A primer to numerical simulations: Changelog

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Legend

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Page 2, starting line 9

This was already demonstrated in the "Schülerakademie Teilchenphysik", where this course was tested successfully on two groups consisting in total of 24 German high school students from 10th to 13th grade in 2015 and 2017. As an example on how the course can be implemented in practice, we describe our own experiences with it in the second to last section.

Page 2, starting line 26

2. Trajectories, velocities, accelerations and Newton's second law

A ~~physical~~ classical physics system is said to be understood, if the assumed forces acting on it lead to the observed trajectories.

Page 3, equation (4), (5) and text below

Analogously we get

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}(t)}{\Delta t} =: \frac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t) \, \cancel{,} = \cancel{\frac{d^2\vec{r}(t)}{dt^2} =} \ddot{\vec{r}}(t) \, , \tag{4}$$

where we introduced the second derivative in the ~~second line~~last step.

~~It was Newton who observed that, if a body is at rest, it will remain at rest, and if it is in motion it will remain in motion at a constant velocity in a straight line, unless it~~

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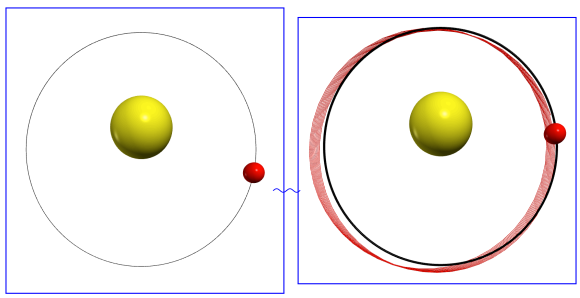
~~is acted upon by some force — this is known as~~ The dynamics is controlled by Newton's ~~first law. Formulated differently: a force $\vec{F}$ expresses itself by changing the motion of some object. This is quantified in Newton's~~ second law

$$\vec{F}(\vec{r}, t) = \frac{d}{dt}(m\vec{v}) \ . \tag{5}$$

Page 3, line 28 and below

Clearly to initiate the procedure at some time t_0 both $\vec{r}(t_0)$ as well as $\vec{v}(t_0)$ must be ~~known~~ fixed — the trajectories depend on these initial conditions §.



(a) (b)

Figure 1: Different Mercury orbits for the purely Newtonian gravity (8) and $\Delta t = \Delta t_0/20$ for (a) and $\Delta t = \Delta t_0 \times 2$ for (b), where the black line represents the orbit of (a) and the red line is the orbit for the larger time steps. The reference time step Δt_0 is defined via inequality (7) as explained in the text. The images are screenshots of the simulation that is described below (with modified colors).

t This procedure can only work if Δt is sufficiently small. One way to estimate whether Δt is small enough is to verify whether the relation

$$|\vec{v}(t)| \gg \frac{1}{2}|\vec{a}(t)|\Delta t = \frac{1}{2m}|\vec{F}(\vec{r}(t))|\Delta t \tag{7}$$

holds. This follows from Eq. (1) where the first term that ~~we~~ was neglected reads $(1/2)a(t)(\Delta t)^2$. The impact of the size of Δt on the simulation is illustrated in Fig. 1 where only Newtonian gravity was used. While for the calculations for the left panel Δt is chosen significantly below $\Delta t_0 = 2m|\vec{v}(0)|/|\vec{F}(\vec{r}(0))|$ (*cf.* Eq. (7)), for the right panel it was chosen twice as large. Accordingly, only the trajectory in the left panel shows the characteristic feature of a $1/r^2$–force of an ellipse fixed in space. The trajectory in the right panel does not reproduce itself in successive revolutions.

§ In general, a differential equation of n^{th} degree (where the highest derivative is order n) needs n initial conditions specified. For $n = 2$ those are often chosen as location and velocity at some starting time, but one may as well pick two locations at different times.

Page 19, before previous section 8. Summary

8. Own experiences with the implementation of the course

Work on the subject began with a project work of one of the authors (JH). The project was presented at the Mädchen-Gymnasium Jülich (a German high school) in 11$^{\text{th}}$ grade in 2014. Based on this study, the course was developed as outlined in Sec. 4 and used in the Schülerakademie Teilchenphysik in 2015, which aims at high school students from 10$^{\text{th}}$ to 13$^{\text{th}}$ grade. This academy is financed by the German Research Society (DFG) as the outreach branch of a research grant focussing on basic research in particle and nuclear physics. One of the authors (CH) is acting as a Principal Investigator for the outreach activities within this scheme. The Schülerakademie Teilchenphysik runs biennially at the Science Center Overbach in Jülich for four days and has about 25 participants coming from various parts of Germany. The academy comprises lectures, a tour to the particle accelerator COSY and the high performance computing facility at the Research Center Jülich as well as one full day of practical work. Given the setting, the participating students are highly motivated and eager to understand.

The course presented in this paper is offered as one out of three hands-on project options. A lecture to the whole group explaining the basics of numerical simulations laid down some foundations in addition to what is presented in this paper. Given the positive to enthusiastic feedback we have received from the participants on the course after its first installation in 2015, we decided to make the course a permanent part of the academy. In 2017 we received similarly positive feedback. About one half of the participants of each year chose to work on the numerical project — out of those about one half had previous programming experience. Most of the participants had already learned about derivatives and vector calculus in school.

The numerical course was allocated for one work day (seven hours) and the students worked in pairs. Each group worked on a laptop provided by us, where the necessary software was pre-installed. Four of the authors (IH, CK, CM, JLW) acted as supervisors for the students although not all at the same time. Each instance of the course was supervised by one to three people. The course was initiated with a brief summary of previously presented methods, motivations and objectives of the simulation. In the first two hours the students were introduced to `Python` and `VPython`. In the following part the students worked with an initial code template (see [6]) and were motivated to come up with own solutions to fulfill the objectives. A large part of the material covered during this day is presented in Sec. 4. With the exception of some younger students, the participants where able to simulate and visualize the motion of mercury including forces from GR. Only a few participants started to implement the quantitative extraction of the perihelion motion, uncertainty estimates were not a subject as the time frame was too short. Once the basic problem was tackled some students deviated from the suggested path and for instance studied the problem with one additional planet or different starting parameters. This is a very interesting problem by itself for there are parameter ranges where the classical three-body system develops chaotic features. We encouraged such explorations as well. We include the additional material in the paper because we want to provide teachers with sufficient material to fill a working group outside the general curriculum running for a longer period than one day.