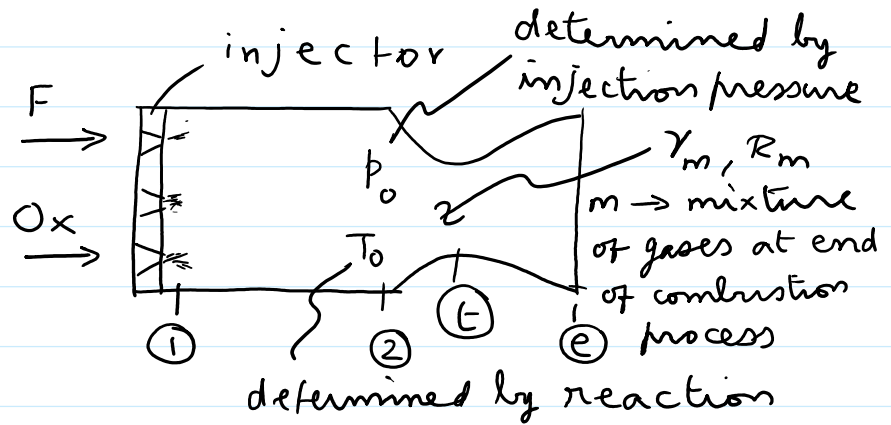* A LPRE Thrust chamber requires propellant feed at high pressure. Unlike in the SRM, there is no increase in p_0 in the combustion chamber.

Recall: $\overline{M}_m = \sum x_i \overline{M}_i$, $\overline{c}_{pm} = \sum x_i \overline{c}_{pi}$, $\gamma_m = \dfrac{\overline{c}_{pm}}{\overline{c}_{pm} - \overline{R}}$

For the mixture, $R_m = \dfrac{\overline{R}}{\overline{M}_m}$

Ex

An ideal hydrogen-oxygen rocket Thrust chamber has $r = 5$ Fuel (H_2) and oxidizer (O_2) are injected at 68 atm, and 298.15 K. The temperature (T_0) and The composition (mole fractions x_i) of the product mixture are given in Table 12.2. Assuming fully-expanded jet at SL, calculate I_{sp}. $\gamma_{MON} = 1.67$, $\gamma_{DI} = 1.29$, $\gamma_{TRI} = 1.17$

$p_0 = 68 \text{ atm} = 68(0.101325) = 6.89 \text{ MPa}$

From Table 12.3,

$T_0 = 3429 \text{ K}$

$X_{H_2} = 0.3607, \quad X_{O_2} = 0.977(10^{-3}), \quad X_{H_2O} = 0.5744,$

$X_{OH} = 0.0260, \quad X_O = 0.0015, \quad X_H = 0.0363$

$\{ \overline{M}_H = 1, \quad \overline{M}_O = 16 \}$

$$\overline{M}_m = \Sigma(X_i \overline{M}_i) = 0.3607\,(2) + 0.977(10^{-3})(32) +$$
$$0.5744\,(18) + 0.0260(17) +$$
$$0.0015\,(16) + 0.0363(1) = 11.59$$

$$R_m = \frac{\overline{R}}{\overline{M}} = \frac{8314.3}{11.59} = 717.4 \,\frac{J}{kg\text{-}k}$$

$$\overline{c}_{pm} = \Sigma(X_i \overline{c}_{pi})$$

$$\overline{c}_{p_H} = \overline{c}_{p_O} = \frac{\gamma_{MON}\,\overline{R}}{\gamma_{MON} - 1} = 2.493\,\overline{R}$$

$$\overline{c}_{p_{H_2}} = \overline{c}_{p_{O_2}} = \overline{c}_{p_{OH}} = \frac{\gamma_{DI}\,\overline{R}}{\gamma_{DI} - 1} = 4.448\,\overline{R}$$

$$\overline{c}_{p_{H_2O}} = \frac{\gamma_{TRI}\,\overline{R}}{\gamma_{TRI} - 1} = 6.882\,\overline{R}$$

$$\overline{c}_{pm} = \Sigma(X_i \overline{c}_{pi}) = 5.772\,\overline{R}$$

$$\gamma_m = \frac{\overline{c}_{pm}}{\overline{c}_{pm} - \overline{R}} = \frac{5.772\,\overline{R}}{5.772\,\overline{R} - \overline{R}} = \frac{5.772}{4.772} = 1.21$$

Calculation of I_{sp} is as discussed earlier (ch 11)

$$P_{oe} = P_o = 68 \text{ atm}$$

$$P_e = P_a = 1 \text{ atm} \underbrace{}_{\text{sea level}}$$
$$\underbrace{}_{\text{fully expanded}}$$

$$\frac{P_{oe}}{P_e} = \frac{68}{1} = 68$$

$$\left(1 + \frac{\gamma_m - 1}{2} M_e^2\right)^{\frac{\gamma_m}{\gamma_m - 1}} = 68 \Rightarrow M_e = 3.21$$

$$T_{oe} = T_o = 3429 \text{ K}$$

$$T_e = \frac{T_{oe}}{1 + \frac{\gamma_m - 1}{2} M_e^2} = 1647 \text{ K}$$

$$u_e = M_e \sqrt{\gamma_m R_m T_e} = 3.21 \sqrt{(1.21)(717.4)(1647)}$$

$$= 3838 \text{ m/s}$$

$$u_{eq} = u_e + \underbrace{\frac{RT_e}{u_e} \left(1 - \cancel{\frac{P_a}{P_e}}^{1}\right)}_{0} = u_e = 3838 \text{ m/s}$$

$$I_{sp} = \frac{u_{eq}}{g_e} = \frac{3838}{9.81} = 391 \text{ s}$$

$$\frac{A_e}{A_t} = \frac{A_e}{A_e^*} = \frac{1}{M_e} \left[\frac{2}{\gamma_m + 1}\left(1 + \frac{\gamma_m - 1}{2} M_e^2\right)\right]^{\frac{\gamma_m + 1}{2(\gamma_m - 1)}} = 8.73$$