Chaewon Kim 3251-14

Assignment 4

Q. 4.20 Age at first marriage, Part II. Exercise 4.16 presents the results of a 2006 - 2010 survey showing that the average age of women at first marriage is 23.44. Suppose a social scientist believes that this value has increased in 2012, but she would also be interested if she found a decrease. Below is how she set up her hypotheses. Indicate any errors you see.

$$H_0: \bar{\bar{x}} = 23.44 \text{ years old}$$
$$H_A: \bar{\bar{x}} > 23.44 \text{ years old}$$

A. The hypotheses she set up is a one-sided test indicating that she is only interested in showing that 23.44 is an underestimate. Here the interest is in only one direction. However, it is said that she would also be interested if she found a decrease. Since we would also be interested if the data showed strong evidence that 23.44 was an overestimate, then the test should be two-sided. Thus, the following is the corrected hypotheses:

$$\mathrm{H_0}: \mu = 23.44 \text{ years old}$$
$$\mathrm{H_A}: \mu \neq 23.44 \text{ years old}$$

Q. 4.30 Testing for food safety. A food safety inspector is called upon to investigate a restaurant with a few customer reports of poor sanitation practices. The food safety inspector uses a hypothesis testing framework to evaluate whether regulations are not being met. If he decides the restaurant is in gross violation, its license to serve food will be revoked.

A.

(a) Write the hypotheses in words.

$$\mathrm{H_0}: \text{Regulations are met by the restaurant}$$
$$\mathrm{H_A}: \text{Regulations are not met by the restaurant}$$

Type 1 Error is rejecting the null hypothesis when H_0 is actually true. A Type 2 Error is failing to reject the null hypothesis when the alternative, H_A, is actually true.

(b) What is a Type 1 Error in this context?

Concluding that the regulations are not met when they actually are met.

(c) What is a Type 2 Error in this context?

Concluding that the regulations are met when they are actually not met.

(d) Which error is more problematic for the restaurant owner? Why?

Type 1 error is more problematic for the restaurant because the inspector will decide the restaurant is in gross violation and its license to serve food will be revoked.

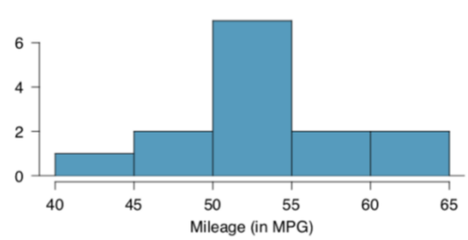
(e) Which error is more problematic for the diners? Why?

Type 2 error is more problematic for diners because they will expect to dine in a clean environment, but they will dine at a restaurant with poor sanitation practices.

(f) As a diner, would you prefer that the food safety inspector requires strong evidence or very strong evidence of health concerns before revoking a restaurant's license?

As a diner, I'd prefer that the food safety inspector requires strong evidence rather than very strong evidence of health concerns before revoking a restaurant's license. If the restaurant is able to provide strong evidence, then I can be confident about the hygiene of the restaurant and dine without speculation. For example, the inspectors could run two tests. The first test would require strong evidence of good hygiene and those who don't pass will go through a second test with more observations. This would give good, clean, restaurants an opportunity to avoid punishment.

Q. 5.8 Fuel efficiency of Prius. Fueleconomy.gov, the official US government source for fuel economy information, allows users to share gas mileage information on their vehicles. The histogram below shows the distribution of gas mileage in miles per gallon (MPG) from 14 users who drive a 2012 Toyota Prius. The sample mean is 53.3 MPG and the standard deviation is 5.2 MPG.



a) We would like to use these data to evaluate the average gas mileage of all 2012 Prius drivers. Do you think this is reasonable?

This may not be reasonable since the sample may not be random, it is a convenience sample. The information shared on fueleconomy.gov are from drivers who volunteer to share their gas mileage information on the website. More specifically, they might be those that are getting much lower or much higher than the gas mileage estimated by the EPA.

b) The EPA claims that a 2012 Prius gets 50 MPG (city and highway mileage combined). Do these data provide strong evidence against this estimate for drivers who participate on fueleconomy.gov?

First, we construct the hypotheses:

$$\mathrm{H_0}: \mu = 50 \text{ MPG}$$
$$\mathrm{H_A}: \mu \neq 50 \text{ MPG}$$

Before calculating the test statistic, we should evaluate the conditions for the test:

1. Independence: Independence: Our sample is a convenience sample, which is a red flag regarding the independence of observations (even when limiting our population to be those who participate on the fueleconomy.gov website). When reporting these results to others, we should volunteer this information and note that our results rely on the assumption that the observations are independent.
2. Sample size: The sample size is less than 30, therefore we will use the t distribution.
3. Skew: The distribution is approximately symmetric and there is no evidence that it is not nearly normal, though checking this condition is difficult for a small sample.

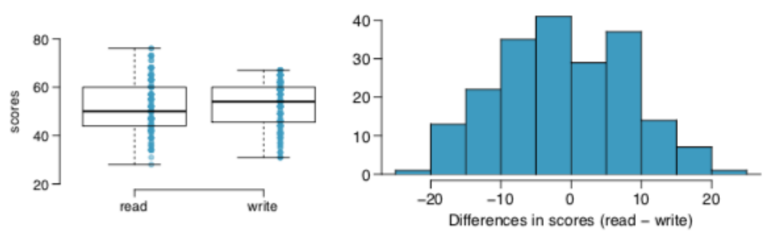
$$\mathrm{SE} = \frac{5.2}{\sqrt{14}} \approx 1.39, \;\; df = 14 - 1 = 13$$
$$\mathrm{T} = \frac{53.3-50}{1.39} \approx 2.37, \;\; 0.02 < p \; value < 0.05$$
$$\therefore \text{ Since } p \; value \text{ is smaller than } 0.05, \text{reject } \mathrm{H_0}$$

(c) Calculate a 95% confidence interval for the average gas mileage of a 2012 Prius by drivers who participate on fueleconomy.gov.

Looking in the column where two tails is 0.05 (for a 95% confidence interval) and row df = 13, we identify $t_{13}^{*} = 2.16$

$$\bar{\bar{x}} \pm t_{13}^{*} SE = 53.3 \pm (2.16 \times 1.39) = (50.3, 56.3)$$
$$\therefore \text{ We are 95\% confident that a 2012 Prius gets on average 50.3 to 56.3 MPG}$$

5.20 High School and Beyond **The National center of Education Statistics conducted a survey of high school seniors, collecting test data on reading, writing and several other subjects. We examine a simple random sample of 200 students from this survey.**



a) Is there a clear difference in the average reading and writing scores?

No. We can only see that the average writing score is slightly higher than that of reading.

b) Are the reading and writing scores of each student independent of each other?

For each observation in one data set, there is exactly one specially-corresponding observation in the other data set for the same student. Thus the data are paired.

c) Hypotheses for "is there an evident difference in the average scores of students in the reading and writing exam?"

$$H_0: \mu_{read-writing} = 0$$
$$H_A: \mu_{read-writing} \neq 0$$

d) Conditions required to complete this test

We examined a simple random sample of 200 students (the sample is at least 30), and they represent less than 10% of all possible high school seniors. By looking at the histogram above we can see that the differences in scores is not strongly skewed.

e) The average observed difference in scores is $\bar{\bar{x}}_{read-write} = -0.545$, and the standard deviation of the differences is 8.887 points. Do these data provide convincing evidence of a difference between the average scores on the two exams?

$$\mathrm{SE} = \frac{8.887}{\sqrt{200}} \approx 0.628$$

$$T = \frac{-0.545 - 0}{0.628} \approx -0.868$$

The degrees of freedom are $df = 200 - 1 = 199$. If we examined Appendix B.2 on page 430, we would see that this value is smaller than any in the 150 df row (we round down for df when using the table), meaning the two-sided $p - value > 0.2$. Because $p - value > 0.05$, we fail to reject the null hypothesis. We have not found convincing evidence a difference between the average reading and writing exam scores.

f) What type of error might we have made?

Since we failed to reject the null hypothesis, there's a possibility that we might have made a type 2 error. This means we've concluded that there is no difference between the average reading and writing exam scores when actually, there is.

g) Based on the results of this hypothesis test, would you expect a confidence interval for the average difference between the reading and writing scores to include 0?

Yes, since we failed to reject H_0, which had a null value of 0.

6.4 Young Americans, Part II. About 25% (success) of young Americans have delayed starting a family due to the continued economic slump. Determine if the following statements are true or false and explain your reasoning.

a) The distribution of sample proportions of young Americans who have delayed starting a family due to the continued economic slump in random samples of size 12 is right skewed.

$$\text{Success: } 12 \times 0.25 = 3, \qquad \text{Failure: } 12 \times (1 - 0.25) = 9$$

The statement is true. The success-failure condition is not satisfied. In most samples we would expect $\hat{p}$ to be close to 0.25, the true population proportion. While $\hat{p}$ can be much above 0.25, it is bound below by 0, suggesting it would take on a right skewed shape.

b) In order for the distribution of sample proportions of young Americans who have delayed starting a family due to the continued economic slump to be approximately normal, we need random samples where the sample size is at least 40.

The Statement is true. The sampling distribution for $\hat{p}$, taken from a sample of size n from a population with a true proportion p, is nearly normal when

1. the sample observations are independent and
2. we expected to see at least 10 successes and 10 failures in our sample, i.e. $np \geq 10$ & $n(1-p) \geq 10$

$$\text{Success: } n \times 0.25 \geq 10 \rightarrow n \geq 40$$
$$Failure: n \times 0.75 \ \geq 10 \rightarrow n \geq 13.3$$

(c) A random sample of 50 young Americans where 20% have delayed starting a family due to the continued economic slump would be considered unusual.

The statement is False.

$$SE = \sqrt{\frac{0.25 \times 0.75}{50}} \approx 0.06123$$

$Z = \frac{0.2 - 0.25}{0.06123} \approx -0.8165, \therefore 0.8165$SEs away from the mean, which would not be considered unusual.

(d) A random sample of 150 young Americans where 20% have delayed starting a family due to the continued economic slump would be considered unusual.

The statement is False.

$$SE = \sqrt{\frac{0.25 \times 0.75}{150}} \approx 0.03536$$

$Z = \frac{0.2 - 0.25}{0.03536} \approx -1.4140, \therefore 1.4140$SEs away from the mean, which would not be considered unusual.

(e) Tripling the sample size will reduce the standard error of the sample proportion by one-third.

The statement is False. It decreases the SE by a factor of $\frac{1}{\sqrt{3}}$.

6.16 Is college worth it? Part I. Among a simple random sample of 331 American adults who do not have a four-year college degree and are not currently enrolled in school, 48% said they decided not to go to college because they could not afford school.

(a) A newspaper article states that only a minority of the Americans who decide not to go to college do so because they cannot afford it and uses the point estimate from this survey as evidence. Conduct a hypothesis test to determine if these data provide strong evidence supporting this statement.

The survey was a simple random sample that includes fewer than 10% of American adults who do not have a four-year college degree and are not currently enrolled in school. In a one-proportion hypothesis test, the success-failure is checked using the null proportion, which is $p_0 = 0.5$, in this context: $np_0 = n(1 - p_0) = 331 \times 0.5 = 165.5 > 10$.

$$H_0: p = 0.5$$
$$H_A: p < 0.5$$
$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(0.5)}{331}} \approx 0.02748$$
$$Z = \frac{point\ estimate - null\ value}{SE} = \frac{0.48 - 0.5}{0.02748} \approx -0.73$$
$$p - value = 0.2327$$

We fail to reject the null hypothesis. We do not have strong evidence supporting that only a minority of Americans who decide not to go to college do so because they cannot afford it.

b) Would you expect a confidence interval for the proportion of American adults who decide not to go to college because they cannot afford it to include 0.5?

Yes, I would expect a confidence interval to include 0.5 because:

95% Confidence Interval $= 0.48 \pm 1.96SE = 0.48 \pm 0.0539 = [0.426, 0.534]$