Phys 243 HW #1

Problem 2.1

$$M = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$$

$$\text{Det}(M) = (1)\left[(1)(-2) - (-2)(-2)\right] - (-4)\left[(-4)(-2) - (2)(-2)\right] + 2\left[(-4)(-2) - (1)(2)\right]$$

$$= -6 + 48 + 12 = 54.$$

$$M^T = \begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$$

Inv. for M

$$\left[\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ -4 & 1 & -2 & 0 & 1 & 0 \\ 2 & -2 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \times 4 + R_2 \to R_2 \\ R_1 \times 2 - R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & -15 & 6 & 4 & 1 & 0 \\ 0 & -6 & 6 & 2 & 0 & -1 \end{array}\right]$$

$$\begin{array}{l} R_2 \div -15 \\ R_3 \div 6 \end{array} \left[\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2/5 & \frac{-4}{15} & \frac{-1}{15} & 0 \\ 0 & -1 & 1 & \frac{1}{3} & 0 & \frac{-1}{6} \end{array}\right] \xrightarrow[R_2 + R_3 \to R_3]{R_2 \times 4 + R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2/5 & \frac{-1}{15} & \frac{-4}{15} & 0 \\ 0 & 1 & -2/5 & \frac{-4}{15} & \frac{-1}{15} & 0 \\ 0 & 0 & 3/5 & \frac{1}{15} & \frac{-1}{15} & \frac{-1}{6} \end{array}\right]$$

$$R_3 \times \frac{5}{3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2/5 & -1/15 & -4/15 & 0 \\ 0 & 1 & -2/5 & -4/15 & -1/15 & 0 \\ 0 & 0 & 1 & 1/9 & -1/9 & -5/18 \end{array}\right] \begin{array}{l} R_3 \times \frac{-2}{5} + R_1 \\ R_3 \times \frac{2}{5} + R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/9 & -2/9 & 1/9 \\ 0 & 1 & 0 & -2/9 & -1/9 & -1/9 \\ 0 & 0 & 1 & 1/9 & -1/9 & -5/18 \end{array}\right] \quad \begin{bmatrix} -\frac{1}{9} & \frac{-2}{9} & \frac{1}{9} \\ \frac{-2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{-1}{9} & \frac{-5}{18} \end{bmatrix}$$

2.2 $\begin{vmatrix} 1-\lambda & -4 & 2 \\ -4 & 1-\lambda & -2 \\ 2 & -2 & -2-\lambda \end{vmatrix} = (1-\lambda)[(1-\lambda)(-2-\lambda)-(-2)(-2)] + 4[(-4)(-2-\lambda) - 2(-2)]$
$+2[(-4)(-2)-2(1-\lambda)$

$= (1-\lambda)[-2-\lambda+2\lambda+\lambda^2-4] + 4[8+4\lambda+4] + 2[8-2+2\lambda]$
$= (1-\lambda)(\lambda^2+\lambda-6) + 4(4\lambda+12) + 2(6+2\lambda)$
$= \lambda^2+\lambda-6-\lambda^3-\lambda^2+6\lambda+16\lambda+48+12+4\lambda$
$= -\lambda^3+27\lambda+54 = -(\lambda^3-27\lambda-54) = -(\lambda+3)(\lambda^2-3\lambda-18)$
$= -(\lambda+3)(\lambda+3)(\lambda-6)$ $\quad \lambda=-3,6$

For $\lambda_1=-3$. $\left[\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ -4 & 4 & -2 & 0 \\ 2 & -2 & 1 & 0 \end{array}\right]$ $R_1 \div 4$ $\left[\begin{array}{ccc|c} 1 & -1 & \frac{1}{2} & 0 \\ -4 & 4 & -2 & 0 \\ 2 & -2 & 1 & 0 \end{array}\right]$

$R_1 \times 4 + R_2 \to R_2$
$R_1 \times (-2) + R_3 \to R_3$ $\begin{bmatrix} 1 & -1 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 - x_2 + \frac{1}{2}x_3 = 0$
$x_1 = x_2 - \frac{1}{2}x_3$. $\begin{bmatrix} x_2 - \frac{1}{2}x_3 \\ x_2 \\ x_3 \end{bmatrix}$.

Let $x_2 = 1, x_3 = 0$ $\quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$x_2 = 0, x_3 = 1$ $\quad v_2 = \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix}$

For $\lambda_2 = 6$ $\begin{vmatrix} -5 & -4 & 2 \\ -4 & -5 & -2 \\ 2 & -2 & -8 \end{vmatrix}$ $R_1 \div 5$ $\begin{vmatrix} 1 & \frac{4}{5} & \frac{-2}{5} \\ -4 & -5 & -2 \\ 2 & -2 & -8 \end{vmatrix}$ $\begin{array}{l} R_1 \times 4 + R_2 \\ R_1 \times (-2) + R_3 \end{array}$ $\begin{vmatrix} 1 & \frac{4}{5} & \frac{-2}{5} \\ 0 & -\frac{9}{5} & -\frac{18}{5} \\ 0 & -\frac{18}{5} & -\frac{36}{5} \end{vmatrix}$

$\begin{array}{l} R_2 \times (-\frac{5}{9}) \\ R_3 \times (-\frac{5}{36}) \end{array}$ $\begin{vmatrix} 1 & 4/5 & -2/5 \\ 0 & 1 & 2 \\ 0 & \frac{1}{2} & 1 \end{vmatrix}$ $R_3 \times 2 - R_2$ $\begin{vmatrix} 1 & 4/5 & -2/5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$ $R_2 \times (\frac{-4}{5}) + R_1$ $\begin{vmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$

$$x_1 - 2x_3 = 0 \Rightarrow x_1 = 2x_3$$
$$x_2 + 2x_3 = 0 \qquad x_2 = -2x_3 \qquad X = \begin{pmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{pmatrix}$$

Let $x_3 = 1$ $\quad V_3 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

2.3. $\frac{\partial f}{\partial x_{11}} = 2x_{11} + 1 \quad \frac{\partial f}{\partial x_{22}} = 1 \quad \frac{\partial f}{\partial x_{33}} = -2x_{33}$

$$\nabla_A = \begin{pmatrix} 2x_{11}+1 \\ 1 \\ -2x_{33} \end{pmatrix}$$

2.4. $g_x(x,y,z) = 3x^2y + yz\cos x + y^2z^5 \qquad g_{xx} = 6xy - yz\sin x$

$g_y = x^3 + z\sin x + 2xyz^5 \qquad g_{yy} = 2xz^5$

$g_z = y\sin x + 5xy^2z^4 \qquad g_{zz} = 20xy^2z^3$

$g_{xy} = g_{yx} = 3x^2 + z\cos x + 2yz^5 \qquad g_{xz} = g_{zx} = y\cos x + 5y^2z^4$

$g_{yz} = g_{zy} = \sin x + 10xyz^4$

$$H = \begin{bmatrix} 6xy - yz\sin x & 3x^2 + z\cos x + 2yz^5 & y\cos x + 5y^2z^4 \\ & 2xz^5 & \sin x + 10xyz^4 \\ y\cos x + 5y^2z^4 & & 20xy^2z^3. \end{bmatrix}$$