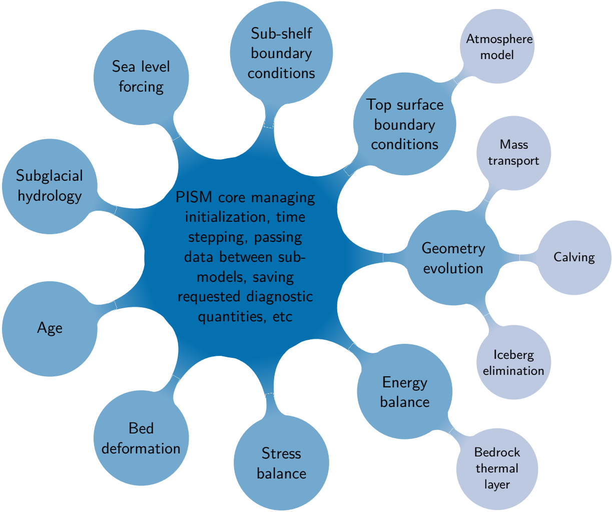
PISM: What does it do? And how does it work?

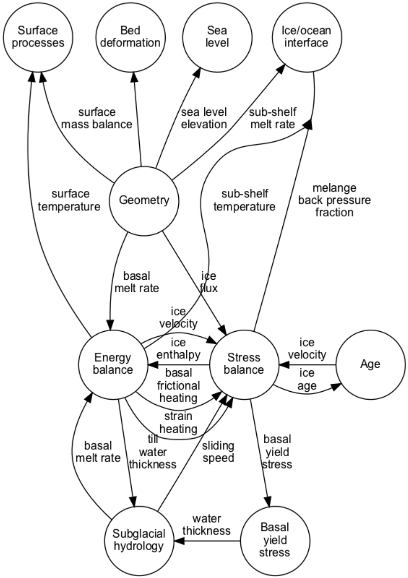
C. Khrulev

University of Alaska Fairbanks

PISM consists of...



Interaction between sub-models



Sub-models use quantities shown on the left.
Also,

- all sub-models use ice thickness,
- or a combination of
    - ice thickness,
    - bed elevation,
    - sea level elevation.

Interaction between sub-models

Which, at a first glance, looks very much like this.

Interaction between sub-models

This is why implicit time-stepping is hard.

...but we don't have to use explicit (or implicit) time-stepping for *all* its components!

Explicit time stepping in an ice sheet model

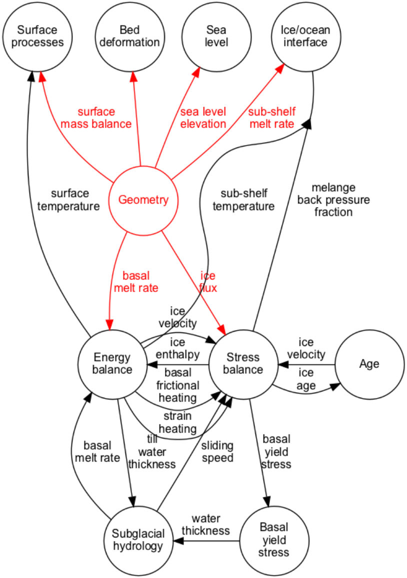
1. use stress balance to compute velocity
   - often: first get (u, v), then w from incompressibility
2. update models of other physical processes: thermodynamics, basal melt, calving, etc
3. decide on time-step Δt
   - using adaptive time stepping (SIA diffusivity, the CFL criterion, etc), or
   - select a fixed time-step
4. from velocity, surface mass balance, and basal balance perform a time-step of the mass continuity equation to get $\frac{\partial H}{\partial t}$
5. update surface elevation: $h \leftarrow h + \frac{\partial H}{\partial t} \Delta t$
6. repeat at 1.

PISM's time step

Update...

1. basal yield stress
2. stress balance (ice velocities)
3. the time step length using stability criteria
4. age of the ice
5. energy balance
6. ice geometry due to flow
7. sea level
8. sub-shelf boundary conditions
9. top-surface boundary conditions
10. ice geometry due to surface and basal mass balance
11. subglacial hydrology
12. bed deformation

Geometry evolution model



PISM's geometry evolution model:

- mass transport (modeling flow)
- calving
- iceberg elimination

Mass transport

$$\frac{\partial H}{\partial t} = S - \nabla \cdot (Q + \mathbf{U}_b H)$$

- uses the finite-volume approach
- tailored to take advantage to PISM's split between "diffusive" and "advective" ice flow
- ice flow is diffusive, so numerical diffusivity does not cause much trouble
- surface and basal mass balance may be negative and extra care is needed to preserve non-negativity of ice thickness

Stability considerations

The SIA flow is diffusive, so preserving staility of explicit time stepping forces us to respect diffusion-related time step restrictions.

Recall that

$$
\begin{aligned}
\frac{\partial H}{\partial t} &= S - \nabla \cdot (Q + \mathbf{U}_b H), \\
Q &= -D \nabla (b + H).
\end{aligned}
$$

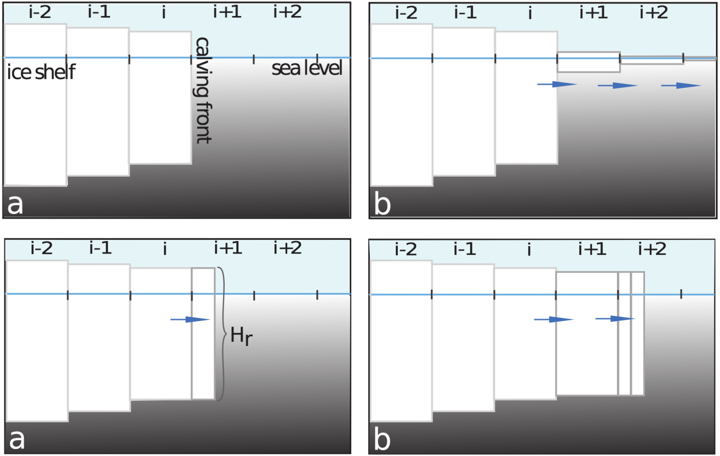
This applies to other models, even if they don't use the shallow ice approximation.

$$\Delta t \leq \Delta x^2 / D_{\text{max}}.$$

First-order upwinding for the advective part of the flux gives

$$\Delta t \leq \frac{1}{\max |u| / dx + \max |v| / dy}.$$

Free boundary at the calving front



See T. Albrecht, M. Martin, M. Haseloff, R. Winkelmann, and A. Levermann. Parameterization for subgrid-scale motion of ice-shelf calving fronts. *The Cryosphere*, 5:35–44, 2011 for details.

Calving

- Geometric criteria
  - ice thickness threshold
  - floating ice
  - prescribed maximum shelf extent
- Stress-based
  - Eigen calving
    $$c = K\, \dot{\epsilon}_+\, \dot{\epsilon}_- \quad \text{and} \quad \dot{\epsilon}_\pm > 0\,.$$
    See *Kinematic first-order calving law implies potential for abrupt ice-shelf retreat* by Levermann et al.
  - von Mises
    $$c = |\mathbf{u}|\frac{\tilde{\sigma}}{\sigma_{max}},$$
    See *Modeling of Store Gletscher's calving dynamics, West Greenland, in response to ocean thermal forcing.* by Morlighem et al.

Iceberg elimination

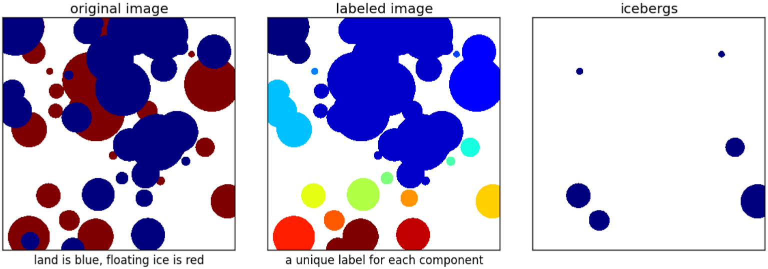
Why do we need this?

- The SSA stress balance determines the sliding velocity of patches of floating ice *up to rigid body motions* (see C. Schoof, *A variational approach to ice stream flow*)
- We need to keep track of an ice sheet's discharge into the ocean.

Where would an "iceberg" (isolated patch of floating ice) come from?

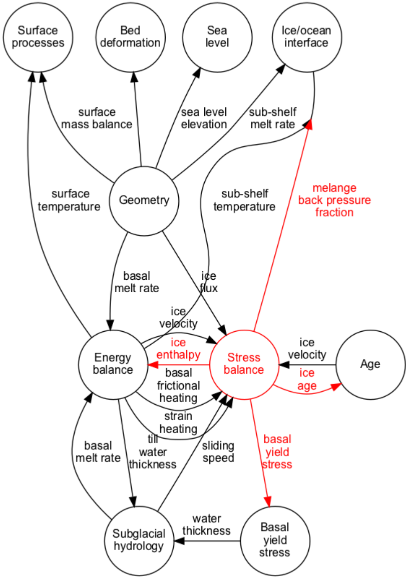
- calving
- sea level changes

Iceberg elimination: technical details



- uses a 2-scan connected component labeling algorithm (serial code)
- computationally cheap

Stress balance model



PISM's "hybrid" stress balance:

1. basal sliding (SSA)
2. deformational flow (SIA)

Vertical velocity is approximated using incompressibility of ice.

PISM supports Glen-like flow laws *that have a viscosity form*:

$$\sigma'_{ij} = 2\nu(D, T, \omega, P, d)\, D_{ij}.$$

Shallow ice approximation (SIA)

$$
\begin{aligned}
\frac{\partial H}{\partial t} &= S - \nabla \cdot (Q + \mathbf{U}_b H),\\
Q &= -D \nabla (b + H),\\
D &= 2 \int_b^h F(z) P(z) (h - z) dz.
\end{aligned}
$$

Can be computed at each grid point independently (using neighboring points for FD approximations).

Schoof's parameterized bed roughness technique

Modifies (reduces!) the diffusivity of the SIA flow.

- is an improvement compared to SIA
- reduces diffusivity, making longer time steps possible

See the User's Manual and *The effect of basal topography on ice sheet dynamics* by C. Schoof. for details.

Shallow shelf approximation (SSA)

$$
\begin{aligned}
-\left[2\bar{\nu}H\left(2u_x+v_y\right)\right]_x-\left[\bar{\nu}H\left(u_y+v_x\right)\right]_y-\tau_{(b)1}&=-\rho g H h_x\\
-\left[\bar{\nu}H\left(u_y+v_x\right)\right]_x-\left[2\bar{\nu}H\left(u_x+2v_y\right)\right]_y-\tau_{(b)2}&=-\rho g H h_y
\end{aligned}
$$

Here both $\bar{\nu}$ and τ_b are nonlinear functions of the ice velocity.

At lateral boundaries we have:

$$
t\big|_{\mathrm{cf}}\cdot\mathbf{n}=-p_{\mathrm{o}}\mathbf{n}.
$$

Note: basal (sliding) velocity is *not* prescribed.

Basal strength: sliding laws

- Coulomb

$$|\boldsymbol{\tau}_b| \leq \tau_c \quad \text{and} \quad \boldsymbol{\tau}_b = -\tau_c \frac{\mathbf{u}}{|\mathbf{u}|} \quad \text{if and only if} \quad |\mathbf{u}| > 0.$$

- Pseudo-plastic

$$\boldsymbol{\tau}_b = -\tau_c \frac{\mathbf{u}}{u_{\text{threshold}}^q |\mathbf{u}|^{1-q}},$$

Basal strength: yield stress models

- constant
- Mohr-Coulomb

$$\tau_c = c_0 + (\tan\phi)\, N_{till},$$

$$N_{till} = \min\left\{P_o, N_0 \left(\frac{\delta P_o}{N_0}\right)^s 10^{(e_0/C_c)(1-s)}\right\}.$$

See *Basal mechanics of Ice Stream B, West Antarctica 1. Till mechanics* by Tulaczyk et al.

Basal strength: inputs and outputs

Inputs

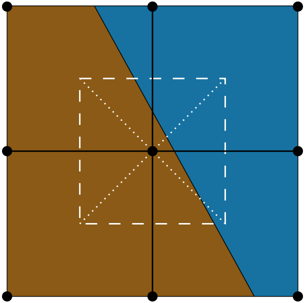
- geometry
- till water thickness
- subglacial water thickness

Outputs

- basal material yield stress

Grounding line treatment

Looking at a 2D cell at the grounding line (in the map plane):



Grounding line treatment

… very similar to SEP1 in *Hydrostatic grounding line parameterization in ice sheet models* by Seroussi et al.

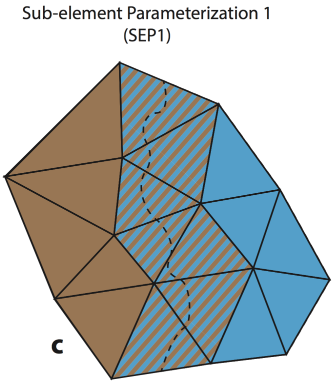


Figure 1. Grounding line discretization. G

SSA: implementation details

PISM uses Picard iteration to solve the non-linear system corresponding to the discretization of the SSA system of equations:

$$A(U^{(k)})U^{(k+1)} = b.$$

Note: the system solved by PISM has the same size independent from the current ice extent.

Challenges:

- stopping criterion
- robustness
- computational cost

Stress balance: inputs and outputs

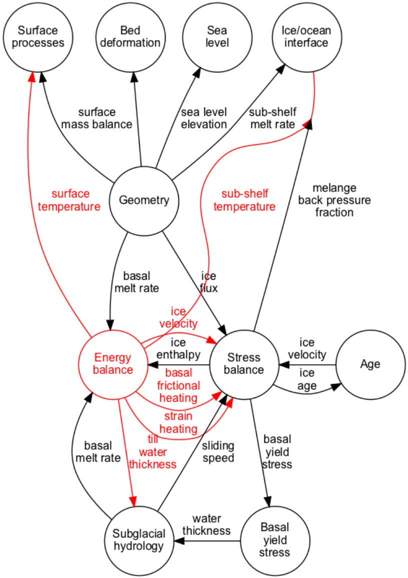
Inputs

- geometry
- basal yield stress
- melange back pressure fraction
- ice enthalpy
- age of the ice

Outputs

- 2D advective velocity
- 2D diffusive flux
- maximum diffusivity
- max time-step allowed by the CFL criterion
- u, v, and w components of the ice velocity
- basal frictional heating
- volumetric strain heating

Energy balance model



PISM's energy balance model

- uses enthalpy-based (or temperature-based) conservation equations
- has a bedrock thermal layer

Energy balance in the ice volume

- *shallow* conduction-advection equation

$$\rho_i \left(\frac{\partial E}{\partial t} + w \frac{\partial E}{\partial z} \right) - \frac{\partial}{\partial z} \left(K(E) \frac{\partial E}{\partial z} \right) = Q - \rho_i \left(u \frac{\partial E}{\partial x} + v \frac{\partial E}{\partial y} \right).$$

- non-linear conductivity (function of enthalpy)
- implicit in the vertical dimension, explicit in horizontal
- first order upwinding for advecion
- has a CFL time step restriction using 3D u and v components of the ice velocity

Energy balance: initialization difficulties

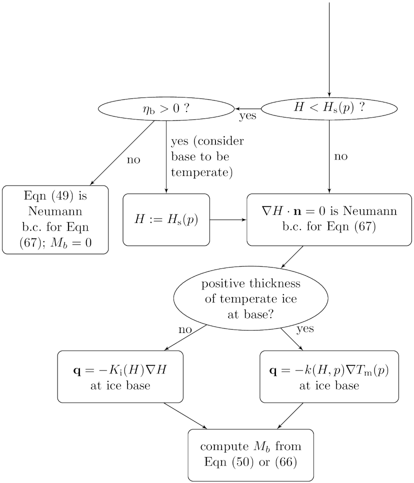
Available measurements of the ice temperature and water content are *very* sparse both in time and in space.

Possible work-arounds:

- fill using heuristics (PISM has two)
- use a spin-up procedure to improve on heuristics by including ice dynamics and the past climate
- data assimulation (read: use fields provided by a different model)

Energy balance: boundary conditions

- top: Dirichlet
- bottom: Dirichlet or Neumann



Energy balance model: inputs and outputs

Inputs

- geometry
- basal frictional heating
- ice shelves: basal temperature
- grounded areas
  - upward heat flux from the bedrock thermal layer
  - till water thickness
- top surface boundary conditions (enthalpy or temperature and liquid water fraction)
- ice velocity

Outputs

- ice enthalpy
- grounded areas: basal melt rate

Bedrock thermal layer

Models the temperature in a thin (≈ 1000 km) layer a the top of the Earth's crust.

- Assumes that this layer is *shallow*: horizontal conduction terms are ignored, i.e. columns of bedrock are treated as independent.
- Uses a regular grid in bedrock columns, fully implicit, no time-step restriction.

Bedrock thermal layer: inputs and outputs

Inputs

- geothermal flux
- temperature of the top surface of the layer

Outputs

- upward heat flux through the top surface of the layer

Bed deformation

- pointwise isostasy

$$b(t,x,y) = b(0,x,y) - f\left[H(t,x,y) - H(0,x,y)\right].$$

- Lingle-Clark model (elastic plate lithosphere over viscous half-space plus purely-elastic, spherical, layered, self-gravitating earth)

  Note: most of simulations use only the first component (elastic plate lithosphere over viscous half-space).

Lingle-Clark bed deformation model

- the fundamental equation includes a pseudo-differential operator define using Fourier transform
- Fourier spectral collocation method with periodic boundaries on an extended domain
- no time step restriction
- can be initialized using measured uplift rates
- relatively cheap computationally
- serial code

See E. Bueler, C. S. Lingle, and J. A. Kallen-Brown. Fast computation of a viscoelastic deformable Earth model for ice sheet simulation. *Ann. Glaciol.*, 46:97–105, 2007

Bed deformation: inputs and outputs

Inputs

- ice thickness
- sea surface elevation

Outputs

- bed elevation
- bed uplift

Environmental forcing

Sub-models providing boundary (usually climatic) forcing

- Top surface
  - mass flux
  - temperature
  - liquid water fracion
- Bottom surface
  - geothermal flux
- Shelf base
  - shelf base temperature
  - shelf base mass flux
- Sea level

Age

$$\frac{\partial \tau}{\partial t} + u\frac{\partial \tau}{\partial x} + v\frac{\partial \tau}{\partial y} + w\frac{\partial \tau}{\partial z} = 1$$

Because the velocity field is incompressible, $\nabla \cdot (u, v, w) = 0$, and so

$$\frac{\partial \tau}{\partial t} + \nabla\left((u, v, w)\tau\right) = 1.$$

- numerical diffusivity is a concern
- implementation details are the same as in the energy balance code.
- boundary conditions: 0 at the top surface and at the bottom surface with freezing conditions

Subglacial hydrology

- "Null" transport: water stored in till, no water conservation
- "Routing" model: water is stored in till; excess is transported along pressure gradient (water pressure assumed equal to overburden)
- "Distributed" model: same as "routing," but with a physical model of the water pressure

Subglacial hydrology: inputs and outputs

Inputs

- geometry
- water input rate due to drainage from the top surface of the ice
- basal melt rate
- ice sliding speed

Outputs

- till water thickness
- subglacial (transportable) water thickness
- overburden pressure

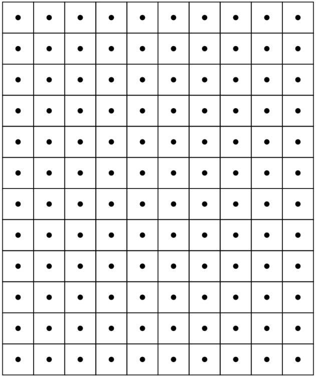
Subglacial hydrology: challenges

- vastly different time scale compared to ice flow
- poorly constrained model parameters (observations are sparse)
- the theory is not mature (no *continuum* model of subglacial water transport in channels)

Technical details

PISM

- uses a uniform cell-centered Cartesian grid
- and a parallel domain decomposition to avoid running out of memory
- dividing the grid into smaller sub-grids.
- Access to “ghost” points requires parallel communication.



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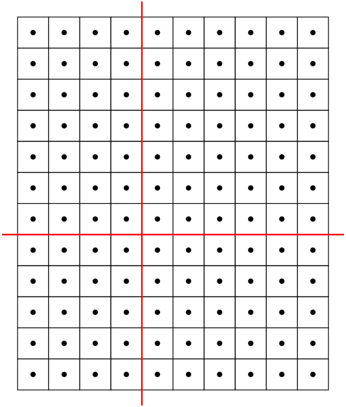

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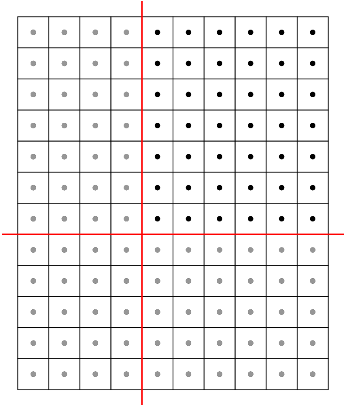

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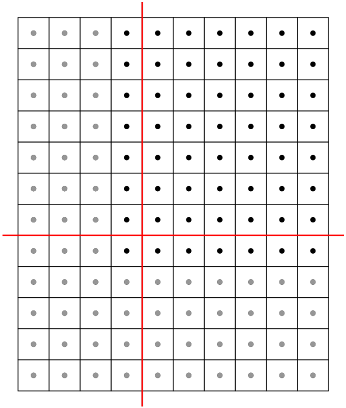


Free boundaries

Both ice thickness and its extent can change, meaning that PISM has to deal with two kinds of free boundaries:

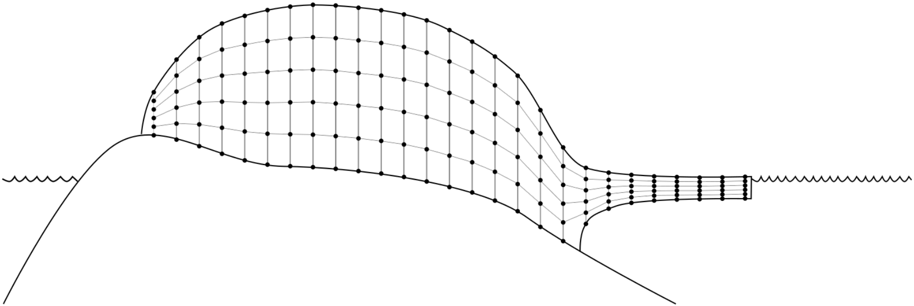
- at the top surface
- lateral (in the map plane)

Free boundary at the top surface

There are two common approaches:

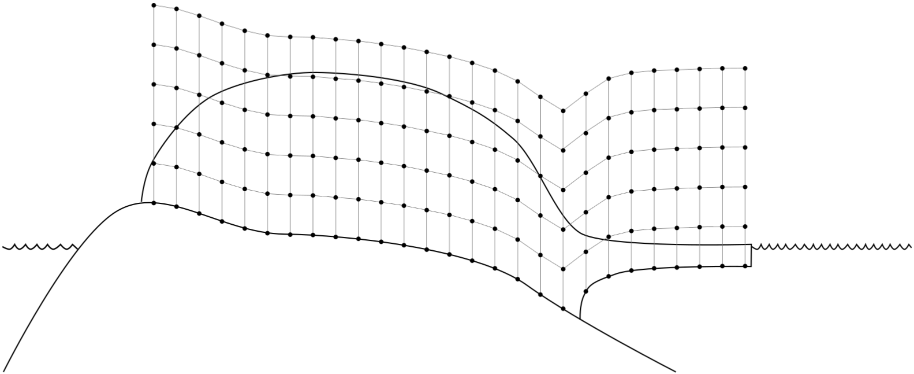
- sigma-coordinate
- "immersed" top boundary plus a transformed vertical coordinate

Vertical sigma coordinate



This approach is used by some other models (e.g. CISM, SICOPOLIS), but not by PISM.

PISM's vertical grid



- does not affect numerical stability
- negative effects can be reduced by increasing grid resolution

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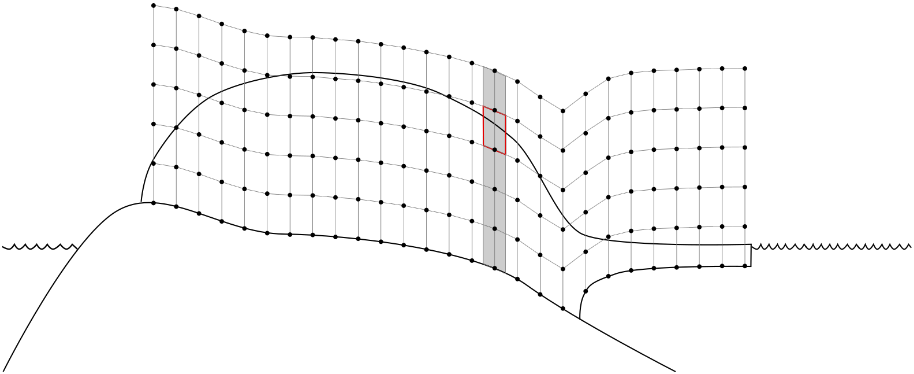

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