Neural Network Verification for Gliding Drone Control: A Case Study

SAIV 2025

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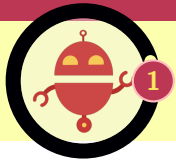
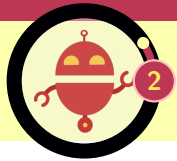
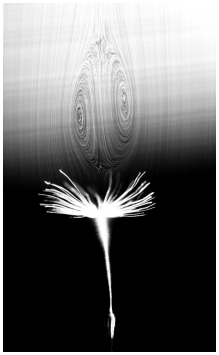
Verification of neural network-controlled bioinspired drones



Motivation - gliding seeds



- Certain plant species use aerodynamics to disperse seeds over large distances [1,2]
- Flying robots with similar characteristics could be used for environmental monitoring and meteorology

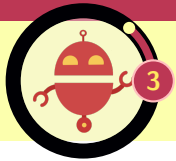


[1]

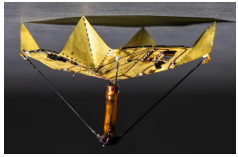
[2]

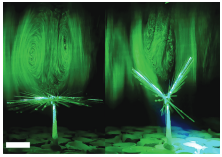
1. Cummins et al, 2018. *A separated vortex ring underlies the flight of the dandelion*. Nature
2. Certini, 2023. *The flight of Alsomitra macrocarpa*. Phd thesis, University of Edinburgh

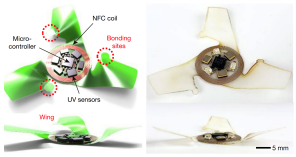
Gliding seed inspired robots



Actuation, sensing, and control methods are the focus of other research:


[3]


[4]

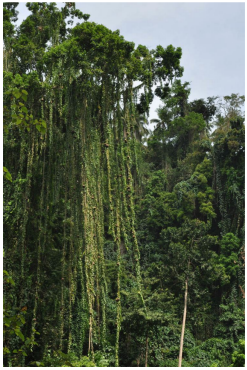


[5]

We will focus on modelling as a basis for verification

3. Johnson et al, 2023. *Solar-powered shape-changing origami microfliers*. Science Robotics
4. Yang et al, 2022. *Dandelion-Inspired, Wind-Dispersed Polymer-Assembly Controlled by Light*. Advanced Science
5. Kim et al, 2021. *Three-dimensional electronic microfliers inspired by wind-dispersed seeds*. Nature

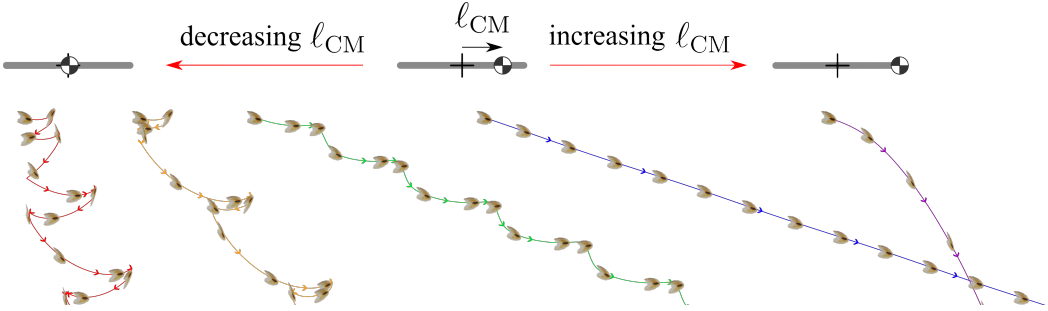
Alsomitra Macrocarpa



[2]

Modelling *Alsomitra*

Alsomitra seed flight can be considered 2D, and is affected by the CoM position:

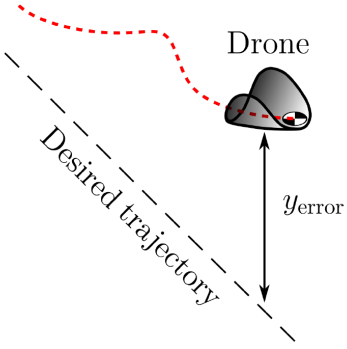


We can therefore employ a quasi-steady low-order model [6], using a controller to actuate the CoM position

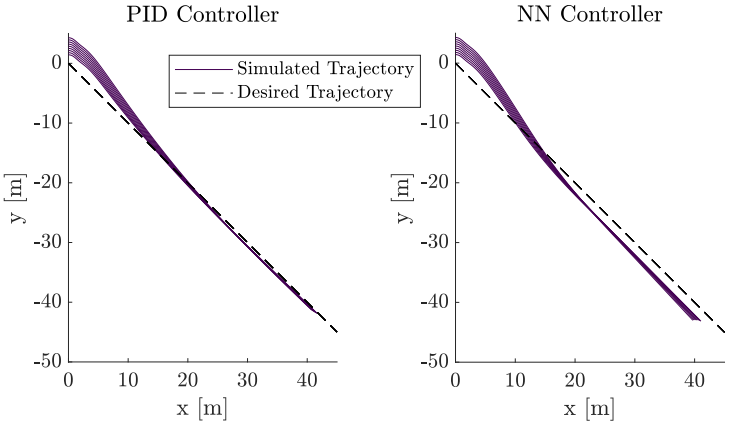
6. Li et al, 2022. *Centre of mass location, flight modes, stability and dynamic modelling of gliders*. Journal of Fluid Mechanics

Control Problem

Our 2D *Alsomitra* model is used as the basis of a feedback control system. For verification, we consider a basic trajectory-following control problem:



Controlling Alsomitra



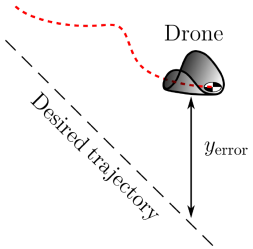
Verification of neural network-controlled bioinspired drones

Ideal Verification Goal

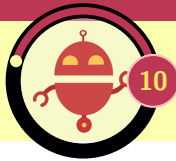
For any starting state $x_1, ..., x_6(0)$, after some time t^* the trajectory of the drone will always be within some small distance y^* of the target trajectory ($x_6 = -x_5$):

$$\forall t \geq t^*, \forall x_1, ..., x_6(0) \in \mathbb{R} : |x_6(t) + x_5(t)| \leq y^* \quad (1)$$

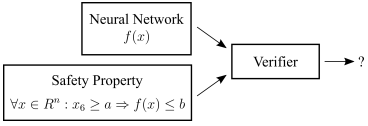
However, this is too complex for current tools to handle

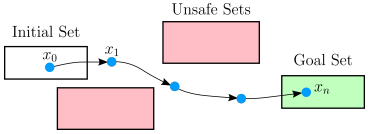


Verification Approaches

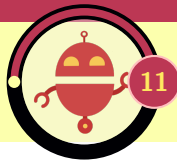


	Neural Network Verification	Reachability Verification
Tools	Marabou, αβ-CROWN, PyRAT	CORA, JuliaReach, POLAR-Express
Benchmarks	VNN-COMP	ARCH-COMP AINNCS
Advantages	large parameter space	dynamics considered
Disadvantages	dynamics not considered	finite time, limited complexity





Lipschitz Robustness



We focus on two notions of robustness, *standard* and *Lipschitz*. Given $x^* \in \mathcal{D}$ and constants $\epsilon, \delta, L \in R$,

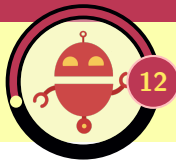
$$\forall x \in R^n : \|x - x^*\| \leq \epsilon \implies \|f(x) - f(x^*)\| \leq \delta \tag{2}$$

$$\forall x \in R^n : \|x - x^*\| \leq \epsilon \implies \|f(x) - f(x^*)\| \leq L\|x - x^*\| \tag{3}$$

Since the latter has been proven to be strictly stronger [7], we implemented a form of PGD training with a Lipschitz loss function, and later verify a version of standard robustness.

7. Casadio et al, 2022. *Neural network robustness as a verification property: A principled case study*. Computer Aided Verification

Verification of neural network-controlled bioinspired drones



Vehicle setup

- Global safety properties
  1. *If the drone is above the line by some threshold y^*, the network output will always make the drone pitch down*

$$x_6 \geq -x_5 + y^* \Rightarrow f(x) \geq 0.187 \tag{4}$$

  2. *If the drone is below the line by some threshold y^*, the network output will always make the drone pitch up*

$$x_6 \leq -x_5 - y^* \Rightarrow f(x) \leq 0.187 \tag{5}$$

- Local (pseudo Lipschitz) robustness
  5. *For any given input point x, the network output $f(x^*)$ of any perturbed point x^* within an ϵ-ball around x, will have a distance less than or equal to $L^*\epsilon$ to $f(x)$*

$$\forall x \in R^n : \|x - x^*\| \leq \epsilon \implies \|f(x) - f(x^*)\| \leq L^*\epsilon \tag{6}$$

Vehicle Implementation - Property 1

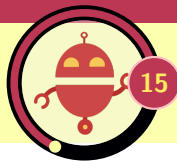
1. *If the drone is above the line by some threshold y^*, the network output will always make the drone pitch down*

$$x_6 \geq -x_5 + y^* \Rightarrow f(x) \geq 0.187$$

```
droneFarAboveLine : UnnormalisedInputVector -> Bool
droneFarAboveLine x =
        x ! d_y >= - x ! d_x + ystar

@property
property1 : Bool
property1 = forall x . validInput x and droneFarAboveLine x =>
  alsomitra x ! e_x >= 0.187
```

Vehicle Results - Global Properties



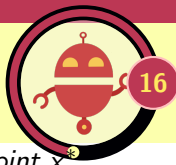
Critical y^* values for Properties 1&2, for naive and adversarially trained networks. A lower y^* indicates a controller that better adheres to the target trajectory:

Property	Naive	Adversarial
1	46	30
2	42	42

$$x_6 \geq -x_5 + y^* \Rightarrow f(x) \geq 0.187$$
$$x_6 \leq -x_5 - y^* \Rightarrow f(x) \leq 0.187$$

Network performance is generally poor, although adversarial training improves P1 result, which could be further improved with PDT.

Vehicle Implementation - Local Robustness



5. *For any given input point x, the network output $f(x^*)$ of any perturbed point x^* within an ϵ-ball around x, will have a distance less than or equal to $L^*\epsilon$ to $f(x)$*

$$\forall x \in R^n : \|x - x^*\| \leq \epsilon \implies \|f(x) - f(x^*)\| \leq L^*\epsilon$$

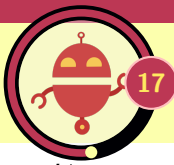
```
boundedByEpsilon : InputVector -> Bool
boundedByEpsilon x = forall i in myList . -epsilon <= x ! i - x ! i + 6 <= epsilon

standardRobustness : InputVector -> OutputVector -> Bool
standardRobustness input output = forall pertubation .
          let perturbedInput = input - pertubation in validPerturbation pertubation and
        validInput perturbedInput and boundedByEpsilon perturbedInput =>
        (output ! e_x - alsomitra perturbedInput ! e_x2) <= Lipschitz * epsilon and
        alsomitra perturbedInput ! e_x2 - output ! e_x <= Lipschitz * epsilon

@property
property4 : Vector Bool n
property4 = foreach i . standardRobustness (trainingInputs ! i) (trainingOutputs ! i)
```

*requires onnx workaround to double network.

Vehicle Results - Local Robustness



Verification success rates (%) of Property 5 for naive and adversarial networks, per L^* values and ϵ, with respect to the training dataset:

Naive

ϵ \ $L^*\epsilon$	10^{-3}	10^{-2}	10^{-1}	10^{0}
10^{-5}				100
10^{-4}			96.1	100
10^{-3}		17.0	98.5	100
10^{-2}	-	-	-	100

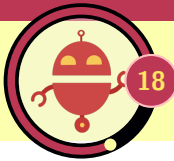
Adversarial

ϵ \ $L^*\epsilon$	10^{-3}	10^{-2}	10^{-1}	10^{0}
10^{-5}				100
10^{-4}			99.7	100
10^{-3}		23.9	99.7	100
10^{-2}	0	13.6	92.5	100

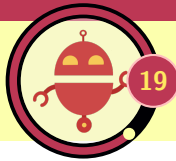
$$\forall x \in R^n : \|x - x^*\| \leq \epsilon \implies \|f(x) - f(x^*)\| \leq L^*\epsilon$$

Network performance degrades with decreasing $L^*\epsilon$ (stricter RHS) and increasing ϵ (larger perturbation region). Adversarial training improves robustness.

Verification of neural network-controlled bioinspired drones



Reachability Goal



Ideal Verification Goal:

$$\forall t \geq t^*, \forall x_1, ..., x_6(0) \in \mathbb{R} : |x_6(t) + x_5(t)| \leq y^*$$

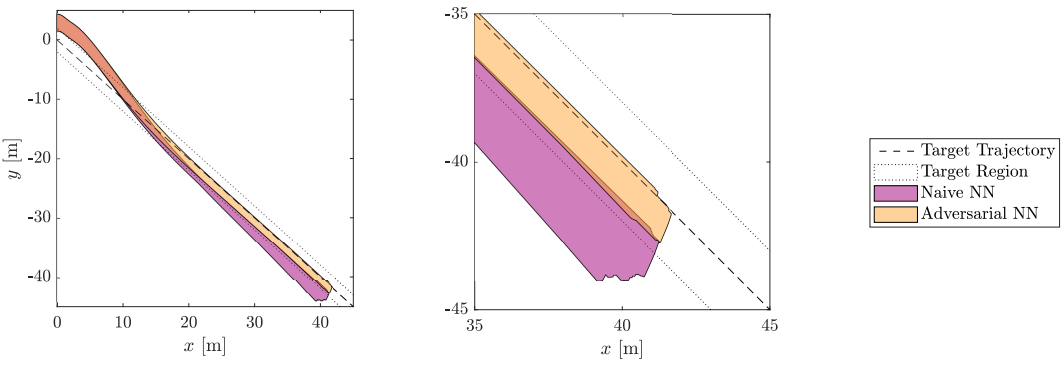
With a reachability tool like CORA, we can verify the entire system - but only for a small initial region and limited time horizon. We define the initial set identically to the training simulations:

$$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, x_6 \in [1.43, 4.29] \tag{7}$$

The goal is for the drone to always be within a distance $y^* = 2$ of the target trajectory after 20 s.

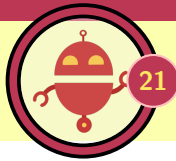
Reachability Result

Reachable regions in x_5 and x_6 for naive and adversarial networks:

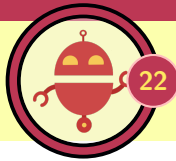


*requires simplified equations, divided initial set, and onnx normalisation workaround.

Verification of neural network-controlled bioinspired drones



Conclusions



- Gliding seed-inspired robots are a novel application for verification
- Verification of such neural-network controlled robots has been demonstrated using two approaches:
  1. Neural network (controller) verification with Vehicle/Marabou
  2. Full system reachability with CORA
- Both approaches are limited in terms of complexity, scalability, and ease of use
- Our results give insights into controller safety, robustness performance, and the effects of adversarial training