Problem 1

Linearize the nonlinear equation $z = x^2 + 7xy - 3y^2 - 3x$ in the vicinity of $x_e = 1$, $y_e = 2$.

Find z_e:

$$z_e = (1^2) + 7(1)(2) - 3(2^2) - 3(1)$$

$$z_e = 1 + 14 - 12 - 3, \quad z_e = 0$$

$$x = x_e + \delta x = 1 + \delta x$$

$$y = y_e + \delta y = 2 + \delta y$$

$$z = \cancel{f(x_e, y_e)}^{\,0} + \frac{\partial f(x_e, y_e)}{\partial x}\delta x + \frac{\partial f(x_e, y_e)}{\partial y}\delta y$$

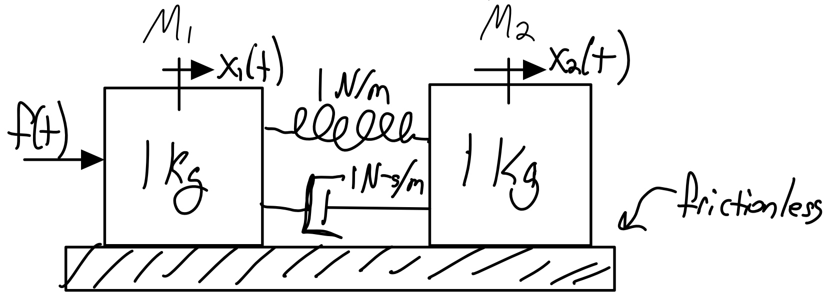
$$z = 0 + (2x_e + 7y_e - 3)\delta x + (7x - 6y)\delta y$$

$$z = (2(1) + 7(2) - 3)\delta x + (7(1) - 6(2))\delta y$$

$$\delta z = 13\delta x - 5\delta y$$

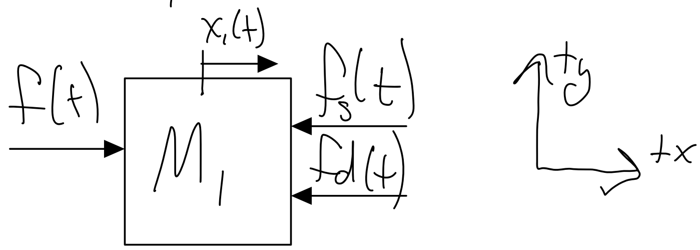
Problem 2

Find the transfer function $G(s)\ \frac{X_2(s)}{F(s)}$



1.) Draw FBD of each mass independently

For m_1:



where $f(t)$ is the input force, $f_s(t)$ is the spring force, & $fd(t)$ is the damper force.

$$f_s(t) = K\left(x_1(t) - x_2(t)\right)$$

$$fd(t) = b\left(\dot{x}_1(t) - \dot{x}_2(t)\right)$$

Write the E.O.M. for m_1

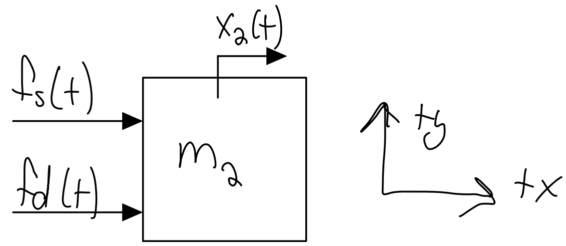
$$\vec{F} = m_1\vec{a} \Rightarrow \sum \vec{F} = m_1\ddot{x}(t)$$

$$f(t) - K\left(x_1(t) - x_2(t)\right) - b\left(\dot{x}_1(t) - \dot{x}_2(t)\right) = m_1\ddot{x}(t)$$

Solve for $\ddot{x}(t)$

$$\ddot{x}(t) = \frac{f(t)}{m_1} - \frac{k}{m_1}\left(x_1(t) - x_2(t)\right) - \frac{b}{m_1}\left(\dot{x}_1(t) - \dot{x}_2(t)\right)$$

For m_2:



where $f(t)$ is the input force, $f_s(t)$ is the spring force, & $f_d(t)$ is the damper force.

$$f_s(t) = K\left(x_1(t) - x_2(t)\right)$$
$$f_d(t) = b\left(\dot{x}_1(t) - \dot{x}_2(t)\right)$$

Write the E.O.M. for m_2

$$\sum \vec{F} = m_2 \vec{a} \Rightarrow \sum \vec{F} = m_2 \ddot{x}_2(t)$$

$$f_s(t) + f_d(t) = m_2 \ddot{x}_2(t)$$

$$K\left(x_1(t) - x_2(t)\right) + b\left(\dot{x}_1(t) - \dot{x}_2(t)\right) = m_2 \ddot{x}_2(t)$$

Solve for $\ddot{x}_2(t)$:

$$\ddot{x}_2(t) = \frac{K}{m_2}\left(x_1(t) - x_2(t)\right) + \frac{b}{m_2}\left(\dot{x}_1(t) - \dot{x}_2(t)\right)$$

Write the Laplace Transform for each case:

For m_1:

$$s^2 X_1(s) = \frac{F(s)}{m} - \frac{k}{m_1}\left(X_1(s) - X_2(s)\right) - \frac{b}{m_1}\left(sX_1(s) - sX_2(s)\right)$$

Simplify:

$$s^2 X_1(s) = \frac{F(s)}{m} - \frac{K}{m_1}\left(X_1(s) - X_2(s)\right) - \frac{b}{m_1} s\left(X_1(s) - X_2(s)\right)$$

Plug in the values for m_1, K, b

$$s^2 X_1(s) = \frac{F(s)}{1} - \frac{1}{1}\left(X_1(s) - X_2(s)\right) - \frac{1}{1} s\left(X_1(s) - X_2(s)\right)$$

$$= s^2 X_1(s) = F(s) - \left(X_1(s) - X_2(s)\right) - s\left(X_1(s) - X_2(s)\right)$$

Solve for $X_1(s)$:

$$X_1(s) = \frac{1}{s^2 + s + 1} F(s) + \frac{s+1}{s^2 + s + 1} X_2(s)$$

For m_2:

$$s^2 X_2(s) = \frac{k}{m_2}\left(X_1(s) - X_2(s)\right) + \frac{b}{m_2}\left(sX_1(s) - sX_2(s)\right)$$

simplify:

$$s^2 X_2(s) = \frac{K}{m_2}\left(X_1(s) - X_2(s)\right) + \frac{b}{m_2} s\left(X_1(s) - X_2(s)\right)$$

Plug in the values for m_1, K, b

$$s^2 X_2(s) = \cancel{\frac{1}{1}}\left(X_1(s) - X_2(s)\right) + \cancel{\frac{1}{1}} s\left(X_1(s) - X_2(s)\right)$$

$$= s^2 X_2(s) = \left(X_1(s) - X_2(s)\right) + s\left(X_1(s) - X_2(s)\right)$$

Solve for $X_2(s)$:

$$X_2(s) = \frac{s+1}{s^2+s+1} X_1(s)$$

Find $G(s) = \frac{X_2(s)}{F(s)}$

Solve $X_1(s)$ for $F(s)$ in terms of $X_2(s)$

substitute $X_1(s)$ into $X_2(s)$ & solve for $F(s)$:

$$F(s) = (s^2+s+1) - (s+1)(s-1)$$

$$G(s) = \frac{\frac{s+1}{s^2+s+1}}{(s^2+s+1)-(s+1)(s-1)} = \frac{s+1}{(s^2+s+1)^2-(s+1)(s-1)}$$

$$G(s) = \frac{s+1}{s^4+2s^3+2s^2}$$