3D Shapes from 2D Curve Fitting – A Survey and Analysis

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Abstract—**Computer vision has long been a hotspot for research in computer science. Among the more difficult problems being looked at by scientists are surface registration, mesh alignment and 3D reconstruction. This technical document is a survey and analysis of ten academic white papers that each contain algorithms used to reconstruct 3D objects from multiple surfaces or other pixelated data. The papers are given a score based on seven assessment metrics and those scores are summed together to get an overall score that indicates how suited that work is for a developer to implement and expand upon.**

Keywords—Surface registration, mesh alignment, shape generation, 3D reconstruction

I. Introduction

The alignment of 2D surfaces to reconstruct 3D shapes is a technique in computer vision that has many applications. One of these applications is stitching together point cloud data obtained through LiDAR scans. This technology is used for creating terrain maps or 3D models of objects and locations. Another application of this technology is in the medical field. Often times, imagery of organs, cells or genetic materials are collected from multiple sensors or on multiple passes and must be aggregated to one image for proper analysis. One last example of the many applications of 3D construction is in Aerospace. Space vehicles will collect imagery from various mission and the transmission of that data back to earth is segmented due to transmission limitations and this data is reconstructed on the back end. Shape description and surface registration is not an easy problem to solve and there is ongoing research to create new algorithms and make old ones more efficient and effective. This paper presents the summary of five techniques in this domain.

II. Summaries

A. Solution 1

Using a technique that generates Non-Uniform rational B-splines (NURBS), an image acquisition system creates a cloud of 3D points (CoP) that map to the surface of an object [1]. These points are translated into a geometric model of the object. Each point in the CoP maps to an x-y-z coordinate location; meaning no topological information can be extracted from them. Normally, it would be impossible to model such an ambiguous system. However, we can create a computational geometric model that allows us to process the system further. Specifically, CoPs can be broken down into either image acquisition systems or computational algorithms. Often, models of 3D space can be extended into Euclidean 4D space using NURBS; a similar operation can be carried out from 2D space into 3D. In order to create a 3D mesh from 2D data, a four step strategy is employed; first the initial data processing, then curve, surface, and finally volume mesh generation. The primary algorithm creates a series of test points and compares their distance to analogous segments of the polygon; meanwhile, a secondary algorithm reduces the total amount of points in the CoP. The final result can be used to print almost any 3D object, such as artificial human organs.

B. Solution 2

There are existing image technologies such as MRIs and CAT scans that can create 3D data from multiple stacks of 2D slices [2]. However, the process of automatically identifying 3D data segments within these slices is tenuous at best. We can take sets of parallel contours, and use their data to interpolate smooth 3D models. The first step of this process involves creating something called an "isosurface" from calculating distances of fields in each 2D slice. Spline interpolation is used to blend slices by associating similar areas of pixels in different slices. Data is initially fed to the algorithm as a set of binary images, where white pixels are treated as contour curves. The centers of clusters of contour pixels are treated as points in a 2D plane, as well as a Multi-Level Partition of Unity (MPU) implicit curve. By utilizing multiple mathematical techniques, we can create a 2 dimensional Euclidean field that can then be extended into 3D space by combining a volume dataset with spline interpolation across distance fields. The MPU uses a special subdivision process with two parameters: one for the support radius and one for the minimum number of points within that radius. MPU curves can also tell us which points are inside the object because their spatial values are positive. Finally, after applying 2D distance field filtering, the 3D representation of the slices can be created from the

original 2D slices. From here, a polygonal surface can be assigned to this result.

C. Solution 3

With current technology, it is possible to relate a 2D image of a person to a 3D mesh model in a similar pose. [3]However, this is difficult without multiple camera sources and a static set of conditions for the image. While optimization-based approaches have been more common, learning-based approaches are more widely used today due to their ability to deduce the correct shape quickly, without good initialization. Network input for these approaches includes surface landmarks, silhouettes, and even raw pixels. While the SMPL model is often used for pose generation, it is possible to make a model independent of it. The method used in this study takes the template mesh from SMPL and changes its regression target to the vertices of the mesh themselves. Using a graph-based convolutional neural network (Graph-CNN), we can extract features directly from the simulation. After calculating each vertex, there is a neighborhood averaging operation that forcibly smooths neighboring vertices. The Graph-CNN can also regress camera parameters, eliminating the need for multiple camera views. The output of the Graph-CNN is better than existing optimization-based approaches, and achieves excellent performance when compared to model-based pose estimation approaches.

D. Solution 4

Current mesh generation technology only creates piecewise linear or quadratic meshes; ways to create curvilinear meshes are in development [4]. A simple case can be the generation of a polynomial curve from a straight sided surface mesh, which can be treated as a case of curve fitting. In this case, the curve is normally not fitted to the surface, but rather to a set of samples that were extracted from the surface itself. More complex curves, like B-splines, are calculated by computing an error functional alongside an energy measure for regularization. Problematically, the energy functional and the error functional are opposed to each other and keep the surface from converging. Many scientists have developed methods to fit curves by calculating the weighted average of error and energy. Using spectral element methods, each face intersecting the domain boundary consists of a polynomial surface patch. With an initial straight-sided mesh, a curvilinear mesh is built from the following: first, construction of boundary curves representing the edges of the faces; second, creation of patches defining these faces; and third, creation of the volume elements. Other techniques are detailed, including a screw surface defined by an analytical incremental algorithm, and also the surface triangulation of a rabbit aorta. By minimizing the amount of energy expended calculating curves, it is

TABLE 1: Code Description for Assessment Metrics

Code	Description
A	The level of success presented in the experimentation and/or results section of the paper. Was the full solution implemented as outlined in the paper? Were the results produced as expected? A high score indicates a high level of success.
B	The level of robustness in the solution. Does the algorithm generate acceptable results under abnormal conditions? How does it handle atypical input? A high score indicates a high level of robustness.
C	The level of reliability in the solution. Does the algorithm perform well under normal conditions? Does it generate expected results from typical input? A high score indicates high reliability.
D	The level of difficulty for implementing the solution. Is there sufficient material provided in the form of mathematical equations, pseudo code, or low-level algorithmic steps? A low score indicates a high level of difficulty.
E	The level of novelty. Is the algorithm original to the authors? Are the equations borrowed from other works? A high score indicates a completely novel idea.
F	The level of prospective future work. Do the authors lay out how to improve the solution? Are there obvious open ends to the research? A high score indicates a high prospect.
G	The level of complexity of the solution. How many steps are in the algorithms? Do they rely on other complex algorithms from previous work? How much computational resources does the implementation require? A low score indicates high complexity

easier to see that the resultant curves are shorter and straighter than they would otherwise be.

E. Solution 5

The Discrete Element Method (DEM) of modeling is becoming more commonplace with regards to granular media [5]. Existing processing methods are rather slow when dealing with large numbers of particles. To combat this, researchers are investigating geometric-based algorithms, such as using polyhedral meshes to densely pack spherical particles. To start with, our sample domain can be described by polygonal Voronoi cells, meant to be treated as particles. To separate these cells, we can generate a set of seeding nodes, then connect neighboring nodes to each other, and finally find a set of bisectors located between every pair of seeding nodes. Next, cell size distribution is normalized by using a weight modification, making the output similar to the final values. Then, cubic-polynomial-curve fitting is used to create a 3D design featuring complex convex granular particles.

G. Solution 7

In this paper [7], the authors present a framework based on Game-Theoretic Matching (GTM). GTM as applied in this research is based on two underlying conditions. The first condition is that there must exist the capability of modeling a matching feature as a strategy in a non-cooperative game. The second condition is that there must exist a reward function that is satisfactory in measuring the compatibility between two matches. If these two conditions are met, ideal strategies can be isolated. Once the strategies are identified the author lend the approach that now the matches can compete with each other in an isometry-enforcing alignment game. Since the goal in the registration of two different surfaces. Strategies then match points from one surface to points in another surface using the nearest descriptor in Euclidean space. The payoff function is then used in conjunction with the competing strategies to undergo an evolutionary matching process.

TABLE 2: Assessment Scores for Each Solution

Solution	A	B	C	D	E	F	G
1	9	3	3	3	2	1	2
2	9	3	7	5	4	3	5
3	8	5	5	2	2	4	1
4	9	4	6	3	4	5	3
5	9	6	3	2	5	5	1
6	9	2	7	2	6	3	3
7	9	6	7	2	5	2	2
8	9	6	8	5	5	4	4
9	9	4	9	6	4	5	5
10	10	5	5	6	3	6	4

F. Solution 6

Research laid out in this publication [6] propose a method to locate all of the points on a non-rigid surface by identifying a geodesic coordinate system and a geodesic distance. The first step in this process is the initial feature extraction where graphs are built to match critical points. These graphs are high level shape descriptors that are used for shape matching in large datasets. Once the graph is constructed, global geodesic distance and coordinates are determined for matching points. One issue identified by the authors is the ambiguity of points local to a surface. The solution to this is a mapping process that recursively populates sets of points to calculate a geodesic map. Finally, for the surface alignment, Markov Random Field (MRF) energy formation is used with a divide and conquer methodology.

H. Solution 8

The authors of this paper [8] propose methodologies that are based on a framework composed of three elements. The first method in this framework, the basis, is key point detection. The input data, objects from a dataset represented by a point cloud is first analyzed point by point. For every point, a neighborhood is defined as a sphere around that point with some defined radius and center. Using some methods proposed by other research, the authors extract a local reference
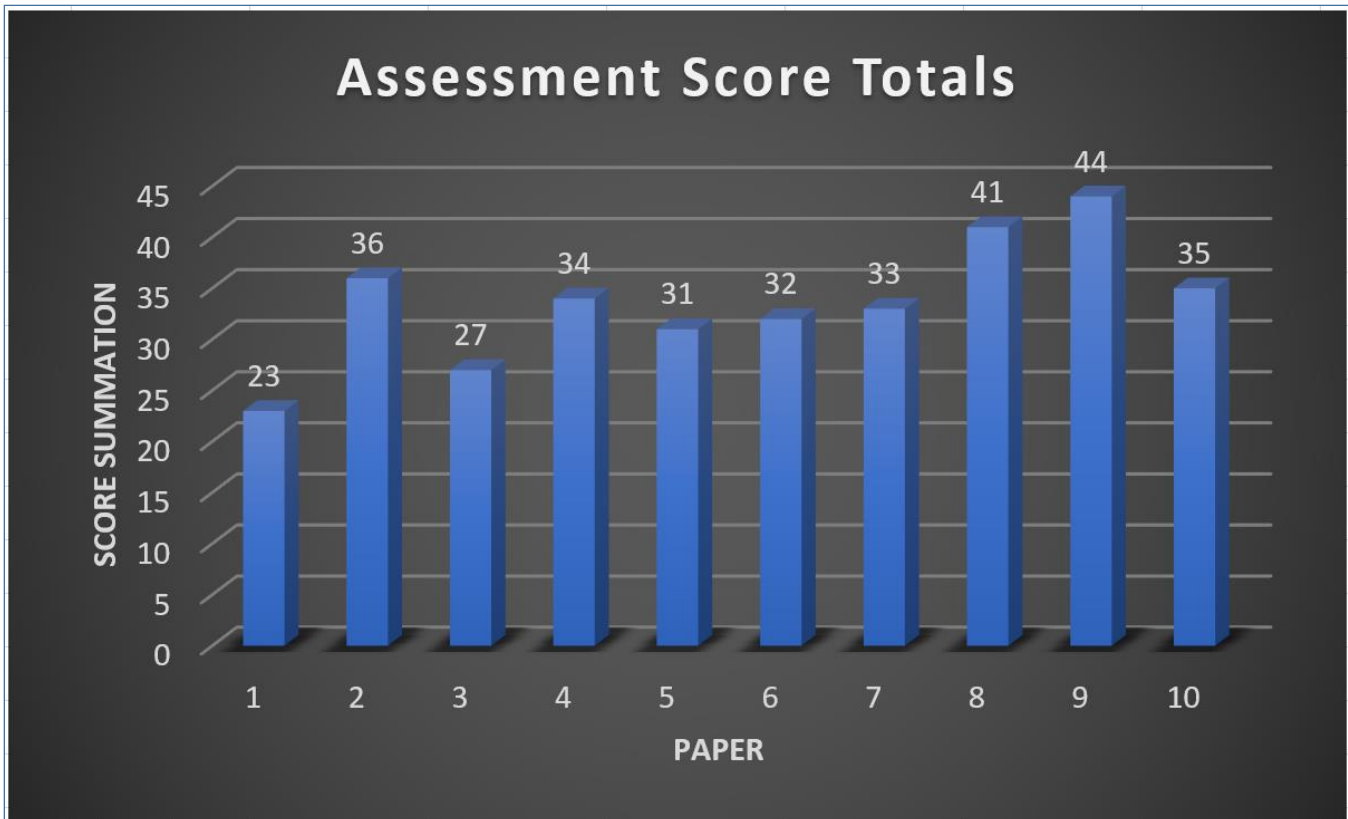
Fig.1: This graph represents the data from Tale 2 added together for an overall score for each paper.

frame (LRF) from each neighborhood and then use principal component analysis to transform each point from a given neighborhood into a locally aligned neighborhood. They then provide the mathematical equations in the next steps then describe the calculations of the surface variation index that indicated how much variation there is around each point in a local neighborhood. Then, the covariance matrix is calculated which is then used to find the eigen values. The second part of the algorithm is the core, feature descriptor generation. First a feature selection function is defined for a point in the point cloud a covariance matrix descriptor is defined. This is a "description" of the information about a neighborhood for each point. Finally, the description of a key point is enhanced by defining a multi-scale covariance matrix descriptor. The third and final step in this methodology is the feature matching where two multi-scale covariance matrices are compared to fins the similarity. Several sub-methods are mentioned for finding these similarities such as nearest neighbor distance ratio, either way, the comparison of these matrices will identify feature matching pairs that will be used for surface matching.

I. Solution 9

The authors of this research [9] claim that they employ a novel method for obtaining LRF, using feature transformation for their creation. The process that they identify for this strategy is to first calculate a spherical neighborhood for each point in a model and selected orthogonal vectors x, y, and z. A covariance matrix is calculated and the eigen value that is smallest is set as the z-axis. The x-axis is constructed from a combination of seven weights that are derived from a weighting function. Finally, the y-axis is calculated from the cross product of the z-axis and the x-axis. The authors then describe the method of scale strategy for computing a scale factor for when the two surfaces are obtained at different stages. Once the scale factor is found, the LRF and feature descriptors are used for feature matching.

J. Solution 10

The research presented in this document [10] proposes an algorithm for 3D surface reconstruction from stereo matching. During the first stage of the algorithm matching cost computation (MCC) is used to calculate the matching differences for every pixel. This is accomplished by using the Sum of Gradient Magnitude (SG) to find the direction along the pixel values. The second stage is the cost aggregation (CA) used for noise reduction, cleaning output from the first stage. Stage three represents a location with a grayscale pixel value encoding 3D data through a process called disparity selection and optimization (DSO). The final step is for post processing and refinement where invalid disparity values are located and filled in. Finally, triangulation is used for the surface reconstruction.

III. Assessment Metric

Evaluation criteria of seven key metrics are used to measure the quality of the papers from section II. Each quality metric is associated with a letter code for ease of reference (Table 1). The letter codes themselves have no particular significance; they were assigned to the assessment metrics randomly. From the each of the evaluation criterion, the solutions are given a score from one to ten (Table 2). For metrics D and G, a low score indicates a high level of measurement. For the rest of the metrics a high score represents a high measurement.

IV. Conclusion

This technical document present ten research articles pertaining to computer vision; specifically, 3D object registration or shape regeneration. The articles were evaluated against each other by scoring them in seven areas of assessment. These scores are from the perspective of a developer, as that viewpoint supports the purposes of the authors of this survey. The scores were then added together to get an overall score by which to compare the papers (Fig. 1). Solution 9 [9] ranked the highest with a score of 44 with Solution 8 [8] only three points behind at 41. Both of these research papers are peer reviewed and current and through the analysis presented herein, they have been identified as being the most mature.

V. Acknowledgment

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