# AE 434 – Spacecraft Control

## Chapter 1:
## Mathematical modeling

### 1.2 Transfer Functions

(Dorf & Bishop: *Ch.2.5*)

**Spring 2023**

# Course overview

- **Modeling a physical system**
  - Linearization of non-linear models
  - ODEs and their solutions – with Laplace transforms
  - ➢ **Transfer functions**
  - Block diagrams
  - State-space models
- **Analysis**
  - Time response
  - Steady-state error and PID control
  - Stability: Routh-Hurwitz criterion
- **Design control laws**
  - Root Locus
  - Frequency response
  - Pole placement
  - Introduction to Optimal control

Lead/Lag compensators

# Final Value Theorem

- It is usually desired to determine the steady-state or final value of the response
- Final value theorem: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$ **iff** $\lim_{t\to\infty} f(t)$ exists

***Example :*** $F(s) = 1 * \dfrac{1}{s+1}$ *Impulse input (∗ 1)*

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s * \frac{1}{s+1} = 0$$

*The system returns to zero after being disturbed by a short impulse.*

- Initial value theorem: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

# Transfer Functions

Modeling approach

Physical system $\Rightarrow$ Linearized ODE $\xrightarrow{\mathcal{L}}$ Transfer function

$$\ddot{x} + 2\dot{x} + 5x = 3$$

for $x(0) = \dot{x}(0) = 0$.

$$X(s) = \frac{3}{s} \cdot \frac{1}{s^2 + 2s + 5}$$

# Transfer Functions

The **transfer function** of a ***linear time-invariant (LTI) ODE*** is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function).

Consider the LTI system defined by the following differential equation:

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

$$G(s) = \left. \frac{\mathscr{L}[\text{output}]}{\mathscr{L}[\text{input}]} \right|_{\text{zero initial conditions}} = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

$$X(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

# Transfer Functions

- In the derivation of the transfer function, it is assumed that **the initial conditions of the ODE are null**.

- The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system.
  - *The transfer functions of physically different systems can be identical.*

- If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system.

# In-class examples: Transfer Functions

1. Find TF $$\frac{d^3c}{dt^3}+3\frac{d^2c}{dt^2}+7\frac{dc}{dt}+5c=\frac{d^2r}{dt^2}+4\frac{dr}{dt}+3r$$

2. Find governing equations from TF $$G(s)=\frac{2s+1}{s^2+6s+2}$$