Final

1. a)

The Bay of Fundy is a long channel that resembles a rectangular basin. In these situations, it is important to consider resonance, as waves or tides are reflected by the head of the bay or basin. The waves combine with their reflected waves to increase in height; commonly this phenomenon is known as constructive interference. Maximum resonance and wave height occurs when the length of a channel is a quarter of the wave or tidal length. The equation that can be used to express this is

$$T = \frac{4L}{\sqrt{gh}}$$

where T is the tidal period, L is the length of the channel, h is the depth of the basin, and g is gravity. Using Google Earth, I estimated the length of the Bay of Fundy to be around 250 km long. I also found the average depth of the bay to be about 75 m. Plugging these values into the equation above gives

$$T = \frac{4 * 250000}{\sqrt{9.81 * 75}} = 36866.75\ s = 10.24\ hours$$

Since this calculated tidal period based on maximum resonance of the Bay of Fundy is close to the actual semidiurnal tidal period of 12.42 hours, this shows that **the length of the Bay of Fundy is near the optimal conditions for tidal resonance**, contributing towards its very large tidal range.

b)

The power generation, P, of a tidal turbine is given as

$$P = \frac{1}{2}\rho u^3 A\eta$$

where ρ is water density, u is the speed of the flow, A is the rotor swept area of the turbine, and η is the efficiency. The speed of the flow in a tide varies with water depth, with the slower moving water at the bottom and faster moving water at the top. This can be shown in the following equation

$$u = \frac{u^*}{\kappa}\ln(\frac{z}{z_o})$$

where u is the speed of the flow, u* is the friction velocity, k is Von Karman's coefficient, z0 is the bottom roughness, and z is the depth. From the power generation equation, one can see that the power is highest when u is maximized. And in the next equation, u is highest when z is maximized. Therefore, the optimal vertical location to put a tidal turbine would be as high as it can reasonably be, which would allow the turbine to capture the energy from the fastest moving water. Assuming the depth throughout the bay is 75 m, a tidal range of 20 m, and a rotor radius of 20 m, **the best place to put the turbine would be at 45 m above the floor**. This is so that at low tide (when the depth is 65 m), the entire turbine would still be completely submerged underwater.

Plugging in 45 m as the z component, and assuming values of z0 = .001, u* = 0.1, and k = 0.4, we get a velocity of

$$u = \frac{0.1}{0.4}\ln\left(\frac{45}{0.001}\right) = 2.68\ \frac{m}{s}$$

Now solving for power given η = 0.3 and ρ = 1029 over a time of 6 hours,

$$P = \frac{1}{2}(1029)(2.68)^3(\pi * 20^2)(0.3)\left(\frac{1}{6*60*60}\right) = \mathbf{172.6\ W}$$

2. a) i)

Longshore sediment transport is given as

$$q_l = \frac{K}{g(\rho_s - \rho)(1-p)}P_1,$$

where K is an empirical constant, g is gravity, ρs is the sediment density, ρ is water density, p is sediment porosity, and P1 is longshore wave power, expressed as

$$P_1 = c_g\frac{1}{8}\rho g H^2 \sin\theta\cos\theta$$

where cg is group velocity, H is wave height, and θ is the angle of incidence. In order to find wave power, wave height, group velocity, and θ must be found. First, wave number and speed will be found, which will help the rest of the calculations.

$$k = \frac{2\pi}{\lambda}\,,\ \lambda = \frac{gT^2}{2\pi}\text{ , and } c_o = \sqrt{\frac{g}{k}}$$

Plugging in 15s for T, k = 0.018 and c0 = 23.3 m/s. Now, the wave height will be assumed to be the breaking wave height, given as

$$H_b = \left(\frac{\gamma}{g}\right)^{1/5}\left(\frac{H_o^2 c_o\cos(\theta_0)}{2}\right)^{2/5}$$

By plugging in the values of γ = 0.78, g = 9.81, Ho = 1, c0 = 23.3, and θo = 20, Hb = 1.56 m. Using the simple relation of Hb = γhb, hb = 2 m.

Now that hb is known, cg can be solved for,

$$cg = \sqrt{gh_b} = 4.4\,m/s$$

Using Snell's law,

$$\frac{sin\theta s}{cs} = \frac{sin\theta o}{c0}$$

Plugging in θo = 20, c0 = 23.3, and cs = cg = 4.4, θs = 3.7°.

Wave power and sediment transport can now be solved for. Using the equation for P1, and setting cg = 4.4, ρ = 1029, g = 9.81, H = 1.56, and θ = 3.7,

P1 = 881.52 Wm^2

Solving for q1, where K = 0.77, ρs = 2650, ρ = 1029, and p = 0.4,

q1 = 0.07 m3/s.

This entire process must be repeated for ii with sea waves of height .5 m, period of 7s, and angle of 45. Using the same calculations and assumptions,

P1 = 208.2 Wm2

q1 = 0.0165 m3/s

The sediment transport from the two waves are in opposite directions, and are assumed to occur seasonally (each occurring for 6 months). Therefore the annual net sediment transport is the difference between the two, multiplied by the time over which the transportation happens (6 months).

0.07 – 0.0165 = 0.0535 m3/s * 60 * 60 * 24 * 365.25 * .5 = **844000 m3 North toward Abbey Island.**

b)

Groins would create a volume of space for the sediment to be stored in. From a bird's eye view, this can be estimated as a right triangle whose width is the groin length, x, and whose length is x/tan(3.7) based on trigonometry. The volume that each groin could contain is then the area of this triangle multiplied by the depth. The depth will be assumed to be 0.5 times the deepest depth of the triangle. The equation for the volume is as follows.

$$\frac{1}{2}(x)\left(\frac{x}{tan3.7}\right)(0.01x)\frac{1}{2} = 0.038x^3$$

To find the number of groins needed, the maximum sediment transport will be divided by the volume each groin can hold. The max sediment transport is the sediment transport from i only, and is calculated as

$$0.07 * 60 * 60 * 24 * 365.25 * .5 = 1104000\ m3$$

So, the number of groins, G, needed is

$$G = \frac{1104000}{0.038x^3}$$

However, the groin length is arbitrary and could be a variety of lengths. In addition, the distance between each groin must be at least the length of the triangle created by the groin, (x/tan(3.7)), to ensure that the volume that each groin holds is not cut off by the next groin. Using Google Earth, the estimated distance between these two islands is 30000 m. The distance between each groin is therefore 30 km divided by the number of groins, and must be equal or greater than (x/tan(3.7)). An equation for this is

$$\frac{30000}{G} = \frac{x}{\tan(3.7)}$$

G can be substituted from the equation before, yielding

x = 121 m

Plugging this in to solve G,

G = 17

So, ideally there will be **17 groins**, each with a length of 121 m, and all evenly spread between both islands (distance between each is 30000/17 = **1765 m**).