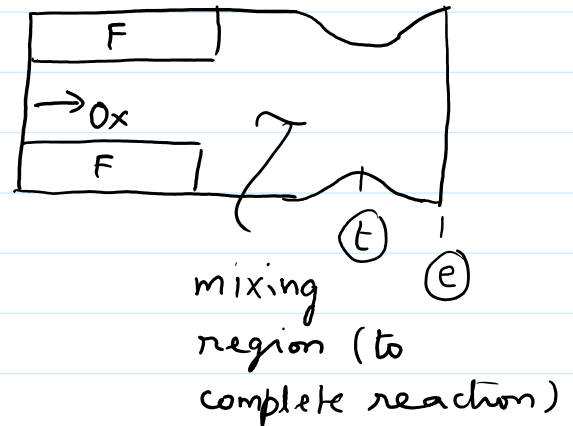


Hybrid Rocket Engines (HRE)

In the HRE, one propellant is liquid and the other propellant is solid. Typically, the oxidizer is the liquid and the fuel is the solid. HRE with liquid fuel and solid oxidizer are referred to as reverse hybrids.

Advantages:

- ① Safer handling
- ② Throttling capability (start, stop, restart)
- ③ Allows use of variety of fuels and oxidizers
- ④ Reduced temperature sensitivity
- ⑤ Higher I_{sp} than SRM, and higher bulk density than LPRE
- ⑥ While cracks and voids in the solid result in unpredictable operation, they are not catastrophic as in SRM.



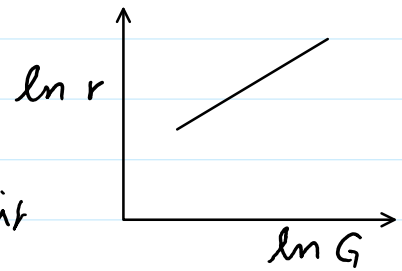
Disadvantages:

- ① Oxidizer-fuel ratio changes during burn
- ② Low regression rate (an order of magnitude smaller than the solid propellant). This results in complicated fuel geometry to generate significant thrust. This also offsets the bulk density advantage to some extent.
- ③ Lower combustion efficiency.

In the typical operating condition of the hybrid rocket engine, the regression rate (r) of the fuel is empirically found to be governed by the mass flux (G) $\left\{ G \stackrel{\text{def}}{=} \frac{\dot{m}}{A} \right\}$

$r = a' G^{n'}$,
where a' and n' are constants.

Since G includes both fuel and oxidizer mass fluxes $\{G = (G_f + G_{ox})\}$, it is more convenient to use



$r = a G_{ox}^n$, ①
with a and n empirically found.

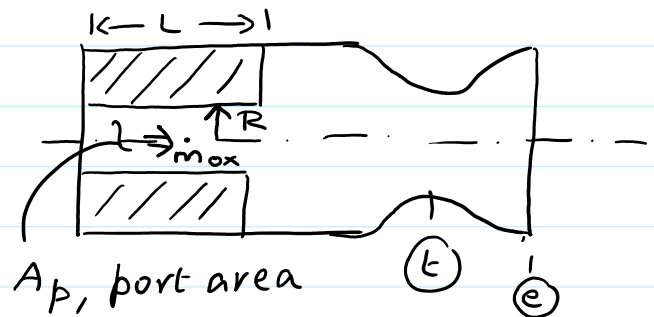
$$\dot{m} = \rho_f r A_b = \rho_f a G_{ox}^n A_b \quad \{A_b - \text{fuel burn area}\}$$

Consider the simple cylindrical arrangement.

$$A_p = \pi R^2, \quad A_b = 2\pi R L$$

$$r = a G_{ox}^n = a \left(\frac{\dot{m}_{ox}}{A_p} \right)^n$$

$$= a \left(\frac{\dot{m}_{ox}}{\pi R^2} \right)^n$$



$$\frac{dR}{dt} = r = a \left(\frac{\dot{m}_{ox}}{\pi R^2} \right)^n = a \frac{\dot{m}_{ox}^n}{\pi^n} R^{-2n}$$

$$R^{2n} dR = a \left(\frac{\dot{m}_{ox}}{\pi R^2} \right)^n dt$$

Integrate from $t = 0 \{R = R_i\}$ to $t = t \{R = R\}$:

$$\frac{R^{2n+1} - R_i^{2n+1}}{(2n+1)} = a \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t$$

$$\frac{1}{2n+1}$$

$$\Rightarrow R = \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1}{2n+1}} \quad (2)$$

$$\dot{m}_f = \rho_f A_b r = \rho_f (2\pi RL) a \left(\frac{\dot{m}_{ox}}{\pi R^2} \right)^n \quad (3)$$

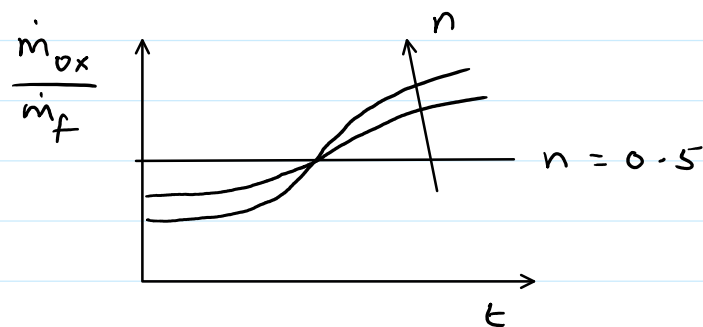
Substituting for R from (2) in (3),

$$\dot{m}_f = 2a\pi^{1-n} \rho_f L \dot{m}_{ox}^n \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1-2n}{2n+1}} \quad (4)$$

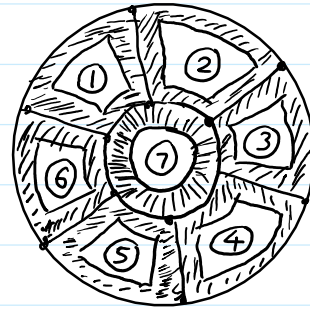
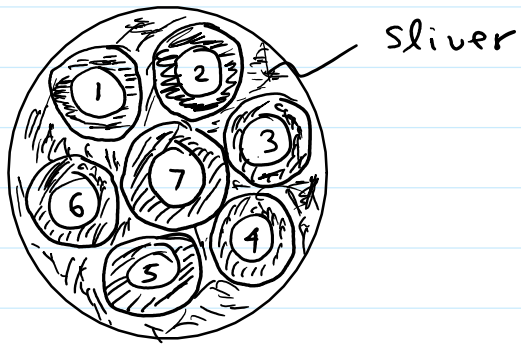
Observe that \dot{m}_f is constant if $n = 0.5$

$$\frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{1}{2\rho_f a L} \left(\frac{\dot{m}_{ox}}{\pi} \right)^{1-n} \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{2n-1}{2n+1}}$$

If $n > 0.5$, \dot{m}_f decreases with time. Assuming \dot{m}_{ox} is constant, $\left(\frac{\dot{m}_{ox}}{\dot{m}_f} \right)$ will increase with time.



To generate large thrust, multiple ports are required (recall the low regression rates of hybrid rocket propellants). An example, seven-port ($N=7$) configuration is shown below.



Typical fuels: HTPB, Paraffin, cellulose, carbon

Typical oxidizers: LOX, N_2O , N_2O_4 , RFNA