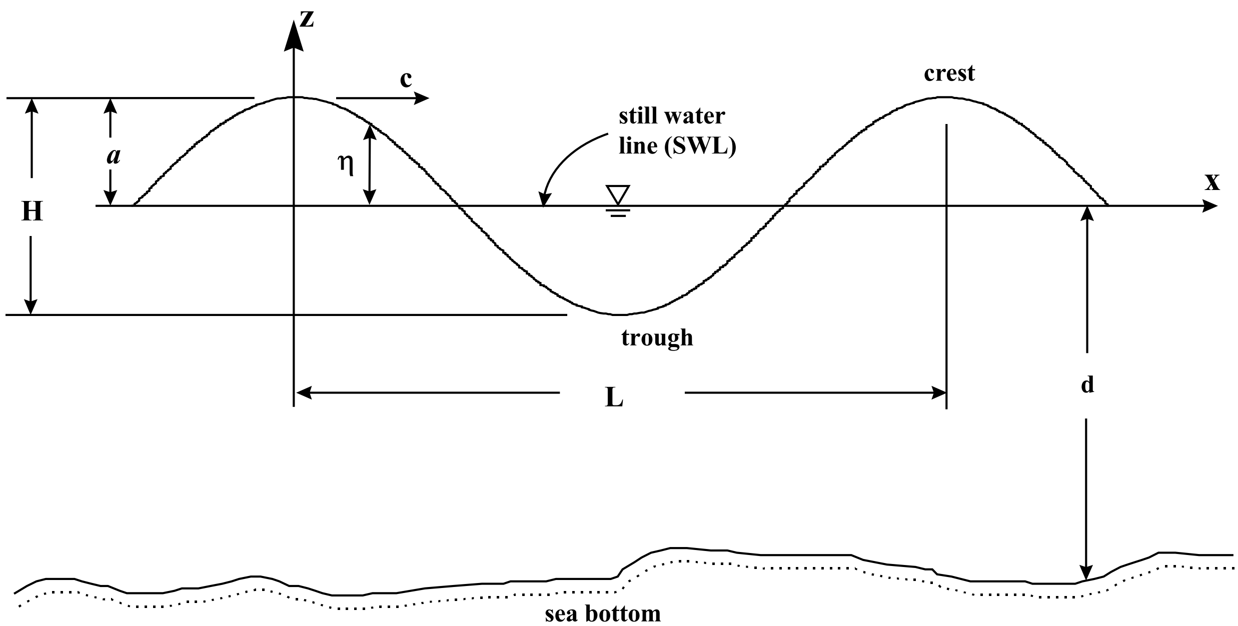
Regular Ocean Waves (Linear Waves)



η	=	displacement of the water surface, varies with time and location, (meters, feet, etc.)
H	=	wave height, constant with time at a given location, (m, ft)
a	=	wave amplitude (m, ft) ($a = H/2$ for linear waves)
L	=	wave length (m, ft)
T	=	wave period, time required for one complete wave to pass a fixed point (seconds, minutes, hours)
f	=	wave frequency = 1/T (cycles/sec = Hertz, Hz)
ω	=	wave circular or radian frequency = $2\pi/T = 2\pi f$ (radians/sec)
c	=	wave celerity or wave speed, c = L/T (m/sec, ft/sec, etc.)
c_g	=	wave group velocity (m/sec, ft/sec, etc.)
d	=	water depth (m, ft, fathoms (fm) = 6 ft)
k	=	wave number = $2\pi/L$ (m^{-1}, ft^{-1}, etc.)
u	=	water particle velocity in the x (horizontal) direction (m/sec, ft/sec, etc.)
w	=	water particle velocity in the z (vertical) direction (m/sec, ft/sec, etc.)
a_x	=	water particle acceleration in the x (horizontal) direction (m/sec^2, ft/sec^2, etc.)
a_z	=	water particle acceleration in the z (vertical) direction (m/sec^2, ft/sec^2, etc.)

Wave Displacement $\qquad \eta = a\cos(kx - \omega t)$

Wave Number $\qquad k = 2\pi/L$

Radian Frequency $\qquad \omega = 2\pi/T$

Dispersion Relation $\qquad \omega^2 = gk\tanh(kd)$

	Shallow Water d/L < 1/20, 0.05 $d/(gT^2) < 0.0025$	**Intermediate** 0.05 < d/L <1/2	**Deep Water** d/L > 1/2 $d/(gT^2) > 0.08$
Wave Speed	$c = \frac{L}{T} = \sqrt{gd}$	$c = \frac{L}{T} = \frac{\omega}{k}$ $= \sqrt{\frac{g}{k}\tanh(kd)}$	$c = c_O = \frac{L}{T} = \sqrt{\frac{gL}{2\pi}}$
Wave Length	$L = T\sqrt{gd}$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = \frac{gT^2}{2\pi}$
Wave Period	$T = \frac{L}{\sqrt{gd}}$	$L = \sqrt{\frac{2\pi L}{g\tanh(2\pi d/L)}}$	$T = \sqrt{\frac{2\pi L}{g}}$
Group Velocity	$c_g = c$	$c_g = \frac{c}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]$	$c_g = \frac{c}{2}$

Horizontal Water Particle Velocity

$$u = \frac{agk}{\omega}\frac{\cosh(kz+kd)}{\cosh(kd)}\cos(kx-\omega t) = a\omega\frac{\cosh(kz+kd)}{\sinh(kd)}\cos(kx-\omega t)$$

Vertical Water Particle Velocity

$$w = \frac{agk}{\omega}\frac{\sinh(kz+kd)}{\cosh(kd)}\sin(kx-\omega t) = a\omega\frac{\sinh(kz+kd)}{\sinh(kd)}\sin(kx-\omega t)$$

Horizontal Water Particle Acceleration $\quad$ $a_x = agk\frac{\cosh(kz+kd)}{\cosh(kd)}\sin(kx-\omega t)$

Vertical Water Particle Acceleration $\quad$ $a_z = -agk\frac{\sinh(kz+kd)}{\cosh(kd)}\cos(kx-\omega t)$

Energy per unit surface area $\quad$ $E = \frac{1}{2}\rho g a^2 = \frac{1}{8}\rho g H^2$

Energy per unit wave crest width $\quad$ $E_T = \frac{1}{2}\rho g a^2 L = \frac{1}{8}\rho g H^2 L$

Energy flux per unit wave crest width $\quad$ $\dot{E} = Ec_g = \frac{1}{2}\rho g a^2 c_g = \frac{1}{8}\rho g H^2 c_g$