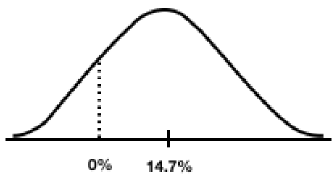
Chaewon Kim 3251-14

Assignment 3

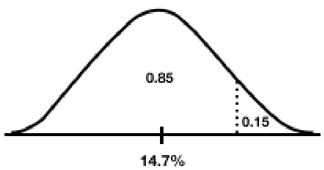
3.8 CAPM The Capital Asset Pricing Model (CAPM) is a financial model that assumes returns on a portfolio are normally distributed. Suppose a portfolio has an average annual return of 14.7% (i.e. an average gain of 14.7%) with a standard deviation of 33%. A return of 0% means the value of the portfolio doesn't change, a negative return means that the portfolio loses money, and a positive return means that the portfolio gains money.

1. What percent of years does this portfolio lose money, i.e. have a return less than 0%?



$$\mu = 14.7, \quad \sigma = 33$$
$$Z = \frac{0 - 14.7}{33} \approx -0.45$$
$$P(X < 0) = P(Z < -0.45) = 0.3264 = 32.64\%$$

2. What is the cutoff for the highest 15% of annual returns with this portfolio?



$$Z = 1.04$$
$$\frac{X - 14.7}{33} = 1.04$$
$$X = (1.04 \times 33) + 14.7 = 49.02\%$$

3.17 Scores on stats final Below are final exam scores of 20 Introductory Statistics students.

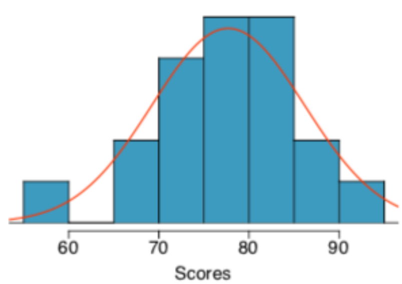
57, 66, 69, 71, 72, 73, 74, 77, 78, 78, 79, 79, 81, 81, 82, 83, 83, 88, 89, 94

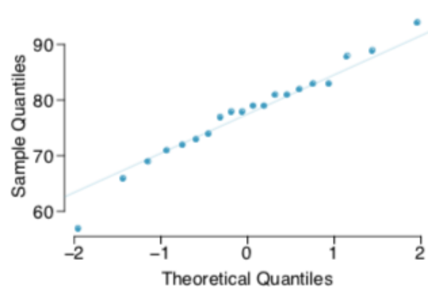
1. The mean score is 77.7 points. with a standard deviation of 8.44 points. Use this information to determine if the scores approximately follow the 68-95-99.7% Rule. ($\mu = 77.7, \quad \sigma = 8.44$)

$$ex. range\ calculation{:}\ \mu \pm 1\sigma$$
$$\mu - \sigma = 77.7 - 8.44 = 69.26$$
$$\mu + \sigma = 77.7 + 8.44 = 86.14$$

Rule	1 – SD, 68%	2 – SD, 95%	3 – SD, 99.7%
Range	$69.26 < X < 86.14$	$60.82 < X < 94.58$	$52.38 < X < 103.02$
Count	$\frac{14}{20}$	$\frac{19}{20}$	$\frac{20}{20}$
Percent	70%	95%	100%

(b) Do these data appear to follow a normal distribution? Explain your reasoning using the graphs provided below.





The distribution is unimodal and symmetric. The superimposed normal curve seems to approximate the distribution pretty well. The points on the normal probability plot also seem to follow a straight line. There is one possible outlier on the lower end that is apparent in both graphs, but it is not too extreme. We can say that the distribution is nearly normal.

Chaewon Kim 3251-14

3.31 Game of dreidel. A dreidel is a four-sided spinning top with the Hebrew letters nun, gimel, hei, and shin, one on each side. Each side is equally likely to come up in a single spin of the dreidel. Suppose you spin a dreidel three times. Calculate the probability of getting

This is a Binomial Distribution: $p = 0.25$ & $n = 3$

$$\binom{n}{k} p^k \times (1-p)^{n-k}$$

(a) at least one nun?

$$1 - P(No\ nun) = 1 - (1 - 0.25)^3 = 0.578$$

(b) exactly 2 nuns?

$$n = 3, k = 2$$
$$Single\ scenario: P(nun) \times P(nun) \times P(no\ nun)$$
$$\#\ of\ Scenarios\ \times P(single\ Scenario)$$
$$\binom{3}{2} 0.25^2 \times 0.75^1 = 0.1406$$

(c) exactly 1 hei?

$$n = 3, k = 1$$
$$\binom{3}{1} 0.25^1 \times 0.75^2 = 0.4219$$

(d) at most 2 gimels?

$$P(0\ gimels) + P(1\ gimels) + P(2\ gimels)$$
$$\binom{3}{0} 0.25^0 \times 0.75^3 + \binom{3}{1} 0.25^1 \times 0.75^2 + \binom{3}{2} 0.25^2 \times 0.75^1 = 0.9844$$

3.41 Sampling at School. For a sociology class project you are asked to conduct a survey on 20 students at your school. You decide to stand outside of your dorm's cafeteria and conduct the survey on a random sample of 20 students leaving the cafeteria after dinner one evening. Your dorm is comprised of 45% males and 55% females.

(a) Which probability model is most appropriate for calculating the probability that the 4th person you survey is the 2nd female?

Negative binomial distribution is the most appropriate model in this scenario since the last observation, 4th person, is fixed and we have to observe the unknown order of first 3 observations. So $\mathrm{n} = 4$ and $\mathrm{p} = 0.55$, where a success is defined as a female student.

$$\binom{n-1}{k-1} p^k \times (1-p)^{n-k}$$

(b) Compute the probability from part (a)

$$n = 4, k = 2, p = 0.55$$
$$\binom{4-1}{2-1} 0.55^2 \times 0.45^2 = \binom{3}{1} 0.55^2 \times 0.45^2 = 0.18376$$

(c) The three possible scenarios that lead to 4th person you survey being the 2nd female are

{M,M,F,F},{M,F,M,F},{F,M,M,F}

One common feature among these scenarios is that the last trial is always female. In the first three trials there are 2 males and 1 female. Use the binomial coefficient to confirm that there are 3 ways of ordering 2 males and 1 female.

$$\binom{4-1}{2-1} = \binom{3}{1} = 3$$

(d) Use (c) to explain why the negative binomial coefficient is $\binom{n-1}{k-1}$ while binomial coefficient is $\binom{n}{k}$

In the binomial case, we typically have a fixed number of trials, n, and instead consider the number of successes, k. In the negative binomial case, we examine how many trials it takes to observe a fixed number of successes and require that the last observation be a success. In other words in Negative Binomial Distribution we observe how many ways could we arrange $k-1$ successes in $n-1$ trials.

3.44 Stenographer's typos. A very skilled court stenographer makes one typographical error (typo) per hour on average.

(a) What probability distribution is most appropriate for calculating the probability of a given number of typos this stenographer makes in an hour?

$$\frac{\lambda^k e^{-\lambda}}{k!}$$

Poisson distribution is most appropriate for this scenario with $\lambda = 1$ representing the average number of typographical error per hour.

(b) What are the mean and the standard deviation of the number of typos this stenographer makes?

$$\mu = \lambda, \ \ \sigma = \sqrt{\lambda}$$

$$\therefore \mu = 1, \ \ \sigma = \sqrt{1} = 1$$

(c) Would it be considered unusual if this stenographer made 4 typos in a given hour?

$Z = \frac{4-1}{1} = 3$, Since 3 is still within 3 standard deviations of the mean, it would not generally be considered unusual. Note that we often use this rule of thumb even when the normal model does not apply.

(d) Calculate the probability that this stenographer makes at most 2 typos in a given hour.

$$P(0\ typos) + P(1\ typos) + P(2\ typos)$$

$$\frac{1^0 e^{-1}}{0!} + \frac{1^1 e^{-1}}{1!} + \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{1} + \frac{e^{-1}}{1} + \frac{e^{-1}}{2} = 0.9197$$