

Industrial Policies for Multi-stage Production: The Battle for Battery-powered Vehicles

Keith Head
UBC

Thierry Mayer
Sciences Po

Marc Melitz
Harvard

Chenying Yang
SMU

LSE T&U Seminar
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Motivation for multi-stage industrial policy

Facility Location Choices and Industrial Policies

How do subsidies and tariffs shape global supply chains?

- Industries targeted by industrial policies often share key features
 - ▶ There is a core input that drives costs of the downstream product
 - ▶ High fixed costs of production dictate a limited number of facilities
- The design of policies to capture these value chains is challenging and may be antithetical to other goals
- We develop a framework to quantify the effects of the new industrial policy regime and apply it to the battery electric vehicle (BEV) industry.

Main Takeaway:

- Endogenizing facility location choices matters for policy evaluation
- Local production requirements and environmentalism are not complements

Subsidies under the US Inflation Reduction Act (IRA)

Consumer subsidy for passenger vehicles up to \$7,500 (\approx 15%-20%) if

1. final assembly in North America,
2. at least 40% (80% by 2027) of **battery critical minerals** sourced from the US or FTA country,
3. at least 50% (100% in 2029) of **battery components** manufactured or assembled in North America,
4. no battery components or critical minerals sourced from a “foreign entity of concern,” e.g., China, Iran, North Korea, and Russia.

Tax credit for US battery production: \$45 per kWh, \approx 20-30% of battery cost, \approx \$ 10 bn. over life of plant)

In addition to IRA incentives, tariffs on China-made EVs (25% under 2018 section 301 → 100% after Biden 2024 review)

Why approach a global problem with locational restrictions?

1. National security takes precedence
2. Political feasibility constraints
3. “Subsidy leakage effect” (Bown, Snyder, & Staiger vaccine paper)
4. By promoting local production, **lower** delivered marginal cost, and hence increase EV adoption.

Explanations 1–3 grant that “clean” subsidies are preferable from a pure emissions-reduction perspective.

The last motivation is possible, but far from certain. Depends on parameters.

A new framework for evaluating industrial policy

What are the technical challenges in modelling the impact of industrial policies on global supply chains?

- Multi-stage production with trade costs \Rightarrow vertical interdependencies
- Increasing returns to scale (high fixed investment costs) \Rightarrow interdependent *paths* of supply
- Higher dimension with endogenous **multi-product** and **multi-market** entry \Rightarrow interdependent product-market offering
- This paper applies a new (to us/ old to Operations Research) method for solving this “**MMM**” uncapacitated facility location problem (**MMM-UFLP**).

Outline of Talk

1. Setting up and solving the **MMM-UFLP**
2. Features of the BEV industry that make it fit into the MMM-UFLP framework
 - 2.1 High investment costs
 - 2.2 Active extensive margin
 - 2.3 Upstream cost share is large
 - 2.4 Proximity matters
 - 2.5 Single-sourcing is prevalent
3. Estimate **variable cost** parameters: Worldwide sourcing and production of batteries and vehicles
4. Estimate **fixed costs**: Simulated method of (intercontinental) moments
5. **Policy counterfactuals** inspired by IRA

Positioning this paper in the GVC/Plant location literature

		Number of production stages	
		Single ($K = 1$)	Multiple ($K > 1$)
Constant returns?	yes	ARRY 2018 Head & Mayer 2019	Antràs & de Gortari 2020 Tyazhelnikov 2022 Johnson & Moxnes 2023 †
	no	Tintelnot 2017 AFT 2017, AES 2024 Oberfield et al 2024 Castro-Vincenzi 2024 [†]	de Gortari 2020 note AFFT (2024a, 2024b) This paper *

features capacity constraints, other non-CRS have fixed costs

* also incorporate multi-product and multi-market entry (stage $K + 1$)

Substitutes or Complements?

Combinatorial discrete choice solutions in literature

- Jia (2008) reduces configurations needed for evaluation if the problem is supermodular
- Arkolakis, Eckert and Shi (2024) extend Jia to allow either super- or sub-modular

The MMM-UFLP problem is **neither super- or sub-modular**

- Within production stages, plants substitute for each other
- Across stages,
 1. production plants across different stages complement each other
 2. distribution facilities at consumer markets complement production plants

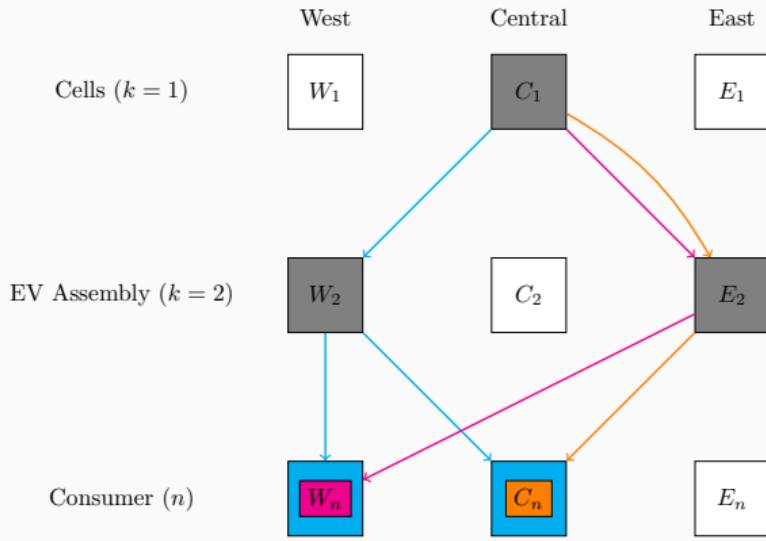
Our approach specifies firms' problem as a **mixed integer linear programming problem (MILP)**, and solves using tools from the OR literature.

Solving the Multi-product Multi-market Multi-Stage Uncapacitated Facility Location Problem

Motivating MILP

- Facility location choices are hard combinatorial problems
- As choice sets grow to realistic numbers, the number of alternatives to evaluate explodes.
- The economics literature going back to Jia (2008) reduces the alternatives by exploiting either super- or sub-modularity (Arkolkakis, Eckert and Shi)
- Multi-stage problems feature substitution (**within stages**) and complementarity (**across stages**): calls for a different approach.
- Mixed integer linear programming (MILP) solves the problem via **constraints on paths** and exploiting fast commercial LP software.

Optimization over Paths with Constraints



- Firm chooses which models m to sell to each market n (market access) subject to market entry cost $\phi_{nm} \rightarrow z_{nm} = \{0, 1\}$
- Firms chooses which facilities to open at each stage of production k subject to facility fixed cost $\phi_{\ell k} \rightarrow y_{\ell k} = \{0, 1\}$
- Firm chooses optimal path $\ell_{nm} \forall n, m$ through open facilities
- The **cyan**, **magenta**, **orange** lines: paths chosen by 3 car models

From Paths to Profits

- Given path ℓ_{nm} , delivered MC $c(\ell_{nm})$ is determined by estimated parameters (local production costs, trade costs including distance, tariffs, etc.)
- Quantity demanded, q , is determined by firm's $c(\ell_{nm})$ (quality-adjusted by ξ_{nm}) and aggregate index of those costs C_n^{EV} across all models
- Variable profits for tuple (n, m) if path ℓ_{nm} is chosen:

$$\pi(c(\ell_{nm}), C_n^{\text{EV}}) = (\mu_{mn} - 1)c(\ell_{nm})q(c(\ell_{nm}), C_n^{\text{EV}})$$

- We use CES demand, Cobb-Douglas production, and monopolistically competitive firms.
- Many Extensions Possible:* Logit demand, Leontief production, production “trees”, oligopoly... Key requirement: **variable profit can be written as a function of paths and a single market aggregator**

MMM UFLP: objectives, variables (x, y, z), & constraints

$$\max \sum_{m \in M_f} \sum_{n \in N} \sum_{\ell_1 \in L_1} \sum_{\ell_2 \in L_2} \pi(c(\ell_{mn})) x_{mn\ell_1\ell_2} - \sum_{k=1}^2 \sum_{\ell_k \in L_k} \phi_{\ell_k} y_{\ell_k} - \sum_{m \in M_f} \sum_n \phi_{mn} z_{mn}$$

subject to

$$\sum_{\ell_1 \in L_1} \sum_{\ell_2 \in L_2} x_{mn\ell_1\ell_2} \leq z_{mn}, \quad n \in N, m \in M_f \quad (1)$$

$$\sum_{\ell_1 \in L_1} x_{mn\ell_1\ell_2} \leq y_{\ell_2}, \quad n \in N, m \in M_f, \ell_2 \in L_2 \quad (2)$$

$$\sum_{\ell_2 \in L_2} x_{mn\ell_1\ell_2} \leq y_{\ell_1}, \quad n \in N, m \in M_f, \ell_1 \in L_1 \quad (3)$$

$$x_{mn\ell_1\ell_2} \geq 0, \quad n \in N, m \in M_f, \ell_k \in L_k, k = 1, 2 \quad (4)$$

$$y_{\ell_k} \in \{0, 1\}, \quad \ell_k \in L_k, k = 1, 2 \quad (5)$$

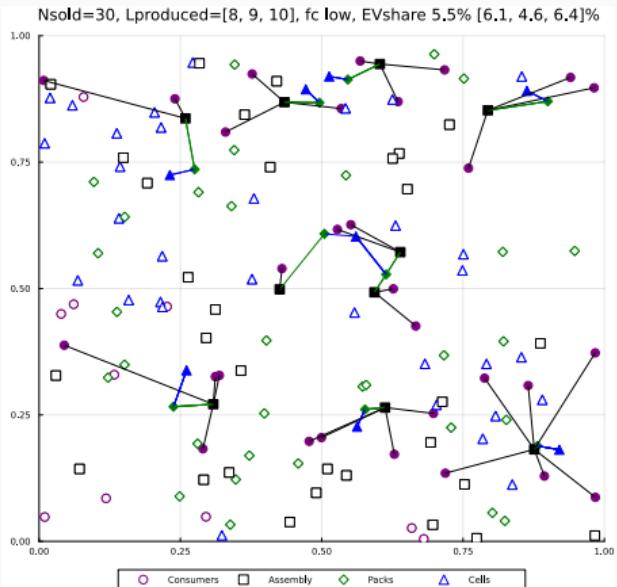
$$z_{mn} \in \{0, 1\}, \quad n \in N, m \in M_f. \quad (6)$$

Simulation: “Flat World”

“Flat World” Simulation Setup

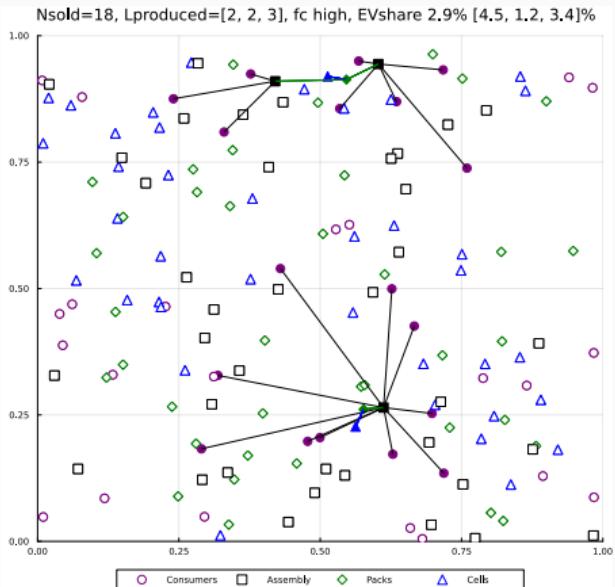
- 1 firm and 1 model
- 3 stages of production + 1 stage of sale
- 40 locations per stage: coordinates are randomly drawn from a unit interval
- Delivered marginal costs depend only on Euclidean distances
- Identical facility fixed costs at all stages across all locations
 - ▶ We simulate 2 levels of fixed costs, low and high, to match range of fixed costs to revenue ratios based on data
- Random market entry fixed costs across consumers
- Northern consumers have higher demand
- Subsidies lower consumer prices

Simulation: No Interventions



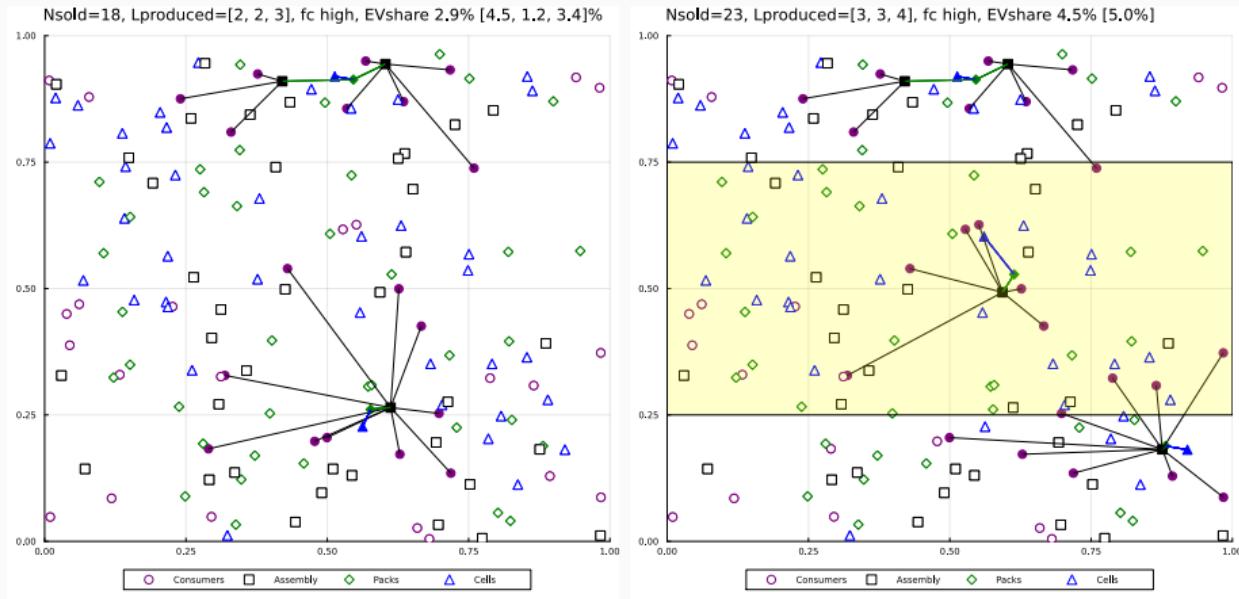
Low Fixed Cost

- More plants at higher demand North
- Peripheral and Southern consumers less likely to be served
- Higher fixed cost: # markets 30 ↘ 18, # plants 27 ↘ 7



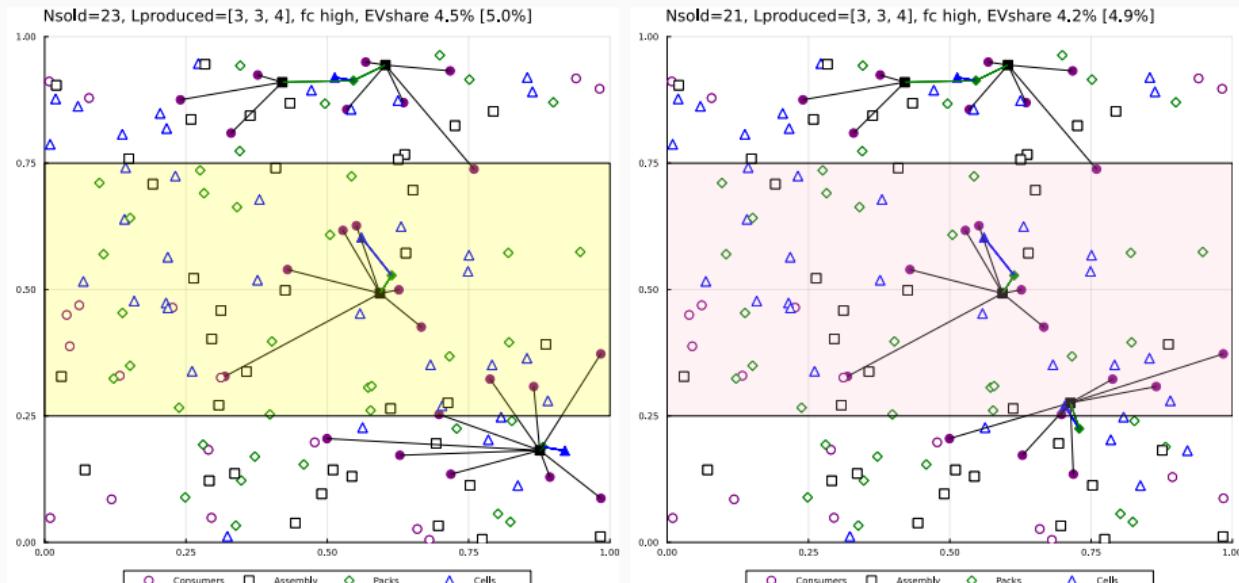
High Fixed Cost

\$5,000 Subsidies in Central: Unconditional



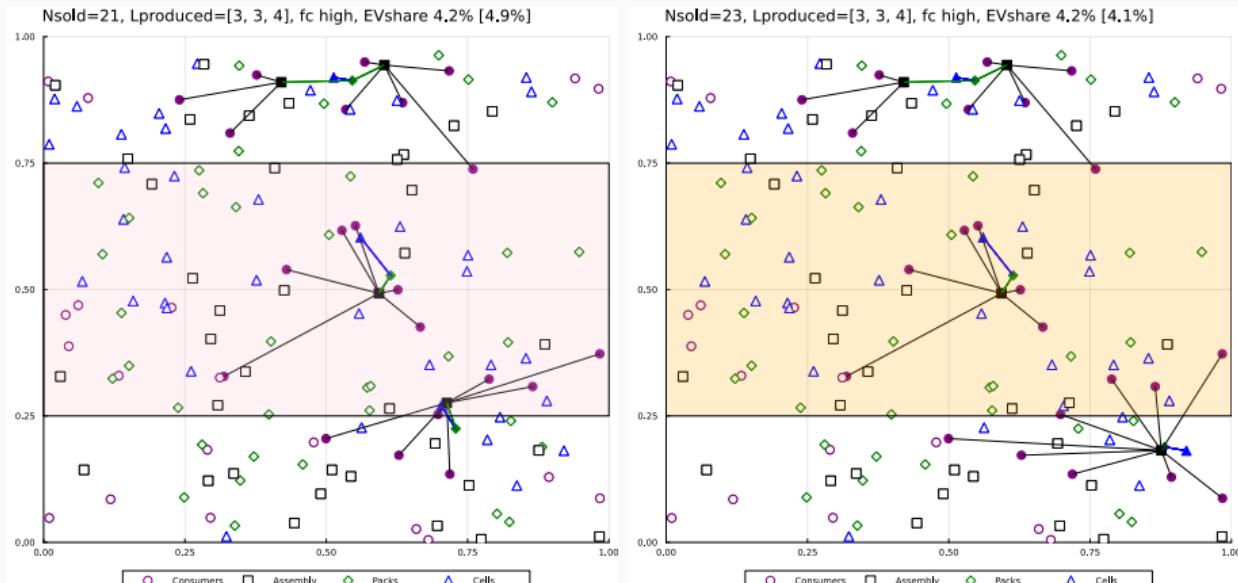
- New assembly in South serves both Central and Southern locations, and existing Central assembly is more “central”
- # markets ↗ by 5, # plants ↗ by 3. Overall EV Share: 2.9% ↗ 4.5%

Central Subsidy Conditional on Final-assembly Location



- Restriction “moves” new Southern assembly to Central → supply chain reconfigured → Lower delivered MC for some Central, and higher for some Southern markets
- # markets in South ↘ by 2, # plants same. EV Share: 4.5% ↘ 4.2%

Central Subsidy Conditional on Full Supply Chain Locations



- Non-compliance decision for Central location: move back to South
 - ▶ Same configuration as uncond. subsidy but lower EV share (some Central consumers no longer get subsidy and buy fewer EVs)
- Content restrictions are counterproductive!

Comments on simulation results

- More centralized at upstream
- Assembly concentrates in countries with high demand
- Subsidy shifts supply chains towards the subsidized region
- It has spillover effects on producers and consumers in nearby regions
- Many possibilities; need to constrain with realistic geography and parameters.

What range of problems does our method handle?

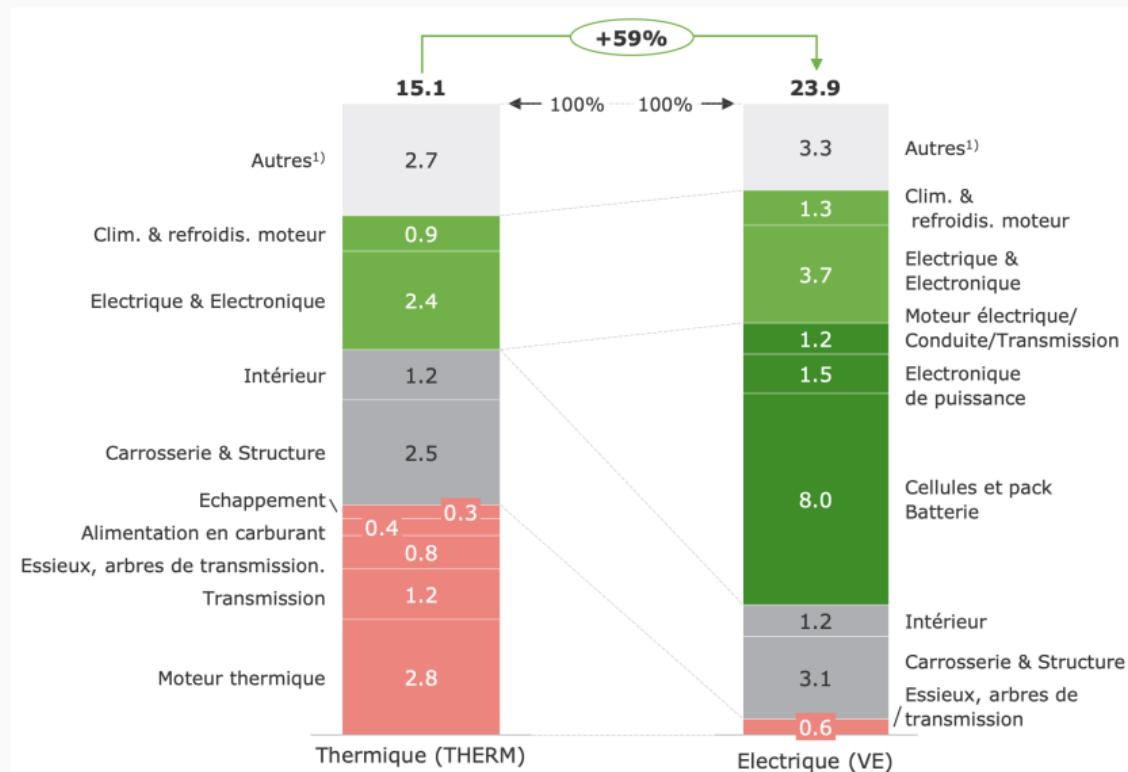
1. no restrictions on complementarity or substitution between facilities
(super vs sub modularity)
2. multi-product firms
3. endogenous market entry to multiple markets up to $K = 3$ production stages are feasible in relatively quick times,
4. plan to extend to handle inter-dependent demand, oligopoly. **Current focus is on supply side; market interactions are de-emphasized.**

Necessary industry characteristics for this framework

1. Inputs from different plants are perfect substitutes if all dimensions of the product are specified \Rightarrow highly disaggregated sourcing data needed (ours comes from IHS Markit)
2. constant marginal costs; plants are “uncapacitated”: no long run capacity constraints
3. together 1+2 \Rightarrow single sourcing from the least-delivered-cost plant
4. Fixed costs should be low enough that extensive margin is active

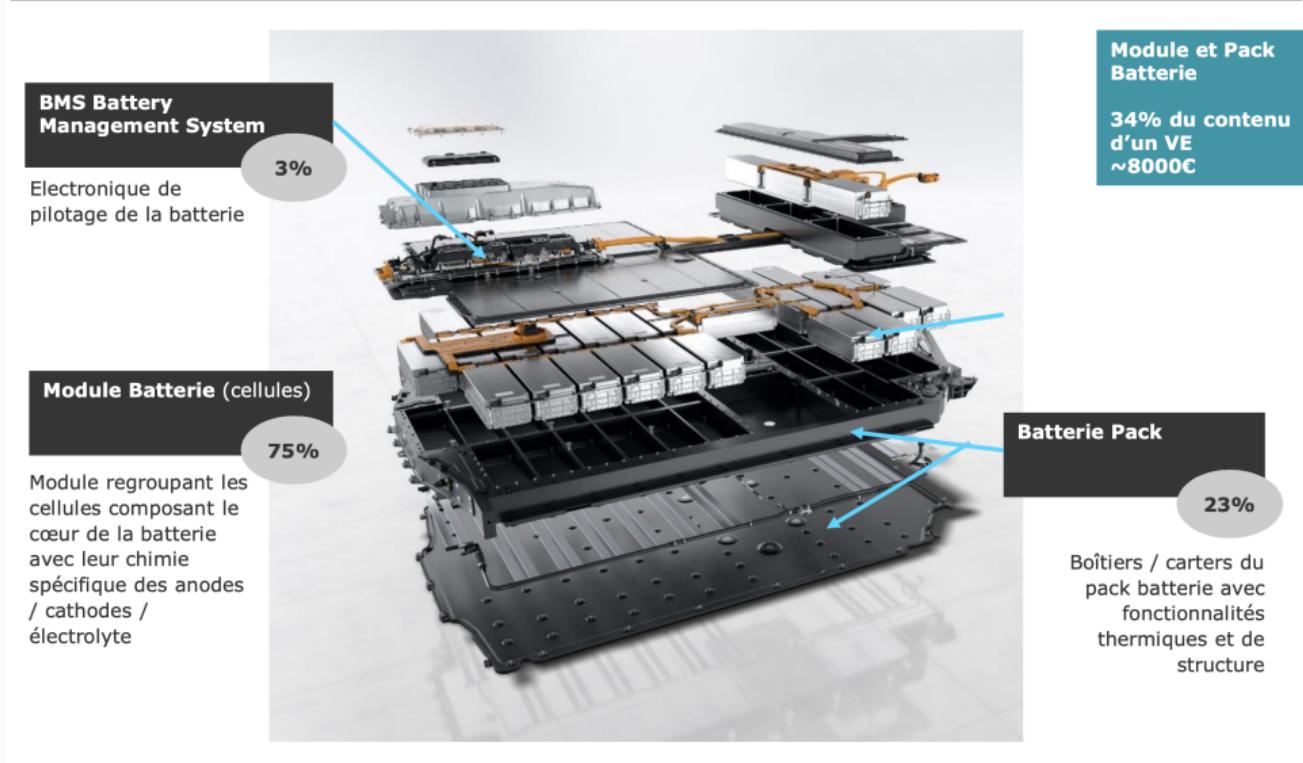
Application to the BEV Industry

Fact 1a: Batteries (upstream) are expensive



Batteries (€ 8k) cost twice as much as gas engine + transm. (€ 4k)

Fact 1a: Cells are the most expensive part of the battery



Battery cells represent > 1/4 of EV cost

Fact 2: High up-front investment costs, especially for batteries

Stage	Cost (\$US bn)		Article Count
	Mean	Std. Dev.	
Cells	2.53	2.60	83
Packs	0.67	0.71	20
Vehicle Assembly	0.79	0.94	198
Battery and Vehicle	4.09	3.54	13

Source: news articles (2007–2023)

Despite the high investment costs, the new plant margin for vehicles and cells has been active on the 3 continents.

SK On's pair of Commerce, GA cell plants



SK On prepares the 2nd factory in Commerce, GA



LG Energy's new Michigan cell factory extension

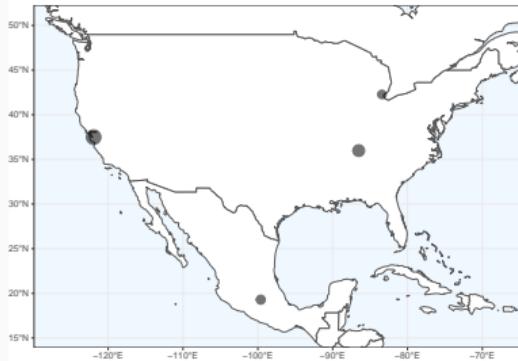


ACC (Stellantis+Mercedes-Benz+Saft JV) battery plant



Fact 3a: Active extensive margin for EV Assembly

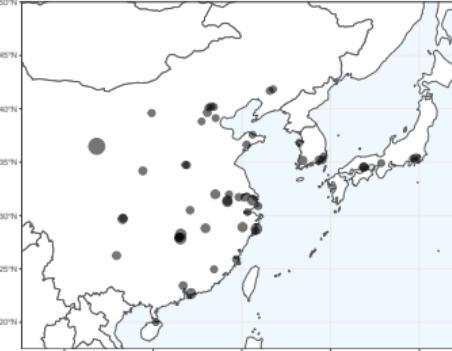
4 Assembly Plants in 2015, Total 75k BEVs



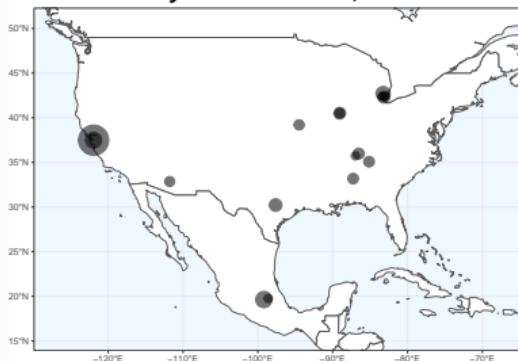
16 Assembly Plants in 2015, Total 80k BEV



52 Assembly Plants in 2015, Total 251k BE



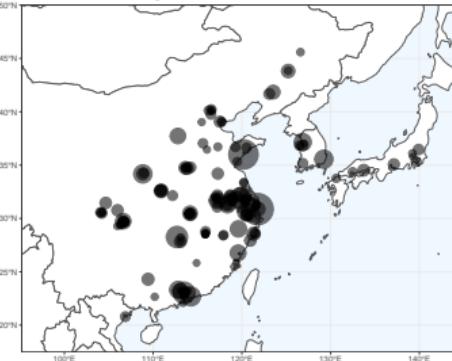
17 Assembly Plants in 2022, Total 812k BE



52 Assembly Plants in 2022, Total 1392k B

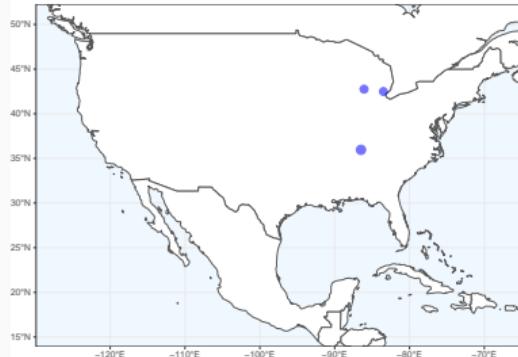


170 Assembly Plants in 2022, Total 5893k I



Fact 3b: Active extensive margin for Cells

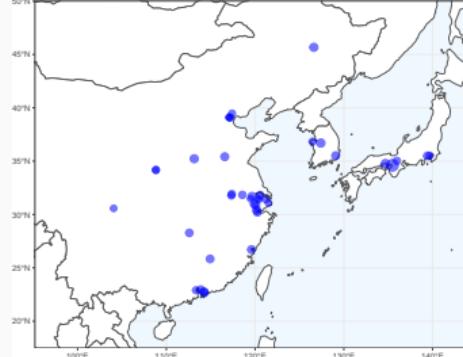
3 Cell Plants in 2015, Total 0.6GWh



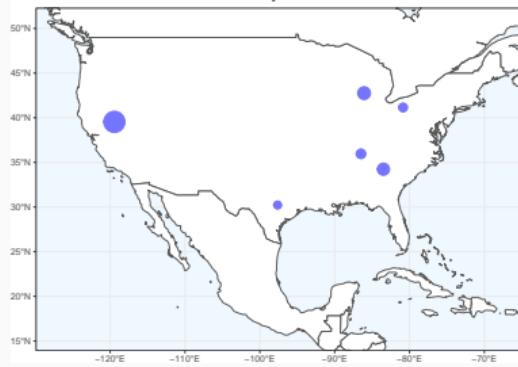
3 Cell Plants in 2015, Total 0.6GWh



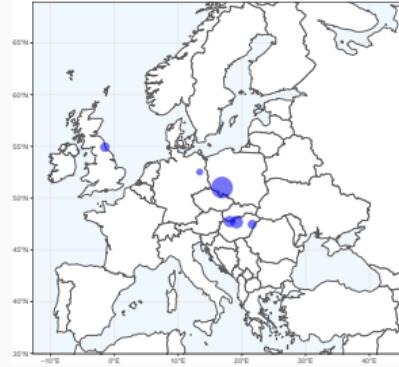
39 Cell Plants in 2015, Total 10.9GWh



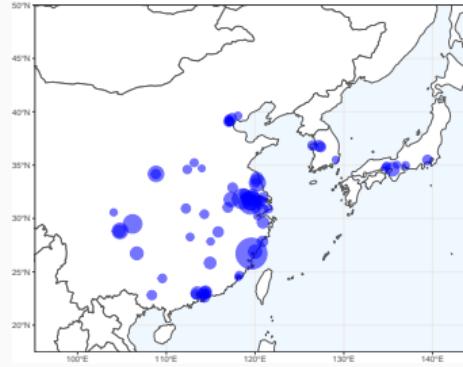
6 Cell Plants in 2022, Total 34.1GWh



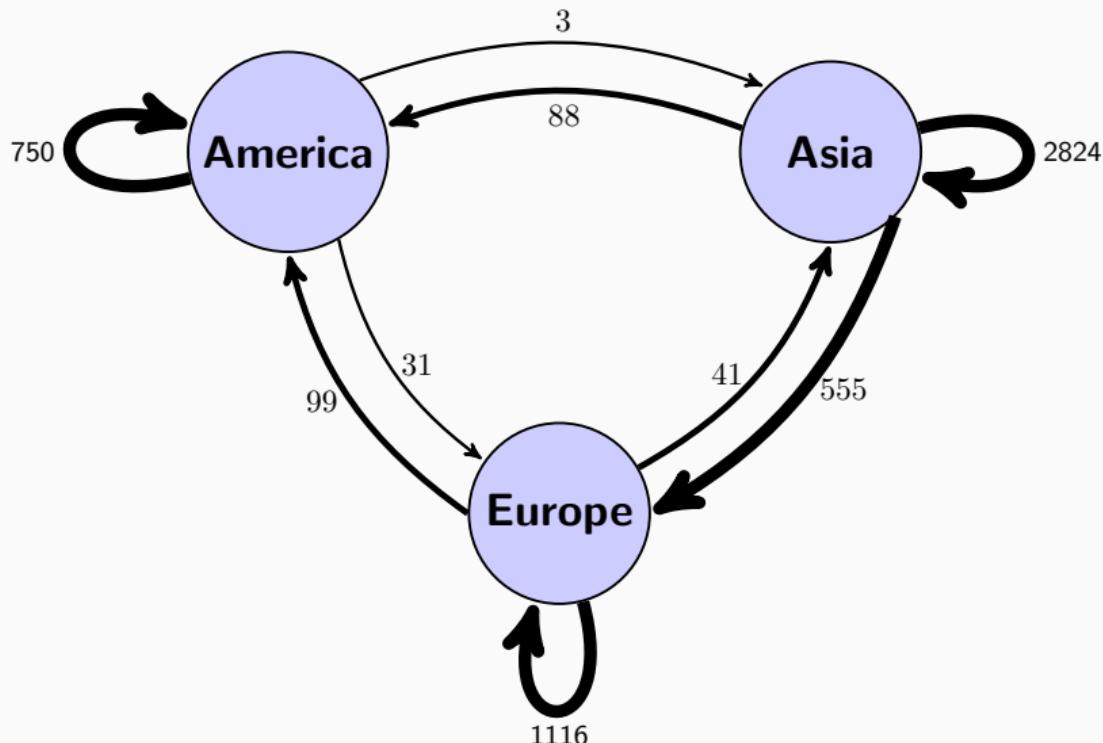
6 Cell Plants in 2022, Total 64.8GWh



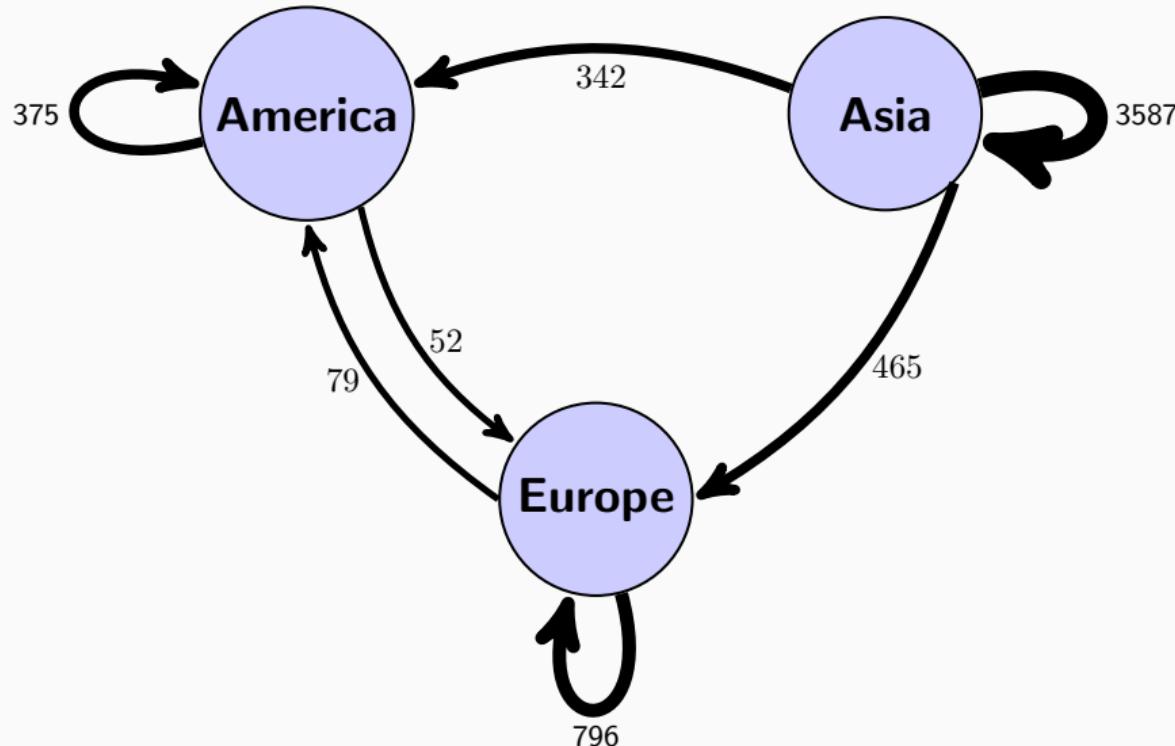
74 Cell Plants in 2022, Total 374.2GWh



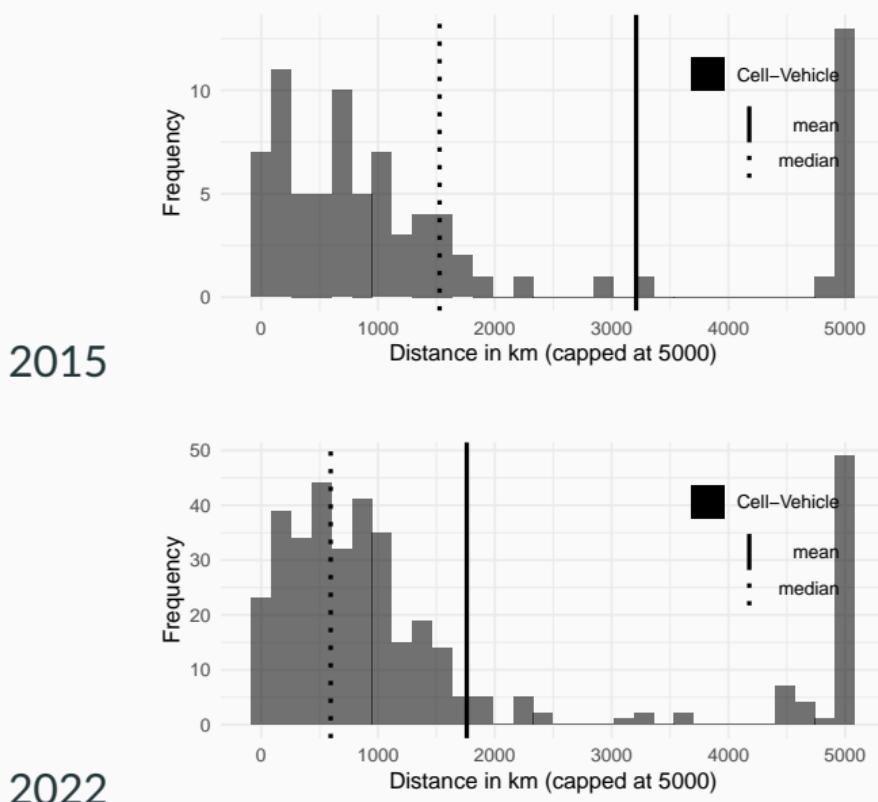
Fact 4a: Vehicle trade is mostly intra-continental



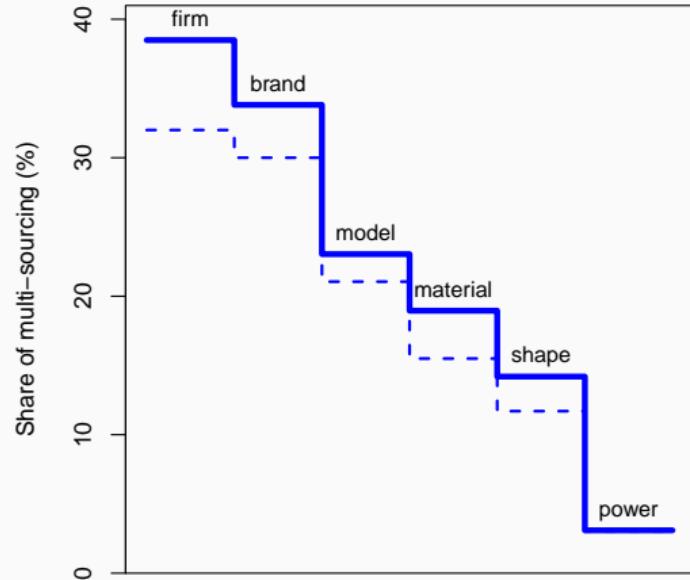
Fact 4b: Cell trade also mainly intra-continental, but Asia exports



Fact 4c: Mainly short distances between stages—with exceptions

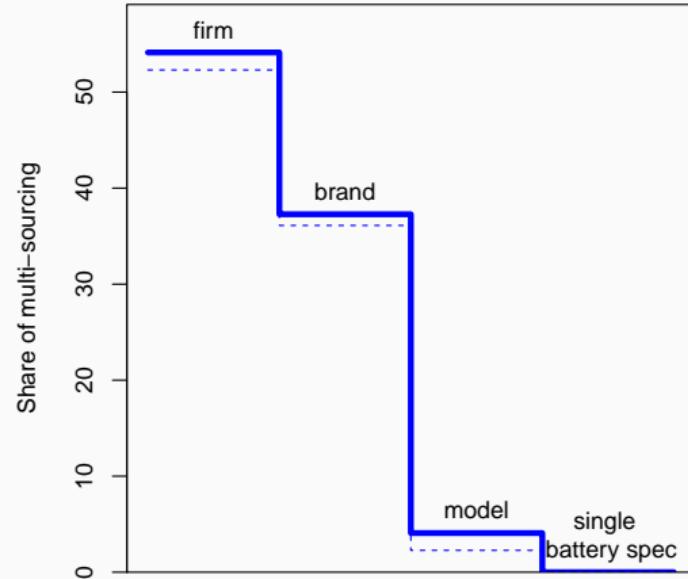


Fact 5a: Multi-sourcing is rare for narrowly defined inputs



Detail in defining chooser

Cells



Detail in defining chooser

Electric Vehicles

Rarity of multi-sourcing \Rightarrow rarity of capacity constraints, LoV

Estimation of the model applied to BEV value chain

Estimation Roadmap

Step 1: Conditional on active facilities (y, z) for each firm, estimate discrete choice assignment (x) problem via sequential nested logit

- Equivalent to minimizing log path costs, C-D \Rightarrow linear in parameters
- Sourcing decisions reveal edge costs as function of observables (geography, tariffs, productivity)
- The **supplier country fixed effects** reveal marginal cost differences

Step 2: Calibrating demand elasticity to the literature, and car relative appeal ξ_{mn} using model-market level prices and sales

Step 3: Estimating fixed cost parameters by matching moments of the data

Variable costs to variable profits

Cells costs: $c_{m\ell_1\ell_2}^1 = w_{\ell_1}^1 \tau_{\ell_1\ell_2}^1 \varepsilon_{m\ell_1\ell_2}^1$,

Vehicle costs: $c(\ell_{mn}) = (w_{\ell_2}^2)^{\alpha_{22}} (c_{m\ell_1\ell_2}^1)^{\alpha_{12}} \tau_{\ell_2 n}^2 \varepsilon_{m\ell_2 n}^2$,

Path costs:

$$\ln c(\ell_{mn}) = \alpha_{22} \ln w_{\ell_2}^2 + \ln \tau_{\ell_2 n}^2 + \alpha_{12} (\ln w_{\ell_1}^1 + \ln \tau_{\ell_1\ell_2}^1) + u(\ell_{mn}),$$

Variable profits:

$$\pi(c(\ell_{mn}), C_n^{\text{EV}}) = \left(\frac{c(\ell_{mn})/\xi_{mn}}{C_n^{\text{EV}}} \right)^{1-\eta} \frac{R_n^{\text{EV}}}{\eta}.$$

Discrete choice approach to variable profits

Stage 1 (cells) estimating equation is

$$\mathbb{P}_{\ell_1|\ell_2}^1 = \exp [\text{FE}_{\ell_1}^1 + \text{FE}_{\ell_2}^1(m) + \beta_D^1 \ln D_{\ell_1\ell_2} + \beta_t^1 \ln (1 + t_{\ell_1\ell_2}^1)],$$

Stage 2 (vehicle assembly) estimating equation is

$$\mathbb{P}_{\ell_2|n}^2 = \exp [\text{FE}_{\ell_2}^2 + \text{FE}_n^2 + \beta_D^2 \ln D_{\ell_2n} + \beta_t^2 \ln (1 + t_{\ell_2n}^2) + \beta_\Phi^2 \text{FE}_{\ell_2}^1(m)],$$

“Inclusive cost”: $\text{FE}_{\ell_2}^1(m) = -\ln \sum_{\ell \in L_1(m)} \exp[\text{FE}_\ell^1 + \mathbf{X}'_{\ell\ell_2} \boldsymbol{\beta}_\tau^1]$

Log path costs as a function of discrete choice estimates ($\boldsymbol{\beta}$), shocks, and SMM parameters (κ):

$$\begin{aligned} \ln c(\ell_{mn}) = & -\kappa_2 \left\{ \text{FE}_{\ell_2}^2 + \beta_D^2 \ln D_{\ell_2\ell_3} + \beta_t^2 \ln (1 + t_{\ell_2\ell_3}^2) \right. \\ & \left. + \kappa_1 [\text{FE}_{\ell_1}^1 + \beta_D^1 \ln D_{\ell_1\ell_2} + \beta_t^1 \ln (1 + t_{\ell_1\ell_2}^1)] \right\} + u(\ell_{mn}). \end{aligned}$$

$\kappa_1 \leftarrow \beta_\Phi^2$, κ_2 scales the variable cost relative to the fixed cost and the variance of the unobserved u shocks

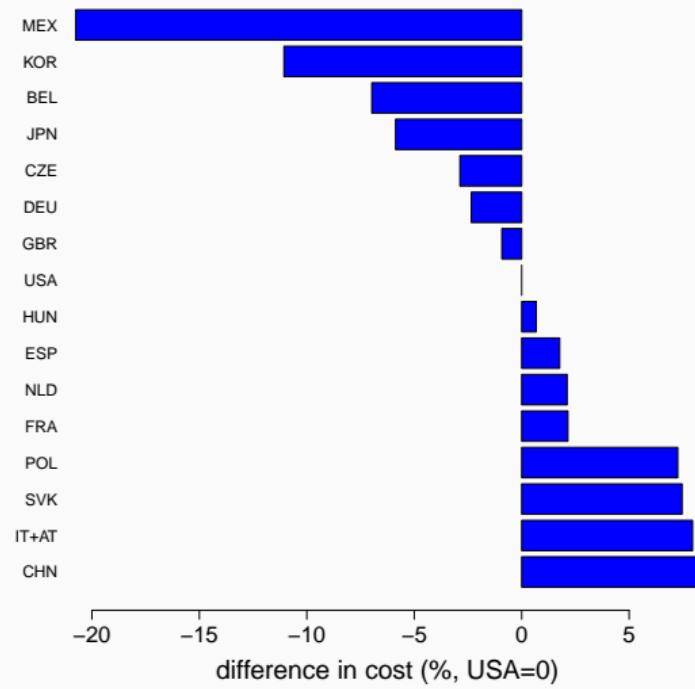
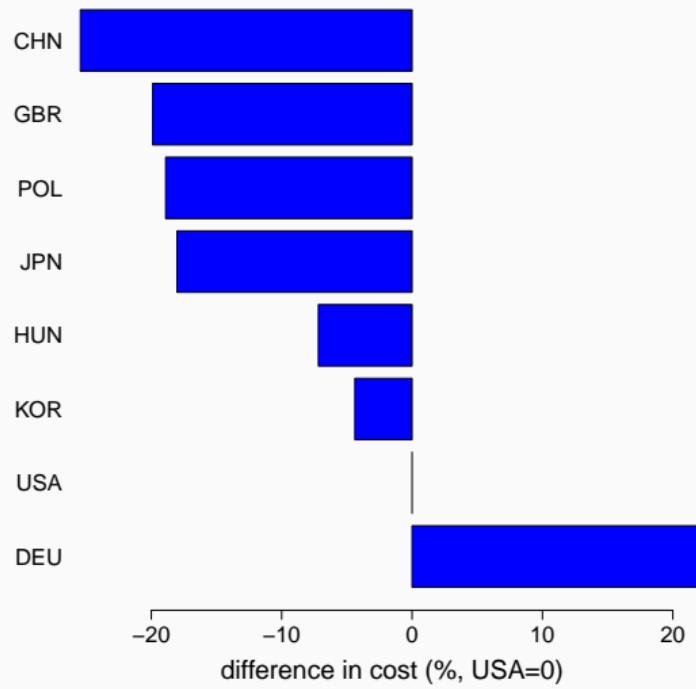
Sequential Nested Choice estimation of variable costs

- Stage $k = 1$: Battery cell sourcing
 - ▶ Chooser: Assembly plant in ℓ_2 for model m
 - ▶ Choice: Battery cell plant in ℓ_1
 - ▶ Choice set: Any cell plant from whom the car maker procures cells
 - ▶ Determinants: dyadic trade costs, fixed effects of supplier countries
- Stage $k = 2$: Vehicle sourcing
 - ▶ Chooser: dealership network in country n sourcing model m
 - ▶ Choice: EV assembly plant in country ℓ_2
 - ▶ Choice set: Any plant that assembles that car maker's EVs
 - ▶ Determinants: dyadic trade costs, fixed effects of supplier countries, plus inclusive cost from cell stage $\rightarrow -\kappa_1$

Nested Logit Sourcing Results

	Cells	Vehicles
border	-0.746 ^b (0.347)	-0.426 ^a (0.139)
log distance	-0.287 ^b (0.121)	0.099 (0.062)
intraplant	0.129 (1.02)	
Inclusive cost of cells		-0.217 ^a (0.083)
log(1+tariff)	-5.45 ^b (2.15)	-10.3 ^a (1.65)
cross-continent	-1.35 ^b (0.571)	-0.634 ^a (0.192)
log GDP per capita	0.594 ^a (0.179)	0.116 (0.079)
Squared Correlation	0.165	0.080
Pseudo R ²	0.156	0.095

Cost differences: Batteries and Cars



Cells

BEV Assembly

Remaining steps to calibration

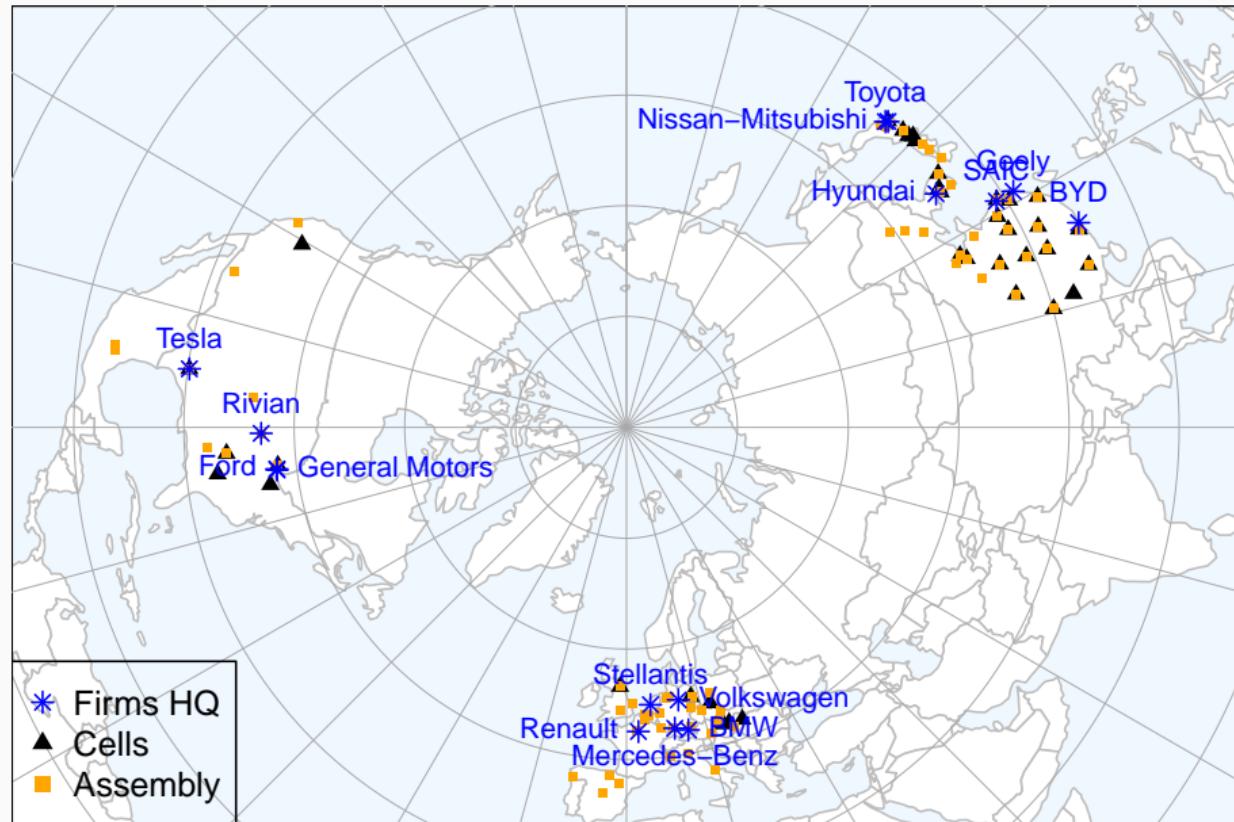
- Fixed costs: calibrate the means distribution fixed costs by matching moments
 - ▶ Fixed costs assumed log-normal with mean shifted by continent (Asia, Europe, Americas) and distance to firms' headquarter.

$$\ln \phi_{\ell_k} \sim \mathcal{N}(\rho_{\text{As/Eu/Am}}^k + \rho_{\text{HQ}}^k \ln \text{dist}_{\ell_k}, \sigma_k)$$

- ▶ Distance to HQ \implies firms' factories concentrate near home
- Filters to speed up computation
 - ▶ 15 large MNCs (138 models)
 - ▶ GAUL1 plant locations in 24 countries

Stage	Americas	Asia	Europe	Total	Configurations
1: Cells	6	24	6	36	69bn
2: Assembly	10	33	29	72	4.7e+21

The top 15 EV makers and their location alternatives



15 top firms in 2022

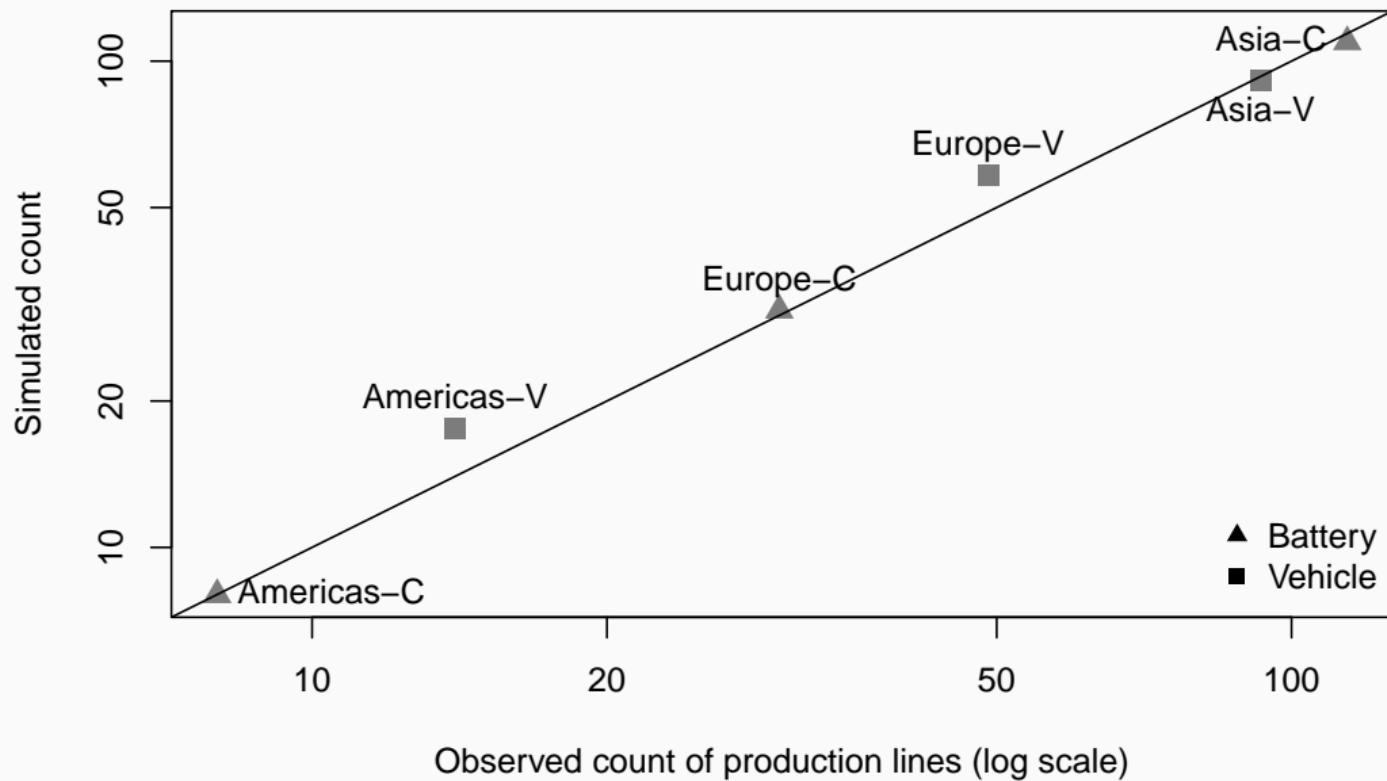
No.	Manufacturer	# Markets	# Models	Production	
				Cum. Share (%)	Sales-exCHN Cum. Shr (%)
1	Tesla	23	4	20.5	28.0
2	Volkswagen	24	23	30.0	42.4
3	Hyundai	23	15	35.6	54.1
4	Stellantis	18	19	40.3	63.7
5	BMW	24	7	44.0	69.7
6	Renault	19	6	46.7	75.3
7	Mercedes-Benz	24	9	49.3	80.0
8	Geely	24	15	55.6	84.4
9	Ford	22	4	57.5	88.3
10	Nissan-Mitsubishi	23	8	60.1	91.9
11	SAIC	17	21	72.8	95.0
12	General Motors	7	7	74.1	96.6
13	Toyota	23	8	74.9	97.8
14	Rivian	3	3	75.3	98.6
15	BYD	10	14	89.6	98.9

SMM Parameter Estimates

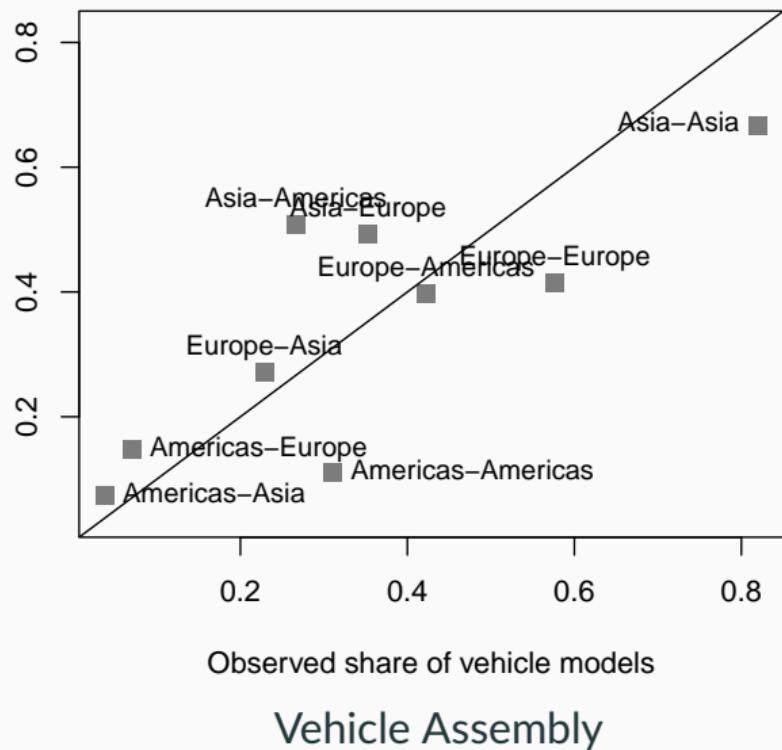
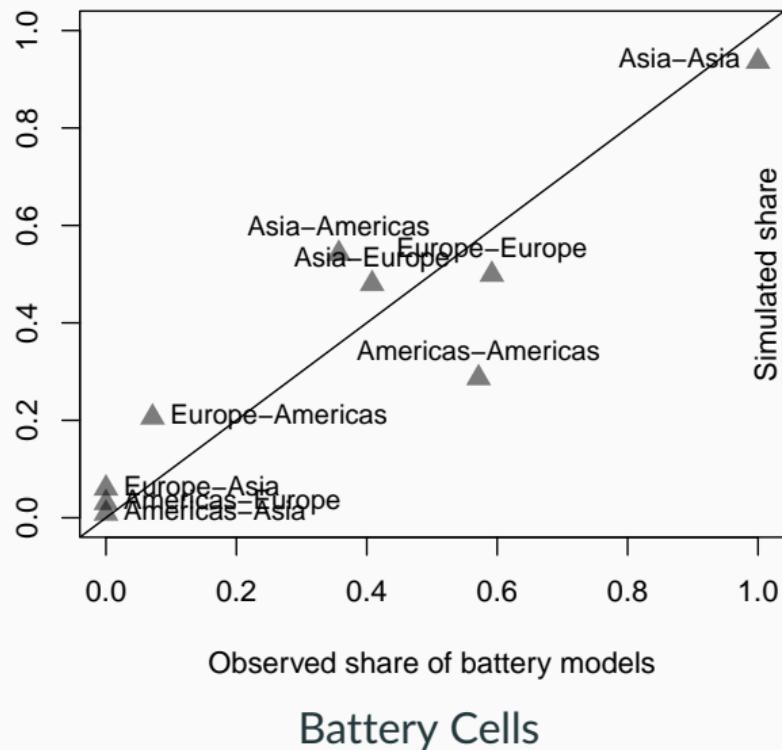
Parameter	Description	Estimate
κ_2	Variable cost (VC) weight	0.19
ρ_1 (Asia)	Fixed cost of cell plant	0.17
ρ_1 (Europe)	(by continent)	0.13
ρ_1 (Americas)		0.42
ρ_2 (Asia)	Fixed cost of assembly plant	0.16
ρ_2 (Europe)	(by continent)	0.18
ρ_2 (Americas)		0.45
ρ_1 (HQ)	FC HQ-dist. elas. (cells)	0.26
ρ_2 (HQ)	FC HQ-dist. elas. (assembly)	0.56
α_c	Marginal cost of quality	0.85

No SEs yet, estimation takes ≈ 24 hours with 100 draws (parallelized)

Calibrated Fit to Data: Production Lines by Continent

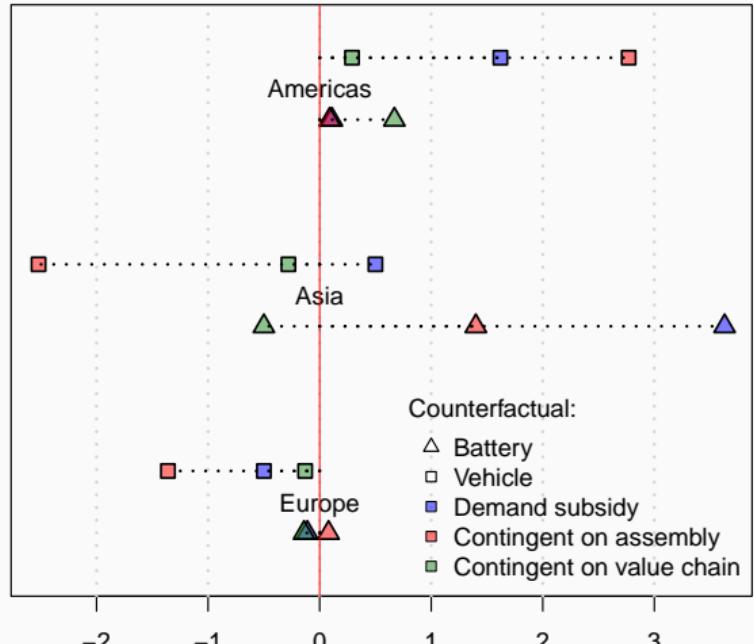


Calibrated Fit to Data: Inter-Continental Flows



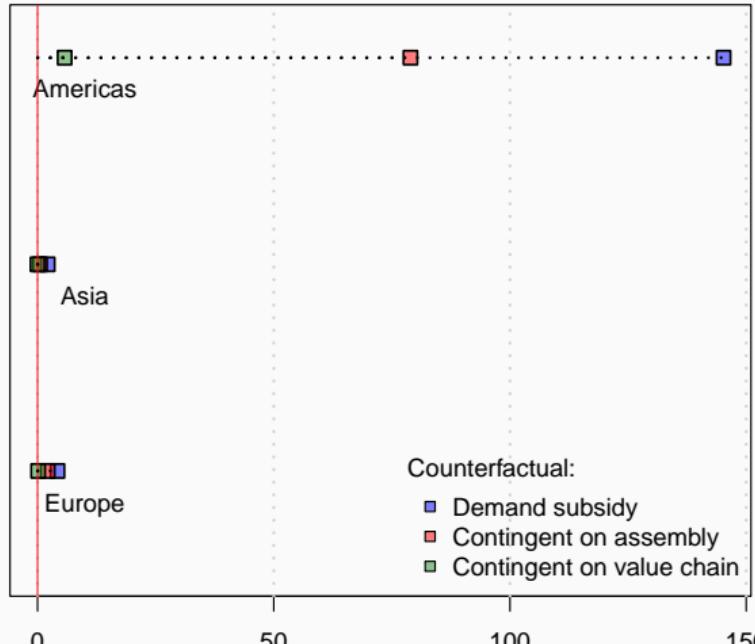
Counterfactual: BEV Policies

Predicted Impact of North American BEV Subsidies



Change in number of production lines wrt baseline

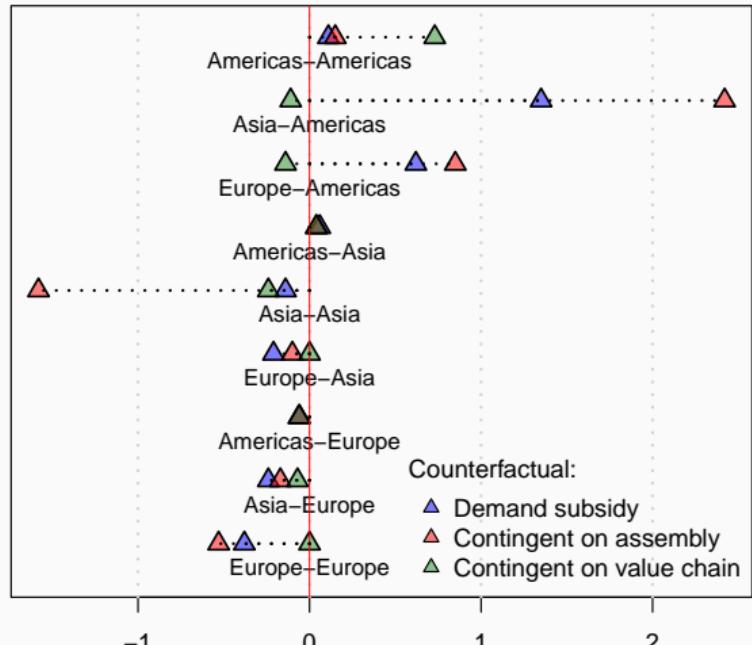
Production Locations



% change total expenditure on EVs wrt baseline

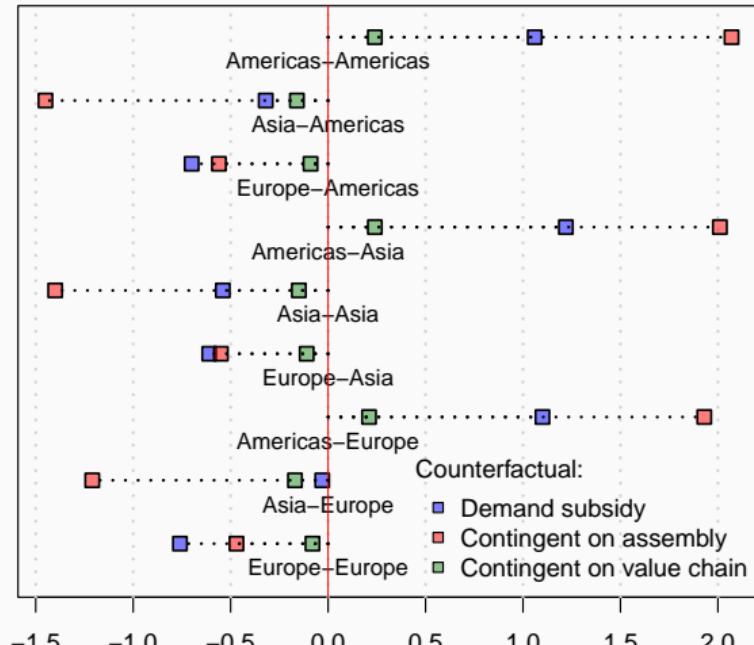
BEV Expenditure Shares

Predicted Effects for Cross-Continental Flows



Change in number of battery models wrt baseline

Battery Cells



Change in number of vehicle models wrt baseline

Vehicle Assembly

Contributions of subsidies and cost reductions to EV sales

Policy	Share Elig.	Cost index redn.			Tot. Exp. increase
		subsidy	costs	total	
1: Unconditional	100	20	5.8	24.6	145.2
2: Continental V	14.7	8.2	4.4	12.4	78.9
3: Continental V+C	2.6	1.6	-0.2	1.5	5.7

Contributions to cost changes (in %) in other continents

Policy	Share Elig.	Cost index redn.			Tot. Exp. increase
		subsidy	costs	total	
Europe					
1: Unconditional	0	0	1.3	1.3	4.2
2: Continental V	0	0	0.7	0.7	2.2
3: Continental V+C	0	0	0.0	0.0	0.1
Asia					
1: Unconditional	0	0	0.7	0.7	2.1
2: Continental V	0	0	0.2	0.2	0.5
3: Continental V+C	0	0	-0.0	-0.0	-0.0

What we've learned so far

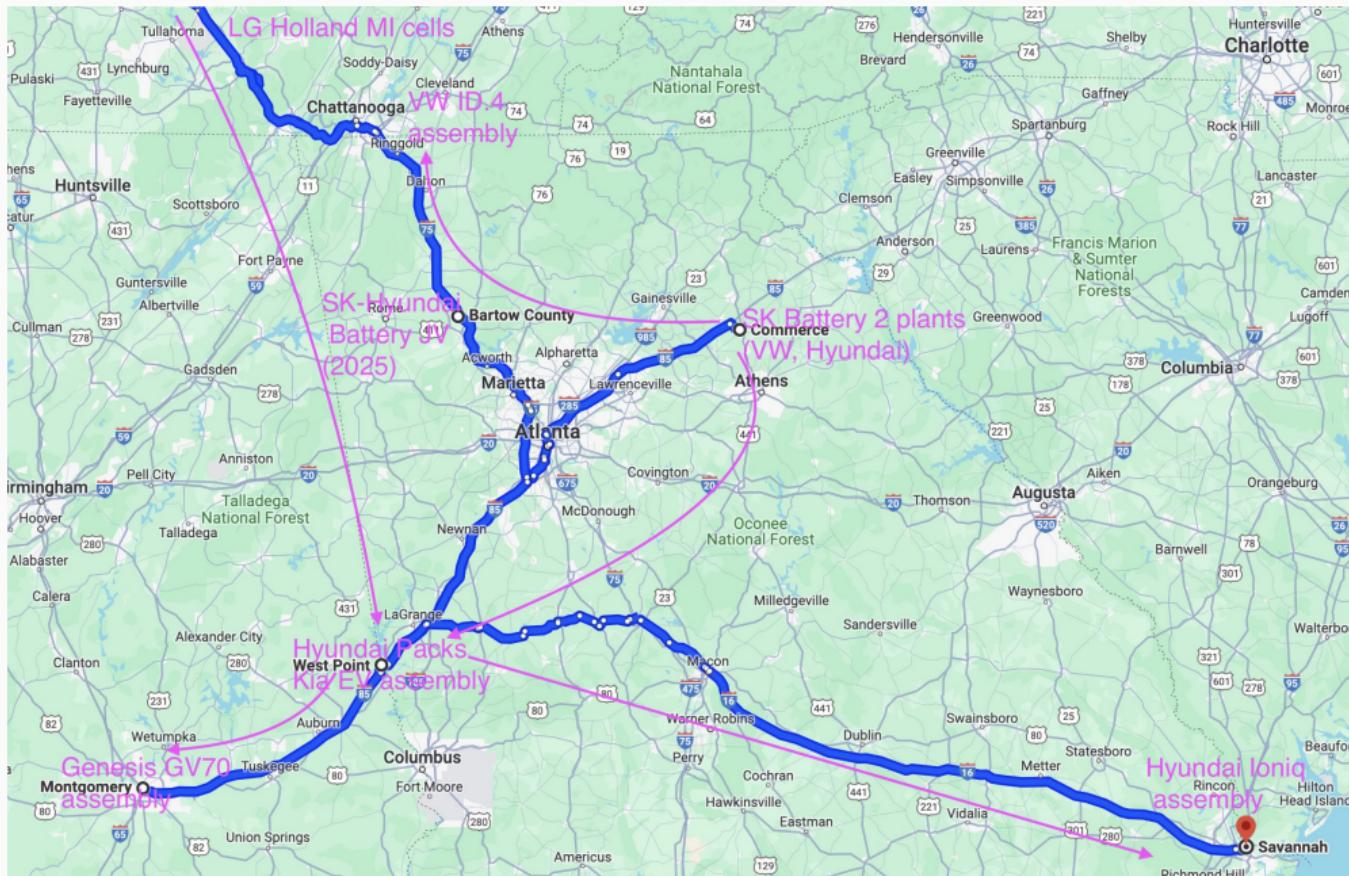
- Algorithm for solving the optimization problems that combine substitution and complement patterns across locations
- Relevance for evaluating the impact of large industrial policies that induce discrete change in locations
- Increasing returns (fixed costs) and multiple production stages are empirically relevant for many sectors
- Preliminary counterfactual results show that IRA subsidies increase EV expenditure, but restrictions on location of cell production are counter-productive
- With IRS, protectionist policies potentially align with emissions goals. Based on our estimates, a **clean consumer credit** would have done more to promote EV adoption

Epilogue: Post IRA investments

Top 10 models excluded by Rule 1 from the IRA buyer credit

Rank	Brand	Model	2022 Sales (US)	Assembly location announcements
1	Hyundai	Ioniq 5	23741	Adds from Korea to Savannah, GA
2	Kia	EV6	21978	Adds EV9 from Korea to West Point, GA
3	BMW	i4	11462	Stays in Germany
4	Polestar	2	8758	Polestar 3 to Ridgeville, SC
5	Porsche	Taycan	8425	Stays in Germany
6	BMW	iX	7394	Stays in Germany
7	Kia	Niro	7262	Stays in Korea
8	Audi	e-tron	7233	Stays in Belgium
9	Hyundai	Kona	4719	Stays in Korea
10	Volvo	C40	4693	Stays in Belgium

Korean BEV cluster: new paths for some IRA-affected models



Hyundai Ioniq Annabell, GA plant (December 2022)



Hyundai Ioniq Annabell, GA plant (October 2024)



Hyundai EV Plant, Elizabethtown, Georgia

October 21, 2024 | 4:39 p.m.

BLACKSKY

Appendix

UFLP is a limit case of AES, allows comparison to MILP

- AES cost function for variety ω (general case)

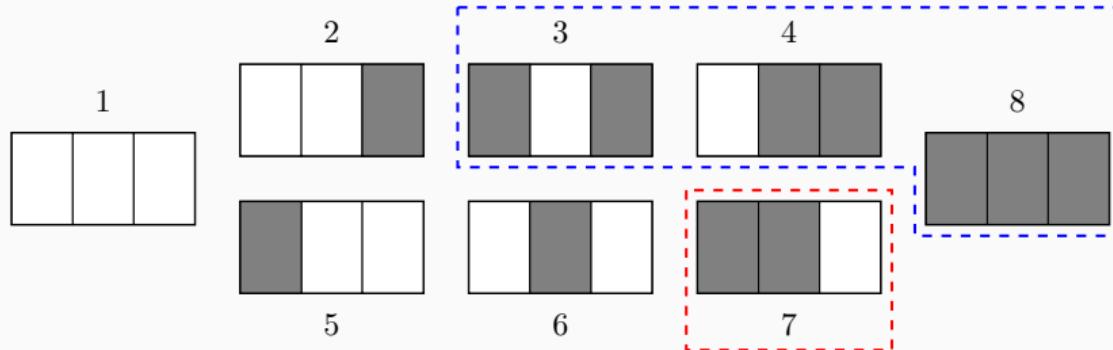
$$c_n(\mathcal{L}, \omega) = \left[\sum_{\ell \in \mathcal{L}} c_{\ell n}(\omega)^{-\theta} \right]^{-1/\theta}, \quad \text{where} \quad c_{\ell n} = w_\ell \tau_{\ell n}(\omega) / z_\ell(\omega).$$

- Limit case as $\theta \rightarrow \infty$: $c_n(\mathcal{L}, \omega) = \min_{\mathcal{L}} c_{\ell n}(\omega)$.
- Perfect substitution implies single-sourcing (least cost plant)
- Plug minimized cost into the profit function (taking P_n as given).

$$\pi(c_{\ell n}) \propto c_\ell^{1-\sigma}$$

- Firm chooses \mathcal{L} to maximize the sum of all destination-specific profit
- Randomness: cost shocks (w_ℓ) and locations on grid ($\tau_{\ell n}$)
- Simulate L potential locations, N consumer markets

How submodularity can reduce evaluations



1. Configurations $i = 1 \dots 8$ of $[y_i^W, y_i^C, y_i^E]$
2. Evaluate 1 and 2: If $\pi_2 < \pi_1$ then position $y_i^E = 0$ for all i .
3. No need to evaluate $i = 3, 4, 8$
4. Evaluate 5 and 6: Whichever is higher will dominate 7.
5. Select larger of π_1 and winner of 5 vs 6.

⇒ only need to evaluate 4 options (50% of all possibilities)

MILP formulation of the UFLP

$$\max \sum_{n \in N} \sum_{\ell \in L} \pi(c_{\ell n}) x_{\ell n} - \sum_{\ell \in L} \phi_\ell y_\ell$$

subject to

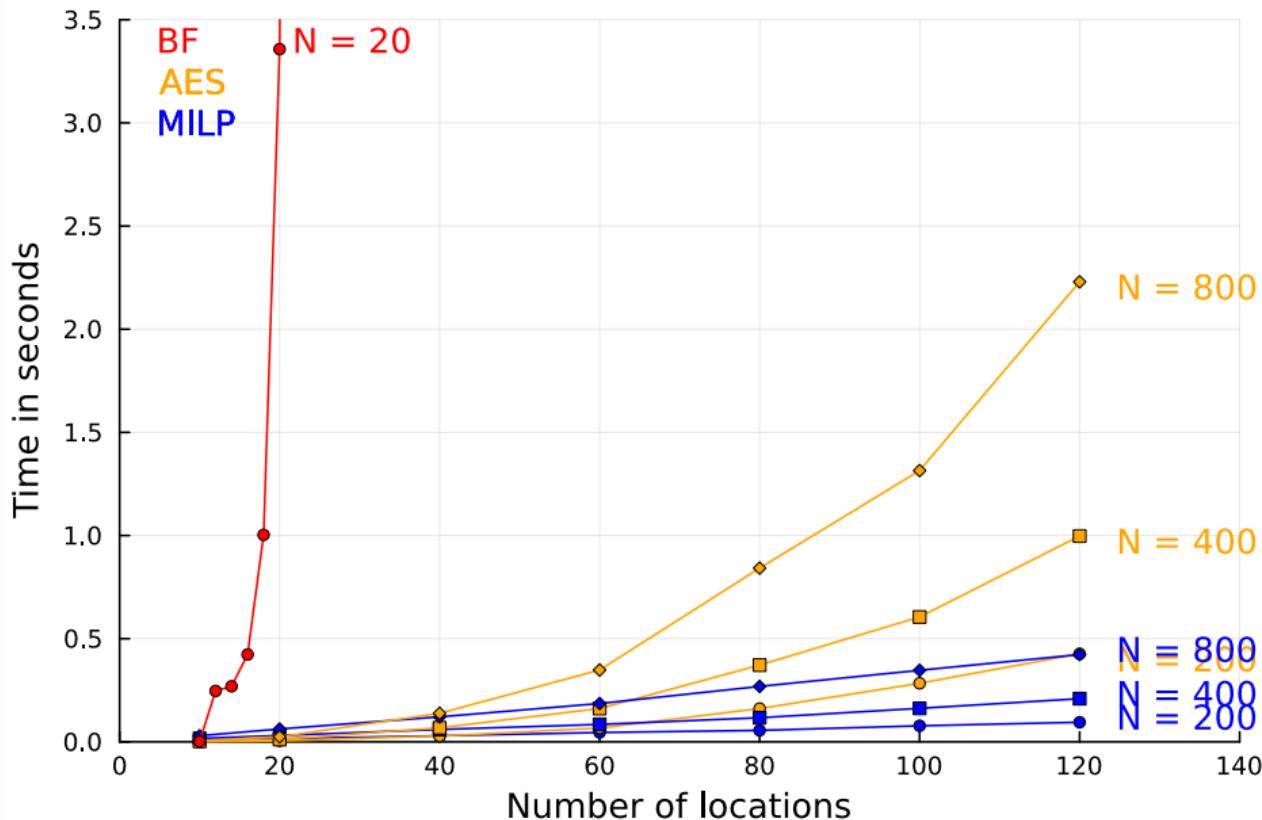
$$\sum_{\ell \in L} x_{\ell n} = 1, \quad n \in N$$

$$x_{\ell n} \leq y_\ell, \quad n \in N, \ell \in L$$

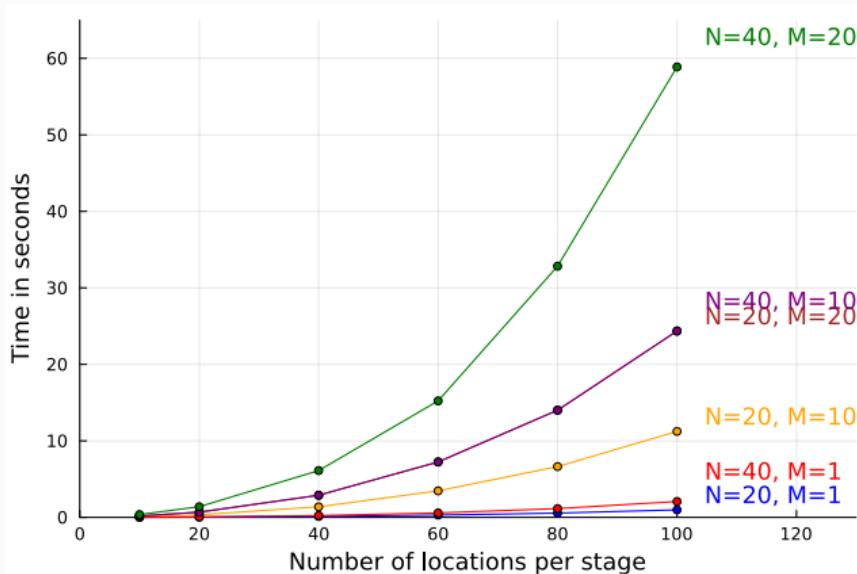
$$x_{\ell n} \geq 0, \quad n \in N, \ell \in L$$

$$y_\ell \in \{0, 1\}, \quad \ell \in L$$

Comparing AES, MILP, and brute force



Computational performance for $K = 2$, MILP



When $K = 2$, possible path for each model-market mn is L^2 . Time increases exponentially with the potential number of locations per stage.

N and M are equivalent in characterizing the problem dimension.

▶ $K = 3$

Computation perform for $K = 3$, MILP

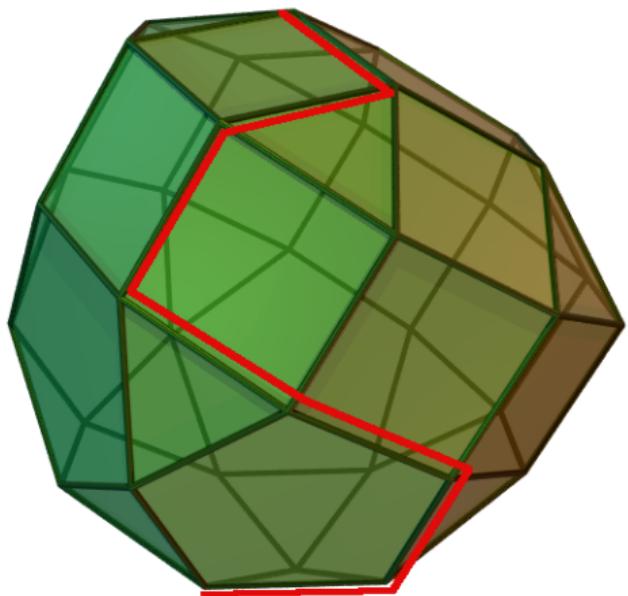
Panel (a): $K = 2$

L_k	N = 20			N = 40		
	M = 1	M = 10	M = 20	M = 1	M = 10	M = 20
10	0.01	0.09	0.18	0.02	0.17	0.37
20	0.03	0.34	0.68	0.06	0.68	1.39
40	0.12	1.38	2.89	0.23	2.89	6.12
60	0.3	3.46	7.25	0.57	7.24	15.22
80	0.56	6.64	14.01	1.14	14.0	32.83
100	0.98	11.24	24.35	2.07	24.36	58.88

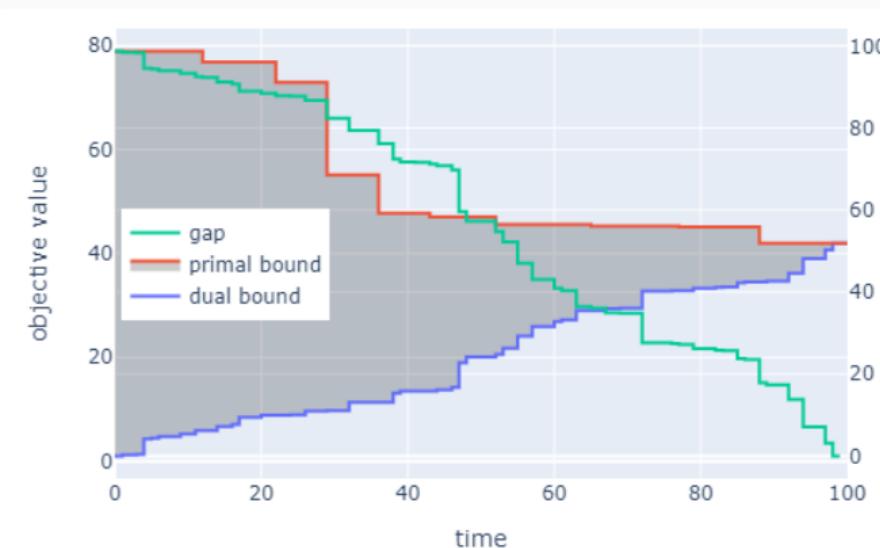
Panel (b): $K = 3$

L_k	N = 20			N = 40		
	M = 1	M = 10	M = 20	M = 1	M = 10	M = 20
10	0.07	0.89	1.86	0.15	1.88	4.08
20	2.02	26.52	57.34	4.15	56.9	130.14

Solving the linear programming problem



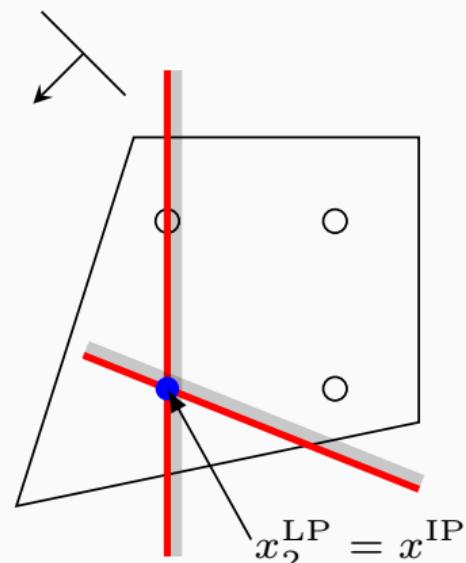
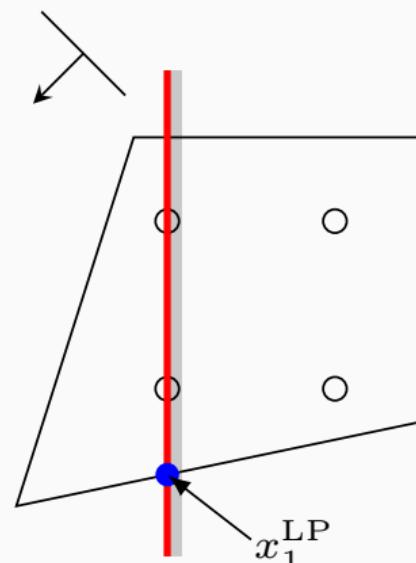
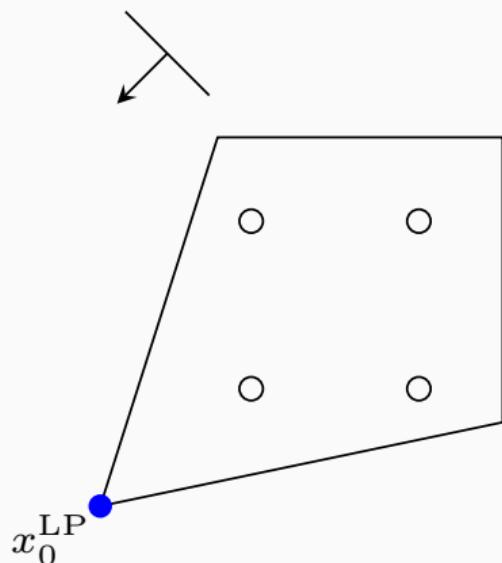
(a) The simplex method



(b) The MIP Gap and duality

Cutting planes and the linear relaxation: an example

Decreasing cost



How Gurobi does it

- The solver is commercial and the code is not public. Best open source alternative (HIGHS) is 5–10 times slower.
- The key component one can track is the “linear relaxation.” LP is really fast even for large problems.
- Gurobi tells us when the LP gives binary y results.
- When the linear relaxation is integral, we don’t need the time-consuming methods: cutting planes, heuristics, branch & bound (smart, exhaustive search).
- Our experience, confirming past OR lit, is that the LP relaxation worked in the vast majority of cases. But no guarantees outside special cases.

Calibrating CES Demand

- CES demand: $q_{mn} = R_n^{\text{EV}} (P_n^{\text{EV}})^{\varepsilon-1} \xi_{mn}^{\varepsilon-1} p_{mn}^{-\varepsilon}$
- EV Price index $n \in N$ consumer countries/markets

$$P_n^{\text{EV}} = \left[\sum_m y_{mn} \left(\frac{p_{mn}}{\xi_{mn}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

- Expenditure share on EVs:

$$\frac{R_n^{\text{EV}}}{R_n} = \frac{(P_n^{\text{EV}})^{1-\varepsilon}}{(P_n^{\text{EV}})^{1-\varepsilon} + (P_n^{\text{ICE}})^{1-\varepsilon}} = \left(\frac{P_n^{\text{EV}}}{P_n} \right)^{1-\varepsilon}$$

- Model appeal (ξ_{mn}) relative to market mean appeal ($\tilde{\xi}_n$) inversion:

$$\frac{\xi_{mn}}{\tilde{\xi}_n} = \left(\frac{p_{mn}}{\tilde{p}_n} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{q_{mn}}{\tilde{q}_n} \right)^{\frac{1}{\varepsilon-1}}$$

Own-price elasticities from the auto literature

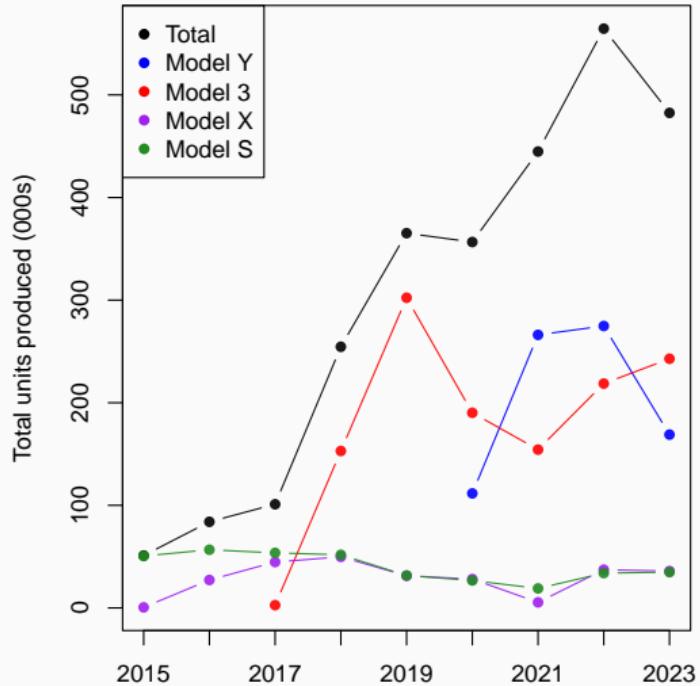
EV-only	elas.	Mainly ICEV	elas.
Barwick et al. (2024)	4.2	Beresteanu and Li (2011)	8.4
Kwon (2023)	4.4	Castro-Vincenzi (2024)	4.3
Li et al. (2022)	3.7	Colon and Gortmaker (2020)	3.9
Li (2023)	3.7	Coşar et al. (2018)	14.9
Li et al. (2017)	1.3	Goldberg (1995)	3.3
Linn (2022)	5.3	Goldberg and Verboven (2001)	5.2
Muehlegger and Rapson (2022)	2.1	Grieco et al. (2024)	5.4
Springel (2021)	1.8	Head and Mayer (2019)	3.9
Xing et al. (2021)	2.8	Li (2018)	9.5
Median	3.7	Median	5.2
Overall Median	4.0		

The 15 top markets of BEVs in 2023

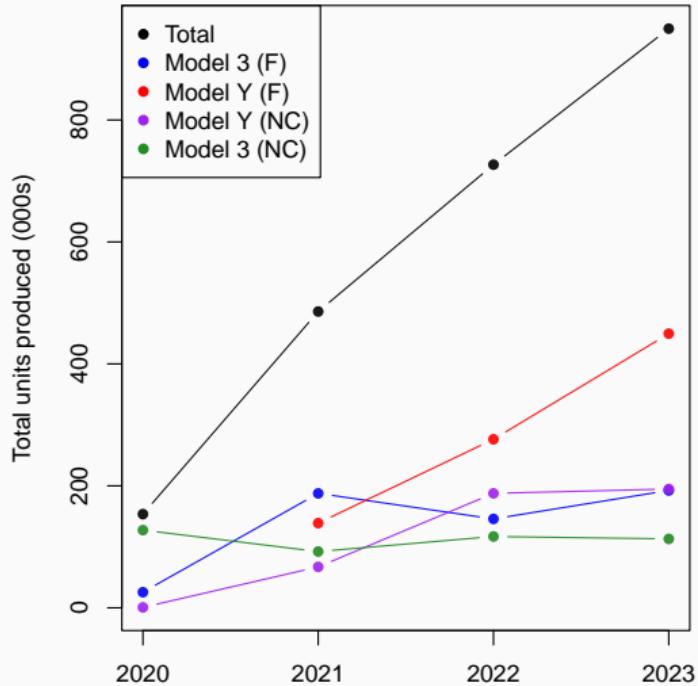
Rank	Country	# Models	# Firms	Sales (000')	EV share (%)
1	China	206	38	5353.0	21.0
2	United States	57	16	1264.3	8.1
3	Germany	124	25	625.8	20.2
4	United Kingdom	104	20	389.7	17.3
5	France	108	20	383.6	17.9
6	South Korea	48	10	181.8	10.7
7	Canada	53	15	160.3	9.4
8	Netherlands	120	24	147.3	33.3
9	Sweden	111	23	143.7	43.1
10	Norway	113	26	132.0	83.7
11	Belgium	111	21	109.4	20.0
12	Japan	49	13	101.0	2.1
13	Australia	51	16	99.8	8.4
14	Italy	105	18	90.5	5.1
15	Thailand	30	13	80.8	10.7
16	Rest Of World	38	11	743.6	7.2

Note: RoW reports averages across 59 countries for # Models and EV % (sum for Sales).

Fact 5b: Capacity can increase rapidly



(a) Tesla-Fremont



(b) Tesla-Shanghai

Fact 5c: Multi-sourcing is sometimes transitory (German Tesla sourcing)

