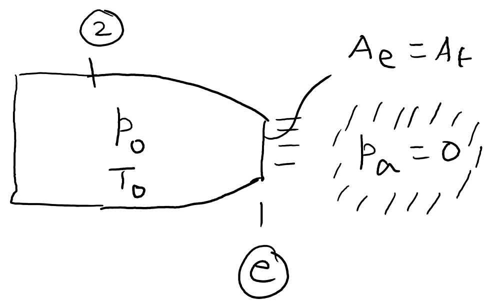
Example : Plm 11.2

An ideal rocket thrust chamber has $p_0 = 10$ MPa and $T_0 = 3000$ K. $\gamma = 1.2$, $\bar{M} = 25$, $A_t = 0.1\ m^2$. Calculate M_e, u_e, p_e and $\dot{m}$, and c^* and C_T for each of the following cases:

1. converging nozzle, $p_a = 0$
2. c-d nozzle ($A_e = 4.06\ m^2$), $p_a = 0$
3. converging nozzle, $p_a = 0.1$ MPa
4. c-d nozzle ($A_e = 4.06\ m^2$), $p_a = 0.1$ MPa

$$R = \frac{\bar{R}}{\bar{M}} = \frac{8314.3}{25} = 332.6 \frac{J}{kg\text{-}K}, \quad \gamma = 1.2$$

Case 1



Let $p_e = p_a = 0$, $\underset{\text{isen}}{p_{oe}} = p_0 = 10$ MPa $\Big\} \Rightarrow \frac{p_{oe}}{p_e} \to \infty$

$$\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}} \to \infty \Rightarrow M_e \to \infty$$

Conv. noz: $\frac{A_e}{A_t} = 1$

$$\underset{\substack{\text{conv}\\\text{noz.}}}{M_e \not> 1} \Rightarrow M_e = 1$$

$$p_e = \frac{p_{oe}}{\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}} = 5.645 \text{ MPa}$$

$M_e = 1$

$$T_e = \frac{T_{oe}}{1 + \frac{\gamma - 1}{2} \underset{\leftarrow 1}{M_e^2}} = 2727\,K \quad \left\{ \underset{\text{isen}}{T_{oe} = T_o} = 3000\,K \right\}$$

$$u_e = M_e \sqrt{\gamma R T_e} = 1043\ m/s$$

$$\rho_e = \frac{p_e}{R T_e} = \frac{5.645\,(10^6)}{(332.6)(2727)} = 6.224\ \frac{kg}{m^3}$$

$$A_e = A_t = 0.1\ m^2$$

$$\dot{m} = \rho_e u_e A_e = 649.2\ kg/s$$

$$T_{CONV} = \underset{649.2}{\dot{m}}\,\overset{1043\,m/s}{u_e} + (\underset{5.645}{p_e} - \underset{0}{p_a})\,\overset{0.1}{A_e}$$

$$= (0.6771 + 0.5645) = 1.242\ MN$$

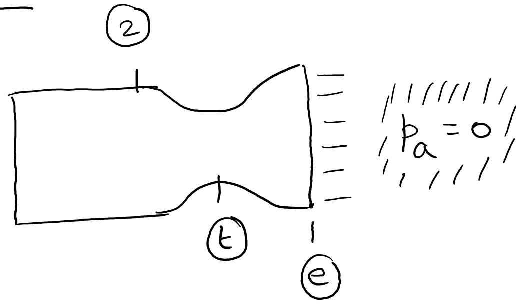
$$c^* = \frac{p_o A_t}{\dot{m}} = \frac{(10)(10^6)(0.1)}{649.2} = 1540\ m/s$$

$$\left\{ \text{From Eqn (11.9)},\ c^* = 1540\ m/s \right\}$$

$$C_{T_{CONV}} = \frac{T_{CONV}}{p_o A_t} = \frac{1.242}{(10)(0.1)} = 1.242$$

$$\left\{ \text{From Eqn (11.12)},\ C_{T_{CONV}} = 1.24,\ p_a = 0 \right\}$$

Case (2)



$A_e = 4.06\, m^2$

$\frac{A_e}{A_t} = \frac{4.06}{0.1} = 40.6$

$\frac{A_e}{A^*} = \frac{A_e}{A_t} = 40.6$

$M_e = 1$

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Trial-and-error, or online calculator $\Rightarrow M_e = 4.25$

$p_0 = 10$ MPa

$$p_e = \frac{\overset{\text{isen}}{p_{oe}}}{1+\frac{\gamma-1}{2}M_e^2} = 0.020476 \text{ MPa}$$

$T_0 = 3000$ K

$$T_e = \frac{\overset{\text{isen}}{T_{oe}}}{1+\frac{\gamma-1}{2}M_e^2} = 1069 \text{ K}$$

$$u_e = M_e\sqrt{\gamma R T_e} = 2776 \text{ m/s}$$

$$\rho_e = \frac{p_e}{R T_e} = \frac{0.020476(10^6)}{(332.6)(1069)} = 0.05759 \frac{kg}{m^3}$$

$$\dot{m} = \rho_e u_e A_e = 649.1 \text{ kg/s}$$

$$c^* = \frac{p_0 A_t}{\dot{m}} = \frac{10(10^6)(0.1)}{649.1} = 1540 \text{ m/s}$$

$(11.9) \Rightarrow c^* = 1540$ m/s

$$\tau = \dot{m} u_e + (p_e - p_a) A_e$$

$$= 1.802 + 0.083 = 1.885 \text{ MN}$$

$$C_\tau = \frac{\tau}{p_0 A_t} = \frac{1.885}{(10)(0.1)} = 1.885$$

$$(11.11) \Rightarrow C_\tau = 1.885$$

$$\frac{C_\tau}{C_{\tau\,conv}} = \frac{1.885}{1.242} = 1.52$$ $\Big\{$ increase in thrust due to divergence $(p_a/p_0) = (0/10) = 0$

<u>Case ③</u>

Conv. noz.: $p_a = 0.1 \text{ MPa}, \; p_0 = 10 \text{ MPa}$

$p_{0e} \xrightarrow{\text{isen}} = p_0 = 10 \text{ MPa}$.

Let $p_e = p_a = 0.1 \text{ MPa}$

$$\Rightarrow \frac{p_{0e}}{p_e} = \frac{10}{0.1} = 100$$

$$\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}} = 100$$

$$\Rightarrow M_e = 3.40$$

$M_e \not> 1 \Rightarrow M_e = 1$

All the exit conditions are the same as in case ①.

$\dot{m} = 649.2 \text{ kg/s}, \; p_e = 5.645 \text{ MPa}, \; u_e = 1043 \text{ m/s}$

$$\tau = \dot{m} u_e + (p_e - p_a) A_e = 0.6771 + 0.5545 = 1.23 \text{ MN}$$

$c^* = 1540$ m/s , $C_{T_{CONV}} = \dfrac{T}{p_o A_t} = \dfrac{1.23}{(10)(0.1)} = 1.23$

$(11.9) \Rightarrow c^* = 1540$ m/s, $(11.12) \Rightarrow C_{T_{CONV}} = 1.23$

Case (4)

$A_e = 4.06 \text{ m}^2$, $\dfrac{A_e}{A_t} = 40.6 \Rightarrow M_e = 4.25$

All exit conditions (M_e, u_e, p_e and $\dot{m}$) are the same as in case (2)

$p_e = 0.020476$ MPa

$c^* = 1540$ m/s

$(11.9) \Rightarrow c^* = 1540$ m/s

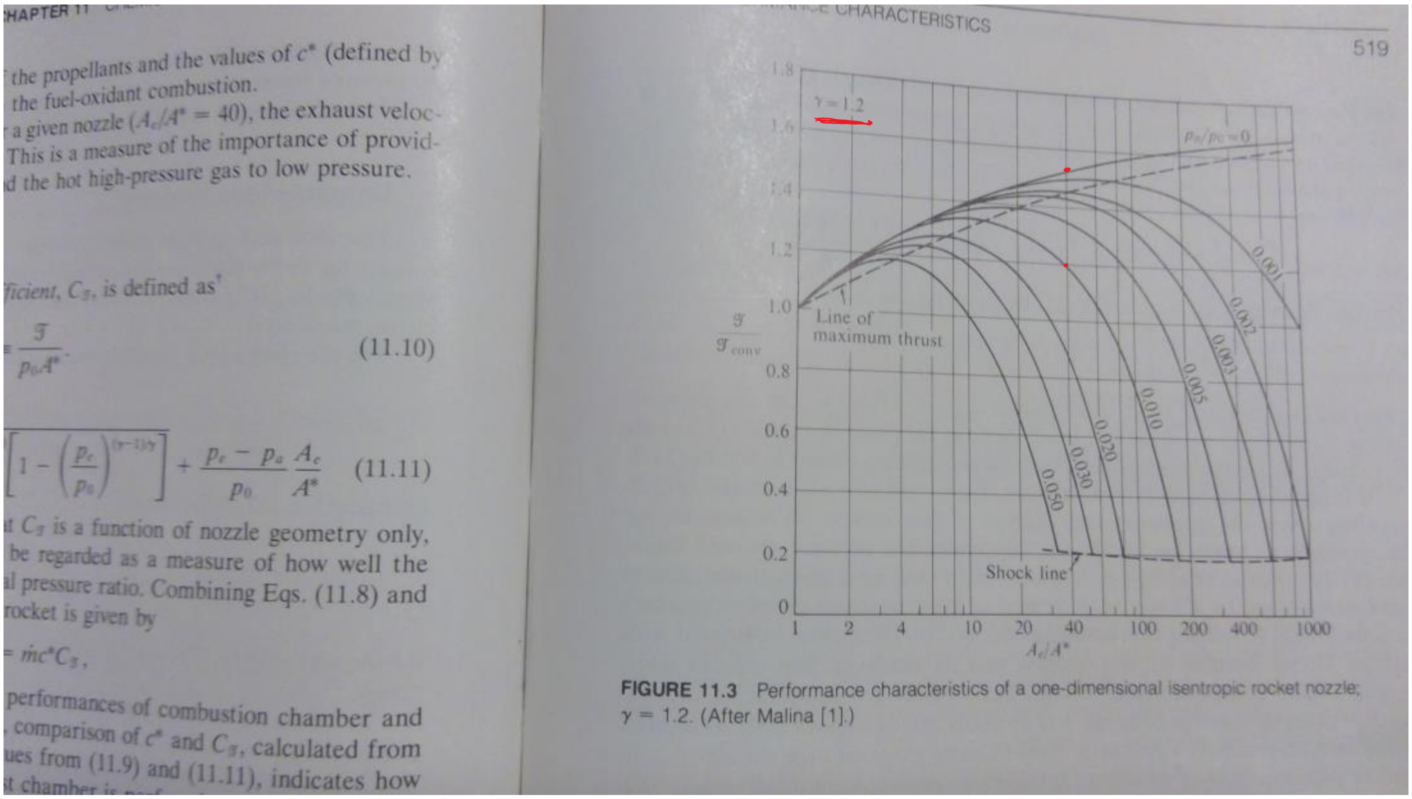
$$T = \dot{m} u_e + (p_e - p_a) A_e$$

$$= 1.802 - 0.323 = 1.479 \text{ MN}$$

$$C_T = \frac{T}{p_o A_t} = \frac{1.479}{(10)(0.1)} = 1.479$$

$(11.11) \Rightarrow C_T = 1.479$

$$\frac{C_T}{C_{T_{conv}}} = \frac{1.479}{1.23} = 1.20 \quad \left\{ \begin{array}{l} \text{increase in thrust} \\ \text{due to divergence} \\ (p_a/p_o) = (0.1/10) = 0.01 \end{array} \right.$$



CHAPTER 11

the propellants and the values of c^* (defined by the fuel-oxidant combustion. a given nozzle ($A_e/A^* = 40$), the exhaust veloc- This is a measure of the importance of provid- d the hot high-pressure gas to low pressure.

ficient, $C_{\mathcal{T}}$, is defined as[†]

$$= \frac{\mathcal{T}}{p_0 A^*}. \quad (11.10)$$

$$\left[1 - \left(\frac{p_e}{p_0}\right)^{(\gamma-1)/\gamma}\right] + \frac{p_e - p_a}{p_0}\frac{A_e}{A^*} \quad (11.11)$$

t $C_{\mathcal{T}}$ is a function of nozzle geometry only, be regarded as a measure of how well the l pressure ratio. Combining Eqs. (11.8) and rocket is given by

$$= \dot{m} c^* C_{\mathcal{T}},$$

performances of combustion chamber and comparison of c^* and $C_{\mathcal{T}}$, calculated from ues from (11.9) and (11.11), indicates how

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FIGURE 11.3 Performance characteristics of a one-dimensional isentropic rocket nozzle; $\gamma = 1.2$. (After Malina [1].)

Compare the results with those depicted in Figure 11.3, text.