In a chemical rocket, the propellant also serves as the energy source. Thus, these vehicles are energy-limited.

In a nuclear rocket, nuclear power is used to energize the propellant. The attendant temperature change is limited by material considerations, and these devices are also energy-limited.

Electrical rockets impart kinetic energy to the propellant without necessarily increasing its temperature, and thus enables attainment of high propellant energy levels and associated I_{sp}. The mass of the power plant, however, increases with the power generated, and, therefore, the performance of these devices is also limited.

Thruster efficiency, $\eta \overset{\text{def}}{=} \dfrac{\frac{\dot{m} u_e^2}{2}}{P}$ (10.40)

Thruster specific mass, $\alpha \overset{\text{def}}{=} \dfrac{M_P + M_S}{P}$ (10.39)

$\dot{m}$ - propellant mass flow rate

u_e - propellant exhaust velocity

$M_{\mathcal{P}}$ – mass of power plant

M_s – mass of the structure

$$M_0 = M_L + (M_{\mathcal{P}} + M_s) + M_p$$

$$M_b = M_L + (M_{\mathcal{P}} + M_s)$$

$$I_{sp} = \frac{u_e}{g_e}$$

$$\Delta u_b = u_e \ln \frac{M_0}{M_b}$$

$$\frac{M_b}{M_0} = e^{-\frac{\Delta u_b}{u_e}} \quad (10.41)$$

Relating $\frac{M_L}{M_0}$ to u_e (or I_{sp})

$$\frac{M_b}{M_0} = \frac{M_L}{M_0} + \frac{(M_{\mathcal{P}} + M_s)}{M_0} \overset{\text{(Eq 10.39)}}{=} \frac{M_L}{M_0} + \frac{\alpha \mathcal{P}}{M_0} \overset{\text{Eq (10.40)}}{=} \frac{M_L}{M_0} + \frac{\alpha}{2\eta} \frac{\dot{m} u_e^2}{M_0}$$

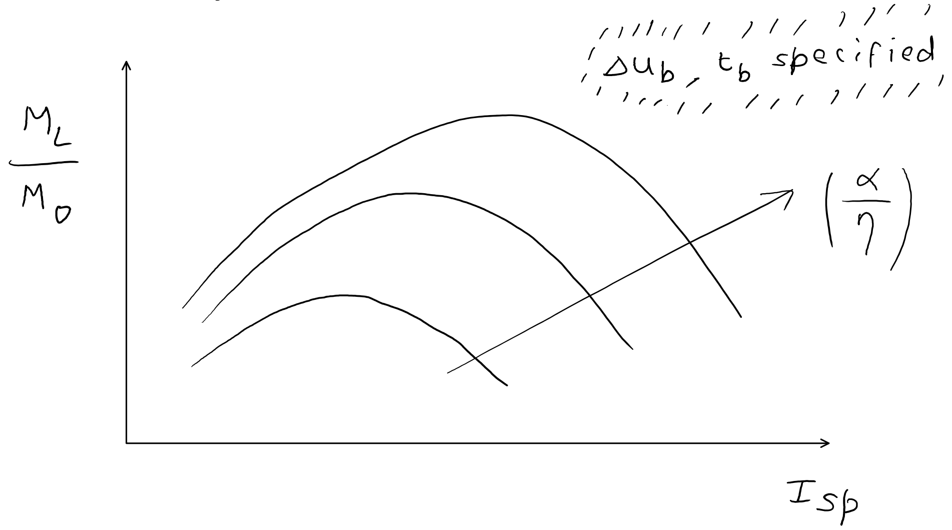
$$\text{(or)} \quad \frac{M_b}{M_0} = \frac{M_L}{M_0} + \frac{\alpha}{2\eta} \overbrace{\left(\frac{M_p}{t_b}\right)}^{\dot{m}} \frac{u_e^2}{M_0} = \frac{M_L}{M_0} + \frac{\alpha u_e^2}{2\eta t_b} \frac{M_p}{M_0}$$

$$= \frac{M_L}{M_0} + \frac{\alpha u_e^2}{2\eta t_b} \left(1 - \frac{M_b}{M_0}\right) \quad \{M_p = (M_0 - M_b)\}$$

$$\Rightarrow \left(\frac{M_b}{M_0}\right)\left(1 + \frac{\alpha u_e^2}{2\eta t_b}\right) = \frac{M_L}{M_0} + \frac{\alpha u_e^2}{2\eta t_b}$$

$$\text{(or)} \quad \underbrace{e^{-\frac{\Delta u_b}{u_e}}}_{\text{Eq (10.41)}} \left(1 + \frac{\alpha u_e^2}{2\eta t_b}\right) = \frac{M_L}{M_0} + \frac{\alpha u_e^2}{2\eta t_b}$$

$$\Rightarrow \frac{M_L}{M_0} = e^{-\frac{\Delta u_b}{u_e}}\left(1+\frac{\alpha u_e^2}{2\eta t_b}\right) - \frac{\alpha u_e^2}{2\eta t_b} \qquad (10.43)$$



* For a given mission, that is given Δu_b and t_b, there exists an optimum I_{sp} that maximizes the $\left(\frac{M_L}{M_0}\right)$ ratio. Unlike a chemical rocket, the optimum is not the highest I_{sp}.
* Due to long burns, the Δu_b that an electrical rocket vehicle has to generate for a mission will be much greater than the corresponding $\Delta u_{b,imp}$ (see Fig 10.22). The vehicle can be advantageously employed, despite this penalty, due to its low propellant consumption (low $\frac{M_p}{M_0}$).

Ex An electric thruster is used to raise a satellite orbit:

$I_{sp} = 2000\ s$, $T = 0.20\ N$, $t_b = 4$ weeks,

$M_L = 100\ kg$, $\eta = 0.5$, $\alpha = 10\ \frac{kg}{kW}$. Find

the mass of propellant required, the power required, the mass of power plant and structure, and the initial mass, the burnout mass and the $\left(\frac{M_L}{M_0}\right)$ of the vehicle. Can the vehicle raise the payload from LEO to GEO?

$$u_e = I_{sp}\, g_e = (2000)(9.81) = 19620\ m/s$$

$$\dot{m} = \frac{T}{u_e} = \frac{0.20}{19620} = 1.02\,(10^{-5})\ kg/s$$

$$t_b = 4 \text{ weeks} = (4)(7)(24)(3600) = 2.4192\,(10^{6})\ s$$

$$M_p = \dot{m}\, t_b = (1.02)(10^{-5})(2.4192)(10^{6}) = 24.68\ kg$$

$$\mathcal{P} = \frac{\frac{1}{2}\dot{m}\, u_e^2}{\eta} = \frac{\frac{1}{2}(1.02)(10^{-5})(19620)^2}{0.5} = 3926\ W$$

(or) $\mathcal{P} = 3.93\ kW$

$$\alpha = \frac{M_{\mathcal{P}} + M_s}{\mathcal{P}} = \overset{kg/kW}{10} \Rightarrow (M_{\mathcal{P}} + M_s) = \alpha \mathcal{P}$$

$$= (10)(3.93) = 39.3\ kg$$

$$M_0 = M_L + (M_{\mathcal{P}} + M_s) + M_p = (100 + 39.3 + 24.68) = 164\ kg$$

$$M_b = M_L + (M_p + M_s) = 139.3 \text{ kg}$$

$$\frac{M_L}{M_0} = 0.61$$

$$\Delta u_b = u_e \ln \frac{M_0}{M_b} = 19620 \ln \frac{164}{139.3} = 3203 \frac{m}{s} = 3.2 \frac{km}{s}$$

Table 10.8 $\Rightarrow (\Delta u_b)_{req} = 4.2$ km/s

No, the vehicle cannot raise the payload from LEO to GEO.