# Risk Analytics - Take home assignment

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# Information

This assignment is designed to individually assess your data analysis skills. You can consult your colleagues for help, but your code and report MUST BE individually elaborated. Should this rule not be respected, the student will be penalised. Furthermore, any emails regarding how to develop the assignment will be ignored. Programming Language and Document Layout: The student MUST use 'Jupyter Notebook' and Python 3.x language. Submission: Your work must be submitted in **YourIDnumber.ipynb** extension via Canvas on 13/12/2018 at 4.59 pm.

# Project:

In this project you will analyse the dataset "excessReturns.txt", which can be downloaded from Canvas. The dataset includes the monthly excess returns (01/01/1990 - 31/12/2003, 168 time observations) of the following 14 stocks:

| Tick | Company | Tick | Company |
|---|---|---|---|
| AA | Alcoa | KMB | Kimberly-Clark |
| AGE | A.G. Edwards | MEL | Mellon Financial |
| CAT | Caterpillar | NYT | New York Times |
| F | Ford Motor | PG | Procter Gamble |
| FDX | FedEx | TRB | Chicago Tribune |
| GM | General Motors | TXN | Texas Instrument |
| HPQ | Hewlett-Packard | SP5 | S&P 500 Index |

Table 1: Excess returns

**Question 1 (10 Points)**
Download from Canvas the dataset "excessReturns.txt" and upload the file on Jupyter.
**Question 2 (20 points)**
One of the most important macroeconometric model is the Market Model introduced by Sharpe (1970), which is a single factor model and it reads

$$Y_{168\times 13} = X_{(168\times 2)}B^T_{2\times 13} + E_{(168\times 13)} \tag{1}$$

where $Y$ is the collection of all excess returns expect 'SP5', $X = (\vec{1}, SP5)$ is a block matrix composed of $\vec{1} \in \mathbb{R}^{168\times 1}$ and the excess returns of 'SP5', $B = (\vec{\alpha}, \vec{\beta})$ with $\vec{\alpha}, \vec{\beta} \in \mathbb{R}^{13\times 1}$, while $E$ is the matrix of unobserved perturbations. In order to apply the model, you should minimize the quantity

$$\min_B \mathbb{E}\Big(E^T E\Big) \tag{2}$$

and obtain analytically the argument $B$ that minimises the function $\mathbb{E}[E^T E]$, where $\mathbb{E}[\cdot]$ denotes the expected value operator. Show your work in Jupyter using LaTex style.
(*Hint:* The solution to this question is $\hat{B}^T = (X^T X)^{-1}(X^T Y)$, please use it to confirm your work or to continue with the next questions.)
**Question 3 (30 points)**
1) Now, compute numerically $\hat{B}^T = (X^T X)^{-1}(X^T Y)$ in order to estimate the coefficients $\vec{\alpha}$ and $\vec{\beta}$ of equation 1.

2) Calculate analytically the sample covariance matrix of matrix $\hat{E} = Y - \hat{B}^T X$. Once obtained the formula, implement the covariance matrix of $\hat{E}$ in python. Show all your work. (*Hint:* Be careful, you should not divide by 168, but by $168 - 2$.)
3) Additionally, compute for every $\hat{\beta}_i$ the corresponding variance $\sigma_i^2$, knowing that the formula is

$$\sigma_i^2 = [Cov(\hat{E})]_{i,i}, \tag{3}$$

for $i = 1, \dots, 13$.
3) Furthermore, compute the $R^2$ for every stock of the 13 stock in $Y$, knowing that the corresponding formula is

$$R_i^2 = 1 - \frac{[\hat{E}^T \hat{E}]_{i,i}}{[Y^T Y]_{i,i}} \tag{4}$$

with $i = 1, \dots, 13$.
4) Present your results using a similar format of table 2.

| Tick | $\hat{\beta}$ | $\sigma$ | $R^2$ |
|---|---|---|---|
| AA | 1.292 | 7.69405 | 0.347 |
| AGE | ... | ... | ... |

Table 2: Results Factor Analysis

**Question 4 (15 points)**
1) Interpret the results of question 3, point 4.
2) Compute the correlation matrix of the residuals ( $Corr(\hat{E})$ ) and display it.
3) Are the residuals uncorrelated? Check the model assumptions and discuss if the model is valid.
**Question 5 (15 points)**
1) Compute the sample covariance of the stocks in matrix $Y$.
2) Calculate analytically the covariance of model $Y = B'X + U$. Show your work.
3) Compute numerically the formula obtained in point 2) of the current question. Display both covariance matrices and clarify if the model approximates adequately the covariance among the stocks.
**Question 6 (10 points)**
In finance, it is a common practice to consider the global minimum variance portfolio, so the analyst can compare the covariance matrix estimated through the model ( $\Sigma_{mod}$ ) with the sample covariance ($\Sigma_{sample}$) of the stocks in our portfolio $Y$. Additionally, this model contributes also to unveil how much a stock 'weights' in our portfolio. In order to determine the minimum variance portfolio, you should solve the following optimization problem

$$\min_{\vec{\omega}} \vec{\omega}^T \Sigma \vec{\omega} \tag{5}$$

$$\text{such that } \vec{\omega}^T \mathbf{1} = 1 \tag{6}$$

where $\mathbf{1}$ is a vector of ones of length 13, while $\vec{\omega} \in \mathbb{R}^{13 \times 1}$ represent the vector of the stock's 'weight'.
1) Solve analytically the optimization problem and find the $\vec{\omega}$s for $\Sigma_{sample}$ and $\Sigma_{mod}$. Show your work.
2) Implement numerically the formulae obtained in point 1) of the current question and display the $\vec{\omega}$s estimated for both cases.
3) Explain the numerical results.