CAMBERS FOR BELT PULLEYS.

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[*Selected for Publication, with written Discussion.*]

Introduction.—The practice of employing crowned pulleys is resorted to in order to cover certain imperfections in the geometrical arrangement of belt drives. These imperfections generally arise from one or other of four causes :—

(1) Pulley shafts out of parallel, but coplanar.
(2) Pulley shafts twisted out of the common plane.
(3) Pulleys staggered.
(4) Belts or joints not straight.

Apart from this necessary function the presence of a crown cannot be other than detrimental, both to the longevity of the belt and to the frictional qualities of the drive. For this reason it is clearly desirable to employ the smallest camber necessary to fulfil its purpose.

At the present time there is no generally accepted standard for pulley crown. Each belt manufacturer and power transmission engineer puts forward his own recommendations, and these vary not only in degree but also in type. Most are based on pulley widths, a few on pulley diameters, others are arbitrarily assigned irrespective of either. None appear to take account of the class of drive or type of belt, and so far as the Author is aware none is based on anything more definite than general experience. If, as must be presumed, it has been found that each of these various recommendations is adequate in practice, then it is open to argument that among these the smallest camber, being the least objectionable, is the most satisfactory.

But it is clearly desirable that the question of pulley camber should be placed upon some more rational basis. Particularly is this the case in view of the large and increasing use of drives over small pulleys, which are specially sensitive to the detrimental effect of excessive camber. It is therefore proposed to examine the principles which underlie the action of a crown under various kinds of misalinement and to develop a theory of pulley camber which can be checked by means of practical tests. In the light of such a theory modified by practical considerations, and of evidence regarding the degrees of imperfection to be anticipated in practice, it is hoped further to suggest a systematic basis of camber specification which shall be at once reasonable and consistent.

Belt Operation.—When a belt passes over a pulley under normal working conditions the whole of the arc of contact is not employed in the transmission of power. The effective angle of embrace is measured by β where $\frac{T_1}{T_2} = e^{\mu\beta}$, and this occupies the later portion of the whole arc of contact α. The remainder, covering the angle $\alpha - \beta$ from the point of entry, constitutes an " idle arc " over which there occurs no change in tension and no relative slipping between belt and pulley. This idle arc is nevertheless of considerable importance to the running of the belt for, since there is no slip over this entry arc, the direction in which the belt is led on at the point of entry determines the direction and rate of its travel, if any, across the face of the pulley. For conditions of steady running with no lateral travel it is necessary that the belt should be led on to the pulley in a line exactly perpendicular to its axis of rotation. If instead it is led on at some small angle ϕ from this line, then succeeding portions of the belt will enter at this same angle and it will form over the pulley a helical track, with a helical angle ϕ, and will travel across the face of the pulley with a velocity $V\phi$, where V is the belt speed. In short, the general condition that a belt shall run true on any pulley and under any conditions (apart from bodily slip) is that it shall be led on to the pulley in a line exactly perpendicular to its axis. This fact is of course well known; not so, apparently, the reasons underlying it.

The fact that there is no relative slipping between the belt and pulley surface over the idle arc also fixes the distribution of stress and the incidence of bending action across the belt. Since each longitudinal fibre is prevented from slipping during its passage over this idle arc it follows that the actual length of fibre led on during any period is directly proportional to the pulley diameter under the fibre. But for steady running the equivalent length of unstretched fibre led on during any period must be the same at every point across the width of the belt. Hence the distribution of longitudinal strain across the face of the belt is definitely fixed by the pulley profile, and if the belt is elastic the distribution of stress can also be determined.

In Fig. 1 suppose the pulley, diameter D, is "coned" with a semi-vertical angle $\frac{a}{2}$. Then, neglecting permanent set, if E is the effective value of Young's Modulus over the range of tensions in operation, the stress f in any fibre distant x from the centre-line is given by

$$\frac{f-f_0}{E}=\frac{xa}{D}$$

FIG. 1.

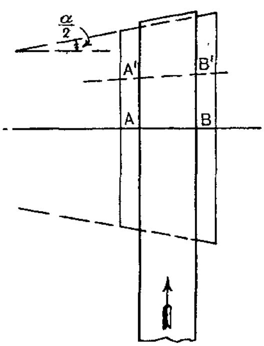


FIG. 2.

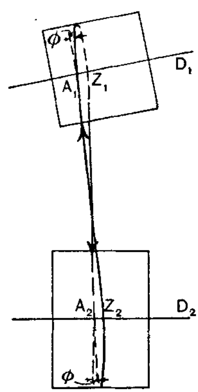
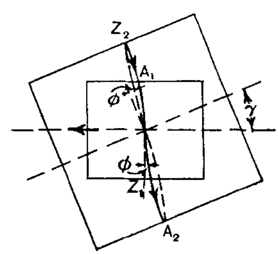


FIG. 3.



where f_0 is the stress at the mid fibre. Hence there will necessarily act at every cross-section such as A′B′ in the idle arc a bending couple

$$C=\int_{-\frac{b}{2}}^{+\frac{b}{2}} ftxdx=\frac{a}{D}\cdot\frac{Eb^3t}{12}=\frac{a}{D}\cdot EI \quad . \; . \; . \quad (1)$$

where t is the thickness and b the breadth of the belt. In particular this couple will act at the point of entry AB and will therefore influence the form and path of the free belt in its passage between the pulleys. In other words, there will act upon any belt as it enters a pulley a lateral bending couple whose magnitude is directly controlled by the profile of the pulley surface, whether it be tapered or crowned in the ordinary way. It is also worthy of note that the same proportional difference in diameter across the face of the belt will give rise to the same stress difference whatever the width of the belt. The same stress differences will therefore be obtained in general if the height of crown is made independent of the width of the belt and proportional to the diameter of the pulley.

Although the bending couple in the belt leading on to the pulley must, for steady running, correspond to the pulley camber this condition does not extend to the point where the belt leaves the pulley. If t_i is the tension in any fibre during its passage over the idle arc, and t_0 the tension in the same fibre when it leaves the pulley, then slip will occur between this fibre and the pulley surface over an arc β where $\frac{t_i}{t_0} = e^{\pm\mu\beta}$ according as the pulley is driver or follower. It is not necessary that either the ratio $\frac{t_i}{t_0}$ or the difference $t_i - t_0$ should be the same for all fibres. Differences in the ratio $\frac{t_i}{t_0}$ will merely result in a variation in the extent of the effective arc β at different points across the face of the belt. Thus, if the residual bending couple in the belt as it leaves the pulley is small, then on the driver the tighter fibres of the belt will commence to slip back before the slacker fibres, since in the ratio $\frac{t_i}{t_0}$, t_0 corresponds to the free tension in the slack belt and greater values of t_i demand greater effective arcs. On the other hand, in the case of the follower the slacker fibres will begin to slip forward first, since t_0 now corresponds to the free tension in the tight belt. The truth of these statements may be confirmed without difficulty if an extensible rubber belt be run slowly over coned pulleys and the relative motion between belt and pulleys be traced along both edges.

It follows from these remarks that although the pulley profile controls the bending couple in the oncoming belt it does not enable us to assign any particular value to the couple acting on the belt as it leaves the pulley. The conditions at this point have to be sought from other premises.

Since over the idle arc the belt has no motion relative to the pulley surface, the form of the belt at any instant over this arc is either a circle in the normal plane of the pulley or, if it has any lateral travel, a helix corresponding to the angle at which the belt is led on. Over the effective arc relative motion does exist between belt and pulley surface, and in accordance with the ordinary laws of friction this slip will take place in the line of the resultant applied force. Now the applied force consists of the tension in the belt, which varies along the belt and from fibre to fibre across its width, together with any lateral force which may be brought into play by the conditions at the far end of the free belt. Of these the longitudinal force always predominates, and except in the case of short twist or cross drives the lateral force is of a lower order. Hence for present purposes it is considered reasonable to neglect any deviation in the track of the belt over the pulley and to assume that it follows the helical path over the whole of the arc of contact. That this assumption is justified as a rule may be demonstrated as before by measurements or observations of an extensible rubber belt. In actual fact the accuracy of this assumption proves to be relatively unimportant with belt drives of ordinary proportions; it will be shown in the analytical work which follows that the exact angle at which the belt leaves the pulley does not normally affect the results to any considerable extent.

It is important at this stage to note also that the helix formed by the belt must have the same angle on each pulley, whatever their relative diameters. When running steadily, i.e. after any transient adjustment due to initial conditions, the belt if it has any lateral travel at all will travel at the same rate on each pulley, and if this rate is v the helical angle of travel will in each case be $\phi = \tan^{-1}\frac{v}{V}$, where V is the belt speed.

In the light of the discussion above it seems reasonable to postulate the following conditions at the end of either length of free belt. If the belt is travelling across the face of the pulleys at a speed $V\phi$ (strictly $V \tan \phi$) then it will take over each pulley the form of a helix of angle ϕ, and at each end the free belt will be displaced at this angle ϕ from the normal plane of the pulley. At that end of the free belt which passes on to a pulley there will be imposed on it a bending couple corresponding directly to the camber of the pulley, but no similar assumption can be made at the other end. These postulates are important, for they lie at the root of the theory of camber action.

The self-adjustment of a belt under normal running conditions depends on the automatic balancing of several factors: on the one hand the obliquity, twist, or misalinement of the drive; on the other the effect of pulley camber and the lateral stiffness of the belt. A clear conception of these various influences is probably best obtained by considering each separately before attacking the general problem. In the first place therefore it is proposed to examine the behaviour on a misalined drive of a belt devoid of any lateral stiffness, a condition nearly reproduced in a very narrow belt or cord.

Cord on Oblique Drive.—Consider a perfectly flexible belt (Fig. 2, page 629) running at velocity V over pulleys of diameter D_1, D_2, at a centre distance l, and having their shafts coplanar but set out of parallel at a small angle β. Assume that when the belt is in steady motion the common helical angle of its track on each pulley is ϕ, measured positive in the sense indicated in Fig. 2. Then the belt will be displaced at this angle ϕ to the normal plane at the point of entry (A_1 or A_2) to each pulley. Further, since the belt is assumed to have no lateral stiffness, its free path from Z_2 to A_1 will be a straight line continuous with the helical track on the pulley D_1, and similarly its path from Z_1 to A_2 will make the same angle ϕ with the normal plane of the pulley D_2. At the points Z_1 and Z_2 the belt will be bent.

Consider that length of belt (A_1Z_1) which is in contact with the pulley D_1 at any instant, together with that length of free belt (Z_2A_1) which is being led on to this pulley. At all points this free portion of the belt makes the same angle ϕ with the normal plane of the pulley D_1. Hence the displacement of the belt between Z_2 and Z_1, when projected along the axis of the pulley D_1, is $l_1\phi$ where l_1 is the length of the belt from Z_2 to Z_1. In the same way the displacement from Z_1 to Z_2 projected in the like direction along the axis of the pulley D_2 is $l_2\phi$, and therefore when projected along the axis of the pulley D_1 is $l_2\phi - l\beta$.

Hence the condition that the belt taken as a whole shall at any instant form a closed track is expressed in the displacement equation $l_1\phi + l_2\phi - l\beta = 0$. It follows that the belt will travel across the face of the pulleys " down " towards the point of convergence with a speed $V\phi = V\beta\frac{l}{L}$ where L is the total length of the belt and l the length of the drive.

Cord on Twisted Drive.—Adopting a similar notation, suppose now that the misalinement is due to a small twist γ of one of the pulley shafts out of the plane of the drive. Then as before, if ϕ is the helical angle of travel, the displacements of the two lengths of belt projected along the axes of the corresponding pulleys will be $l_1\phi$ and $l_2\phi$. Reference to the end view of the drive shown in Fig. 3 will show that the displacement of the length l_2 when projected along the axis of the other pulley D_1 is $l_2\phi - \frac{D_1 + D_2}{2}\gamma$. Hence the displacement equation now takes the form

$$l_1\phi + l_2\phi - \frac{D_1 + D_2}{2}\gamma = 0.$$

It follows that if the belt is running in the direction shown in the figure it will travel across the pulleys in the direction indicated by the arrow at a speed

$$V\phi = V\gamma \, . \, \frac{D_1 + D_2}{2L}.$$

Effect of Camber.—With a perfectly flexible belt the effect of pulley camber is negligible. It merely introduces a small component of frictional force across the face of the pulleys which may alter very slightly the extent of the effective arc, but has no effect on the geometry of the drive. Reference to the analysis above shows that this would be equally valid whether the pulleys were cambered or flat; the pulley profile has no effect on the behaviour of a cord on either an oblique or a twisted drive. In the same way it will be shown as a special case of the analysis which follows that a cord is indifferent to pulley camber when running true. In examining the effect of camber it is therefore necessary at the same time to introduce considerations of lateral stiffness in the belt.

In the analysis which follows immediately the pulleys are assumed to be tapered in one direction only, and to a constant angle, so that the pulley surface forms part of a right circular cone. It will transpire that results obtained with this simple form of profile can be adapted without difficulty to cover various curvilinear profiles, including the symmetrical profiles commonly employed in practice, which are in effect of course merely double-acting tapers.

Consider a drive over pulleys tapered to give semi-vertical angles $\frac{\alpha_1}{2}$ and $\frac{\alpha_2}{2}$ and mounted on truly parallel shafts. Fig. 4 shows the general geometry of the drive and Fig. 5 the conditions at the point of entry of one of the pulleys. In Fig. 5 T and P are the peripheral and axial components of the force acting on the belt as it passes on to the pulley at A, while C is the bending couple at the same point. If the free belt is regarded as a beam subject to these end conditions, its deflexion at various points, measured from the normal plane through A, will satisfy the elastic equation

$$EI\frac{d^2y}{dx^2} = Ty + Px - C.$$

The solution of this equation is

$$y = \frac{C}{T} - \frac{P}{T} x + A \sinh kx + B \cosh kx$$

where $$k^2 = \frac{T}{EI}.$$

At the point A the end conditions are

$$x = 0, \quad y = 0, \quad \frac{dy}{dx} = \phi$$

and at the remote end

$$x = l, \quad \frac{dy}{dx} = \phi.$$

FIG. 4.

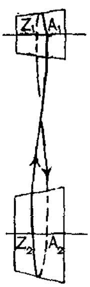


FIG. 5.

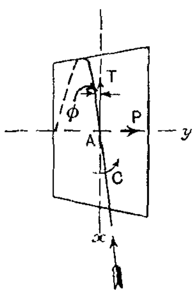
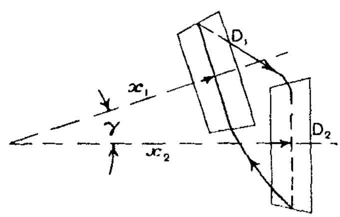


FIG. 6.



These equations enable A, B, P to be determined, and the total deflexion of this length of free belt from the normal plane through A is obtained by writing $x = l$ in the resulting equation, giving

$$\delta = l\phi - \frac{C}{T}\left(\frac{kl \sinh kl}{\cosh kl - 1} - 2\right) = l\phi - \frac{C}{T} klN, \text{ say.}$$

Referring now to Fig. 4, if we write δ_1 for the deflexion corresponding to one pulley D_1 and δ_2 to the other pulley D_2, of a drive, the displacement equation for the whole belt takes the form

$$\left(\delta_1 + \frac{\pi}{2} D_1 \phi\right) + \left(\delta_2 + \frac{\pi}{2} D_2 \phi\right) = 0.$$

This determines the helical angle of travel

$$\phi = \frac{l}{L}\left(\frac{C_1}{T_1} k_1 N_1 + \frac{C_2}{T_2} k_2 N_2\right)$$

where C_1 and C_2 are the bending couples corresponding to the cambers on the two pulleys. Giving C_1 and C_2 the values derived from equation (1) it will be found that

$$\phi = \phi_1 + \phi_2 \text{ where } \phi_1 = \frac{l}{L}\frac{a_1 N_1}{k_1 D_1}$$

and for values of kl common in belt drives

$$\phi = \frac{l}{L}\left\{\frac{a_1}{k_1 D_1}\left(1 - \frac{2}{k_1 l}\right) + \frac{a_2}{k_2 D_2}\left(1 - \frac{2}{k_2 l}\right)\right\}, \text{ nearly.}$$

Since in this result the value of ϕ is necessarily positive when measured in the same direction indicated in Fig. 5, it follows that a belt will always tend to climb towards the crown of the pulleys on a truly parallel drive, at a rate of travel depending principally upon the values of k, $\frac{a}{D}$, and $\frac{l}{L}$, the influences of which will be considered in detail later. When a belt without lateral resilience is employed the values of k become large and the climbing property vanishes.

Having considered separately the more important factors controlling the self-adjustment of belt drives, it remains to examine the more general and practical problem where the effects of camber and misalinement are both operative. In the following sections the belt is assumed to have some lateral resilience and to be running over tapered pulleys on a drive which is either oblique, twisted, or misalined.

Oblique Drive.—When the two shafts are coplanar, but inclined at a small angle β, the general method of the last section may be followed, but the condition at the remote end of the free belt now becomes: $x = l$, $\frac{dy}{dx} = \phi - \beta$, β being reckoned positive when the shafts converge in the same direction as the pulley surfaces. The total deflexion of the free belt is then found to be

$$\delta = l\phi - \frac{C}{T}klN - \frac{\beta}{k}\cdot\frac{\sinh kl - kl}{\cosh kl - 1}.$$

The displacement equation for the whole belt is also modified

$$\left(\delta_1 + \frac{\pi}{2}D_1\phi\right) + \left(\delta_2 + \frac{\pi}{2}D_2\phi\right) + l\beta = 0.$$

It will then be found by substitution that the helical angle of travel is

$$\phi = \frac{l}{L}\left(\frac{a_1}{k_1 D_1} N_1 + \frac{a_2}{k_2 D_2} N_2\right) - \frac{\beta}{2}\frac{l}{L}(M_1 + M_2)$$

where $M_1 = 1 - \frac{2}{k_1 l} \cdot \frac{\sinh k_1 l - k_1 l}{\cosh k_1 l - 1}$ and so for M_2.

For values of kl above about 4, a range which covers the majority of belt drives in practice, the expressions for M, N may be simplified with sufficient accuracy

$$M = N = 1 - \frac{2}{kl}.$$

With flat pulleys the helical angle becomes

$$\phi = \frac{\beta l}{L} \cdot \frac{M_1 + M_2}{2}.$$

When a belt is running without any lateral travel across the pulleys, the angle $\phi = 0$. Hence the neutral setting of the drive is given by

$$\beta = \frac{2}{M_1 + M_2}\left(\frac{a_1}{D_1}\frac{N_1}{k_1} + \frac{a_2}{D_2}\frac{N_2}{k_2}\right)$$

and the tapers necessary to correct for an angle of obliquity β must satisfy the equation

$$\frac{a_1}{D_1}\frac{N_1}{k_1} + \frac{a_2}{D_2}\frac{N_2}{k_2} = \frac{\beta}{2}(M_1 + M_2).$$

It is to be noted that provided the tapers on the two pulleys jointly satisfy this condition, it is theoretically immaterial what proportion of the necessary taper is provided by each pulley.

It will be remembered that the foregoing analysis is based on the assumption that the belt continues its helical track over the effective as well as the idle arc on each pulley. If this assumption is incorrect the orientation of the belt at the remote end will not be $\phi - \beta$ as assumed, but some other angle $\phi - \beta'$, say, where β' is less than β. This alteration affects the final result only in so far as the values of M_1 and M_2 are concerned. The criterion M now becomes

$$M = 1 - \frac{\beta'}{\beta} \cdot \frac{2}{kl}\frac{\sinh kl - kl}{\cosh kl - 1}$$

and its approximate value

$$M = 1 - \frac{\beta'}{\beta} \cdot \frac{2}{kl}.$$

Now in practice the value of kl seldom falls below 5 and normally lies between 10 and 30, so that the second term in this approximation is comparatively small, and since $\frac{\beta'}{\beta}$ will never exceed unity the value of M is not very sensitive to errors in the estimation of β', and any consequent error in the final result is likely to be smaller than discrepancies due to the various practical uncertainties, except perhaps in very short drives.

Twisted Drives.—Fig. 6 represents a twisted drive as viewed from one end in a direction parallel to the common line of the pulley planes. The shafts in this view are inclined at an angle γ and the belt is shown running with its centre-line on the respective pulleys at distances x_1, x_2 from the virtual axis of twisting.

Adopting the same notation as before for the deflexion of either free length of belt, and assuming that the belt is running in its neutral position, so that $\phi = 0$, it is found that

$$\delta_1 = -\frac{C_1}{T_1}\left(\frac{k_1 l \sinh k_2 l}{\cosh k_1 l - 1} - 2\right) = -l\frac{a_1}{D_1} \cdot \frac{N_1}{k_1}.$$

This result is only correct provided the twist of the free belt is insufficient to cause any appreciable change in its lateral stiffness. If the angle of twist is considerable this lateral stiffness will be reduced owing to the rotation of the axis of bending relative to the belt, and the effective value of k will be increased.

There are two displacement equations to be satisfied by the twisted drive

$$x_1 \cos\gamma = x_2 + \delta_2 + \frac{D_1}{2}\sin\gamma$$

$$x_2 \cos\gamma = x_1 + \delta_1 + \frac{D_2}{2}\sin\gamma.$$

If the pulleys are set in such a way that

$$-x_1 \sin\gamma = \frac{D_1}{2}\cos\gamma + D_2$$

$$-x_2 \sin\gamma = \frac{D_2}{2}\cos\gamma + D_1$$

then $\delta_1 = \delta_2 = 0$ identically and the conditions of neutral running are fulfilled without any pulley camber and whatever the lateral stiffness of the belt may be. These conditions correspond to those ordinarily laid down for the setting of twist drives, for Fig. 6 then takes the form shown in Fig. 7.

For steady running in such a way that $x_1 = x_2 = 0$ both pulleys must have specified tapers :

$$\frac{a_1 N_1}{k_1} = \frac{a_2 N_2}{k_2} = \gamma \frac{D_1 D_2}{2l} \text{ for small twists.}$$

If, as will more commonly be the case, it is sufficient that the mean displacement shall vanish ($x_1 + x_2 = 0$), then the tapers must jointly satisfy the condition

$$\frac{a_1}{D_1}\frac{N_1}{k_1} + \frac{a_2}{D_2}\frac{N_2}{k_2} = \frac{D_1 + D_2}{2l} \cdot \gamma.$$

FIG. 7.

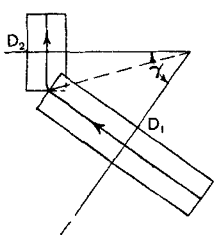


FIG. 8.

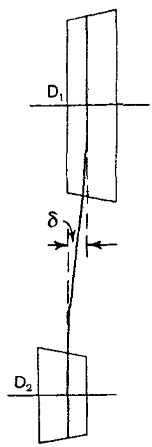
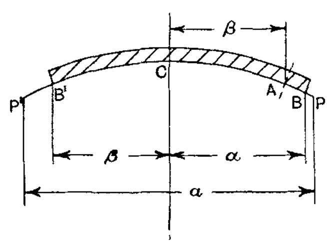


FIG. 9.



In this case the deviation of the belt from the common axis will be

$$x_1 = -x_2 = \frac{l}{2(D_1 + D_2)}\left(\frac{a_1 N_1}{k_1} - \frac{a_2 N_2}{k_2}\right)$$

on each pulley, which as a rule will be quite small in practice.

Staggered Pulleys.—If two pulleys are set out of line on parallel shafts, a belt running over them will tend to straighten its path unless and until opposing cambers on the two pulleys (*see* Fig. 8) provide a neutral position, at which the belt will become stabilized.

In this neutral position the deflexion of each side of the belt will be

$$\delta = l\frac{a_1}{D_1}\frac{N_1}{k_1} = l\frac{a_2}{D_2}\frac{N_2}{k_2}$$

and the necessary taper criteria

$$\frac{a_1}{D_1} = \frac{\delta}{l}\frac{k_1}{N_1} \qquad \frac{a_2}{D_2} = \frac{\delta}{l}\frac{k_2}{N_2}.$$

In these expressions the deflexion δ refers not to the fixed "stagger" of the central planes of the pulleys, but to the residual "stagger" of the belt in its stable position. With pulleys crowned in the ordinary way, as the belt adjusts itself laterally the effective tapers a_1 and a_2 alter as well as the deflexion δ.

The camber necessary to correct the ordinary degrees of misalinement will be found to be small. It is interesting to note, however, that if either of the pulleys is flat, the deflexion is necessarily zero, and it is therefore not possible to correct for misalinement of this kind so long as the shafts remain parallel. But if the shafts can be set out of parallel the necessary correction can be made. Suppose the taper $a_2 = 0$. Then if the drive runs neutral with an obliquity β, the deflexions of the two portions of the belt, in the notation already used, are

$$-\delta_1 = l\frac{a_1}{D_1}\frac{N_1}{k_1} + \beta\frac{l}{2}(1 - M_1), \qquad -\delta_2 = \beta\frac{l}{2}(1 - M_2)$$

and the displacement equation becomes

$$\delta_1 + \delta_2 + l\beta = 0.$$

Hence a misalinement δ may be corrected by setting obliquely either the tapered pulley or the flat pulley.

If the tapered pulley is adjusted

$$\beta = \frac{2\delta}{l(1 - M_2)} \rightarrow \delta k_2.$$

If the flat pulley is adjusted

$$\beta' = \frac{\delta}{l\left(1 - \frac{1 - M_2}{2}\right)} \rightarrow \frac{\delta}{l}.$$

Since β' is usually much smaller than β the adjustment is preferably made by means of the flat pulley.

In either case the necessary taper is fixed for a given residual misalinement δ

$$\frac{a_1}{D_1} = \frac{\delta k_1}{l N_1} \cdot \frac{M_1 + M_2}{1 - M_1} \rightarrow \frac{\delta T_1}{EI},$$

where T_1 is the tension in that side of the belt leading on to the tapered pulley.

Crooked Belts.—When a belt is run over flat pulleys any systematic crookedness or elastic asymmetry will cause unequal stress across the belt, which will induce a transverse bending couple of exactly the same nature as would be produced by a cambered pulley. Hence an opposing camber sufficient to produce a balancing couple will be required if the belt is to run true. Alternatively the effect of crookedness may be considered in terms of the obliquity of drive which would require the same correction.

This latter convention is the more convenient, for any want of truth can then be measured as the amount by which shafts carrying flat pulleys need to be set out of parallel in order that the belt may run true. This provides a simple method of testing the truth and comparing the crookedness of different belts, and is a quantity which can be applied when considering the necessary camber for pulleys. The method of test is as follows.

First the belt is run over flat pulleys at a specified mean tension and the shafts adjusted until the belt runs true. Then the belt is taken off, turned round edge for edge, replaced on the same pulleys and run again. The angular adjustment now required to make the belt run true is measured, and one-half of this will give the "equivalent obliquity" of the belt.

It is to be noted that any want of truth in joints or fasteners will be included in this, or indeed in any measurement of the crookedness of a belt, for it becomes an inherent part of this crookedness. It is also worthy of notice that although any want of lateral symmetry in a belt produces a stress variation such as might result from a cambered pulley, yet nothing inherent in the belt can possibly produce the effect of a pulley crown. The action of a crown is dependent entirely on the fact that a lateral shift of the belt on the pulley induces a *change* in the bending couple acting upon it, and where no change occurs there can be no additional restraint. For this reason a belt construction with a tight centre or tight edges, for example, can have no effect on the action of a crown.

The degree of crookedness to be expected in commercial belting is considered later in reference to pulley design.

Theory of Tapered Pulleys.—The tapers necessary to correct for the three chief forms of pulley misalinement are given by the equations

Obliquity: $$\frac{a_1}{D_1}\frac{N_1}{k_1}+\frac{a_2}{D_2}\frac{N_2}{k_2}=\beta\frac{M_1+M_2}{2}.$$

Twist: $$\frac{a_1}{D_1}\frac{N_1}{k_1}+\frac{a_2}{D_2}\frac{N_2}{k_2}=\gamma\frac{D_1+D_2}{2l}.$$

Stagger: $$\frac{a_1}{D_1}\frac{N_1}{k_1}+\frac{a_2}{D_2}\frac{N_2}{k_2}=\frac{\delta}{l}.2.$$

It will be noticed that the corrective effect of the drive is measured by the same expression for each form of misalinement, with a slight qualification in the case of staggered pulleys. Hence it is possible, and clearly convenient, to refer all forms of misalinement, for purposes of correction, to any one chosen form. Since obliquity of the drive, or belt crookedness, which is conveniently related to it, will usually be the predominant form of misalinement in its effect, it is convenient to choose this as the standard, and to regard any misalinement as an " equivalent obliquity."

The expression representing the corrective effect of a drive is

$$\frac{a_1}{D_1}\frac{N_1}{k_1}+\frac{a_2}{D_2}\frac{N_2}{k_2}.$$

In this the terms $\frac{a}{D}$ depend only on the pulleys, while the terms $\frac{N}{k}$ are concerned only with the belt itself and the length of the drive.

For given belts and pulleys the length of drive only affects the value of $N=1-\frac{2}{kl}$ in this expression. Hence the corrective effect will be greater with long drives, but only to a small extent under normal conditions.

With pulleys of fixed diameter and taper and a given length of drive the corrective effect depends upon the characteristics of the belt in so far as these affect the value of

$$\frac{N}{k}=b\,.\sqrt{\frac{E}{12f}}\left(1-\frac{2b}{l}\sqrt{\frac{E}{12f}}\right),\text{ nearly,}$$

where f is the mean fibre stress in the belt. In this expression the term in brackets usually differs but little from unity and for descriptive purposes may be neglected.

Provided the tension in the belt is proportional to its sectional area it will be seen that the effect of pulley taper is independent of the thickness of the belt, almost directly proportional to its width, and increasing with its elastic modulus. Other conditions being constant the effect will become smaller as the belt tension is increased.

With a given belt and length of drive the effectiveness of a tapered pulley depends directly on the value of $\frac{a}{D}$. Pulleys of different diameters will therefore be equally effective when their taper angles are made proportional to their diameters.

Now the product $\frac{a}{D}\frac{N}{k}$ may be written in the form

$$\frac{d}{D}\sqrt{\frac{E}{12f}}\left(1 - \frac{b}{l}\sqrt{\frac{E}{3f}}\right)$$

where d is the difference in diameter of the pulley measured across the face b of the belt. Neglecting the influence of the term in brackets it will be seen that with any one type of belting the corrective contribution of any pulley is measured by the ratio $\frac{d}{D}$. Hence the same proportional difference in diameter across the width of the belt will provide the same corrective effect whatever the width of the belt. It has also been shown to provide the same stress difference across the belt. From both standpoints therefore the ratio $\frac{d}{D}$ is an important criterion in tapered pulleys. In the absence of any accepted term it will here be referred to as the "ramp" of the pulley.

To obtain a given correction, an unduly small ramp on one pulley of a pair demands an excessive ramp on the other and consequently an excessive stress difference across the belt. Hence it seems clear that the best working conditions will be obtained if the two ramps are made equal

$$\frac{d_1}{D_1} = \frac{d_2}{D_2}.$$

If carried to its logical conclusion this suggestion has obvious practical advantages, for it should be possible to assess a reasonable limit to the probable errors of mounting belt drives and then to determine a ramp which, when referred to its equivalent camber, could be applied to pulleys of all diameters and thereby enable any pulley to be used with any other with confidence that the necessary camber would be provided in the drive without undue stress differences across the belt.

The practice of putting all the camber on to one pulley, and the smaller one in particular, is not only objectionable from the standpoint of stress distribution but it also disturbs the distribution of pressure on that pulley where slipping is most likely to occur, and it robs the drive of its power of self-correction in case the pulleys are staggered.

The recommendation of equal ramps is made at this stage because it leads to certain simplifications in the subsequent discussion. If it is not accepted the general results of this discussion are not rendered invalid, nor are any particular results seriously affected if the ramp is assigned its mean value

$$r = \frac{d}{\mathrm{D}} = \frac{1}{2}\left(\frac{d_1}{\mathrm{D}_1} + \frac{d_2}{\mathrm{D}_2}\right).$$

If the value of $\frac{a}{\mathrm{D}}\,\frac{\mathrm{N}}{k}$ is plotted against the fibre stress f it will be found that for all ordinary values of the quantities involved the curve

$$\mathrm{F}(f) = \frac{d}{\mathrm{D}}\sqrt{\frac{\mathrm{E}}{12f}}\left(1 - \frac{b}{l}\sqrt{\frac{\mathrm{E}}{3f}}\right)$$

falls away as the tension increases, but is concave upwards. In other words the corrective effect decreases as the tension increases, but to a diminishing extent, so that

$$\mathrm{F}(f_1) + \mathrm{F}(f_2) > 2\mathrm{F}\left(\frac{f_1 + f_2}{2}\right).$$

Hence it follows that with equal ramps the corrective effect of a drive transmitting power (when f_1 and f_2 are unequal) is greater than that of the same drive when running light under the same mean tension.

This result can be applied as it stands to twisted drives and staggered pulleys. In the case of an oblique drive the effect is further enhanced by the fact that the curve connecting M with f is convex upwards, so that a smaller correction is needed when power is being transmitted than when the drive is running light.

This result is a convenience from the practical standpoint, for it means that a drive which is lined up to run satisfactorily without load will continue to do so when load is applied at the same mean tension. It is also convenient from the standpoint of testing and design, for it implies that a camber specified on the simple basis of mean belt tension can be safely applied to the conditions of normal running under load.

If this further simplification be combined with that of equal ramps, the equation for oblique drives becomes

$$r = \frac{D}{d} = \beta\sqrt{\frac{3f}{E}}, \text{ nearly,}$$

where f refers to the mean tension in the belt.

Symmetrical Profile of Crown.—The principles of camber action are most conveniently investigated by considering pulleys with constant tapers, since this form of pulley gives a simple expression for the lateral bending moment imposed on the belt while it is passing over the idle arc. But plain conical tapers are of little practical importance for two reasons: first, because the property of self-adjustment is no more inherent in a tapered pulley than in a flat one, depending as it does on *changes* in restoring action as the belt travels across the pulley, and second, because as a rule the tendency of travel in a belt is equally possible in either direction, so that some symmetrical form of pulley profile is necessary.

When a belt rides over a pulley having a symmetrical profile of any ordinary form (*see* Fig. 9, page 638) the couple due to the camber depends upon the position of the belt on the pulley and increases as it moves further from the crown. Moreover, for any given position of the belt the only portion of the camber which is effective is that corresponding to the unbalanced portion of the belt, AB, any camber over the portion AB′ being balanced side for side and therefore inoperative. Once the form of profile is known it is a simple matter to correlate lateral shifts with their camber effect.

But the choice of a suitable form of profile introduces other considerations besides camber efficiency. It is necessary for example to consider convenience in pulley manufacture, the distribution of duty across the belt, and the elimination of unnecessary bending strains. A double cone with a central ridge would be simple to turn or to roll, but the sudden lateral bending over the ridge would tend to " break the back " of the belt and to reduce the effective width of its contact with the pulley. Further than this, the central region of the crown, where the stress is actually greatest in the running belt, takes no part in the cambering action. This last objection would be removed by a pulley with a flat central band and chamfered edges, but the concentration of pressure and acute bending action would continue to operate at the ridges left by the chamfers.

The two forms of profile which seem least objectionable from the general point of view are the circular arc and the flat central band with circular arcs at the edges. Neither of these forms should present any difficulty in manufacture, and neither has the objectionable features of a ridge. The single circular arc is at a disadvantage in that its central portion strains the belt without adding to the camber action, but this is balanced by the better distribution of pressure and the gentler bending action which it produces. The bending near the edges of the curved chamfer form would be more sudden and might be too sharp for a stiff or selvedged belt to follow. If the circular profile is adopted it is shown below that the corrective couple will be directly proportional to the displacement of the belt from its central position on the pulley. On the whole therefore the advantage seems to lie with the plain circular form of profile.

It is not difficult to determine the height of a crown of this form necessary to correct for the same misalinement as a known degree of taper. In Fig. 9 suppose a belt of width b is running over a pulley of width a with a circular (or parabolic) camber of height h_0.

If the stress in the fibre passing over the pulley crown is f_0, the stress in any fibre distant x from this will be

$$f = f_0 - \lambda x^2 \quad \text{where} \quad \lambda = 8\frac{h_0}{\mathrm{D}}\frac{\mathrm{E}}{a^2}.$$

If we denote by α, β the distances from the pulley crown to the two edges of the belt, the bending couple produced across the belt may be written

$$\mathrm{C}_0 = \int_{-\beta}^{\alpha} ft\left(x - \frac{\alpha - \beta}{2}\right)dx = -\frac{\lambda t}{12}(\alpha + \beta)^3(\alpha - \beta).$$

The couple is therefore directly proportional to the displacement ϵ of the belt from its central position

$$\mathrm{C}_0 = -\lambda\frac{b^3 t}{6} \cdot \epsilon.$$

When one edge of the belt is just running on the edge of the pulley the value of this couple becomes

$$\mathrm{C}_0 = -\frac{2}{3}\frac{\mathrm{E}b^3 t}{a\mathrm{D}} \cdot \frac{h_0(a - b)}{a}.$$

In the case of a pulley of straight profile, tapered to a height h across the face of the belt b, the corresponding bending couple would be

$$\mathrm{C} = -\frac{2}{3}\frac{\mathrm{E}b^3 t}{a\mathrm{D}} \cdot \frac{h}{4} \cdot \frac{a}{b}.$$

Hence in order to produce the same couple on a belt running in its limiting position the height of camber for a pulley of circular profile should be

$$h_0 = h \cdot \frac{a^2}{4b(a-b)}$$

and the " specific camber "

$$c = \frac{2h_0}{D} = r \cdot \frac{a^2}{4b(a-b)}.$$

The simplified expression for the mean ramp is

$$r = \beta\sqrt{\frac{3f}{E}}.$$

Hence, if we denote by x the ratio $\frac{a-b}{b}$ of the margin to the width of the belt, the camber equation becomes

$$\frac{\beta}{c} = \frac{4x}{(1+x)^2}\sqrt{\frac{E}{3f}}.$$

If the margin were chosen as one-eighth of the width of the pulley face, a value which appears reasonable, then the ramp equivalent to a specific camber c would be $r = \frac{7}{16}c$ and the camber equation would be

$$\frac{\beta}{c} = \frac{7}{16}\sqrt{\frac{E}{3f}}.$$

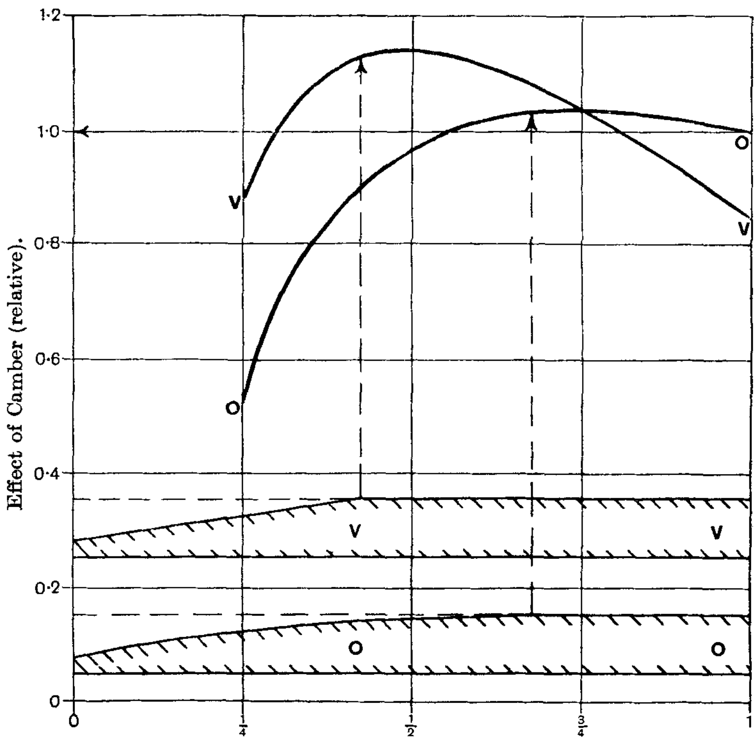
The relative efficiencies of different forms of cambered profile can now be estimated without difficulty. If we consider first pulleys in which the cambered portions take the form of circular arcs, the general case is a pulley of width a, constructed with circular cambers each of width $x \cdot \frac{a}{2}$ connected by a flat central band of width $a(1-x)$. This becomes a single circular camber when $x = 1$.

In the general case it can be shown that the couple produced by the camber on a belt of width $\frac{7}{8}a$ when running in its limiting position is

$$C = \frac{d}{D}Eta^2 \cdot \frac{1}{48x^2}\left(-x^3 + 3x^2 - \frac{23}{32}x + \frac{15}{256}\right).$$

For a given belt and length of drive, this couple is a direct comparative measure of the effectiveness of different proportions of camber. For equal values of the specific camber $\frac{d}{D}$, the effects of pulleys with different widths of flat band are shown by the relevant curve OO in Fig. 10, the effect of a single circular arc being taken as unity. The curve shows that a narrow flat band results in some slight improvement of the camber efficiency, but this never exceeds 4 per cent and disappears when the band extends over half the pulley face. With wider flat bands the camber effect falls away rapidly.

FIG. 10.



It would appear therefore that no appreciable operating advantage is to be gained by the use of flat bands in conjunction with circular cambers.

A similar examination of pulleys with straight tapered cambers connected by a flat central band shows that the corresponding maximum couple produced on a belt of width $\frac{7}{8}a$ is

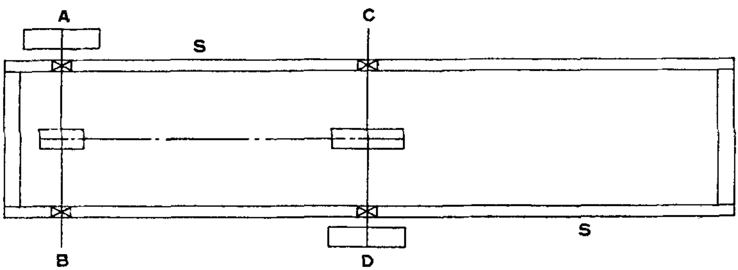
$$C = \frac{d}{D} Eta^2 \cdot \frac{1}{48x}\left(-\frac{3}{2}x^2 + 3x - \frac{23}{64}\right).$$

The relative efficiencies of cambers of this type are shown by the curve VV in Fig. 10, to the same scale as for circular cambers. The simple double taper is some 15 per cent less effective than the circular profile, but with a flat band of half its width it becomes about 14 per cent more effective than this standard.

If this examination is extended to cover other profiles it will be found that, provided the pulley is not made flat over more than half its width, the exact form of the camber, whether circular, tapered, or any intermediate smooth curve, is not a matter of serious moment for purposes of design, and the plain circular form may well be chosen as a representative standard.

Experimental Investigation.—The theory developed in the foregoing sections has been based on the assumption that the belt

FIG. 11.



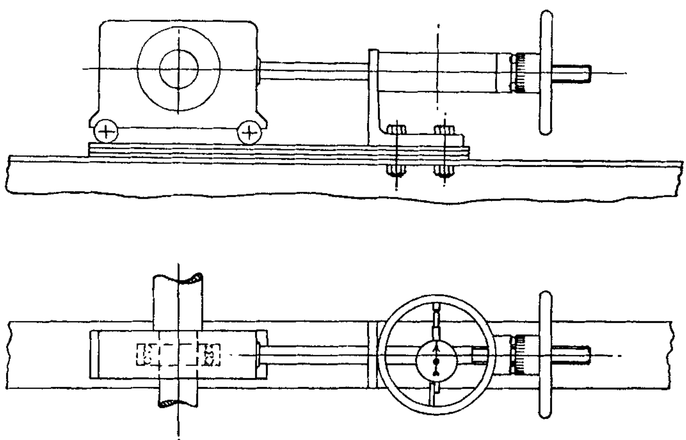
behaves as a homogeneous elastic body. When the theory is applied to the types of belting commonly used this assumption needs qualification.

In the first place belting does not obey Hooke's Law, nor is it easy to assign any effective value to the modulus of elasticity under running conditions. Further than this, belts do not behave elastically in operation. The primary effect of camber is to maintain a certain difference of *strain* across the face of the belt, and this may be taken up either elastically or plastically. It is possible to weave a textile belt so loosely that the whole of any ordinary camber strain appears as permanent set, so that no flexural stresses are induced and the belt entirely fails to respond to camber action. In ordinary belts the strain is partly of this type and partly elastic. Of the permanent set part appears as soon as the belt is strained and part develops over a period, at a rate which gradually diminishes and ultimately vanishes.

Consequently when a belt is run over cambered pulleys it will respond to part only of the actual camber; this response will be greatest at first and will gradually diminish until it reaches a steady residual value after an extended period of running. For this reason it would be futile to expect any close quantitative agreement between camber effects as revealed by tests and those deduced from static tests on the belt. On the other hand experiment should confirm the various general results of the theory and should furnish a few simple constants which would enable the theory to be applied with confidence to practical conditions.

The Experimental Plant.—The general arrangement of the experimental plant is shown in Fig. 11, which is largely self-

FIG. 12.



explanatory. AB is a fixed shaft to which power is transmitted through the overhanging pulley A. CD is an adjustable shaft having at each end a self-alining ball-bearing, housed as indicated in Fig. 12. Each housing is mounted on rollers and retained by means of a screwed tension rod and adjusting wheel, the thrust being conveyed from the wheel through a compression dynamometer to an angle bracket bolted rigidly to the side channel. The compression dynamometer consists of a steel ring with clearance holes for the tension rod and carrying a dial gauge on the transverse diameter to register elastic strains. In this way axial movement of the hand-wheel is reduced to such an extent that adjustments can be measured approximately by using this wheel as a micrometer, while belt tensions are indicated on the dial gauges.

It will be seen that with this plant it is a simple matter to set the drive with any desired amount of obliquity, twist, or stagger. When it is required to transmit power through the test drive a brake drum is mounted at D and vertical loads applied to the brake strap.

During the main systematic tests cast-iron pulleys were employed, two each of diameters 22, 14, 9, and 6 inches, some 8 inches, and others 6 inches wide. The rims were cast specially thick to allow for progressive tapers and cambers. Various types of belting were tested up to 6 inches in width. For the main systematic work nine belts were used: leather, woven cotton, and rubber friction belting of widths $1\frac{1}{4}$, 3, and 5 inches. Fasteners of the "Alligator" type were adopted for convenience of change over.

The systematic experimental work was divided into four sections:

(*a*) Oblique drives over flat pulleys.
(*b*) Twisted drives over flat pulleys.
(*c*) The effect of pulley taper.
(*d*) The effect of pulley camber.

(*a*) *Flat Pulleys: Oblique Drive.*—The helical angle of travel of a belt towards the point of convergence of two shafts set obliquely at a small angle β is theoretically *

$$\phi = \beta\frac{l}{\mathrm{L}} \cdot \frac{\mathrm{M}_1 + \mathrm{M}_2}{2}$$

$$= \beta\frac{l}{\mathrm{L}}\left(1 - \frac{2}{kl}\right) \text{ under light-running conditions.}$$

For a cord or narrow belt this becomes $\beta\frac{l}{\mathrm{L}}$ simply, and with wider belts, as k becomes less, the rate of travel should diminish. Theory gives no indication of any dead angle over which the belt is insensitive to adjustments or errors.

Experiment showed that in all cases belts are extremely sensitive to errors of obliquity. The smallest angular adjustment which it was practicable to make on the testing plant was 1 in 50,000, but this was quite sufficient in every case to cause the belt to travel across the pulleys in one direction or the other. This easily demonstrable fact suggests that the use of flat pulleys is a counsel of perfection quite incapable of realization in practice.

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* *See* page 636.

Experiments in which the rate of travel was timed across the pulleys showed that with cords the theoretical rate of travel was realized very closely. With belts of normal types the rate of travel was less, by amounts which varied with the conditions in the way expected from theory and corresponded to reasonable values of the quantity k. As regards the rate of travel on tapered pulleys, it will be recalled * that in theory although the tapers affect the neutral setting of a drive yet the rate of lateral travel for any angular displacement from this neutral position β_0 should be identical with that for the same displacement from the true position with flat pulleys. For the helical angle of travel may be written

$$\phi = (\beta - \beta_0)\frac{l}{\mathrm{L}}\left(1 - \frac{2}{kl}\right).$$

Ad hoc tests on tapered pulleys confirmed this result in all cases.

(*b*) *Flat Pulleys: Twisted Drive.*—According to theory † the helical angle of travel of a belt on a drive which is twisted through a small angle γ is $\phi = \gamma \,.\, \frac{\mathrm{D}_1 - \mathrm{D}_2}{2l}$ and is therefore independent of the stiffness or width of the belt.

Experimental measurements confirmed this result in all cases; with every drive and belt the measured rate of travel agreed with the formulæ above within the range of experimental error.

In order to ascertain whether the effects of obliquity and twist could be superposed without mutual interference a few representative drives were set in such a way that the measured rates of travel due to obliquity and twist were exactly equal and opposite. In each case it was found that the belt ran true under the balanced action of obliquity and twist. This means that errors in twist may be corrected by changes in obliquity, and vice versa, and it justifies the assumption that twist in a drive may be represented, for purposes of correction, by an "equivalent obliquity."

(*c*) *Effect of Tapered Pulleys.* (1) *Power Transmitted.*—In undertaking an experimental confirmation of the theory of tapered pulleys it is important at the outset to check the theoretical conclusion that the results of tests carried out under light-running conditions err on the safe side when applied to belts transmitting power at the same mean tension. For this purpose it is necessary

* *See* page 636. † *See* page 633.

to prove that the curve showing the obliquity β as a function of the tension T is concave upwards.

Representative tests with several belts at various tensions showed in each case that, as the tension was increased, the obliquity due to a given taper became smaller, and also the rate of diminution. This result was checked in the subsequent systematic tests, which showed in all cases that the mean of the obliquities for $\frac{T}{2}$ and $\frac{3T}{2}$ was greater than that for the mean tension T appropriate for the belt under test.

In the light of this result the scope of the subsequent investigation of pulley taper and camber was reduced to experimental work under light-running conditions, and so far as tapered pulleys are concerned, to an examination of the ramp equation

$$\frac{d_1}{D_1}+\frac{d_2}{D_2}=\beta\,.\,bk\frac{M}{N}.$$

(2) *Length of Drive.*—For the larger values of the product kl common in practice the ratio $\frac{M}{N}$ in this expression is nearly unity, but when kl is small this is not true. In the first place the value of the expression for $\frac{M}{N}$ increases appreciably for values of kl less than about 4. For a given stress in the belt it will be found that kl is proportional to $\frac{l}{b}$. Hence with wide belts on short drives the necessary ramp will be appreciably greater than that given by

$$\frac{d_1}{D_1}+\frac{d_2}{D_2}=\beta\,.\,bk.$$

Moreover under these conditions the effect of changes in the helical angle as the belt leaves the pulley becomes more important, since in

$$\frac{M}{N}=\frac{kl-2\frac{\beta'}{\beta}}{kl-2}$$

the value of the ratio $\frac{\beta'}{\beta}<1$ becomes more important at small values of kl. For both these reasons short drives employing wide belts are likely to require greater camber than longer ones with narrower belts. It is a matter of experiment to ascertain whether this tendency becomes important within the range of common practice.

Experiments in which the length of drive was varied from 4 to 15 feet, the pulley diameters from 6 to 20 inches, and belts from 1¼ to 5 inches wide, confirmed that the effect of a given ramp is least with short drives and wide belts, particularly when a large flat pulley is in use.

It is therefore clear that in order to obtain results which can be applied with safety in general practice it is desirable to carry out the experiments on short drives. Incidentally this is also convenient from the experimental standpoint for it leads to economy in belting material and makes for accuracy in the gauging of obliquities. The bulk of the systematic work was therefore carried out on drives having a centre distance of about 4 feet.

(3) *Permanent Set.*—Preliminary tests were also made to ascertain the effects of permanent set on camber efficiency. In each case it was found that the effect of a given ramp, as measured by the obliquity which it balanced, fell away from its initial value as the drive was left running. This diminution continued for an hour or two, depending on the type and age of the belt, after which a tolerably steady condition was attained. That this was due to the development of permanent set was shown by the curvilinear form of the belt when it was removed from the pulleys and laid out on the floor.

The influence of permanent set was particularly noticeable when the belt, after running for some time on tapered pulleys, was reversed edge for edge and run again. The initial stretch produced by the earlier run made the taper effect appear unduly high immediately after reversal, but in course of time it gradually fell away as before to a steady residual value. The amount of this slowly developing permanent set varied with the age and type of belt, and numerical values would have little general significance. In any event it is only the residual camber effect which is of practical importance.

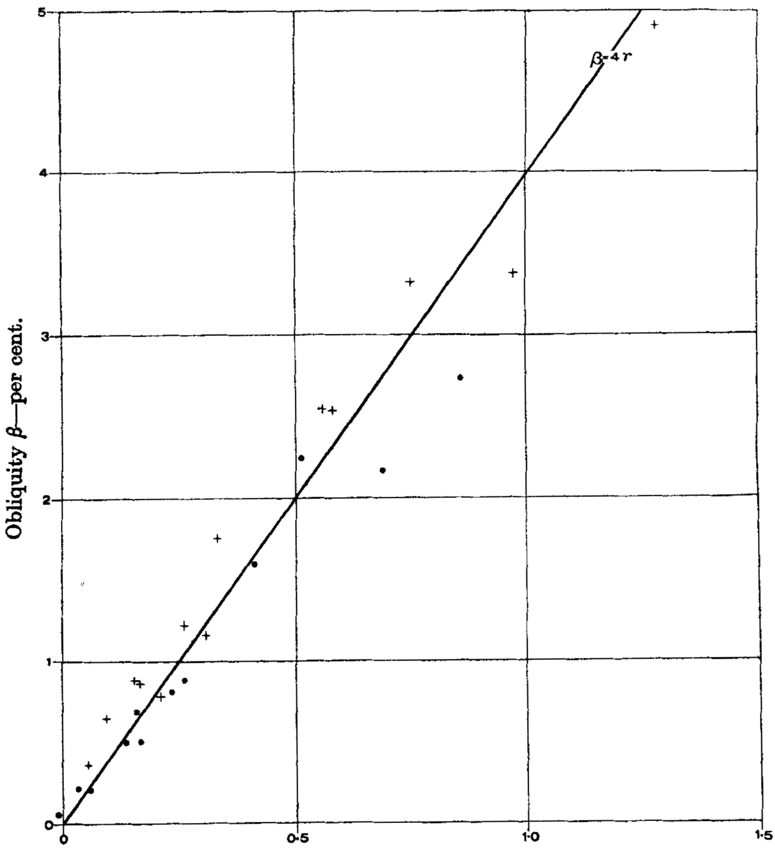
Since it was important that the systematic tests should give results on the safe side, it was decided in the light of these observations to run each belt for two hours at its appropriate mean tension before taking the readings on which the corrective effect was to be based. Moreover, no belt was used again until an interval of three days had elapsed, during which time it was hung up loosely to give full scope for natural recuperation. In order further to eliminate errors due to permanent set or intrinsic crookedness each test was repeated—after the interval of three days—with the belt reversed edge for edge. In this way it was possible to find not only the true effect of the pulley taper but also the intrinsic error of the belt itself.

(4) *Systematic Comparison.*—For these tests the nine belts of widths 1¼ inches, 3 inches, and 5 inches were run at speeds varying

from 1,000 to 2,000 ft. per min. over pulleys set at centre distances of about 4 feet. The pulleys were in some cases flat, in others tapered, to give various ramps, and in each case had a smooth

FIG. 13.—*Tapered Pulleys, Leather Belts.*

+ Both pulleys tapered.
• One pulley flat.



Mean Ramp $r = \frac{1}{2}\left(\frac{d_1}{D_1} + \frac{d_2}{D_2}\right)$ —per cent.

machined finish. The representative mean tensions chosen for the tests were : for the leather belts 40 lb. per inch of width, for the solid woven and rubber friction belts 50 lb. per inch of width.

In some of the tests both pulleys were tapered, in others one was tapered and the other flat. In each case the mean ramp

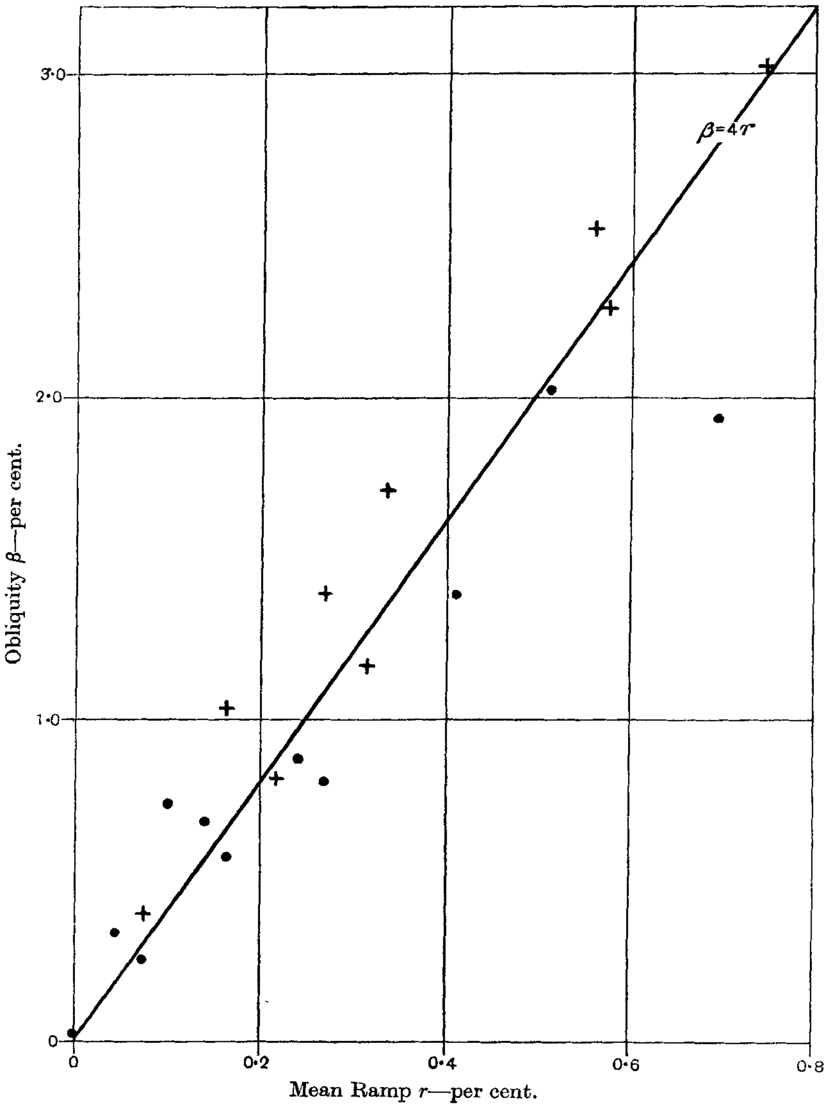
$$r = \frac{1}{2}\left(\frac{d_1}{D_1} + \frac{d_2}{D_2}\right)$$

was calculated and against this was plotted the obliquity β at which the drive ran correctly. Under the same conditions of drive all three types of belting gave very similar results; the rubber friction belts gave somewhat lower obliquities than the others, but the differences (not more than 20 per cent) were insufficient to necessitate the reproduction of separate diagrams. The average results which are plotted in Fig. 14 may therefore be regarded as representative.

It will be noticed in Figs. 13 and 14 that a distinction is made between those results obtained from drives in which both pulleys were tapered and those from drives in which one pulley was flat. It will be noticed moreover that in the latter case the equivalent obliquities are consistently lower than when both pulleys are tapered, particularly at higher obliquities. This fact, which has met with no recognition in the theoretical treatment, is easily accountable in terms of the permanent strain which develops during the period of running-in. Consider a drive between a flat pulley and a tapered pulley. The edge of the belt running nearest to the crown of the tapered pulley, being under the greater tension over this pulley, is permanently stretched more than the other edge. Consequently on the flat pulley this edge is subject to a lower tension than the other, and a reverse couple is induced as though the flat pulley were tapered in the opposite direction to the actual tapered pulley. Hence the flat pulley not only fails to contribute to the corrective action of the drive, but it also detracts from the effect of the ramp on the other pulley with which it is paired. This fact is of course further evidence in favour of the adoption of similar ramps on both pulleys. On the assumption that this principle is conceded, it is reasonable for our present purpose to consider primarily those drives in which both pulleys are tapered.

When due allowance is made for the varying conditions of test and for the several disturbing factors which have been foreshadowed, the results show that for each type of belting the equivalent obliquity β of the drive is approximately proportional to the mean ramp r of the pulleys. Moreover, at the appropriate mean tensions the results are so similar that a single value of the ratio $\frac{\beta}{r}$ may be employed

FIG. 14.—*Tapered Pulleys, Average Results.*



to cover all three types. If all the tests are taken into account the mean value of this ratio is 4·1, while for those tests in which both pulleys were tapered the mean value was 4·7.

On comparison with the theoretical ramp equation

$$\frac{\beta}{r}=\sqrt{\frac{E}{3f}}$$

it will be found that the mean strain in an elastic belt which would give $\frac{\beta}{r} = 4{\cdot}7$ is 1·5 per cent. This is, of course, considerably greater than the strain corresponding to the representative tensions in the belts, which is estimated at about 0·7 per cent on the average. The balance has to be explained by reference to the permanent strain.

If we denote by m the fraction of the actual strain difference across the belt which remains elastic after the period of running-in, then it is easily shown that the residual bending couple at the point of entry to a pulley is $C = \frac{a}{D}mEI$. The lateral resilience of the belt is not affected to any appreciable extent by this slowly developing strain. Hence the effect of the permanent stretch will be virtually to reduce the value of N in the theory to mN, so that the approximate ramp equation becomes

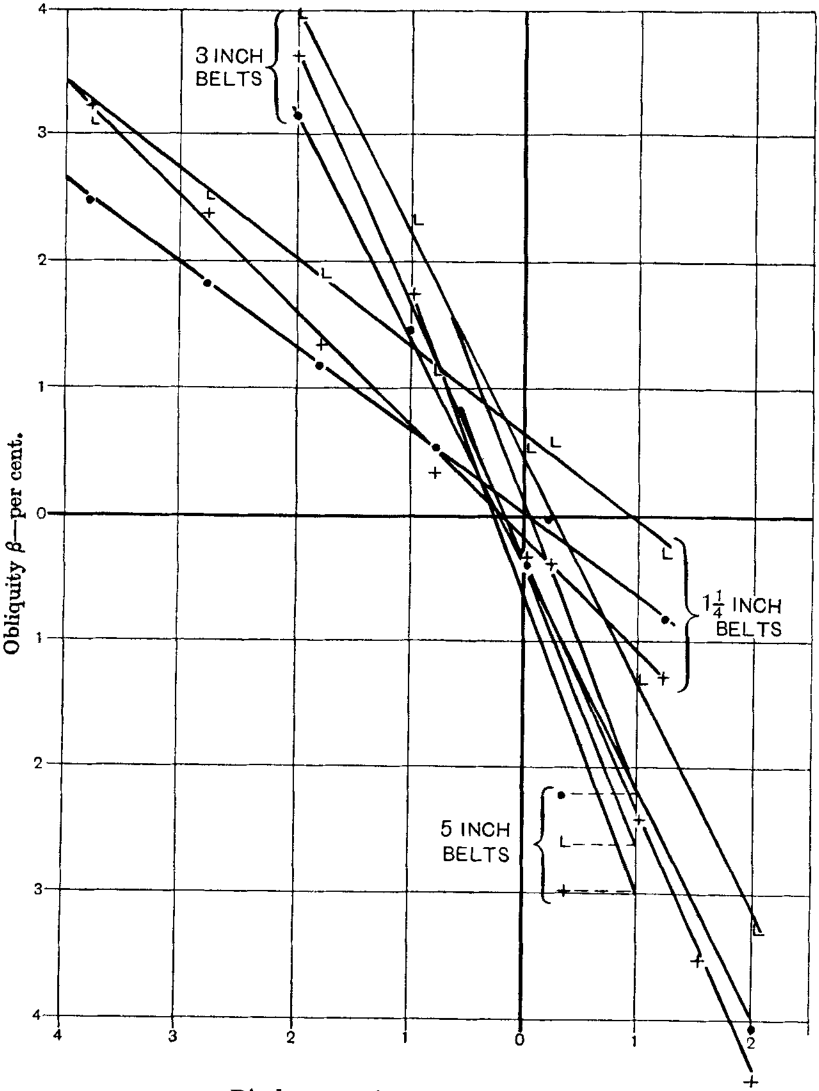
$$\frac{\beta}{r}=m\sqrt{\frac{E}{3f}}.$$

In order to agree with theory the experimental results would therefore require a value of $m = 0{\cdot}68$, which implies that about 32 per cent of the strain induced by the ramp of the pulleys becomes converted ultimately into permanent set. This figure does not appear unreasonable.

(*d*) *Effect of Pulley Camber.*—For the major portion of these tests the same length of drive (about 4 feet) and the same belts were employed as in the tapered pulley experiments. The same pulleys were also used, but were now turned to circular cambers of various required radii and finished in position on the testing plant itself by means of a special grinding attachment giving a geometrically correct profile. As before, the belts were run in at representative tensions before observations were made, and ample intervals were allowed for recuperation between tests. Since in this case the tests were primarily concerned with the *differences* in obliquity caused by changes in the position of the belt across the pulleys, it was not considered necessary to repeat each test with the belt reversed edge for edge.

Following the period of running-in the drive was adjusted until the belt ran steadily at a chosen position on the pulleys. Record

Fig. 15.



Displacement from crown—inches.

was made of the position of the belt and the corresponding obliquity of the drive. A series of readings of this kind was taken at different positions across the pulleys and from this a relation was obtained connecting the obliquity of the drive with the lateral displacement of the belt.

The theoretical treatment leads us to expect that the bending couples induced by the pulleys, and therefore the equivalent obliquity of the drive, will be directly proportional to the lateral displacement of the belt from its neutral position. The tests confirmed this linear relationship in all cases, as illustrated by the typical results plotted in Fig. 15. The slope of the line obtained in this way is a measure of the correcting power of the drive under test per inch of lateral displacement of the belt.

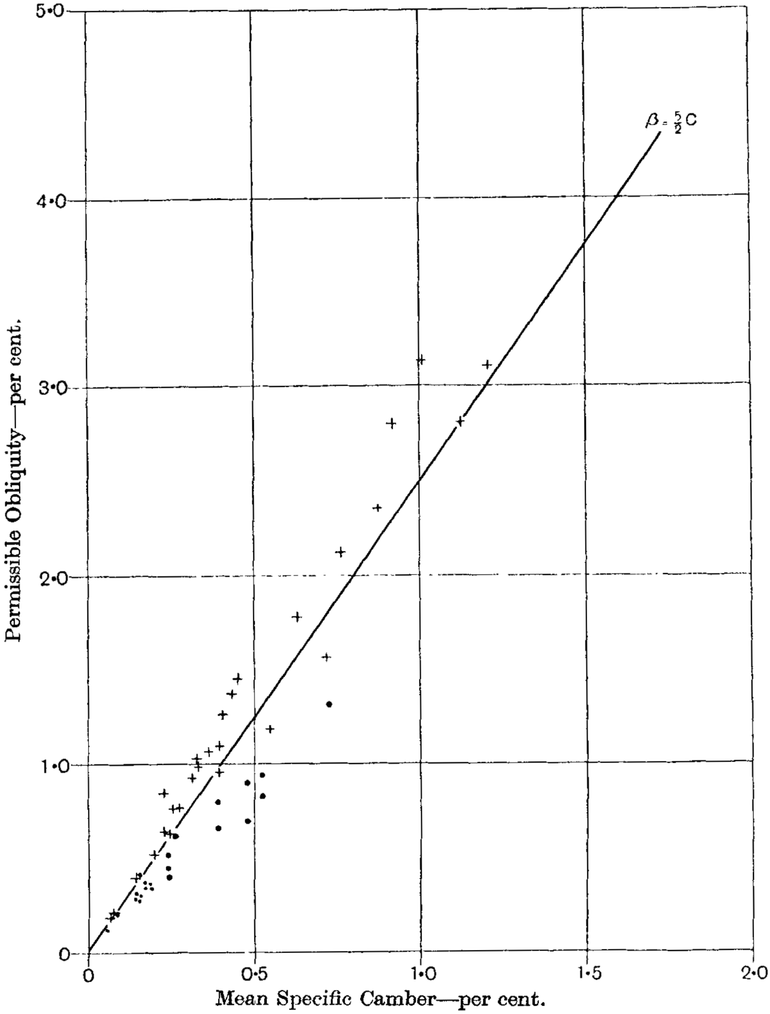
In order to deduce the maximum correcting power of a pulley constructed to carry any particular belt it is necessary to assume some relationship between the width of the belt b and the margin $(a - b)$ for its lateral travel on the pulley. As was foreshadowed above, it is assumed for the purposes of the present discussion that the belt occupies seven-eighths of the width of the pulley, so that the margin $a - b = \frac{1}{8}a$. If a different margin is preferred the method of treatment is still valid, but the correcting power will be modified in direct proportion to the margin adopted. In any case, assuming that the belt is not permitted to override the edges of the pulleys the greatest permissible obliquity is reached when the belt has travelled laterally a distance $\frac{a-b}{2}$ from its central position on the crown.

In the tests under discussion the permissible obliquity was computed for a lateral travel $\frac{b}{14}$ and plotted for various combinations of cambered and flat pulleys against the mean specific camber of the pulleys

$$c = \frac{1}{2}\left(\frac{d_1}{D_1} + \frac{d_2}{D_2}\right).$$

After the experience of the tapered pulley tests, it was anticipated that those drives in which the pulleys had similar specific cambers would give consistently higher values for the permissible obliquity than those in which one of the pulleys was flat. Reference to Figs. 16 and 17 shows that this expectation was fulfilled, there being again a marked distinction between the two classes of drive. In Fig. 16 the results are shown for leather belts covering the whole range of cambers employed. The figures obtained with solid woven and rubber friction belts did not differ from those sufficiently to justify separate diagrams; the rubber friction belts gave on the whole rather lower obliquities than the other two types. Average results for all three types are plotted in Fig. 17 to a larger scale and covering

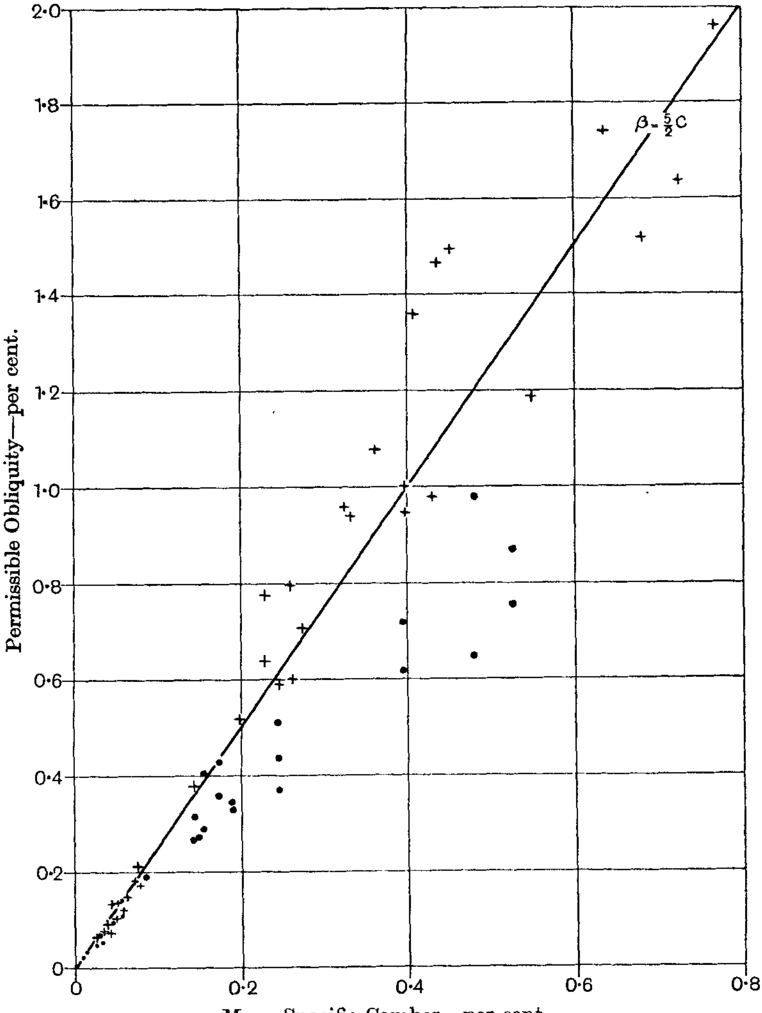
FIG. 16.—*Cambered Pulleys, Leather Belts.*



only the smaller range of specific cambers likely to be required in practice.

Setting aside again those drives in which one pulley is flat, the most satisfactory general relation between camber and obliquity is

FIG. 17.—*Cambered Pulleys, Average Results.*



approximately $\frac{\beta}{c} = \frac{5}{2}$. It will be recalled that the theoretical camber equation is

$$\frac{\beta}{c} = \frac{7}{16}\sqrt{\frac{E}{3f}},$$

so that the mean strain in an elastic belt giving these test results would be about 1 per cent. Making an estimate as before on the basis of permanent set, it is found necessary in this case to assume that about 16 per cent of the strain difference induced by the circular camber becomes lost as permanent stretch. With tapered pulleys the corresponding loss was found to be about 32 per cent.

A possible explanation of this difference may be found in the distribution of strain across the belt. When running over cambered pulleys the maximum difference of strain between fibres is greater than with tapered pulleys of equivalent obliquity, and this difference reaches its maximum across half the width of the belt, since the strain falls from the centre fibres towards both edges. The rapid change in strain between adjacent fibres and the more or less symmetrical distribution of strain produced by the pulleys across the belt probably assists the process of adjustment and equalization among the fibres, which takes place while the belt is passing between the pulleys. Whatever the true explanation, this observed difference will not be sufficient to affect matters of design, and the general agreement between the results appears sufficient to give confidence in the theory of camber as developed above from the theory of taper action.

Application to Pulley Design.—In order to provide a safe basis for design the systematic experimental work described above was carried out under conditions designedly unfavourable to camber action. The tests were made on drives as short in proportion to belt width as any employed in good practice, and it has been shown both in theory and by test that longer drives give greater effectiveness of camber. The tests were also made under light-running conditions, it having been shown that the effect of camber is improved by the transmission of power, provided bodily slip does not occur. Hence the conditions of test were definitely conservative.

Since it is desirable and convenient to adopt similar specific cambers for purposes of design, there seems no need to make allowance for the lower efficiency of drives in which one pulley is flat. But it is clearly necessary to take into account the more unfavourable results obtained with drives in which the pulleys have similar cambers. For this reason the representative relation $\frac{\beta}{c} = \frac{5}{2}$ is not put forward as a safe basis for design. The results of tests show a rather wide divergence from this mean relation, not owing to any intrinsic errors in the methods of testing but to variations in the proportions of drives and in running conditions, which are equally liable to operate in practice.

An examination of the results with pairs of cambered pulleys, amounting to well over a hundred in all, shows that in one test only did the value of the ratio $\frac{\beta}{c}$ fall below 2, and then only slightly and in the case of a very small camber where experimental errors might become relatively large. Hence the safe relation suggested by the tests is $\frac{\beta}{c} = 2$.

In order to test the general validity of this relation, a number of special confirmatory experiments were made with various proportions of drive and conditions of running to cover as wide a practical range as possible. The drives varied from $1\frac{1}{4}$-inch belts over 5-inch pulleys to 8-inch belts over 42-inch pulleys. Some two-dozen tests were made, and in no case did the value of the ratio, based as before on a margin of one-eighth of the pulley face, fall as low as 2. In most cases with longer drives it exceeded 3. In the light of these results it is considered that a safe and reliable basis for the design of pulley camber, with the margin suggested, is to be found in the relation

$$c = \frac{d}{\mathrm{D}} = \frac{1}{2}\beta,$$

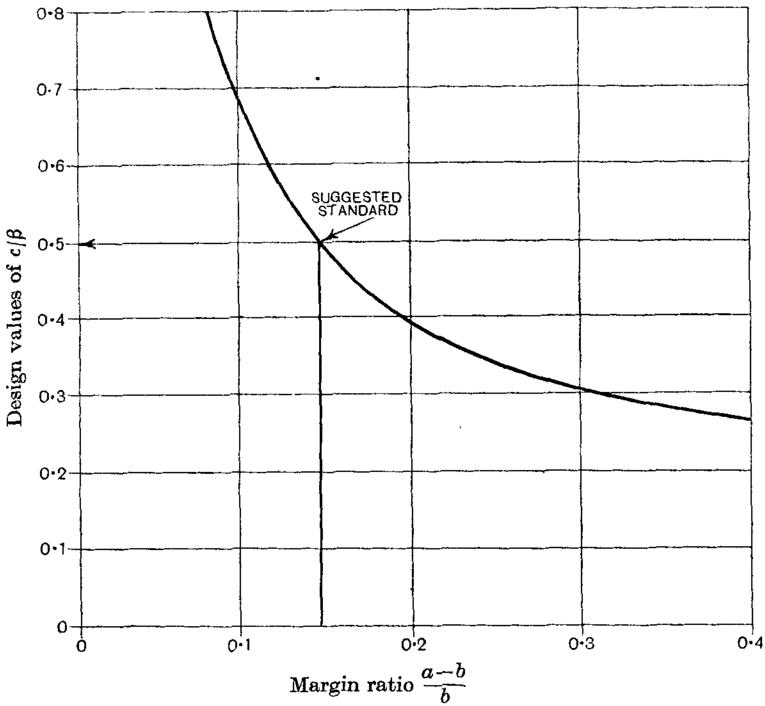
where β is the "equivalent obliquity" which it is reasonable to anticipate in a drive, and which remains to be chosen having regard to inaccuracies in the manufacture of belting and in the alinement of drives.

If some other margin for lateral adjustment is preferred to that adopted above, then the necessary camber is modified, but the corresponding value for the ratio $\frac{c}{\beta}$ is easily determined and may be taken from the curve in Fig. 18, page 664, for any selected value of the margin ratio $\frac{a-b}{b}$, which for convenience in practice is here referred to the width of the belt instead of the pulley. Any combination of margin ratio and specific camber taken from this curve will provide the same effect as the particular one employed in the present Paper.

Estimation of Tolerances.—In the choice of a reasonable value for the "equivalent obliquity" of a drive there are two independent factors to be considered: crookedness or "bias" in the belt and misalinement of the drive. In each case it is largely a matter of experiment to determine a reasonable tolerance.

(*a*) *Belts.*—The procedure adopted to find the equivalent obliquity of various belts followed the lines laid down on page 640. Measurements were taken on several occasions for each of the nine belts used in the systematic tests, and subsequently special tests of bias were made on a number of other belts of various widths from

FIG. 18.—*Cambers for Various Pulley Margins.*



2 inches to 8 inches. It was found in the course of the systematic tests that although variations occurred from time to time, owing no doubt to the recent history of the belt and to atmospheric conditions, there was no general tendency for the bias of the belts either to increase or diminish over a period of service. This observation was supported by the fact that in the later *ad hoc* tests no general difference was noted between old and new belts.

The bias revealed by these tests, covering over thirty belts in all, varied from nil to 0·65 per cent equivalent obliquity. The mean value for all the tests was 0·27 per cent, and there was no evidence that the width of belt or type of belt had any systematic effect on this mean. Actually the mean for the belts over 3 inches wide was somewhat greater than that for narrower belts, but this was due more to large errors in one or two belts than to any general tendency.

The fact that no important difference was recorded between belts of different types—leather, solid woven, and rubber friction—may call for some explanation. Leather belts are naturally less uniform in texture than fabric belts, and over any particular strip are no doubt subject to rather considerable error, but in a properly constructed leather belt this error tends to become balanced between one strip and the next, so that over the length of an ordinary drive it assumes the character of an accidental as opposed to a systematic error. On the other hand, in fabric belts any want of symmetry in the weaving or construction is likely to persist through the whole length of the belt, and consequently, although the belt may appear to the eye to run more truly, such error as is present, being systematic in character, produces a relatively greater resultant bias.

In the light of these tests it is suggested that a reasonable limit to the belt error for which pulley cambers should be expected to compensate is about 0·5 per cent equivalent obliquity. In fact, if the truth of belting is ever made subject to specification—and the test is remarkably simple—this limit would appear to provide a standard reasonable for the user and fair to the manufacturer.

(*b*) *Drives.*—Misalinement of the drive may arise from any of three causes : obliquity, twist, or pulley stagger. In order to form some reliable estimate of the obliquity to which drives are subject in ordinary practice, careful measurements were taken between the shafts of a number of different drives in engineering shops and textile mills. It was not possible to make accurate measurements in cases where one shaft was very short, as in motor or fan drives, but there seems no reason for treating these drives on any more generous basis than others, since some means of adjustment is almost invariably available to correct for misalinement during a trial run of the belt.

In the course of these experiments no marked difference was observed between the errors of vertical, oblique, and horizontal drives. It was found that the three-dozen drives tested, which were chosen as fairly representative in matters of age, length, and probable care in setting, gave a mean obliquity error of approximately 0·15 per cent, the worst being 0·67 per cent out of parallel. In the light of these measurements it is thought reasonable for purposes of camber specification to allow for an obliquity error of twice the observed mean, i.e. 0·3 per cent.

It is natural to expect that the twist error of a drive will be of the same order as its obliquity error; there is no more uncertainty in the method of setting and no more probability of subsequent displacement due to subsidence and other influences. A few check measurements confirmed this expectation. Now it has been shown that in belt drives of ordinary proportions the effect of twist is small compared with that of a similar degree of obliquity. Hence for camber purposes an allowance of 0·1 per cent on the equivalent obliquity should be ample to cover reasonable errors due to twist.

Pulley stagger is also relatively unimportant for purposes of camber design, but for a different reason; it is nearly always a simple matter to adjust one or other of the pulleys of a drive laterally along its shaft so that the belt runs in a similar position on both pulleys and pulley stagger is eliminated. Should there remain some small stagger δ_0 between the planes of the pulleys, it can be shown that the lateral displacement of the belt from the crown of each pulley will be approximately $\delta_0 \Big/ \left(2 + \frac{\beta l}{a - b}\right)$, where β is the total equivalent obliquity for which the cambers are designed. Hence the maximum stagger for which the drive is able to compensate is $a - b + \beta\frac{l}{2}$. This is, of course, always greater than the pulley margin, and since it increases conveniently with the length of the drive, does not introduce difficulties in long drives where alinement is less certain.

When prescribing a tolerance for the setting of a drive under ordinary working conditions all three sources of error have to be considered, but it is highly improbable that all these errors will conspire to give their maximum effect in one direction at the same time. Hence it would appear that a total equivalent obliquity of 0·5 per cent is a generous allowance for any drive whose erection and maintenance have been carried out with reasonable care. To attempt to cover more serious errors would probably be unwise. The increased camber necessary for this would only accommodate the occasional bad drive at the expense of a greater stress difference across all good drives. Moreover, a camber which will only compensate for reasonable errors is a useful means of detection for badly set drives and will therefore lead to their correction. It is clearly bad practice to cover by excessive pulley camber errors which might lead to trouble in other directions; maltreatment of bearings, undue power losses, and vibration.

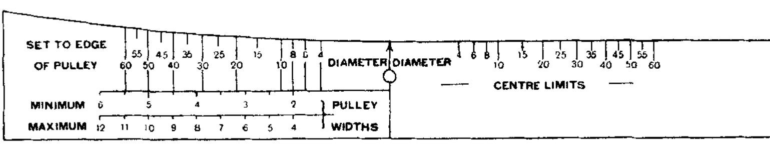
Proposed Specific Camber.—It has been shown that both in the setting of the drive and in the truth of the belt a tolerance of 0·5 per cent equivalent obliquity is a reasonable figure for purposes of design. This leads to a total tolerance of 1·0 per cent. Hence if the margin for lateral adjustment is taken as $\frac{a-b}{a}=\frac{1}{8}$ the relevant camber equation, $\frac{c}{\beta}=\frac{1}{2}$, shows that a satisfactory standard for the specific camber for general use with an open drive would be

$$c=\frac{d}{\mathrm{D}}=\frac{1}{200}.$$

The corresponding specific cambers for other margins may easily be obtained from Fig. 18.

This specific camber would impress on the belt during its passage over the pulleys a difference of strain of ½ per cent across its width.

FIG. 19.



A rough estimate based on average conditions shows that with this camber the stress difference across the belt as it passes on to the driver is about 40 per cent of the mean stress in the tight belt, so that the maximum stress in the belt is increased some 20 per cent. The increase due to other cambers can be roughly estimated in proportion. As compared with the current common practice, whereby cambers are based on belt widths, this suggested camber will be found considerably smaller in cases where the belt is wide in proportion to the pulley diameter, the very cases in which at the present time the injurious effects of excessive camber are most commonly experienced.

The adoption of a standard specific camber should present no greater practical difficulty than the present arbitrary methods. In the first place it is a very simple matter to calculate the necessary camber for any pulley. Further, it has been shown that the effectiveness of a given specific camber is practically the same whether the whole width is cambered to a single circular arc or whether a central band is left flat, provided this band does not exceed half the width of the pulley face. Moreover, it is not a serious matter if the profile curves depart from truly circular arcs provided they follow some smooth curve and do not leave a central band wider than half the pulley face. In cases where templates are employed in the turning of pulley cambers it would be possible to cover the whole of the normal range of pulley diameters and widths by means of a set of three templates of the type shown diagrammatically in Fig. 19, corresponding to camber radii of 6·25, 25, and 100 inches respectively. This set would cover the range from 4-inch pulleys with any face from 1 inch to 5 inches up to 64-inch pulleys with any face from 3 inches to 22 inches.

Crossed Drives.—In a crossed drive each length of the free belt forms half a turn of a helix. If the helices are right-handed the drive is termed a right-hand drive, and vice versa. When the taper theory is applied to a belt of this kind the bending couples acting at entry to the pulleys are unaffected, but owing to the twist in the free belt its effective lateral resilience is substantially reduced. It can easily be shown that the ramp equation in this case becomes

$$\frac{d}{\mathrm{D}} = \beta \frac{b}{2} \frac{k^2}{k_0},$$

where $k^2 = \frac{\mathrm{T}}{\mathrm{EI}}$ refers to the belt as it passes on to the pulley and is therefore the same as for an open drive, while k_0, having some mean value over the free length of twisted belt, is considerably greater than k. It follows that the efficacy of taper as a means of adjustment will be greater with a crossed drive than with a similar open drive, and pulley camber will therefore be more effective as a means of compensating for errors of alinement.

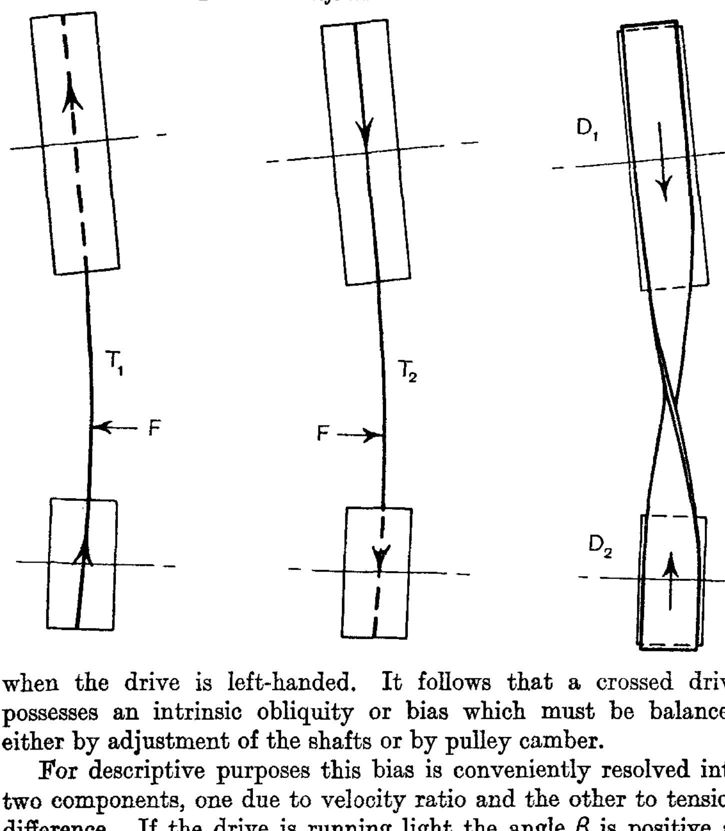
But the geometry of crossed drives introduces a special problem of its own which is quite distinct from misalinement in the ordinary sense. Owing to the fact that the two lengths of free belt cross one another at an angle there is necessarily some mutual interference near the point of crossing.

A geometrical investigation of this interference leads to troublesome equations with no simple solution, but it shows that each length of the belt will be displaced from its free path to an extent which is greatest with wide belts, short drives, and pulleys of very unequal diameter. This interference introduces mutually opposing forces tending to cause axial displacements of each length of the free belt. If these forces, F, are supposed to act between the belts at the point of crossing it can be shown that the drive will run correctly over flat pulleys only if these are set at an obliquity of approximately

$$\beta = \frac{F}{D_1 + D_2}\left(\frac{D_1}{T_2} - \frac{D_2}{T_1}\right)$$

where T_1 is the tension in the length of belt running on to the pulley D_1. The angle β is positive in the sense indicated in Fig. 20 when the force F acts on the belt T_1 in the direction shown, which occurs

FIG. 20.—*Left-hand Crossed Drive.*



when the drive is left-handed. It follows that a crossed drive possesses an intrinsic obliquity or bias which must be balanced either by adjustment of the shafts or by pulley camber.

For descriptive purposes this bias is conveniently resolved into two components, one due to velocity ratio and the other to tension difference. If the drive is running light the angle β is positive as shown if $D_1 > D_2$. Hence a crossed belt between unequal pulleys has an inherent tendency to travel towards the side occupied by the belt running on to the smaller pulley. If the pulleys are of equal diameter this tendency vanishes, but when the belt transmits power the angle β becomes positive as shown if T_1 is the tight side of the belt. Hence as the load becomes greater there is an increasing tendency for the belt to travel towards the side occupied by the slack belt at the point of crossing.

In the general case these two tendencies will be superposed. If the larger pulley is the driver they will act together; if the smaller pulley is the driver they will tend to neutralize one another, and if the tension ratio happens to be the same as the velocity ratio the drive will have no intrinsic bias.

From the standpoint of correction it will be seen that the two tendencies are of different type. If the drive were always working under the same load it would theoretically be possible to set it in such a way that no duty would be thrown on to the pulley cambers, though this would not be practicable in cases where the two shafts must be parallel, as for instance in the normal type of reversing drive with fast and loose pulleys. But in any event changes of load involve changes in the natural bias and can therefore only be compensated by pulley camber.

An adequate study of crossed drives would require a paper to itself. It will, however, be found as a rule that the intrinsic light-running obliquity of a crossed drive will not exceed 1 per cent if the value of the ratio $\frac{b\Delta}{l^2} < \frac{1}{2}$, where b is the width of the belt in inches, Δ the difference in pulley diameters in inches, and l the length of the drive in feet. In a similar way under normal conditions the change in obliquity from no load to full load will not exceed 1 per cent if the ratio $\frac{bD}{l^2} < \frac{1}{2}$, where D is the mean pulley diameter in inches. Taking into account the greater effect of camber on a crossed drive it may be taken as a rough guide that reducing drives in which both these ratios are less than $\frac{1}{2}$, and increasing drives in which they are both less than $\frac{1}{4}$ will fall within the range of the specific camber proposed above for open drives.

On the other hand, badly conditioned crossed drives, judged in terms of these ratios as criteria, will demand cambers considerably in excess of those contemplated. Any attempt to accommodate such drives in a general camber specification would penalize all other drives for the sake of a few which do not conform to good practice. It has therefore been thought best to base this specification on open drives and to point out the conditions under which crossed drives may be expected to come within its scope.

Acknowledgments.—The experimental work described in this Paper was carried out at the Technical College, Bradford. It was made possible by the generous support of Messrs. British Belting and Asbestos, Cleckheaton, and by the valuable co-operation of Mr. F. Sykes, A.M.I.Mech.E., of that company, particularly during the exploratory work. During the extensive systematic tests the Author was fortunate to have the help of Mr. W. B. Yeadon, G.I.Mech.E., Research Assistant at the College, whose zeal and care deserve very special acknowledgment.

Discussion.

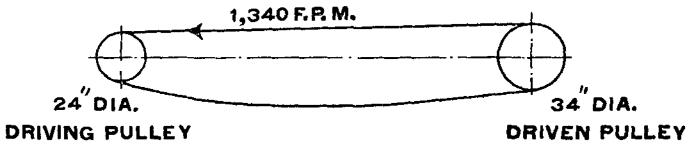
Mr. E. R. DOLBY wrote that, so far as he was aware, there had not hitherto been any generally accepted rule for camber of belt pulleys, though various empirical rules had been put forward by pulley makers. It was satisfactory to find that the theoretical treatment of the subject, so ably set forth by the Author, was well supported by the experimental results obtained. He was in full agreement with the Author that it was desirable to employ the smallest camber necessary to fulfil its purpose. Camber was essential on all pulleys, except on those upon which the belt had to be moved by a belt fork, otherwise there was a risk of the belt running over the edge of the pulley.

In reading the description of the experimental investigation, one must naturally be struck by the very small distance—4 feet—employed between the centres of the two shafts and, to reassure the reader, it needed the definite statement given by the Author that a greater distance between centres would not vitiate the results. The statement on page 647 that " a narrow flat band results in some slight improvement of the camber efficiency, . . . but this disappears when the band extends over half the pulley face," and that on page 648, " provided the pulley is not made flat over more than half its width, the exact form of the camber, whether circular, tapered, or any intermediate smooth curve, is not a matter of serious moment," deserved careful attention.

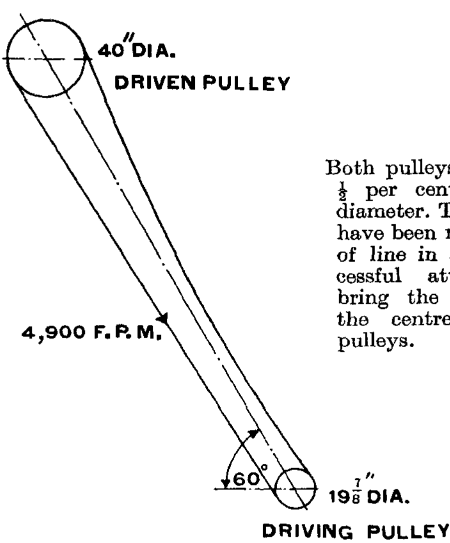
The proposed specific camber on page 667 really summarized the definite results obtained from the theoretical and experimental treatment of the problem, and Fig. 19 showed a template suggested as suitable for use in gauging the camber. He would be interested to learn whether such templates were accepted by pulley makers in general, and considered that it would facilitate the work of the consulting engineer if he could state that the camber of the pulleys required was to be in accordance with the "Swift system of templates," or by whatever name such templates might in future be designated.

FIG. 21.

Belt 9 inches × $\frac{3}{8}$-inch transmitting a maximum of 76 h.p.



Both pulleys crowned $\frac{1}{2}$ per cent on the diameter. The belt runs at least 1 inch out of centre.



12-inch × 10 mm. double leather belt, transmitting peak load of 135 h.p. and a normal load of 110 h.p.

Both pulleys crowned $\frac{1}{2}$ per cent on the diameter. The pulleys have been moved out of line in an unsuccessful attempt to bring the belt into the centre of the pulleys.

Mr. W. C. FENTON (Messrs. British Belting and Asbestos) wrote that for a number of years his firm had been impressed by the fact that belts used on drives where one pulley was small did not in many cases transmit the amount of power they should. They found that in most of these instances the small pulley had a great deal of crown. Invariably when this crown was turned off, the amount of power transmitted by the belt increased. As a result of a number of trials they arrived empirically at a formula in which the diameter of the centre of a pulley was given as 1 per cent greater than at the edges. They had applied belts to a large number of drives in which the large pulley had been crowned to this extent and the small pulley had been flat. They could say without hesitation that the results had been uniformly satisfactory.

Since Dr. Swift had carried out his investigation they had applied two belts under conditions shown in Fig. 21 and regretted to say that though these belts were in a textile mill where a relatively high standard of millwrighting work prevailed, and though the belts had normal characteristics, they would not remain in the centre of the pulleys. They felt, therefore, that further investigation was required, possibly at higher speeds and at longer centres than had been used in Dr. Swift's experiments. In another case where the crown amounted to ½ per cent only on one pulley, continual trouble occurred though the belt was not overloaded.

Mr. J. R. HOPPER referred to the proposal on page 667 for the adoption of a standard specific camber for belt pulley rims. He had noticed a tendency among Continental engineers of recent years to reduce the height of the camber in designing belt pulley rims. This might be due, on the one hand, to the desire to diminish the cross strain caused by the camber in a fast-running belt; on the other hand, more care was now exercised in the manufacture and jointing of belts, which might warrant a reduction in height of camber. The Germans had for many years paid great attention to improvements in belt driving, and he had found in their most excellent engineering handbook, "Hutte," confirmation of what he had noticed in practice. He possessed the 23rd edition of 1920, which on page 803 gave a formula for the height of camber of pulley rims. Taking C for the height of camber and W for width of pulley rim, both in millimetres, the camber recommended was from $C = \frac{\sqrt{W}}{4}$ to $C = \frac{\sqrt{W}}{3}$. The first of these gave the result:—

W, mm. .	100	200	300	400
C, mm. .	2·5	3·5	4·33	5·0

He had compared this with the most recent edition in the library of the Institution, that of 1931. This did not give a formula, but gave the heights of camber in tabular form, as follows:—

W, mm. up to	100	120–170	200–230	260–300	350	400	450–600
C, mm.	1·0	1·5	2·0	2·5	3·0	3·5	4·0

This, curiously enough, agreed almost exactly with the formula mentioned above, $C = \frac{\sqrt{W}}{4}$, but with a deduction of 1·5 mm. from each result; thus the formula would be $C = \frac{\sqrt{W}}{4} - 1\cdot5$. To adapt this tabular camber to English measures, with W and C in inches, it should read $C = \frac{\sqrt{W}}{20} - \frac{1}{16}$-inch, and the following rough tabulation gave the equivalent results :—

W, inches	4	6	8	12	16	20	24	32	40
C, inches	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{5}{64}$	$\frac{7}{64}$	$\frac{9}{64}$	$\frac{5}{32}$	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$

He did not agree with the proposal to turn the middle of the pulley face, not exceeding one-half of the width, flat and the remaining quarter of the width on each side to a radius. This would require more care and time than for turning the whole surface to one radius, and would probably necessitate filing, while in the lathe, the junction lines where the straight portion met the radial slopes, making an unsightly finish. Besides, not every purchaser might be willing to accept the new profile, unless consent had been obtained beforehand.

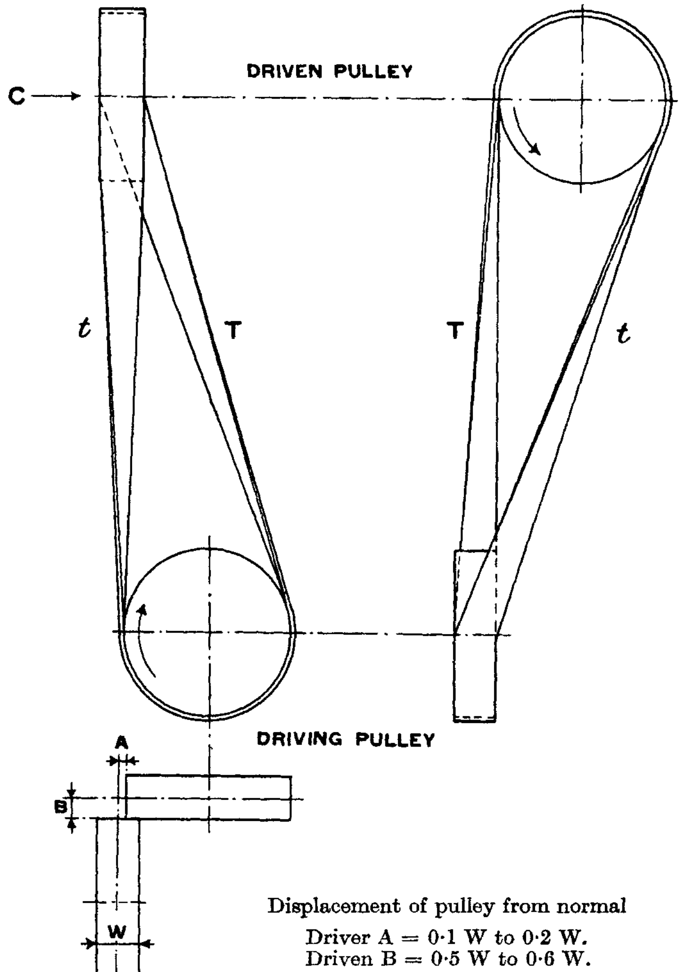
Mr. D. N. HUNT wrote that the general conclusions reached by the Author did not appear to require any criticism; they should be found very useful to pulley manufacturers and users of belts. The references on page 633 to lateral stiffness and on page 635 to lateral resilience should be strongly emphasized as there was at least one type of laminated belt on the market made up of groups of strip fastened together by rivets which were staggered when looking on the face of the belt and which individually did not pass through any great width of the belt. In this type lateral rigidity was almost absent, and such belts always ran towards the low side when pulleys were on converging shafts.

As the Author made special reference to crossed drives it would be interesting if he could say whether the figures showing displacement of pulleys for the quarter-twist drive shown in Fig. 22 came within the range of the limits of the displacement figures which his formulæ would suggest. In trying to ascertain what his figures would have been for such a drive, difficulty had been found in finding what units the various symbols represented, but possibly a longer study of the Paper would make these points clear.

The symbol β seemed to be used for three different factors. It would add to the value of the Paper if some other symbols were employed.

FIG. 22.

Elevation. *Elevation looking in direction C*



Plan.

Mr. NEIL LAWSON (Messrs. Lewis and Tylor) wrote that Dr. Swift was to be congratulated on his careful mathematical analysis of a problem which had long been the subject of wild speculation. This was in itself a considerable contribution to the theory of the transmission of power by flat belting, but the confirmatory tests which he had carried out rendered the work of considerably greater value to industry as a whole, for there were many who, although unable to understand fully the significance of the mathematical analysis, were, none the less, able to appreciate the very practical conclusions deduced from those tests. Besides the obvious result of specifying a rational camber for pulleys, the exposition of the governing principles gave rise to an inviting prospect of what might be achieved by a thorough understanding of the fundamental principles involved in the study of obliquity, twist, and stagger.

Consider for example, the well-known quarter-twist drive. In this drive twist and stagger were self-compensating, but there could be little doubt that the practical genius who first made use of it, did not arrive at it by a mathematical calculation. Armed now with the analysis given by the Author, would it not be possible to design drives with purposely induced, and entirely self-compensating, errors of twist, obliquity, and stagger ? By this means a skew drive could be achieved without recourse to expensive gearing. A drive in which obliquity and twist were equally opposed should be quite as practical a proposal as the quarter-twist drive, in which stagger and twist were the self-compensating factors.

Use might also be made of the corrective effect of opposed stagger and taper for the operation of fast and loose pulleys. The usual method was to have the width of the driving pulley equal to the sum of the widths of the fast and loose pulleys, the belt being moved from one to the other by means of striking gear, and the best practice being to use a double cambered driving pulley in conjunction with cambered fast and loose pulleys. Suppose now a single cambered driving pulley of half the usual width were substituted, floating on a spline on the driving shaft so that it could be moved at will along the axis of the shaft. By this means the belt could be moved from the fast to the loose pulley by moving the driving pulley along its shaft, thereby obviating the destructive wear which occurred when guide forks were used. It was a simple matter, knowing the belt speed, to calculate what was the permissible speed at which the change-over could be effected. Thus the stagger caused by moving the driving pulley along its shaft would be compensated for by the taper (of one side of the camber), so that the belt would tend to keep as near as possible to the centre of the driving pulley, both during the change-over and when in the running position.

There must be many such applications which a complete understanding of the fundamental principles involved would render practicable. The Author was to be thanked for giving a clear exposition of those fundamentals, and it was to be hoped that his recommendations for pulley camber might become a standard. At the same time, it was to be hoped that he would extend his work to embrace crossed drives, and quarter-twist drives, adding thereto a few examples of drives in which obliquity, twist, and stagger might be converted from a necessary evil to a profitable asset.

Mr. T. S. RILEY, M.C., wrote that the Author rightly remarked that his subject had become one of increased importance with the multiplication of drives over small pulleys prescribed by high rotational speeds. The conclusions reached were of the greatest interest, but the real value of his research would depend upon the extent to which these results were applied in the drawing office and workshops. It might be forgiven if some ventured to doubt whether the rule-of-thumb methods at present prevailing in most engineering works would be corrected—as they most certainly should be—in the light of the Author's work.

The belt manufacturer was in this instance not seriously to blame, as, erroneous though many of his recommendations might have been, there was no ground for believing that they had been followed or even considered to any extent by the men called upon to make a pulley, whether as a specialized article or an occasional detail of a machine assembly. The belt manufacturer was, as a rule, only called in when it was required to make the best use of two pulleys which were supposed to agree, but, being as they most frequently were, of different origin, generally agreed only to differ.

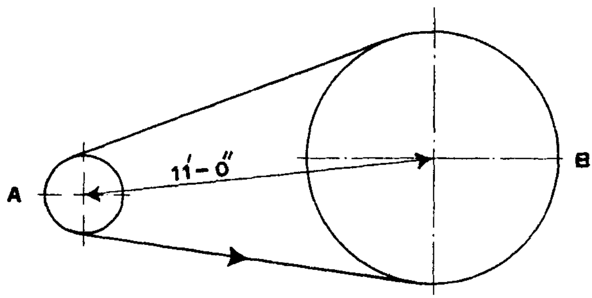
Fig. 23, which represented an actual drive from his own experience, illustrated this point in a striking way. The drive was from the fly-wheel of a high-compression oil-engine, supplied by a leading maker, to the pulley of a dynamo, also by a manufacturer of great repute. It would be noted that while the dynamo pulley was made with a reasonable camber, the fly-wheel, which, in conformity with Dr. Swift's conclusions, should have been made with a crown several times higher, was in fact flat on the face.

In this instance the belt manufacturer was only consulted with regard to minimum centres required, or to be quite accurate, was informed of the maximum centres possible and asked whether he could guarantee his belt to take the drive without a jockey pulley or other tightening gear, which the user wished to avoid. This belt, as he had predicted, showed a certain tendency to wander across the pulley face (especially when a motor which accounted for half the dynamo output was suddenly thrown on or off the line), but not to any very alarming extent. Eventually, after over twelve months' working, it was decided to take the bull by the horns and crown the face of the fly-wheel by re-turning in position. Since this had been done the belt had run with perfect steadiness although the camber on the fly-wheel was, even now, not more than that on the dynamo pulley.

The Author's conclusion that the camber should be dependent on the pulley diameter and nothing else was particularly agreeable to him since his rule for camber of pulleys had long been "$\frac{1}{8}$-inch (in

FIG. 23.

Horse-power (normal rated load), 125.
Belt, laminated leather 16 inches × $\frac{1}{2}$-inch.
A. Dynamo pulley, 2 ft. 6 in. diameter, 750 r.p.m., camber $\frac{3}{8}$-inch (on diameter).
B. Oil-engine fly-wheel, 8 ft. 0 in. diameter, originally flat.



diameter) per foot of pulley diameter irrespective of the width," or, in the notation of the Paper, $\frac{d}{D} = \frac{1}{96}$. (It should be remarked that this rule was intended to apply only to pulleys up to about 48 inches in diameter, and not less than one-sixth of the diameter in width. This covered about 95 per cent of the drives met with in ordinary practice. For main drives and individual drives to large machines of all kinds, in the lay-out of which special attention was paid to alinement and to stiffness of shafts and rigidity of bearings, cambering to the extent suggested by the Author should be adequate for all requirements.)

The actual camber according to the above rule was, it was true, rather more than double that suggested by the Author, but as he had been concerned only with laminated leather belting which possessed great lateral flexibility, his rule had worked out well in practice in the few cases in which anyone had troubled to apply it. Indeed, it was arguable that for drives within the scope indicated above, a figure of, say $\frac{1}{100}$, was not excessive for any type of belting, as cases of accidental or unavoidable error in alinement in excess of the tolerances assumed by the Author occurred much more frequently than he appeared to suppose. The case of a waterside factory on the Thames estuary might be instanced. The conditions were certainly somewhat unusual, as the building and structures had been erected at different dates and were variously on concrete footings, ferro-concrete rafts, and piles. Owing to differences in the rate of subsidence, comparatively large errors in level had been found to occur, and on checking over a 30-foot length of 3-inch shafting recently it was found that one end had sunk 4 inches in relation to the other. Whilst abnormal, it should be remarked that this instance occurred in a factory possessing an adequate and highly qualified maintenance staff, and he had no doubt that in many works where no systematic supervision of plant maintenance was provided, many cases approaching it in degree could be cited.

The Author rightly considered misalinement as arising from three possible causes: obliquity, twist, and pulley stagger. It was remarkable how often, in practice, the second of these was entirely lost sight of as a potential source of error. He had on many occasions been called to investigate the running-off of belts, after being assured that the alinement had been carefully checked over, to find that a slight twist to a machine or motor in the case of a vertical drive, or correction in level of a motor or shaft in the case of a horizontal drive, was all that was necessary to obtain perfectly true running of the belt.

In discussing the precise form of camber the Author concluded that any smooth curve which did not leave too wide a central band would answer well, and this conclusion was borne out in practice. A form of camber which it should be unnecessary to warn against, but which nevertheless was sometimes met with on quite big drives, was that in which each side of the pulley was turned to a conical form, leaving a more or less sharp ridge in the centre.

It might not be irrelevant to mention a drive which came under his notice some time ago, although in this case the cambers were reasonably correct. The pulleys, respectively about 72 inches and 36 inches in diameter, were well proportioned in cast iron but the metal itself was close-grained and very white, having in fact the appearance of chilled iron. The pulley maker seemed to have had difficulty in machining as, instead of finishing with a flat-nosed tool and coarse feed, he had evidently used a sharp-nosed tool and fine feed, with the result that the turning had left a fine-pitched helical groove on the faces. The metal also had a glazed appearance. Although the belt had ample strength for its work, a certain amount of slip was being experienced. The trouble was eventually overcome by giving the belt a sparing but regular dressing of graphite over a considerable period. In his opinion the best pulleys from every point of view were made from cast iron of fairly soft grade showing a reasonable content of graphitic carbon, the faces being turned at the finish by means of a flat tool and coarse feed, and following closely the recommendations of the Paper with regard to form and height of camber.

There could be no doubt that not only belting manufacturers, but engineers and power users generally, owed a very great debt to the Author for the contributions which he had already made to their knowledge, and it was to be hoped that he would be encouraged to attack other phases of the subject which he seemed to have made his own.

Dr. H. W. SWIFT wrote in reply that he appreciated the helpful remarks of those who had contributed to the discussion, and noted with satisfaction the general agreement with the views that pulley cambers stood in need of standardization, and that the cambers employed at the present time generally erred on the side of excess. A general lowering of cambers would improve the performance of belts and relieve stress inequalities in the fibres and so make for longevity.

No criticism had been directed against the analytical treatment, experimental results, or general conclusions contained in the Paper. The Author would naturally like to feel that these were invulnerable, but it seemed that the various writers had been chiefly concerned in viewing the final recommendations in the light of their own practical experience, and in this respect their contributions could be regarded as complementary—and he was gratified to note, in the main complimentary—to his own. The fact that those who spoke from wide and intimate experience of practical belting problems should agree so closely with the conclusions of theory and experiment should assist very materially in the main purpose of the Paper: to inspire confidence in the treatment of belting problems by scientific principles, and to attract the attention of all concerned in the installation or operation of belt drives to the need for an intelligent consideration of pulley camber and of the arrangement of belt drives in general. The belt drive possessed certain inherent properties which for many purposes made it an ideal agent for power transmission, and its advantages could only be fully exploited if its principles of operation were understood and its idiosyncrasies respected. In this way the belt user would find that his drives were more efficient and reliable, and that his maintenance costs were reduced, while the belt manufacturer would find that confidence in his product was enhanced and its sphere of useful application made wider.

The important principle that cambers should be made proportional to the diameter of the pulley might be regarded as definitely established; indeed, the remarks of Major Fenton and Mr. Riley showed that it had already been applied by a few progressive firms. The German formulæ quoted by Mr. Hopper, according to which the camber was made dependent on the width of the pulley, must be discredited as fallacious in principle, as must a number of other arbitrary and quasi-empirical formulæ employed in this country and in America. The question as to the best value of the specific camber d/D for general adoption was more open to discussion. The determining factors were the "equivalent obliquity" to be tolerated, the lateral stiffness of the belting employed, and the pulley margin to be allowed. A resultant equivalent obliquity of 1 per cent was suggested as a reasonable tolerance in the Paper in the light of numerous measurements on belts and industrial drives. The drive referred to by Mr. Riley, with an obliquity of 1 in 90, was exceptional, and represented an error which the Author felt would not be tolerated in good practice. There was much to be said in favour of a low camber which would lead to the detection and correction of such a condition. As regards the lateral stiffness of belts, the tests described in the Paper covered friction rubber and solid woven fabric belts and leather belts of normal construction. Laminated belts were not tested, and the Author could appreciate that these, by reason of diminished lateral stiffness, might require more camber than others for the same corrective effect. On the other hand, from the nature of their construction they should be inherently straighter than other belts, so that the duty of the camber would be largely confined to the accommodation of alinement errors in the drive itself. The Author hoped to extend his tests to cover this type of belting, but in any event it seemed doubtful whether any good purpose would be served by standardizing a higher camber, with its concomitant disadvantages in belts of all types, in order to accommodate belting of a special type. It seemed preferable either to employ a specially high camber—if it were found necessary—with belt drives of this type, or to take special care in the alinement of these drives—a matter which would not present difficulty in most applications of laminated belting. The "uniformly satisfactory" results experienced by Major Fenton on drives with one pulley having a 1 per cent camber and the other flat, and the trouble which occurred when one pulley had a camber of only ½ per cent and the other flat, were evidence that the suggested standard of ½ per cent on each pulley was not unreasonable. The two cases he mentioned in which, although this camber was employed on each pulley, the belts "would not remain in the centre of the pulleys," served to emphasize the need for pulley margins, for in neither case were the pulleys wider than the belt. With pulleys of exactly the same width as the belt, no camber, however severe, would keep a belt central if there was any equivalent obliquity in the belt or drive; it was a fundamental principle that the corrective power of camber only came into action when the belt was displaced from its central position. The Author had been given an opportunity to inspect the two drives in question, and the lateral displacements were found to be ¾-inch and ⅜-inch respectively for the 12-inch and 9-inch belts, neither of which appeared excessive in view of the fact that the pulley margins should have been 1·7 and 1·3 inches respectively.

The short centre distance (4 feet) employed in the bulk of the systematic work to which Mr. Dolby drew attention was adopted (*see* page 653) because theory and experiment showed that results obtained with short drives erred on the side of safety when applied to longer drives. The Author did not wish to press for any particular method of turning pulleys in order to obtain the necessary camber. He suggested the set of three templates as a simple device likely to appeal to manufacturers, but for the sake of any who shared Mr. Hopper's dislike for any special form of profile he purposely pointed out that "it is not a serious matter if the profile curves depart from truly circular arcs provided they follow some smooth curve and do not leave a central band wider than half the pulley face." He shared Mr. Riley's dislike of a central ridge on the face of the pulley, and agreed with his view that the finish of the pulley surface was important, particularly with pulleys of small diameter. He concurred in Mr. Neil Lawson's view that a knowledge of the principles underlying obliquity, twist, and stagger would enable belt drives to be constructed with special geometrical arrangements, but it must be remembered that the use of tapered pulleys in any such arrangement involved an inequality of tension across the belt and consequently an increase of stress and loss of performance. It must also be remembered in contemplating the balancing of twist, stagger, and taper that the effect of a taper varied with the load, mean tension, and period of running of a belt, so that continuous stable running might not always prove as simple as it appeared. It might be well to point out in this connexion that the use of tapered pulleys with twist or skew drives, which was the practice of certain textile machinists, was entirely unnecessary and undesirable. The use of a double cambered driving pulley in conjunction with cambered fast and loose pulleys mentioned by Mr. Lawson was good practice, as it relieved the belt from fork action except during the actual process of striking, but the novel scheme by which Mr. Lawson proposed to strike the belt by moving a single-width cambered driving pulley along a spline would probably be found uncertain in operation except with short drives and wide belts.

The question of units was raised by Mr. Hunt. Throughout the Paper, except on page 670, where special units were used and defined in empirical criteria for crossed drives, any consistent set of units could be applied. The British engineer would probably find it convenient to interpret the various expressions in terms of pounds and inches. All angles were measured in radians, for convenience both in the analytical work and in the final relationships. The symbol β had been used in two connexions, as the effective arc of embrace on a pulley in conformity with an earlier treatment,* and as the angle of obliquity between shafts. The two significations never occurred in juxtaposition, and were so distinct that it was unlikely that confusion could arise from the use of the same symbol.

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* Proc. I. Mech. E., 1928, vol. ii, page 667.