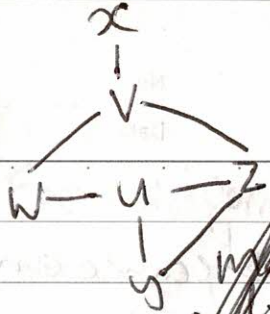
No.

Date. / /

Q1



~~For automorphism f that does not map every vertex to itself, means that a different vertex must be swapped in for each vertex of the graph G. eg u→w~~

Isomorphisms preserve degsequence.

~~Imops~~ Impossible to map x to a different vertex because x is the only vertex with degree of 1 and vertices can't be mapped to itself.

Therefore f cannot exist

~~[illegible]~~ Q2

(A) Euler circuit: deg of every vertex must be even
 middle is odd. Not euler circuit, when middle not odd i.e W_5, other vertices are odd.

(B)  Hamiltonian cycle: Yes because as everything is connected to middle node there can be no closed loop so all vertices can be accessed.

(C) K_3 because K_1 is just dot. K_2 is line ⟶ with no going back but from K_3 every vertex can be accessed from any vertex thus hamiltonian.

(d) From K_3 onwards for any n where n is odd as each vertex is connected to every other which means every vertex has degree $n-1$ and degree must be even to form euler circuit.

$\boxed{Q3}$

(A) 6 dragons, 200 types $= \binom{200}{6}$

1 leader, 6 dragons $= \binom{6}{1}$

so $\binom{200}{6} \times \binom{6}{1} = \frac{200!}{(200-6)!\,6!} \times \frac{6!}{(6-1)!\,1!}$

SEE PAGE 4 FOR JUSTIFICATION OF Q3

$= \frac{200 \times 199 \times 198 \times 197 \times 196 \times 195 \times \cancel{194!}}{\cancel{194!}\,6!} \times \frac{6 \times \cancel{5!}}{\cancel{5!} \times 1}$

$= \frac{200 \times 199 \times 198 \times 197 \times 196 \times 195}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{6}{1} = 4.95 \times 10^{11}$

(B) 20 pizza, 5 types, at least 4 of either margherita or vegetarian,

~~4(M or V) = $\binom{4}{2}$, remainder = 16 pizzas ⟹ $\binom{16}{5}$~~

$4(M \text{ or } V) = \binom{n+k-1}{k-1} = \binom{4+2-1}{2-1} = \binom{5}{1}$

4 picked, 16 remain ⟹ $\binom{16+5-1}{5-1} = \binom{20}{4}$

$\binom{20}{4} \times \binom{5}{1} = \frac{20!}{16!\,4!} \times \frac{5!}{4!\,1!}$

$= \frac{20 \times 19 \times 18 \times 17}{4!} \times \frac{5}{1}$

$= \cancel{[illegible]}\ 24225$

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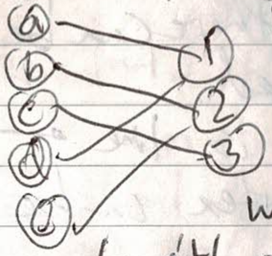
Q3

c) codomain = set Y. Onto = onto every element of codomain. Nonpartial = for every element of domain elements from set X each map to just 1.

~~5 objects, no repetition,~~



Total number of functions (not necessarily surjective) $= |Y|^{|X|}$
$= 3^5$

No. surjective funcs $= 3^5 - |Y_1 \cup Y_2 \cup Y_3|$,
where Y_i = no. functions where nothing mapped to ith element of Y, ie not surjective

$|Y_1 \cup Y_2 \cup Y_3| = (|Y_1| + |Y_2| + |Y_3|) - (|Y_1 \cap Y_2| + |Y_1 \cap Y_3| + |Y_2 \cap Y_3|) + (|Y_1 \cap Y_2 \cap Y_3|)$.

$Y_i = |Y|^{|X|} = 2^5$. $\Rightarrow (|Y_1| + |Y_2| + |Y_3|) = 3 \times 2^5$

$Y_1 \cap Y_2 = \underline{1}$ (nothing mapped to 1 or 2, so a–e must all be mapped to 3)

$Y_1 \cap Y_3 = \underline{1}$, $Y_2 \cap Y_3 = \underline{1}$ $\quad Y_1 \cap Y_2 \cap Y_3 = 0$ bc no functions.

$|Y_1 \cup Y_2 \cup Y_3| = (3 \times 2^5) - (3 \times 1) + 0 = 93$

No. surjective functions $= 3^5 - 93 = \underline{150}$

3d) Last name starts with K.

4 letters: K _ _ _ where 1 slot is occupied by x and other two have choices from 25 types.

$\Rightarrow (25^2 \times 1) \times 3$
$\hookrightarrow x$ can be in 3 diff positions.

5 letters: $(25^3 \times 1) \times 4$

therefore, name starting with K, between 4–5 letters and exactly one x $= (25^2 \times 3) + (25^3 \times 4)$
$= 64375$ choices available.

Q3A) I chose the n!/r!(n-r)! method because order is not important and there is no repetition of dragons as I only have one of each type.

Q3B) I chose the (n+r-1)!/r!(n-1)! method because order is not important but there is repetition

Q3C) For cardinality of Y1, Y2 and Y3, I chose the n^r method because the order was important and there were repetitions

Q3D) For each letter of the word, I chose the n^r method because the order was important and there were repetitions

Date. / /

Q4

For midpoint to have integer coords in

$$\left(\frac{a+x}{2}, \frac{b+y}{2}, \frac{c+z}{2}\right)$$

$(a+x)$ / $(b+y)$ / $(c+z)$ must = even (even $\div 2$ = even, odd $\div 2 \neq$ integer)

To equal odd 1 must be even and other = odd.

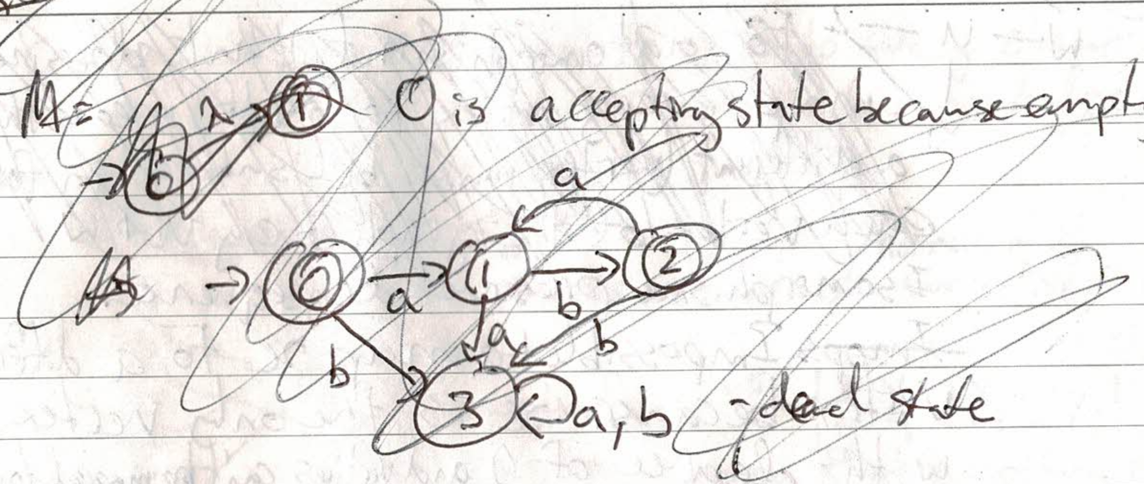
Arrangements = (O = odd, E = even)

Each: (O+E, O+E, O+E)
(O+E, O+E, E+O)
(O+E, E+O, O+E)
(O+E, E+O, E+O)
(E+O, O+E, O+E)
(E+O, O+E, E+O)
(E+O, E+O, O+E)
(E+O, E+O, E+O)

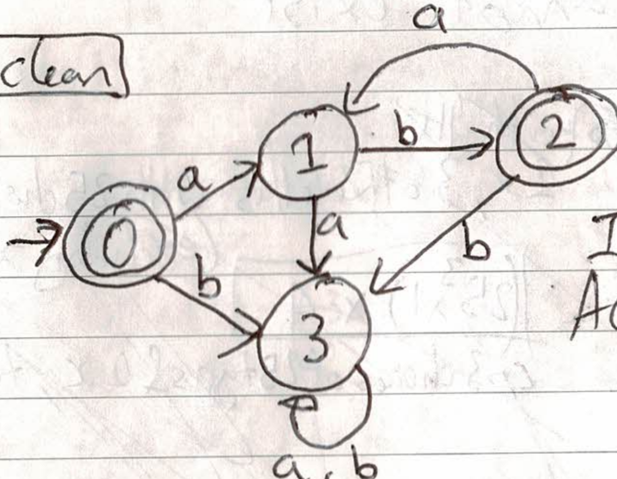
} 8 combinations.

For a set of 9 points in 8 combinations of non-integer, by the pigeonhole principle at least one pair must be integer because one pair will overlap to equal integer.

Q5



Q5 clean



Initial state = 0
Accepting states = {0, 2}

M recognise language L by sorting through valid and invalid inputs. If empty, accepted because state 0 is accepting. If starts with a, because ab must alternate, accepts any number of 'ababab...' with states 1 & 2. But if any part of the string does not alternate, it goes to 3, which can never reach an accepting state and thus always be invalid. Also if start with b it goes to 3 so invalid.