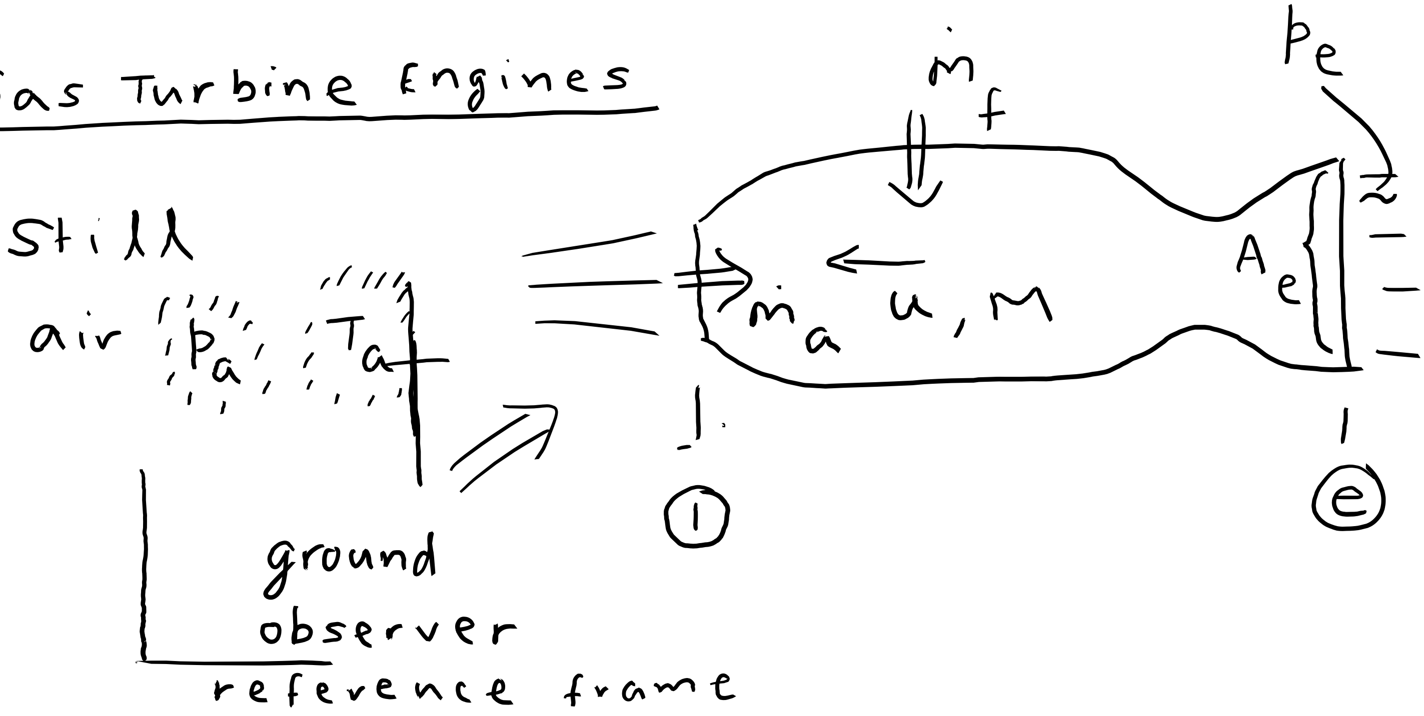
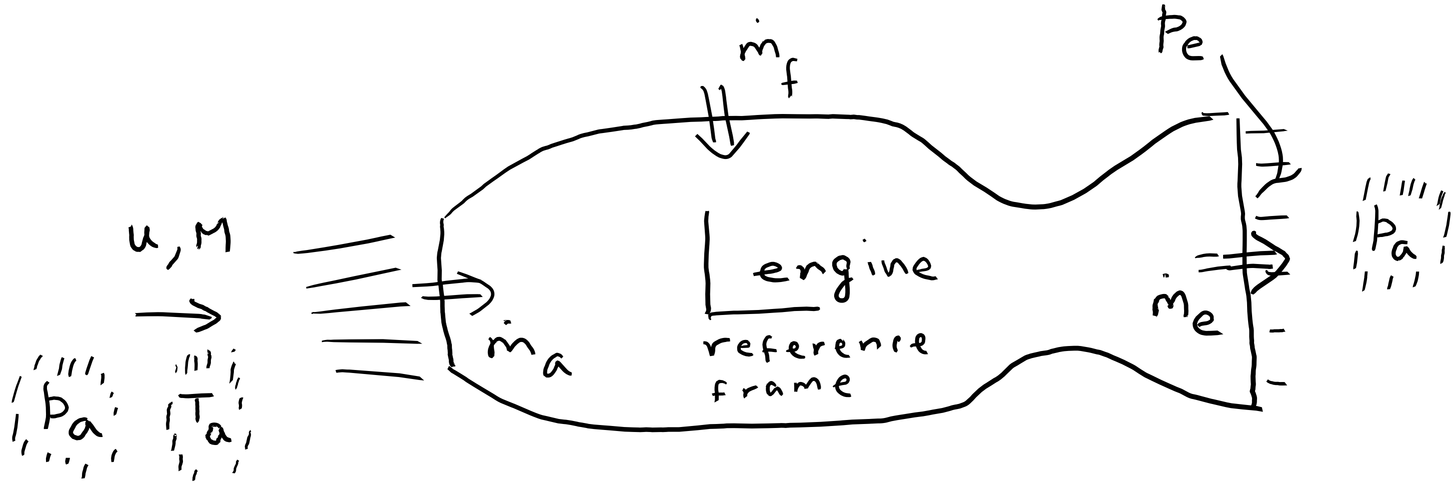
Gas Turbine Engines



Change reference, fix it to engine



$\dot{m}_a$ – propellant (air) mass flow rate (kg/s)

$\dot{m}_f$ – fuel mass flow rate (kg/s) [energizes propellant. The mass of fuel added is a very small fraction of the mass of propellant, since fuels have very large heating values (~ 43000 kJ/kg).

<u>Mass</u>: $\dot{m}_e = (\dot{m}_a + \dot{m}_f) = \dot{m}_a\left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right)$

$f \overset{\text{def}}{=} \frac{\dot{m}_f}{\dot{m}_a}$, fuel-to-air ratio [as described earlier, f is very small; typically about 0.02.]

$\dot{m}_e = \dot{m}_a(1+f)$

<u>Momentum</u>:

$$T = (\dot{m}_e u_e - \dot{m}_a u) + (p_e - p_a) A_e$$

(or) $T = \dot{m}_a (1+f) u_e - \dot{m}_a u + (p_e - p_a) A_e$

$$= \underbrace{\dot{m}_a [(1+f) u_e - u]}_{\text{momentum thrust}} + \underbrace{(p_e - p_a) A_e}_{\text{pressure thrust}}$$

{See Eqn 5.6}

$$T_{gross} \overset{\text{def}}{=} \dot{m}_a (1+f) u_e + (p_e - p_a) A_e$$

$$D_{ram} \overset{\text{def}}{=} \dot{m}_a u$$

$$T = (T_{gross} - D_{ram})$$

Specific thrust is the thrust generated per unit mass flow rate $\left\{ \frac{T}{\dot{m}_a} \right\}$

$$\frac{T}{\dot{m}_a} = [(1+f)u_e - u] + \frac{(p_e - p_a)A_e}{\dot{m}_a}$$

$\left\{ \text{units} : \frac{N}{(kg/s)} , \frac{lb_f}{(lb_m/s)} \right\}$

Thrust-specific fuel consumption:

$$TSFC \overset{def}{=} \frac{\dot{m}_f}{T} = \frac{f\dot{m}_a}{T} = \frac{f}{(T/\dot{m}_a)}$$

$\left\{ \text{units} : \frac{(kg/s)}{N} , \frac{(lb_m/s)}{lb_f} \right\}$

Alternately, $(TSFC)_{wb} \overset{def}{=} \frac{\dot{w}_f}{T} = \frac{\dot{m}_f g_e}{T} = (TSFC) g_e$

$\dot{w}_f$ - weight flow rate of fuel ;

g_e - sea-level, earth, acceleration due to gravity.

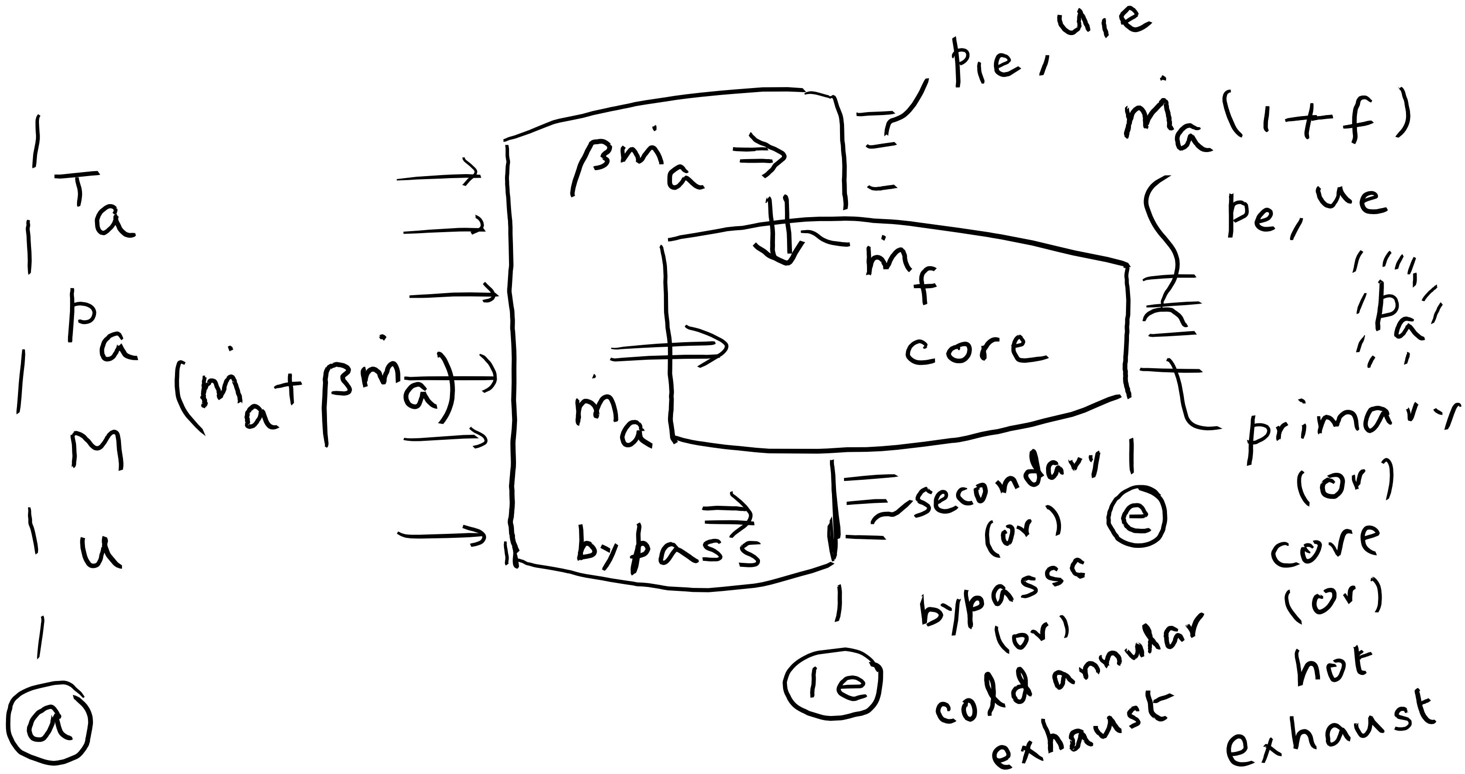
$(TSFC)_{wb}$ has units of $\frac{N/s}{N}$ (or) $\frac{1}{s}$ (or) $\left(\frac{1}{h}\right)$.

Specific thrust is a measure of engine performance. TSFC is a measure of engine efficiency. The reciprocal of $(TSFC)_{wb}$ is known as the specific impulse (I_{sp}) and is used in rocket analysis.

$$I_{sp} \stackrel{def}{=} \frac{T}{\dot{w}_f} = \frac{1}{(TSFC)_{wb}} \quad \left\{\text{units: } \frac{1}{s} \text{ or } \frac{1}{h}\right\}$$

Gas turbine engines used to propel transport aircraft often employ twin exhausts. These engines are known as turbofan engines. The expression for thrust generated by them is different from that given earlier {Eqn. 5 6}. The simplest type of the single-exhaust engine is a turbojet engine.

Turbofan Engines



Bypass ratio $(\beta) \overset{def}{=} \dfrac{\dot{m}_{bypass}}{\dot{m}_{core}}$

$\dot{m}_{core}$ is taken to be $\dot{m}_a$ $\{\dot{m}_{core} = \dot{m}_a\}$

$\dot{m}_{bypass} = \beta\dot{m}_a$

Low-bypass turbofan engines : $\beta < 1$
(LBTF) (used in fighter aircraft)

High-bypass turbofan engines : $5 < \beta < 9$
(HBTF) (used in transports)

$$T_{core} = \dot{m}_a(1+f)u_e - \dot{m}_a u + (p_e - p_a)A_e$$

$$T_{bypass} = \beta \dot{m}_a u_{1e} - \beta \dot{m}_a u + (p_{1e} - p_a)A_e$$

$$T = T_{core} + T_{bypass}$$

$$TSFC = \frac{\dot{m}_f}{T} \text{, (or) } (TSFC)_{wb} = \frac{\dot{m}_f g_e}{T}$$

<u>Typical $\frac{T}{\dot{m}_a}$ and $(TSFC)_{wb}$ values</u>

Engine Type	$\frac{T}{\dot{m}_a}\left(\frac{N}{kg/s}\right)$	$(TSFC)_{wb}\left(\frac{1}{h}\right)$
Turbojet	900	1.3
LBTF	700	1.0
HBTF	100	0.6

Turbofan engine example:

A HBTF engine has the following characteristics:

Flight velocity (u) 250 m/s

Core mass flow rate ($\dot{m}_a$) 100 kg/s

Bypass ratio (β) 6

Core exit velocity (u_e) 550 m/s

Bypass exit velocity (u_{1e}) 370 m/s

Core jet exit pressure (p_e) is equal to ambient pressure (p_a)

Bypass jet exit pressure (p_{1e}) is equal to ambient pressure (p_a)

Fuel-air ratio (f) 0.02

Questions:

(a) What is the engine mass flow rate?
(b) Calculate the fuel mass flow rate.
(c) What is the thrust generated by the core?
(d) What is the thrust generated by the bypass?
(e) What is the total thrust?
(f) What percentage of the thrust is generated by the bypass stream?
(g) What is the engine specific thrust?
(h) Determine the TSFC of the engine.

Answer:

(a) The engine mass flow rate = $\left(\dot{m}_{core} + \dot{m}_{bypass}\right) = (\dot{m}_a + \beta\dot{m}_a) = \dot{m}_a(1+\beta)$

$$= 100(1+6) = 700\ kg/s$$

(b) $\dot{m}_f = f\dot{m}_a = (0.02)(100) = 2\frac{kg}{s} = 7200\ kg/h$

(c) $\tau_{core} = \dot{m}_a(1+f)u_e - \dot{m}_a u + (p_e - p_a)A_e = 100(1+0.02)(550) - 100(250) + 0$
$= 31100\ N$

(d) $\tau_{bypass} = \beta\dot{m}_a u_{1e} - \beta\dot{m}_a u + (p_{1e} - p_a)A_{1e} = (6)(100)(370) - (6)(100)(250) + 0$
$= 72000\ N$

(e) $\tau = \tau_{core} + \tau_{bypass} = 31100 + 72000 = 103100\ N$

(f) % of thrust generated by the bypass stream = (72000/103100)(100) = 69.8%

(g) Engine specific thrust = 103100/700 = 147.3 N/(kg/s)

(h) TSFC = $\left(\dot{m}_f/\tau\right) = (2/103100) = 1.9399(10^{-5})\frac{kg}{s} = 19.399\ \left(\frac{mg/s}{N}\right)$

$(TSFC)_{wb}$ = (TSFC) g_e = 1.9399 (10^{-5}) (9.81) = 1.9030 (10^{-4}) s^{-1} = 0.685 h^{-1}

Efficiencies

Thermal efficiency (η_{th})

Propulsive efficiency (η_{pr})

Overall efficiency (η_o)

Fuel heating value (Q_R) is the ideal energy released under conditions on combustion of unit mass of fuel. As mentioned earlier, $Q_R \sim 43000 \frac{kJ}{kg}$ for gas turbine fuels. The actual heat released is $\eta_b Q_R$, where η_b is said to be the burner (or) combustion efficiency.

Single-exhaust engine

$$\boxed{\dot{m}_f Q_R}$$

(see Eqn 5.13)

$$\eta_o = \frac{Tu}{\dot{m}_f Q_R}$$

$$\eta_{th} = \frac{\dot{m}_e u_e^2 - \dot{m}_a u^2}{2\,\dot{m}_f Q_R}$$

$$\boxed{\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_a u^2}{2}}$$

$$\eta_{pr} = \frac{2Tu}{\dot{m}_e u_e^2 - \dot{m}_a u^2}$$

$$\boxed{Tu}$$

(See Eqn 5.10)

$$\eta_{th} = \frac{\dot{m}_a(1+f)u_e^2 - \dot{m}_a u^2}{2f\dot{m}_a Q_R} = \frac{(1+f)u_e^2 - u^2}{2fQ_R}$$

$$\eta_{pr} = \frac{2Tu}{\dot{m}_a(1+f)u_e^2 - \dot{m}_a u^2} = \frac{2\frac{T}{\dot{m}_a}u}{(1+f)u_e^2 - u^2}$$

$$\eta_o = \frac{Tu}{f\dot{m}_a Q_R} = \frac{\frac{T}{\dot{m}_a}u}{fQ_R}$$ (see Eqn 5.13)

Twin-Exhaust Engine

$$\boxed{\dot{m}_f Q_R}$$

$$\downarrow \quad \eta_{th} = \frac{\dot{m}_e u_e^2 + \beta \dot{m}_a u_{1e}^2 - (1+\beta)\dot{m}_a u^2}{2\dot{m}_f Q_R}$$

$$\boxed{\frac{\dot{m}_e u_e^2}{2} + \frac{\beta \dot{m}_a u_{1e}^2}{2} - \frac{(1+\beta)\dot{m}_a u^2}{2}}$$

$$\downarrow \quad \eta_{pr} = \frac{Tu}{\dot{m}_e u_e^2 + \beta \dot{m}_a u_{1e}^2 - (1+\beta)\dot{m}_a u^2}$$

$$\eta_o = \frac{Tu}{\dot{m}_f Q_R}$$

$$\boxed{Tu}$$

$$\eta_{th} = \frac{(1+f)u_e^2 + \beta u_{1e}^2 - (1+\beta)u^2}{2 f Q_R}$$

$$\eta_{pr} = \frac{2 \dfrac{T}{\dot{m}_a} u}{(1+f)u_e^2 + \beta u_{1e}^2 - (1+\beta)u^2}$$

$$\eta_o = \frac{\dfrac{T}{\dot{m}_a} u}{f Q_R}$$

Range, and its dependence on η_o

(see page 152)

In steady flight,

level flight

$$T = D = \frac{C_D}{C_L} L = \frac{C_D}{C_L} W \quad (1)$$

$$\frac{dW}{dt} = -\dot{w}_f$$

$W \rightarrow$ weight of airplane

$\dot{w}_f \rightarrow$ weight flow rate of fuel

Since $(TSFC)_{wb} = \dfrac{\dot{w}_f}{T}$, $\dot{w}_f = (TSFC)_{wb} T$

$$\Rightarrow \quad \frac{dW}{dt} = -(TSFC)_{wb} T = -(TSFC)_{wb} \frac{C_D}{C_L} W$$

from (1)

$$\frac{dW}{dt} = \frac{dW}{ds} \cdot \frac{ds}{dt} = \left(\frac{dW}{ds}\right) u$$

$\left\{ \begin{array}{l} s \rightarrow \text{distance} \\ u = \dfrac{ds}{dt}, \text{ velocity} \end{array} \right.$

$$\Rightarrow \quad u \frac{dW}{ds} = -(TSFC)_{wb} \frac{C_D}{C_L} W$$

$$\text{(or)} \quad ds = -\frac{u}{(TSFC)_{wb}} \frac{C_L}{C_D} \frac{dW}{W}$$

Integrate to get range $R \left[= \int ds \right]$

$$R = \frac{u}{(TSFC)_{wb}} \frac{C_L}{C_D} \ln \frac{W_{ini}}{W_{fin}} \quad (2)$$

$$\left\{ W_f = W_{ini} - W_{fin} \right\}$$

From (5.13),

$$\eta_o = \frac{\frac{T}{\dot{m}_a} u}{f Q_R} = \frac{u}{(TSFC) Q_R} = \frac{u g_e}{(TSFC)_{wb} Q_R}$$

$$\Rightarrow \frac{u}{(TSFC)_{wb}} = \frac{\eta_o Q_R}{g_e}$$

$$= \frac{L}{D}^{*}$$

$$(2) \Rightarrow R = \frac{\eta_o Q_R}{g_e} \left(\frac{C_L}{C_D} \right) \ln \frac{W_{ini}}{W_{fin}} \quad (3)$$

$\{$ see Eqn 5.19. *Recall $L = C_L q_\infty S$,

$D = C_D q_\infty S \Rightarrow \frac{L}{D} = \frac{C_L}{C_D}$ $\}$

Example:

A transport aircraft has wing area $S = 93\ m^2$ and drag polar $C_D = 0.015+0.05C_L^2$. Its engines have TSFC of $0.6\ h^{-1}$. Calculate the range of the aircraft for flight at u = 250 m/s and altitude 11000 m (density = 0.37 kg/m^3, temperature = 217 K). The aircraft mass at the beginning of cruise is 45000 kg and the cruise fuel available has mass 9100 kg.

$q_\infty = ½\ \rho_\infty u^2 = ½\ (0.37)\ (250^2) = 11563\ N/m^2$

$C_L = L/(\ q_\infty S) = W/(\ q_\infty S) = [(45000)(9.81)]/[(11563)(93)] = 0.41$

$C_D = 0.015+0.05(0.41^2) = 0.023$

$M_{ini} = 45000$ kg, $M_{fin} = (45000-9100) = 35900$ kg

$(W_{ini}/W_{fin}) = (M_{ini}/M_{fin})$

$(TSFC)_{wb} = 0.6\ h^{-1} = 0.6\ (3600^{-1}) = 1.6667\ (10^{-4})\ s^{-1}$

$$R = \frac{u}{(TSFC)_{wb}} \frac{C_L}{C_D} \ln\left(\frac{W_{ini}}{W_{fin}}\right) = \frac{250}{1.6667\ (10-4)} \frac{0.41}{0.023} \ln\left(\frac{45000}{35900}\right)$$

$= 6.040\ (10^6)$ m = 6040 km = 3776 mi = 3283 nm

{The speed of sound $(a_\infty) = (\gamma RT_\infty)^{1/2} = [(1.4)(287)(217)]^{1/2} = 295.3$ m/s at cruise altitude.

Flight Mach number $M = (u/a_\infty) = 0.85$}