 \text{ m/s}, \right.$

$\Delta u_{b,imp3} = 857.7 \text{ m/s}$

$\left. \Delta u_{b,imp} = \sum_{i=1}^{3} \Delta u_{b,imp\,i} = 11023 \text{ m/s} \right)$

* If M_L is maintained at 68000 kg,

$$(M_{01})_{opt} = \frac{M_L}{(M_L/M_{01})_{opt}} = \frac{68000}{0.0294} \simeq 2,313,000 \text{ kg}.$$

Thus the optimized design is capable of imparting the Δu_b of 11023 m/s to the 68000 kg payload with a lighter launch vehicle [2,313,000 kg vs 2,500,000 kg]

* If, on the other hand, M_{01} is maintained at 2,500,000 kg,

$$(M_L)_{opt} = M_{01}\left(\frac{M_L}{M_{01}}\right)_{opt} = 2,500,000\,(0.0294) = 73500 \text{ kg}.$$

Thus the optimized 2,500,000 kg launch vehicle is capable of providing the Δu_b of 11023 m/s to a more massive payload (73650 kg vs. 68000 kg)