Turbojet Engine Cycle Analysis

Recall, from the discussion of isentropic flow with work that the compressor power is given by $\wp_{sc} = \dot{m}c_p(T_{01} - T_{02})$, where subscripts 1 and 2 represent the compressor entry and exit stations. For an ideal compressor the temperature and pressure ratios are related by the isentropic expression:

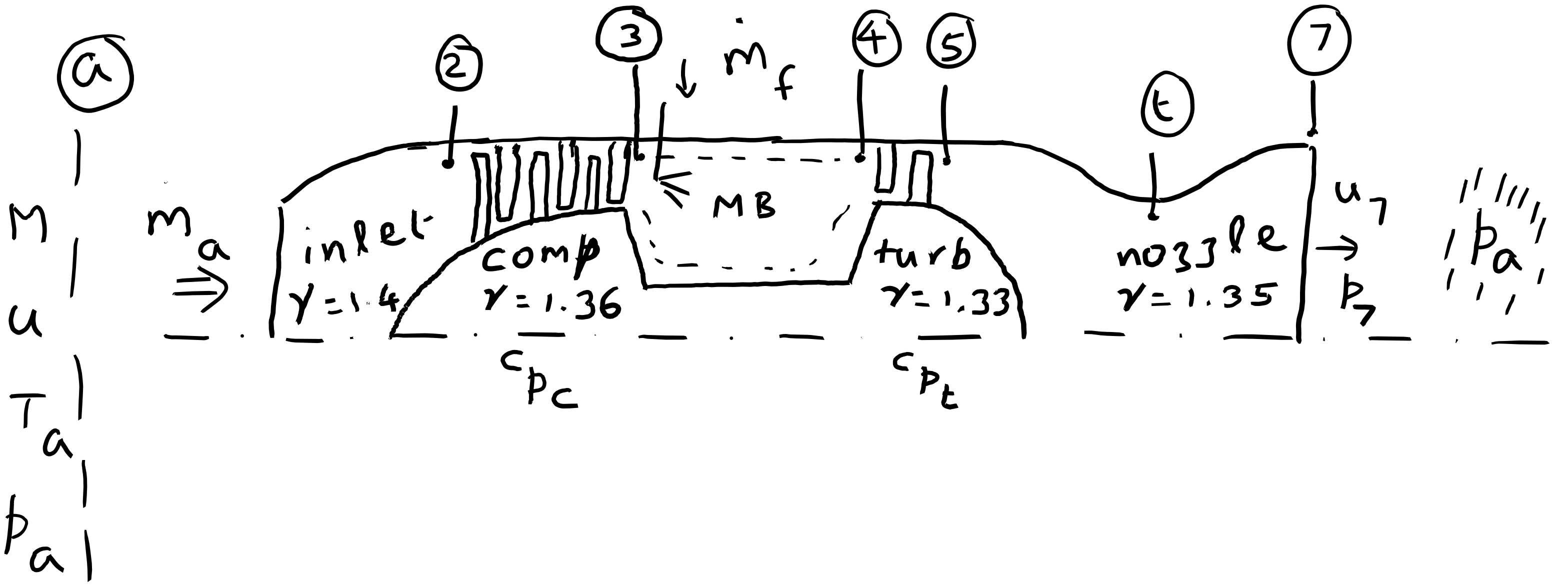
$$\left(\frac{T_{02}}{T_{01}}\right) = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \pi_c^{\frac{\gamma-1}{\gamma}}$$

, where π_c is the compression ratio.

The turbine power is given by $\wp_{st} = \dot{m}c_p(T_{01} - T_{02})$, and for the ideal turbine, the temperature and pressure ratios are related by the isentropic expression:

$$\left(\frac{p_{02}}{p_{01}}\right) = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}.$$

Consider the burner in the gas turbine engine. Fuel, with heating value Q_R, is added at the rate of $\dot{m}_f$. The rate of energy release $(\dot{Q})$ due to combustion of this fuel with air is $\dot{m}_f Q_R \eta_b$, where η_b is the burner (or combustion) efficiency. The mass flow rate through the burner is $(\dot{m}_a + \dot{m}_f)$, and the heat added per unit mass is $q = \frac{\dot{Q}}{\dot{m}_a + \dot{m}_f} = \frac{\dot{m}_f Q_R \eta_b}{\dot{m}_a + \dot{m}_f} = \frac{f Q_R \eta_b}{1+f}$. As mentioned earlier, f is typically small (around 0.02 or 0.03, so it can be ignored in comparison to 1; that is $(1+f) \simeq 1$. Therefore, $q = f Q_R \eta_b$.



Example:

An ideal turbojet has the following characteristics: M = 2, T_a = 217 K, p_a = 19.4 kPa, π_c = 22, T_{04} = 1600 K, fully expanded exhaust jet, Q_R = 43500 kJ/kg.

(i) Perform cycle analysis and find the engine exit stagnation temperature and stagnation pressure. The values of γ across the various components are noted in the Figure.

(ii) Find the specific thrust and TSFC of the engine, and the exhaust nozzle area ratio.

Solution:

(i)

Station (a): M = 2, u = $M\sqrt{\gamma RT}$ = 590.6 m/s

$T_{0a} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right)$ = 390.6 K

$p_{0a} = p_a\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$ = 151.8 kPa

(a) → (2), inlet diffuser:

$T_{02} = T_{0a}$ = 390.6 K

$p_{02} = p_{0a}$ = 151.8 kPa

(2) → (3), compressor:

π_c = 22, or $\left(\frac{p_{03}}{p_{02}}\right)$ = 22 → $p_{03} = 22p_{02}$ = 3340 kPa

$\left(\frac{T_{03}}{T_{02}}\right) = \left(\frac{p_{03}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}} = \pi_c^{\frac{\gamma-1}{\gamma}} = 22^{\frac{1.36-1}{1.36}}$ = 2.2665 → $T_{03} = 2.2286T_{02}$ = 885.3 K

$\wp_{sc} = \dot{m}_a c_{pc}(T_{02} - T_{03})$ = - 5.363(10^5) $\dot{m}_a$ {negative value indicates work done on air passing through the compressor}

{$c_{pc} = \frac{\gamma R}{\gamma-1} = \frac{1.36}{1.36-1}287$ = 1084 J/kg-K}

(3) → (4), main burner:

I law: $(h_{04} - h_{03}) = q - w = fQ_R\eta_b - 0 \rightarrow f = \frac{(h_{04}-h_{03})}{Q_R\eta_b} = \frac{(c_{pt}T_{04}-c_{pc}T_{03})}{Q_R\eta_b}$, or

$f = \frac{(1157)(1600)-(1084)(885.3)}{43500(1000)(1)}$ = 0.0205

$\{c_{pt} = \frac{\gamma R}{\gamma-1} = \frac{1.33}{1.33-1}287$ = 1157 J/kg-K}

$p_{04} = p_{03}$ = 3340 kPa {From Rayleigh line analysis (AE 308), recall that heat addition at zero Mach number entails no loss in stagnation pressure}

(4) → (5), turbine:

Turbine generates power to drive compressor. This observation constitutes the turbine-compressor power compatibility condition.

$|\wp_{st}| = |\wp_{sc}|$ = 5.363(10^5) $\dot{m}_a$, or

$\dot{m}_a(1+f)c_{pt}(T_{04} - T_{05})$ = 5.363(10^5) $\dot{m}_a$ → T_{05} = 1137 K

$\left(\frac{p_{05}}{p_{04}}\right) = \left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$ = 0.2524 → p_{05} = (0.2524)(3340) = 843 kPa

(5) → (7), nozzle:

$T_{07} = T_{05}$ = 1137 K

$p_{07} = p_{05}$ = 843 kPa

↑_______________end cycle analysis________________↑

(ii)

Fully expanded exhaust jet: $p_7 = p_a$ = 19.4 kPa

$\left(\frac{p_{07}}{p_7}\right) = \frac{843}{19.4}$ = 4.3454

$\left(1 + \frac{\gamma-1}{2}M_7^2\right)^{\frac{\gamma}{\gamma-1}}$ = 4.3454 → M_7 = 3.079 {γ = 1.35}

$T_7 = \frac{T_{07}}{\left(1+\frac{\gamma-1}{2}M_7^2\right)}$ = 427.6 K

$u_7 = M_7\sqrt{\gamma R T_7}$ = 1253 m/s

$\tau = \dot{m}_a(1+f)u_7 - \dot{m}_a u + (p_7 - p_a)A_7 \simeq \dot{m}_a(1253 - 590.6)$ = 662.4$\dot{m}_a$

Specific thrust $\frac{\tau}{\dot{m}_a}$ = 662.4 N/(kg/s)

$TSFC = \frac{\dot{m}_f}{\tau} = \frac{f\dot{m}_a}{\tau} = \frac{f}{\left(\frac{\tau}{\dot{m}_a}\right)}$ = (0.0205/662.4) = 3.095(10^{-5}) (kg/s)/N

$(TSFC)_{wb} = (TSFC)g_e$ = 3.095(10^{-5})(9.81) = 13.036(10^{-4}) $s^{-1} \xRightarrow[x3600]{}$ 1.093 h^{-1}

Nozzle area ratio is $(A_7/A_t) = (A_7/A_7^*) = \frac{1}{M_7}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_7^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$ = 5.039

{γ = 1.35}

If the propellant (air) mass flow rate is 100 kg/s,

$\tau = \left(\frac{\tau}{\dot{m}_a}\right)\dot{m}_a$ = (662.4)(100) = 66240 N $\simeq 14892\ lb_f$

$\dot{m}_f = f\dot{m}_a$ = (0.0205)(100) = 2.05 kg/s $\xRightarrow[x3600]{}$ 7380 kg/h $\simeq 16236\ lb_m/h$

$A_7 = \frac{\dot{m}_7}{\rho_7 u_7} = \frac{\dot{m}_a(1+f)}{\rho_7 u_7} \simeq \frac{\dot{m}_a}{\rho_7 u_7}$

$\dot{m}_a$ = 100 kg/s, $\rho_7 = \frac{p_7}{RT_7} = \frac{(19.4)(1000)}{(287)(427.6)}$ = 0.1581 kg/m^3, u_7 = 1253 m/s

$A_7 = \frac{100}{(0.1581)(1253)}$ = 0.5048 m^2 {$D_7 = \sqrt{\frac{4A_7}{\pi}}$ = 0.80 m $\simeq$ 31.5 in.}

$A_t = \frac{A_7}{\left(\frac{A_7}{A_t}\right)} = \frac{0.5048}{5.039}$ = 0.1002 m^2 {$D_t = \sqrt{\frac{4A_t}{\pi}}$ = 0.36 m $\simeq$ 14.2 in.}