Dynamic Probabilistic Models for Latent Feature Propagation in Social Networks

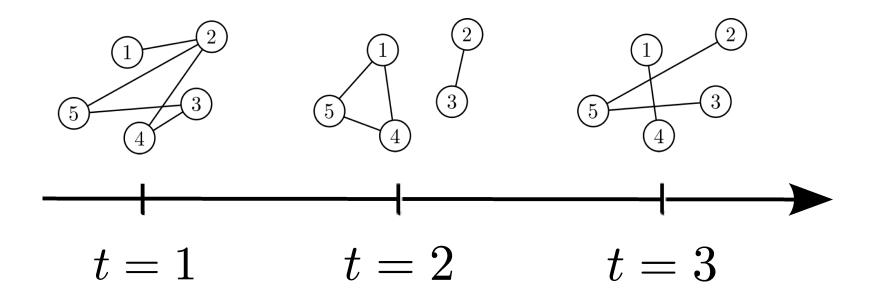
Creighton Heaukulani and Zoubin Ghahramani

University of Cambridge

TU Denmark, June 2013

A Network

Dynamic network data record the link statuses in the network at T time points:



Generative Models of Networks

... encoded as link adjacency matrices $\{\boldsymbol{Y}^{(1)},\boldsymbol{Y}^{(2)},\ldots,\boldsymbol{Y}^{(T)}\}$:

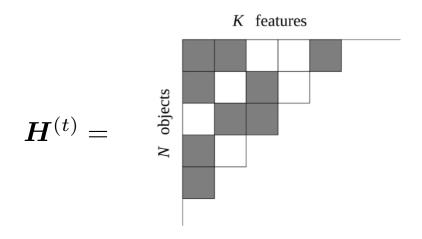
$$m{Y}^{(t)} = \left(egin{array}{ccccc} \cdot & 1 & 0 & 0 & & \\ 1 & \cdot & 1 & 1 & & \\ 0 & 1 & \cdot & 0 & & \\ 0 & 1 & 0 & \cdot & & \\ & & & \cdot & \end{array}
ight)$$

Example: friendship statuses in a social network

- $y_{ij}^{(t)} = 1$: actors i and j are friends at time t;
- $y_{ij}^{(t)} = 0$: actors i and j are not friends at time t.

Latent feature representations

Assume there are K latent features underlying the population. Associate actor n with feature indicators $\boldsymbol{h}_n^{(t)} \in \{0,1\}^K$ at time t:



Interpretation: features represent unobserved hobbies/interests, e.g., if feature k represents "plays tennis", then

- $h_{nk}^{(t)} = 1$: actor n plays tennis at time t;
- $h_{nk}^{(t)} = 0$: actor n doesn't play tennis at time t.

Hidden Markov models

Hidden Markov models assume features evolve independently

$$h_{ik}^{(t)} \mid h_{ik}^{(t-1)} \sim \mathbf{Q}(h_{ik}^{(t-1)}, h_{ik}^{(t)})$$

where Q is a Markov transition matrix. Then the edges are conditionally independent given the latent features

$$y_{ij}^{(t)} \mid \boldsymbol{h}_i^{(t)}, \boldsymbol{h}_j^{(t)} \sim \text{Bernoulli} \left[\sigma \left(\sum_{k\ell} h_{ik}^{(t)} h_{j\ell}^{(t)} v_{k\ell} \right) \right]$$

where V is a feature-interaction weight matrix

$$v_{k\ell} \sim \mathcal{N}(0, \sigma_H^2)$$

An Example

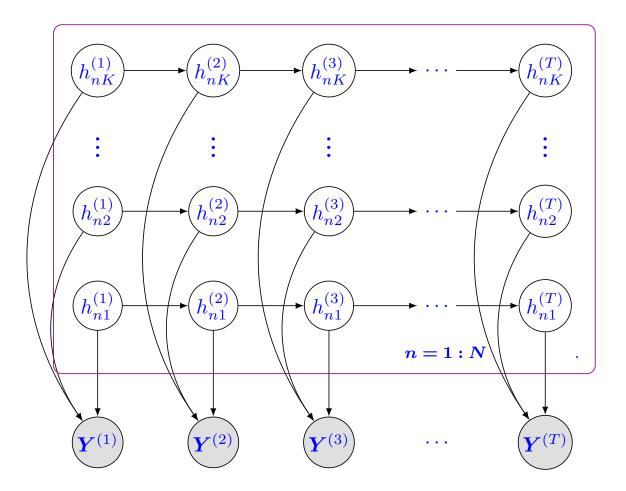
Consider the following example: feature $h_{ik}^{(t)}$ evolves as^a

$$h_{ik}^{(t)} \mid h_{ik}^{(t-1)} \sim \text{Bernoulli}\left(a_k^{1-h_{ik}^{(t-1)}} b_k^{h_{ik}^{(t-1)}}\right)$$

- $a_k \in [0,1]$ controls the probability of feature k switching from off to on
- $b_k \in [0,1]$ controls the persistency of feature k in the off state

^aFinite version of the DRIFT model from ?.

Factorial HMM



Each feature evolves over time independently. The latent feature configuration at a given time step produces the observed network. See ?.

Latent Feature Evolution

But consider the following:

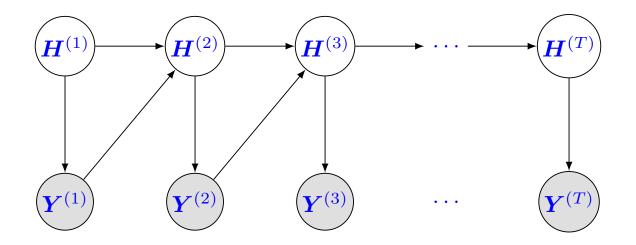
• If my friends enjoy playing tennis, I am likely to start playing tennis

• If a friend gets me to join the tennis team, then I am more likely to be riend other tennis players

We call this phenomenon latent feature propagation

Latent Feature Propagation

Want something more like this:



Network observations influence future latent features; information propagates between the observed and latent structures throughout the network over time

Latent Feature Propagation

We use the following model (?)

$$h_{ik}^{(t+1)} \mid \mu_{ik}^{(t+1)} \sim \text{Bernoulli} \left[\sigma \left(c_k \left[\mu_{ik}^{(t+1)} - b_k \right] \right) \right]$$

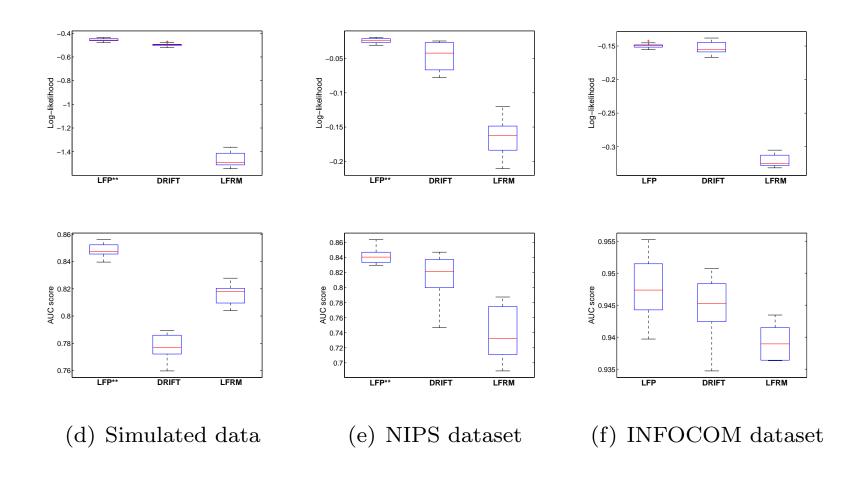
$$\mu_{ik}^{(t+1)} = (1 - \lambda_i) h_{ik}^{(t)} + \lambda_i \frac{h_{ik}^{(t)} + \sum_{j \in \varepsilon(i,t)} w_j h_{jk}^{(t)}}{1 + \sum_{j' \in \varepsilon(i,t)} w_{j'}}$$

- 1. $\lambda_i \in (0,1)$: actor *i*'s susceptibility to the influence of friends; $(1-\lambda_i)$ is the corresponding measure of social independence;
- 2. $w_i \in \mathbb{R}_+$: the weight of influence of person i;
- 3. $c_k \in \mathbb{R}_+$: a scale parameter for the persistence of feature k;
- 4. $b_k \in \mathbb{R}_+$: a bias parameter for feature k.

Latent Feature Propagation

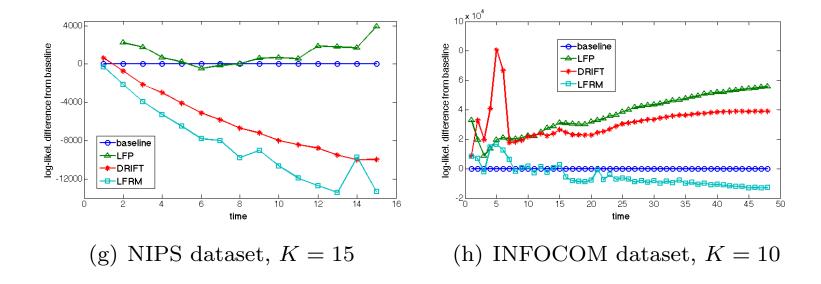
- Inference by MCMC; Forward-Backwards Algorithm from ?
- Datasets:
 - Simulated: N = 50, T = 100, K = 10.
 - NIPS: N = 110, T = 17, K = 15.
 - INFOCOM (?): N = 78, T = 50, K = 10.

Prediction of Missing Links



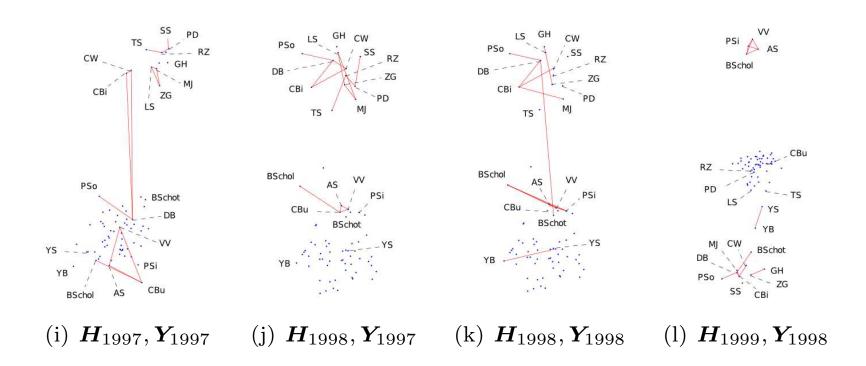
Top: log-likel. of test edges. Bottom: AUC scores for classifying test edges. 10 repeats on different 20% hold-outs. Averaged over 300 samples. Significance indicated by $(\star\star)$ $(\alpha=0.05)$.

Forecasting Future Networks



Forecasting a future unseen network. Differences from a naive baseline of the log-likelihoods of $\mathbf{Y}^{(t)}$ after training on $\mathbf{Y}^{(1:t-1)}$

Visualising Feature Propagation



Visualising feature propagation in the NIPS dataset (K = 15).

Visualising Feature Propagation

We can look at the research interests of a trio of sparsely linked authors between 1997 and 1998 who are nearby in latent space:

Author & year	Topics
Top cluster	"Prior Knowledge in Support Vector Kernels"
Bengio_Y, Singer_Y, et al.	"Shared Context Probabilistic Transducers"
Hinton_G, Ghahramani_Z	"Hierarchical Non-linear Factor Analysis and Topographic Maps"
Bishop_C, Williams_C, et al.	"Regression w/ Input-depend. Noise: A Gaussian Process Treatment"
Sollich_P, Barber_D	"On-line Learning from Finite Training Sets in Nonlinear Networks"
Barber_D, Bishop_C	"Ensemble Learning for Multi-Layer Networks"
Bishop_C, Jordan_M	"Approx. Posterior Distributions in Belief Networks Using Mixtures"

Visualising Feature Propagation

We can also examine the five authors with the largest inferred weights w_n and some of their research interests (1995 - 1999):

Author	w_n (relative)	Topics
Barto_A	1.55	reinforcement learning, planning algorithms
$Rasmussen_C$	1.32	Gaussian processes, Bayesian methods
$Vapnik_{-}V$	1.29	SVMs, learning theory
$Scholkopf_B$	1.28	SVMs, kernel methods
$Tresp_{-}V$	1.26	neural networks, Bayesian methods

References

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