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PAPER

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Simulating satellite and space probe motion at high school with spreadsheets

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Abstract

This paper gives an account of an experiment in which thirty-three high school students of ages 17–19 developed spreadsheet numerical models of satellite and space probe motion. The models are free to download. A survey was carried out to find out the students' opinion of the lessons.

1. Introduction

Satellite motion is a common topic of physics courses, starting at secondary level with Kepler's laws (taught in the first year at high school in the author's country). The proofs to the laws can be found in many textbooks [1–3] and articles [4–8]. However, the laws say nothing about the position of the satellite in time.

The problem is given by a system of two differential equations. The analytical solution is well beyond high school mathematics [2, 4]. That suggests that satellite motion is inappropriate to be studied at high school. However, dynamic systems can be studied also through solving the governing system numerically. The Euler method is the simplest numeric method [9]. It is easy to comprehend and work with even at high school level. Spreadsheets allow the implementation of the method in a transparent way [10, 11].

This paper reports on an experiment in which thirty-three high school students of ages 17–19 developed spreadsheet models that simulated satellite and space probe motion. The governing system of differential equations was solved numerically by the Euler method. The models are free to download. A questionnaire survey was carried out to find out the students' opinion of the lesson. The results are discussed.

2. Theory

Let the centre of Earth be positioned at the centre of the xy coordinate system (figure 1). Let a satellite start moving at time $t = 0\,\mathrm{s}$ from point $(r_0, 0)$ at speed v_0 in the direction of half axis y^+. The satellite is subject to the gravitational force of Earth, the direction of which is towards Earth and the amplitude is [1]

$$F_g = G\frac{mM}{r^2}, \tag{1}$$

where $G = 6.667 \times 10^{-11}\,\mathrm{Nm^2\,kg^{-2}}$ is the gravitational constant, m is the mass of the satellite, $M = 5.972 \times 10^{-24}\,\mathrm{kg}$ is the mass of Earth and $r = \sqrt{x^2 + y^2}$ is the radius, that is, the distance between the satellite and Earth's centre.

Decomposing force $\mathbf{F}_g = \mathbf{F}_{gx} + \mathbf{F}_{gy} = F_{gx}\,\mathbf{i} + F_{gy}\,\mathbf{j}$ and radius vector $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$ into the x and y directions gives two similar right-angle triangles. It holds that

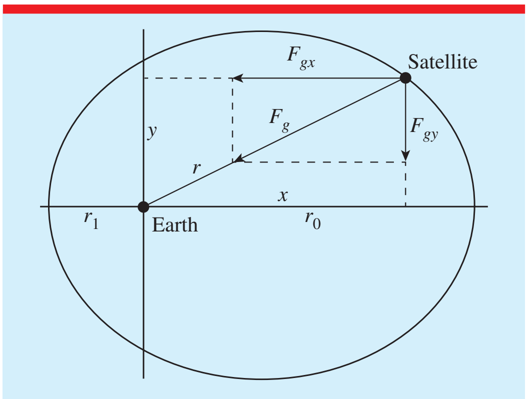

Figure 1. Trajectory, force components and coordinates of a satellite.

$$\frac{|F_{gx}|}{F_g} = \frac{|x|}{r}, \tag{2}$$

$$\frac{|F_{gy}|}{F_g} = \frac{|y|}{r}. \tag{3}$$

Substitution from equation (1) and factoring the sign of the gravitational force components and projectile coordinates in yields

$$F_{gx} = -\frac{GmMx}{(x^2 + y^2)^{1,5}}, \tag{4}$$

$$F_{gy} = -\frac{GmMy}{(x^2 + y^2)^{1,5}}. \tag{5}$$

Applying the formulas $F_x = m\Delta v_x/\Delta t$, $F_y = m\Delta v_y/\Delta t, v_x = \Delta x/\Delta t$ and $v_y = \Delta y/\Delta t$ gives

$$\Delta v_x = -\frac{GMx}{(x^2 + y^2)^{1.5}}\Delta t, \tag{6}$$

$$\Delta v_y = -\frac{GMy}{(x^2 + y^2)^{1.5}}\Delta t, \tag{7}$$

$$v_x = v_x + \Delta v_x,\ v_x(0) = 0, \tag{8}$$

$$v_y = v_y + \Delta v_y,\ v_y(0) = v_0, \tag{9}$$

$$\Delta x = v_x \Delta t, \tag{10}$$

$$\Delta y = v_y \Delta t, \tag{11}$$

$$x = x + \Delta x,\ x(0) = r_0, \tag{12}$$

$$y = y + \Delta y,\ y(0) = 0. \tag{13}$$

Let k be the ratio of the centrifugal and gravitational force at the start, that is,

$$k = \frac{v_0^2 r_0}{GM}. \tag{14}$$

Five types of trajectory exist depending on k [4]:

$0 < k < 1$: ellipse; the satellite first approaches the Earth and then recedes from it.
$k = 1$: circle; the first cosmic speed; the distance between the satellite and Earth is constant.
$1 < k < 2$: ellipse; the satellite first recedes from the Earth and then approaches it.
$k = 2$: parabola; the second cosmic speed; the body recedes only; space probe (not a satellite).
$k > 2$: hyperbola; space probe; after long time it moves along a line, the asymptote.

3. Models

The models are available at www.dropbox.com/s/dijxz45yc2ihozi/benacka_satellite_spaceprobe.zip?dl=0.

The zip file contains file 01_satellite_spaceprobe.xlsm with sheets Satellite (figure 2) and Space probe (figure 3), file 02_examples.xlsm with sheets Sputnik 1, Apollo 11 and Earth, and file 03_kepler_laws.xlsm with sheets Kepler 1, Kepler 2 and Kepler 3. The common features are as follows: the inputs are in the white cells. $R = 6371$ km is the Earth's radius and h_0 is the altitude above Earth at start. Cell C7 contains either period T (figure 2) or flight duration $t_{\max}$ (figure 3). The grey cells contain formulas. Parameter $r_0 = R + h_0$ is calculated in cell G2. Parameter k is calculated in cell G3 by equation (14). Cell G4 contains the last value of y (see the aim below). The time is in range B19:B10019 calculated from 0 to T or $t_{\max}$ in $N = 10000$ steps $\Delta t = T/N$ or $\Delta t = t_{\max}/N$ (cell F12 and G12). The velocity, its components, and the coordinates are calculated in range C19:G10019 by equations (6)–(13). The trajectory is a XY graph made over range F19:G10019.

Range B10:E16 is to simulate the motion. Time T or $t_{\max}$ is divided into $n = 100$ steps Δt (cells B12 and C12). Number i of elapsed steps in cell D12 is governed by the spinbutton. Its properties Max, Min, SmallChange and LinkedCell

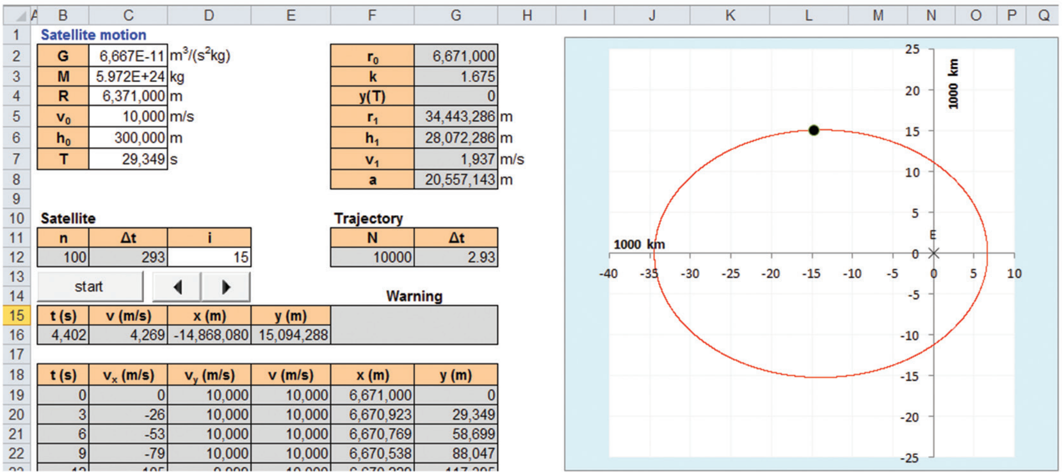

Figure 2. Model of satellite motion.

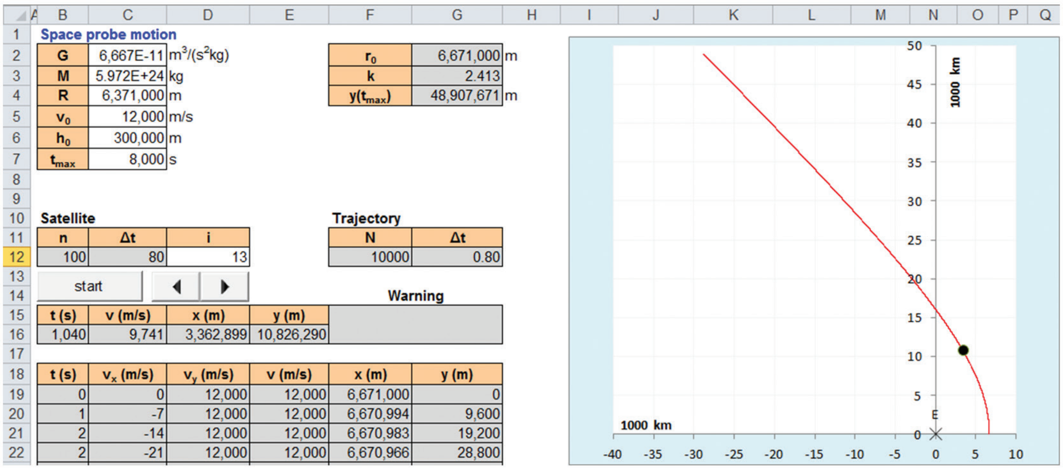

Figure 3. Model of space probe motion.

are set to 100, 0, 1 and D12. Time $t = i \cdot \Delta t$ is calculated in cell B16. The speed and coordinates are returned to cells C16, D16 and E16 by function VLOOKUP. The body is a one-point graph of type XY made over range D16:E16. Clicking and holding down the spinbutton increases number i by 1 and puts the body at new coordinates. A click on button 'start' sets the body in motion via the VBA code

```
Private Sub CommandButton1_Click()
 For I = 1 To 100
  Range("D12") = I
  Calculate
 Next I
End Sub
```

The speed of animation depends on the computer, Excel version and number of open files.

If the user needs to know the position and speed at a given time, it is enough to write it in cell B16 but animating will not work until the formula = C12 ∗ D12 is returned back into cell B16.

Model Satellite holds if $k < 2$. To find period T, the user inputs an estimate in cell C7 so that the trajectory is almost closed. Then he/she applies Goal Seek with parameters Set cell G4, to value 0 and by changing cell C7. Parameter r_1, the other extreme distance, is in cell G5 and the corresponding altitude h_1 above Earth and speed v_1 are calculated in cells G6 and G7. Length $a = (r_0 + r_1)/2$ of the main half-axis is calculated in cell G9. If $k > 1.9$, 10,000 steps is not enough to get a correct solution and warning 'Incorrect ellipse' appears in range F15:G15. If $k \geqslant 2$, then warning 'Not a satellite, use sheet Space probe' appears in range F15:G16.

Model Space probe holds if $k \geqslant 2$. If $k < 2$, then warning 'Not a space probe, use sheet Satellite' appears in range F15:G16. The user inputs flight duration $t_{\max}$ in cell C7 and iterates it until the trajectory ends up near the left or upper border of the chart.

Models Sputnik 1, Apollo 11, Earth, Kepler 1, 2 and 3 were derived from model Satellite. In Sputnik 1, cells B6 and F6 are rewritten as perigee and apogee and ranges B8:D8 and F9:H9 are added for the inputted apogee and the relative error between the inputted and calculated apogee. The user inputs the perigee, period and apogee in cells C6, C7 and C8, and applies Goal Seek with Set cell G4, To value 0 and By changing cell C5 to find starting speed v_0.

Model Kepler 1 is to prove Kepler's first law. The x-coordinate $C_x = r_0 - a$ of ellipse centre C is calculated in cell G18. The x-coordinate $F_x = 2C_x$ of focus F is calculated in cell G19. The distances from the satellite to the foci are calculated in range H24:I10024. The difference between the sum of the distances and $2a$ is calculated in J24:J10024. The maximum is in G20.

Model Kepler 2 is to prove Kepler's second law. Radius r is calculated in range H24:H10024, where r' means the previous value. Distance d between the current and previous position is calculated in range I24:I10024. Semiperimeter $s = (r + r' + d)/2$ of the triangle that the radius sweeps out in interval Δt is calculated in range J24:J10024. Area A of the triangle is calculated in range K24:K10024 by Heron's formula $A = \sqrt{s(s-r)(s-r')(s-d)}$. The maximum and minimum of the area are in G18 and G19. The difference is calculated in G20.

Model Kepler 3 is to prove Kepler's third law. Range B18:D21 contains the reference data (taken from model Satellite). The square of the ratio of the current and reference period is calculated in cell G18. The cube of the ratio of the current and reference semi-major axis is calculated in cell G19. The difference between them is calculated in cell G20.

4. Lessons and survey

Thirty-three high school students aged 17–19 took part in the experiment in three groups. Two 90 min lessons were taught to each group by the author ('teacher' in the following text). The students were familiar with equation (1), Kepler's laws, the first and second cosmic speed, and formulas $F_x = m\Delta v_x/\Delta t$ and $v_x = \Delta x/\Delta t$ from the first year. They were not familiar with the Euler method, Goal Seek, spinbuttons, buttons and programming in VBA.

The lesson started with a discussion on planetary motion. The teacher projected the models in sheets 'Satellite' and 'Space probe' and showed the four types of trajectory. Then, he projected figure 1 and went through the theory. He applied the method of questioning and encouraged the students to recall and derive the formulas. Then, he sent the pdf with the theoretical background to the students' emails.

The students downloaded the template (sheet 'Satellite' without formulas and chart) and started developing the model. The teacher was working along with the students, discussing the steps and projecting his work to give feedback but with a delay so that the students could work individually. The students wrote the data in cells C2:C7 (20 000 for T) and the formulas in ranges G2:G4, F12:G12 and B19:G10019, and created the graph. It was a hit to see that the trajectory was really an ellipse-shaped curve. The teacher showed how to find period T by Goal Seek. The students wrote the formulas in range G5:G8. The lesson was over.

The second lesson started with simulating the motion. The students wrote the formulas in ranges B12:D12 and B16:E16, added the graph into the chart, put the spinbutton in the sheet and set up the parameters. It was a big hit to set the satellite in motion by clicking the spinbutton. Then the teacher explained the VBA code assigned to button 'start'.

The teacher invited the students to experiment with the inputs and get a hyperbolic trajectory. He pointed out that there is no period and the number in cell C7 gives the duration of the motion. The students made a copy of the sheet, renamed it and adjusted as shown in figure 3. The teacher asked the students to find speed v_0 at which the trajectory would be a parabola (Goal Seek, *Set cell* G3,

Table 1. Number of answers to the questionnaire.

	All				All %				Boys %				Girls %			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
A	8	25	0	0	24	76	0	0	35	65	0	0	13	88	0	0
B	15	16	2	0	45	48	6	0	47	47	6	0	44	50	6	0
C	7	24	2	0	21	73	6	0	6	82	12	0	38	63	0	0
D	2	26	5	0	6	79	15	0	6	88	6	0	6	69	25	0
E	20	13			61	39			94	6			25	75		
F	17	16			52	48										

To value 2 and *By changing cell* C5; $v_0 = 10\,927$ m s^{-1} if $h_0 = 300$ km). Then he invited the students to think of how to warn of a periodic solution. Some students made the right suggestion and the class wrote the warning formulas in range F15:G16. The class switched back to sheet 'Satellite' and wrote the warning formulas in range F15:G16. The teacher showed that if k was between 1.9 and 2, the trajectory was drawn incorrectly due to the numeric method. The students adjusted the warning in range F15:G16. Developing sheets 'Satellite' and 'Space probe' was finished.

The teacher invited the students to model the flight of Sputnik 1. The students made a copy of sheet 'Satellite', renamed it 'Sputnik 1' and adjusted it. They took the perigee, apogee and period from Wikipedia and found speed v_0 by Goal Seek. The teacher invited them to model at home the flights of the lunar module of Apollo 11 around the Moon and Earth around the Sun. That was the end of the second lesson. Kepler's laws were not proved due to lack of time. The students filled in the following questionnaire:

(A) *The lesson was (1 = very; 2 = quite; 3 = little; 4 = not) interesting.*
(B) *I understood (1 = all; 2 = most; 3 = little; 4 = nothing) of the algorithm*
(C) *I learned (1 = a lot; 2 = quite a lot; 3 = little; 4 = nothing) new in Excel.*
(D) *I learned (1 = a lot; 2 = quite a lot; 3 = little; 4 = nothing) new in physics.*
(E) *I would like to model the motion of a rocket: (1 = yes; 2 = no).*
(F) *I am a boy (1 = yes; 2 = no).*

The number of answers is in table 1. Answers 1 and 2 in A–D and 1 in E–F are positive ones.

The relative number of positive answers is (all, boys, girls): (A) 100%, 100%, 100%; (B) 94%, 94%, 94%; (C) 94%, 88%, 100%; (D) 85%, 94%, 75% and (E) 61%, 94%, 25%. The result of 100% and 94% in questions (A) and (B) suggest that the lessons were successful. The result of 94% in questions (C) suggests that the students did a considerable step in their Excel skills. The result of 85% in questions (D) is interesting as just the Euler method, which is mathematics and not physics, was new to the students. Creating the models must have given the students the feeling of having extended their physics knowledge. The outcome of 61% in question (E) suggests that the topic was motivating to continue with numeric modelling but considerably more to boys (94%) than girls (25%). The overall result implies that modelling dynamic systems numerically with spreadsheets has the potential to promote STEM to high school students, mainly to boys.

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Jan Benacka graduated in mathematics, physics and informatics. He has 21 years' experience in teaching at secondary school and 13 years at university. His present research is focused on promoting STEM at upper secondary level through computer modelling with Excel, Delphi/Lazarus and Visual C# (www.ki.fpv.ukf.sk/~jbenacka/).