Assignment 2

<2.28 Socks in a drawer>

Q. In your sock drawer you have 4 blue, 5 gray, and 3 black socks. Half asleep one morning you grab 2 socks at random and put them on. Find the probability you end up wearing

(a) 2 blue socks

$$\frac{4}{12}\times\frac{3}{11}=\frac{1}{11}\approx 0.09$$

(b) no gray socks (Blue & Black)

$$\frac{7}{12}\times\frac{6}{11}=\frac{7}{22}\approx 0.318$$

(c) at least 1 black sock

$$1-(No\ Black)=1-\left(\frac{9}{12}\times\frac{8}{11}\right)=1-\frac{6}{11}=\frac{5}{11}\approx 0.454$$

(d) a green sock

$$The\ probability\ of\ grabbing\ \text{"a green sock" is 0 because you are grabbing 2 socks not one.}$$

(e) matching socks

$$P(Blue\ \&\ Blue)+P(Green\ \&\ Green)+P(Black\ \&\ Black)$$
$$=\left(\frac{4}{12}\times\frac{3}{11}\right)+\left(\frac{5}{12}\times\frac{4}{11}\right)+\left(\frac{3}{12}\times\frac{2}{11}\right)$$
$$=\left(\frac{1}{11}\right)+\left(\frac{5}{33}\right)+\left(\frac{1}{22}\right)=\frac{6+10+3}{66}=\frac{19}{66}\approx 0.288$$

<2.34 Ace of club wins>

Q. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade (♠), you win $5. For any club (♣), you win $10 plus an extra $20 for the ace of clubs.

(a) Create a probability model for the amount you win at this game. Find the expected winnings for a single game and the standard deviation of the winnings.

i	Red	♠	♣	A♣	Total
x_i	$0	$5	$10	$30	
$P(X=x_i)$	$\frac{26}{52}$	$\frac{13}{52}$	$\frac{12}{52}$	$\frac{1}{52}$	
$x_i \times P(X=x_i)$	$0	$\frac{13}{52}\times 5 = 1.25$	$\frac{12}{52}\times 10\approx 2.308$	$\frac{1}{52}\times 30\approx 0.577$	$\approx \$4.135$
$(x_i-\mu)$	-4.135	0.865	5.865	25.865	
$(x_i-\mu)^2$	≈ 17.098	≈ 0.748	≈ 34.398	≈ 668.998	
$P(X=x_i)\times(x_i-\mu)^2$	≈ 8.549	≈ 0.187	≈ 7.938	≈ 12.865	$V(x)=29.539$

$$\therefore \sigma=\sqrt{V(x)}=\sqrt{29.539}\approx 5.435$$

(b) What is the maximum amount you would be willing to pay to play this game?

The maximum amount I would be willing to pay is E(x) = $4.13 because if I pay any more than that, I will lose money in the long run.

<2.40> European roulette

Q. The game of European roulette involves spinning a wheel with 37 slots: 18 red, 18 black, and 1 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. Gamblers can place bets on red or black. If the ball lands on their color, they double their money. If it lands on another color, they lose their money.

(a) Suppose you play roulette and bet $3 on a single round. What is the expected value and standard deviation of your total winnings?

$$W = Amount\ won\ by\ betting\ \$3\ on\ Red$$

$$P(R) = \frac{18}{37}$$

W	$P(W)$	$W \times P(W)$	$(W - \mu)^2$	$P(W) \times (W - \mu)^2$
$3	$\frac{18}{37}$	$\frac{54}{37}$	$(3 - \left(-\frac{3}{37}\right))^2 \approx 9.493$	$\frac{18}{37} \times 9.493 \approx 4.618$
$-\$3$	$\frac{19}{37}$	$-\frac{57}{37}$	$(-3 - \left(-\frac{3}{37}\right))^2 \approx 8.520$	$\frac{19}{37} \times 8.520 \approx 4.375$

$$E(W) = \frac{54}{37} - \frac{57}{37} = -\frac{3}{37} \approx \$ - 0.081$$

$$V(W) \approx 8.9934$$

$$\therefore \sigma(w) = \sqrt{8.9934} \approx \$2.999$$

(b) Suppose you bet $1 in three different rounds. What is the expected value and standard deviation of your total winnings?

W_i	$P(W_i)$	$W_i \times P(W_i)$	$(W_i - \mu)^2$	$P(W_i) \times (W_i - \mu)^2$
$1	$\frac{18}{37}$	$\frac{18}{37}$	$(1 - \left(-\frac{1}{37}\right))^2 \approx 1.055$	$\frac{18}{37} \times 1.055 \approx 0.513$
$-\$1$	$\frac{19}{37}$	$-\frac{19}{37}$	$(-1 - \left(-\frac{1}{37}\right))^2 \approx 0.947$	$\frac{19}{37} \times 0.947 \approx 0.486$

$$E(W_1) = \frac{18}{37} - \frac{19}{37} = -\frac{1}{37} \approx -\$0.027$$

$$W = W_1 + W_2 + W_3$$

$$E(W) = 3 \times \frac{-1}{37} \approx \$ - 0.081$$

$$V(W) = 3 \times 0.9992695397 \approx \$2.998$$

$$\therefore \sigma(w) = \sqrt{2.998} \approx \$1.731$$

c) How do your answers to parts (a) and (b) compare? What does this say about the riskiness of the two games?

The Expected Value of total winnings were the same for both parts (a) and (b) at → $E(W) \approx \$ - 0.081$
However, the standard deviation of total winnings for part (a) was $2.998 whereas for part (b) it was $1.731. Thus we can say that it is less risky to bet $1 in three different rounds rather than bet $3 on a single round.

MONTY HALL SIMULATION

In [16]:

```
import random
from random import randint
import matplotlib.pyplot as plt
%matplotlib inline
```

In [17]:

```
simulations = 10000

winS1_counts=0

#scenario 1, player keeps his first choice

for i in range(simulations):
    #select the door randomly which has a car behind
    winning_door = randint(1,3)
    #contestant picks a door
    c_first_door = randint(1,3)
    #host opens a door with a goat behind
    door1 = [1,2,3]
    door1.remove(winning_door)
    if c_first_door != winning_door : door1.remove(c_first_door)

    game_host_opened = random.choice(door1)

    #player keeps his first choice

    if c_first_door == winning_door:
        winS1_counts = winS1_counts + 1

#scenario 2, player changes his firt choice

winS2_counts = 0

for i in range(simulations):
    #select the door randomly which has a car behind
    winning_door = randint(1,3)
    #contestants pick a door
    c_first_door = randint(1,3)
    #host opens a door with a goat behind
    door2 = [1,2,3]
    door2.remove(winning_door)
    if c_first_door != winning_door : door2.remove(c_first_door)

    game_host_opened = random.choice(door2)

    #contestant changes first choice


    door2 = [1,2,3]
    door2.remove(game_host_opened)
    door2.remove(c_first_door)

    c_new_door = door2[0]

    if c_new_door == winning_door:
        winS2_counts = winS2_counts + 1

print("The chance of winning the prize when the contestant sticks to his or her first choice is:", 1.0 * winS1_counts
/ simulations )
print("The chance of winning the prize when the contestant switches his or her first choice is:", 1.0 * winS2_counts /
simulations )
```

```
The chance of winning the prize when the contestant sticks to his or her first choice is: 0.3246
The chance of winning the prize when the contestant switches his or her first choice is: 0.6668
```