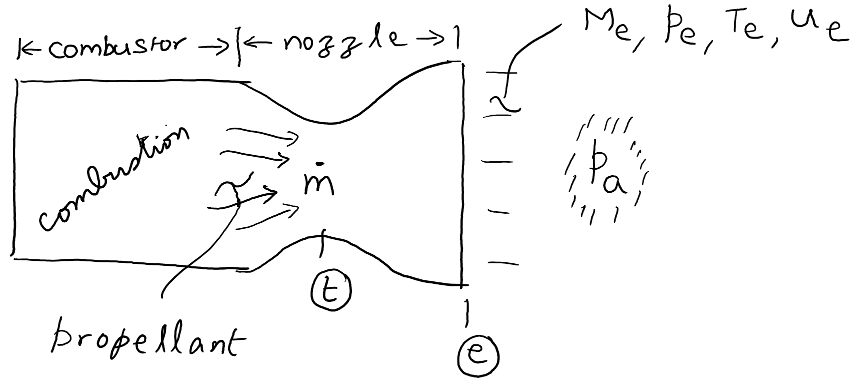
Chemical Rocket Thrust Chamber (Ch 11)

* Consists of the combustion chamber and nozzle.



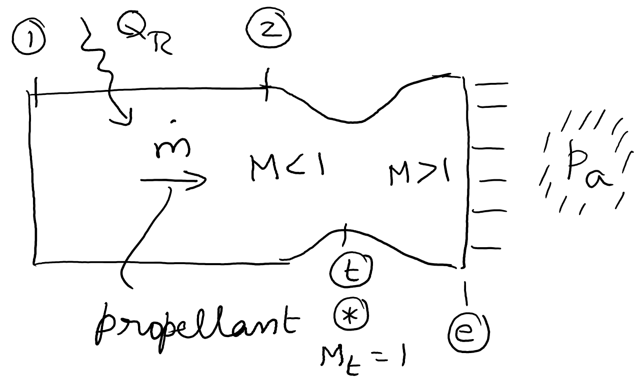
Characteristic velocity $c^* \stackrel{\text{def}}{=} \dfrac{p_0 A_t}{\dot{m}}$

Thrust coefficient $C_T \stackrel{\text{def}}{=} \dfrac{T}{p_0 A_t}$

$T = C_T \, p_0 A_t = \dot{m} c^* C_T$, $\boxed{T = \dot{m} c^* C_T}$

* Model (ideal rocket thrust chamber)

1. Working fluid (propellant) is a perfect gas with constant composition.
2. Combustion is equivalent to ideal, constant-pressure heat addition. ← p_0 constant ($M=0$) ← recall Rayleigh-line result (from AE 308)
3. Nozzle expansion is steady, one-dimensional and isentropic.



Q_R – heat added per unit propellant mass

I law across combustor:

$$\dot{m} c_p T_{01} + \dot{m} Q_R = \dot{m} c_p T_{02}$$

$$\Rightarrow \boxed{T_{02} = T_{01} + \frac{Q_R}{c_p}} \qquad (11.1)$$

$$\boxed{p_{02} = p_{01}}$$

T_{02} and p_{02} are referred to as chamber conditions, and are often represented as just T_0 and p_0. T_{02} is also known as the flame temperature.

The Mach number of the flow at the nozzle exit is found from <u>either</u> of the following stipulations:

① If nozzle area ratio is specified, this will determine M_e:

$$\frac{A_e}{A_t} = \frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$M_e = 1$

(2) If the jet pressure at nozzle exit is known, this will determine M_e.

$$\frac{p_{oe}}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

For isentropic nozzle (third assumption in the model),

$T_{oe} = T_{o2}$ and $p_{oe} = p_{o2}$.

I law across nozzle : $h_{oe} = h_{o2} + \cancel{q}^{\,0} - \cancel{w}^{\,0}$

(or) $h_e + \frac{u_e^2}{2} = h_{o2} \Rightarrow c_p T_e + \frac{u_e^2}{2} = c_p T_{o2}$

(or) $u_e = \sqrt{2 c_p (T_{o2} - T_e)}$ $\left\{ T_e = \frac{T_{o2}}{1 + \frac{\gamma-1}{2} M_e^2} \right\}$

<u>Comments</u>

(1) $c_p = \frac{\gamma R}{\gamma - 1} = \frac{\gamma}{\gamma-1} \frac{\bar{R}}{\bar{M}} \Rightarrow u_e = \sqrt{\frac{2\gamma}{\gamma-1} \frac{\bar{R}}{\bar{M}} (T_{o2} - T_e)}$

Low molar mass ($\bar{M}$) yields higher u_e.

(2) $T_{o2} = T_{o1} + \frac{Q_R}{c_p} = T_{o1} + \frac{1}{c_p} \frac{\bar{Q}_R}{\bar{M}}$

Higher $\left(\frac{\bar{Q}_R}{\bar{M}}\right)$ yields higher T_{o2}, and, hence, higher u_e.

Both $\bar{Q}_R$ and $\bar{M}$ depend on fuel-oxidizer ratio of the reactant mixture.

For the ideal rocket thrust chamber model,

$$c^* = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{\bar{R}\, T_{02}}{\bar{M}}} \qquad (11.9)$$

{recall T_{02} is the same as T_0}

c^* is primarily dependent on combustion chamber properties (temperature, molar mass), and, therefore, it is a performance measure of the combustor part of the thrust chamber.

For the ideal rocket thrust chamber model,

$$C_T = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \frac{p_e - p_0}{p_{02}}\,\frac{A_e}{A^*} \qquad (11.11)$$

{recall p_{02} is the same as p_0}

C_T is dependent on nozzle characteristics and so, it is a measure of the performance of the nozzle part of the thrust chamber.