# Population wide Medical Image and Shape Analysis

**Felix Ambellan (ZIB)** and **Christoph von Tycowicz (FUB/ZIB)**

2nd Tandem Tutorial within the
MATH+ Thematic Einstein Semester 21/22 on
Mathematics of Imaging in Real-Word Challenges

# Outline

+ Statistical Shape Modeling (Primer)
+ Segmentation
+ Riemannian Shape Spaces
+ Shape Analysis

Practical part



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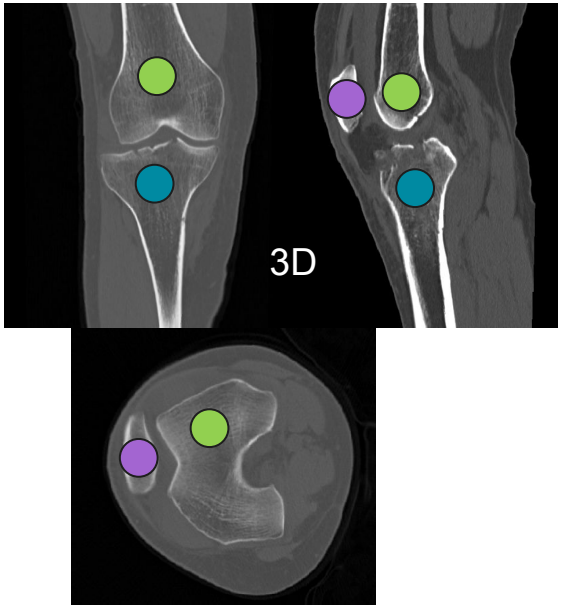
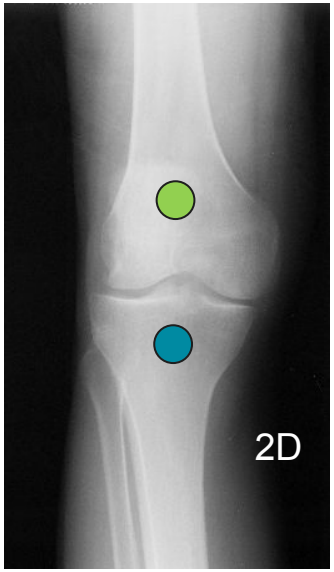
# Medical Images to Assess Shape

Computed Tomography (CT)

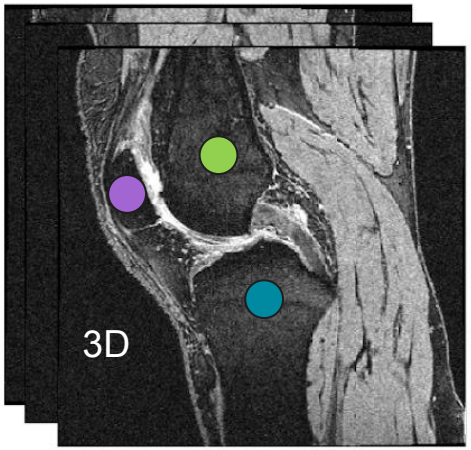


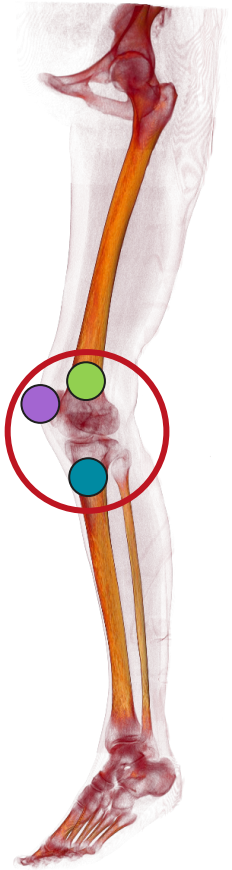

X-Ray




Grey valued function sampled on regular grid

Magnetic Resonance Imaging (MRI)





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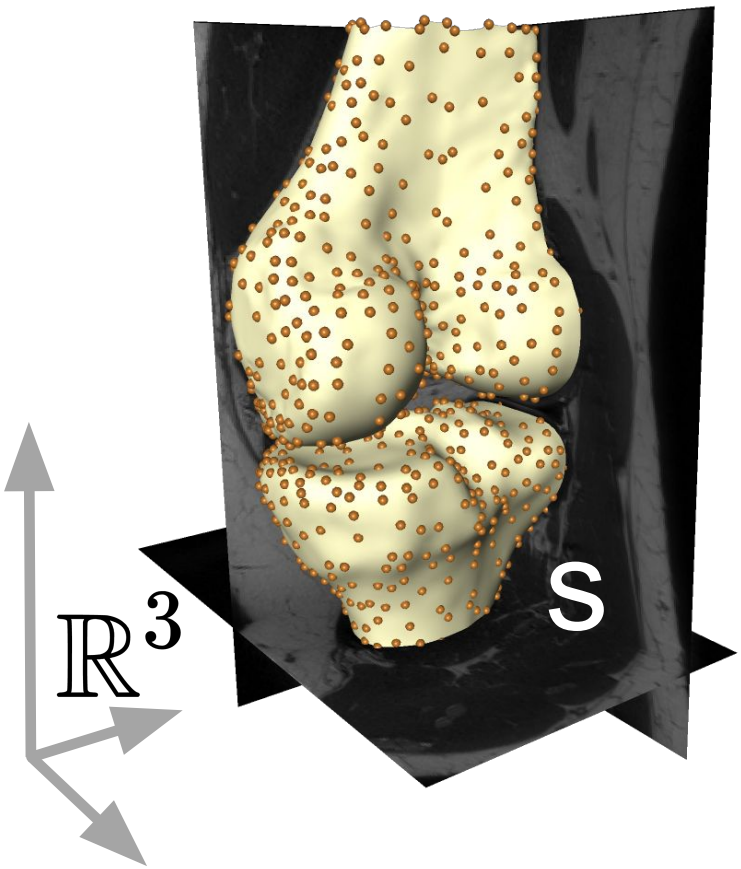
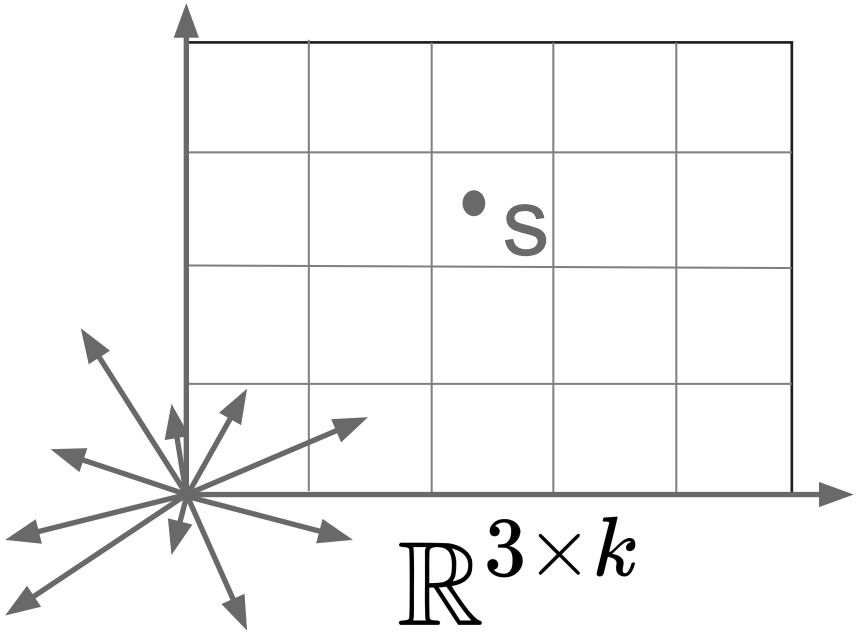
# Digital Shape Representation

$k$ vertices




Configuration Space



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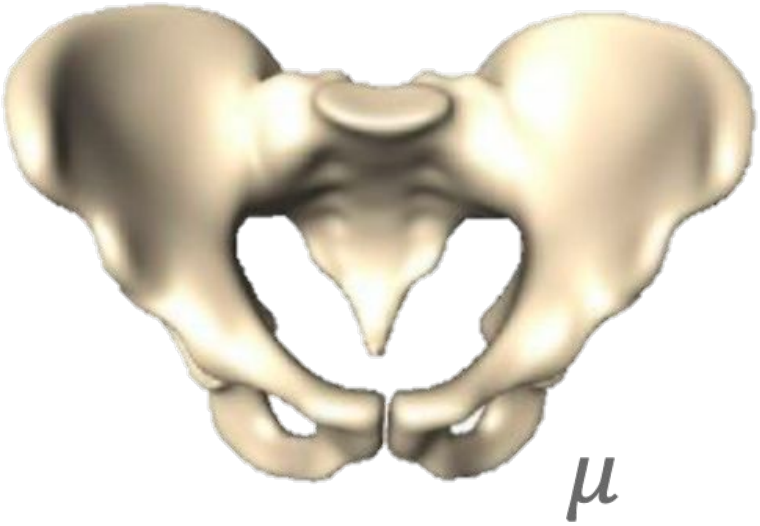
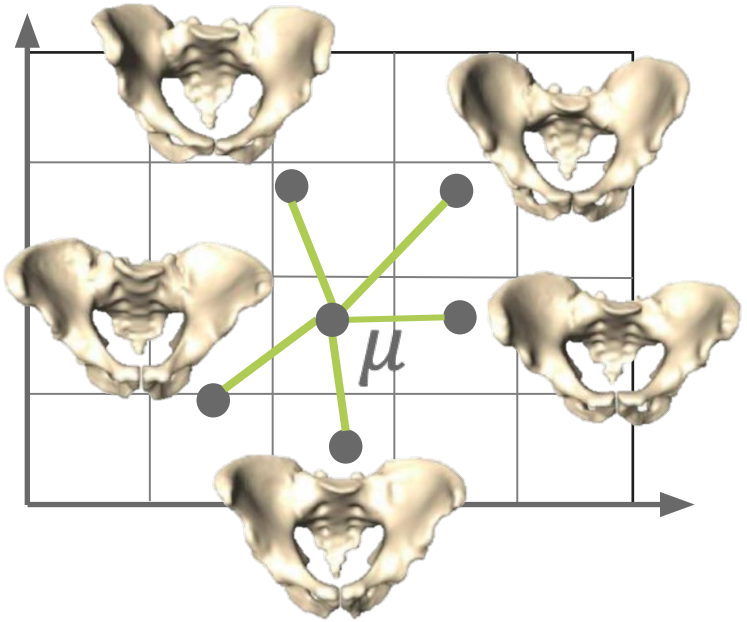
# Statistical Shape Model (SSM)

Mean Shape



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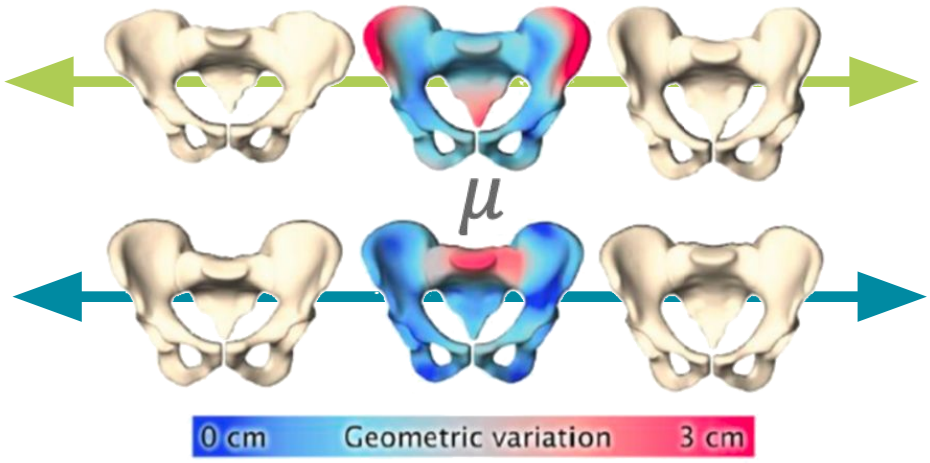
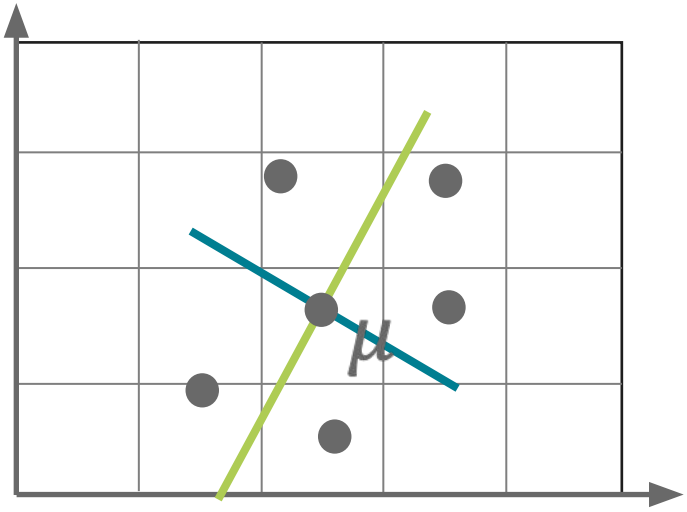
# Statistical Shape Model (SSM)

Principal Modes of Variation






[Cootes et al. 1995, Comput Vis Image 61(1)]

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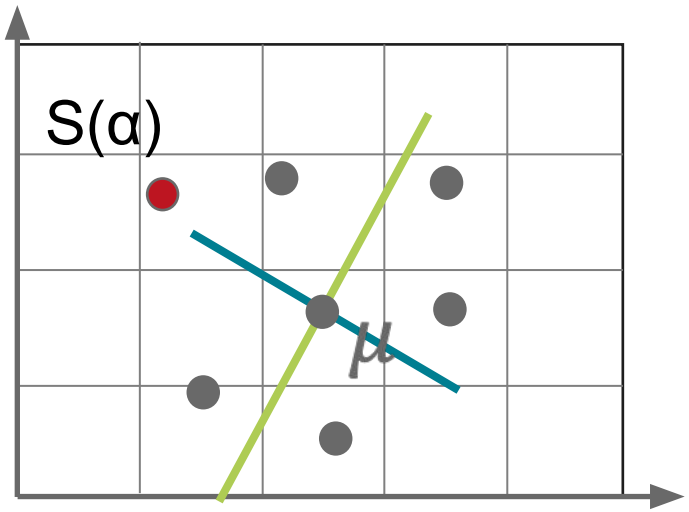
# Generation of Shapes

$$S(\alpha) = \mu + \alpha_1 + \alpha_2$$

An SSM can be adjusted to a given (unknown) configuration.



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# SSM: Point Distribution Model (PDM)

Shape Representation

- Landmark coordinates (three per vertex)

  \+ Simple vector space structure, i.e. fast

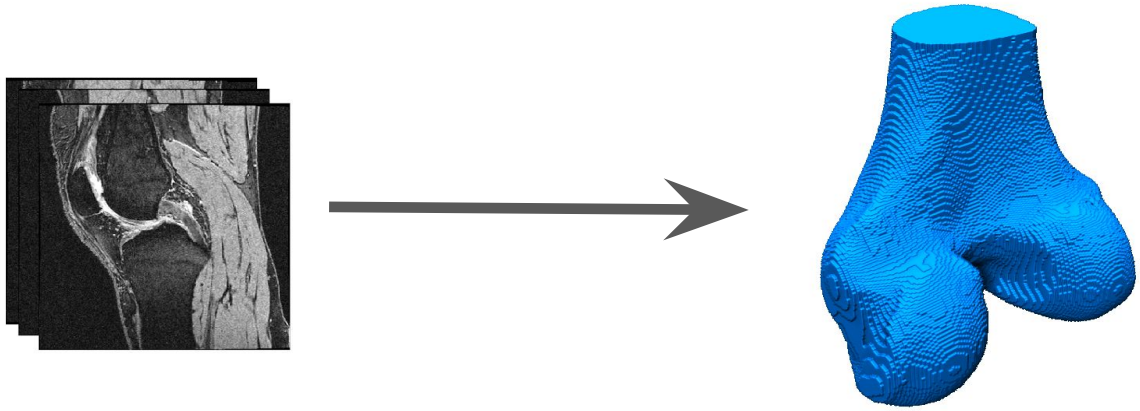
  \- Bias due to pose and orientation

  \- Cannot handle large (rotational) deformations

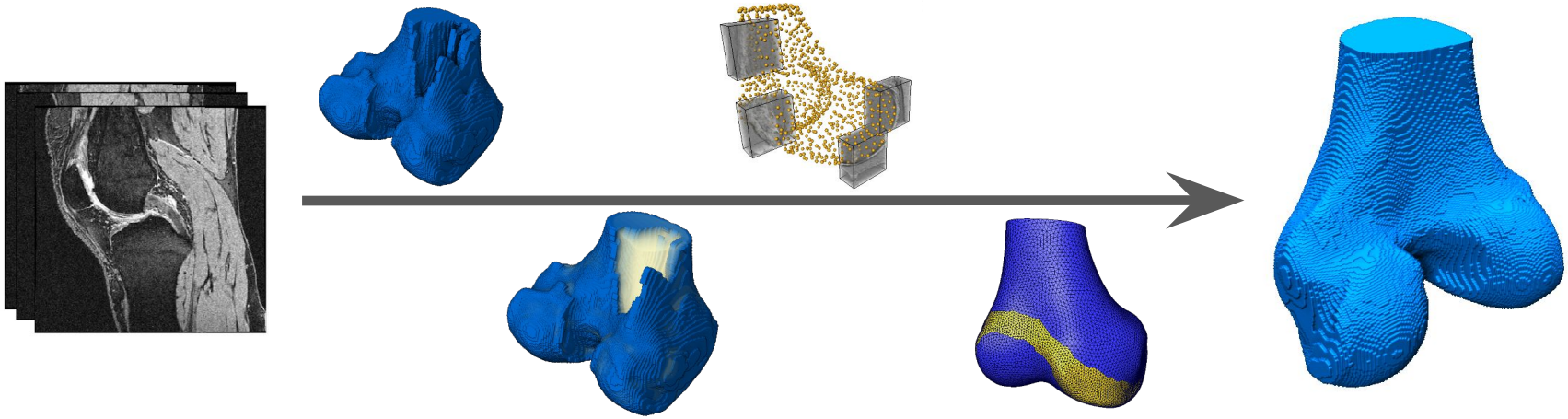
Generative model

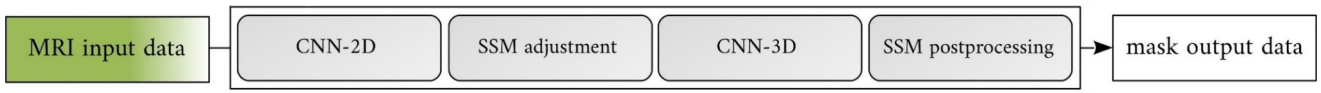
- Map representation to landmarks (inverse problem): Direct and in closed form

# Segmentation of Anatomical Structures



# Segmentation





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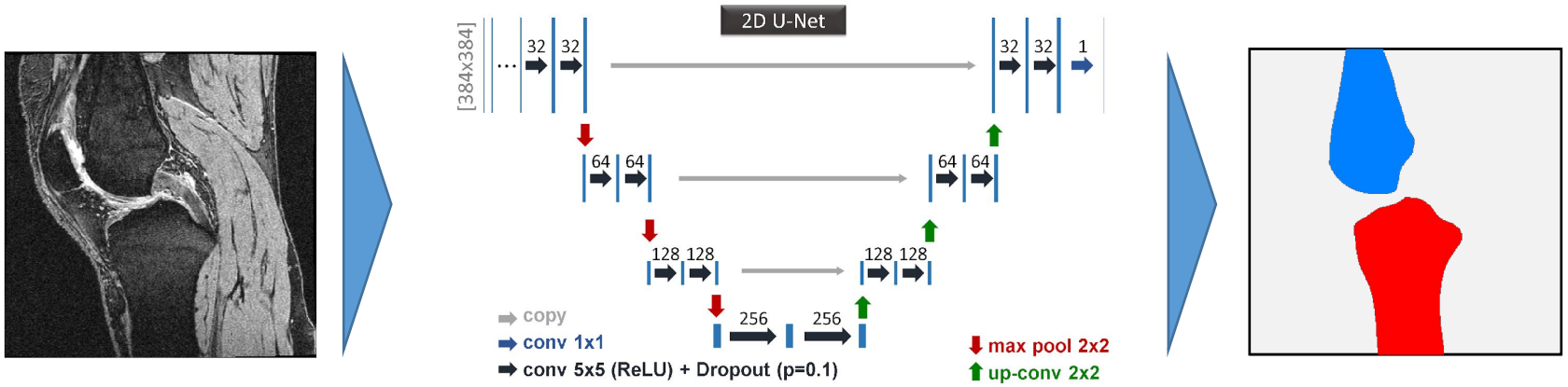
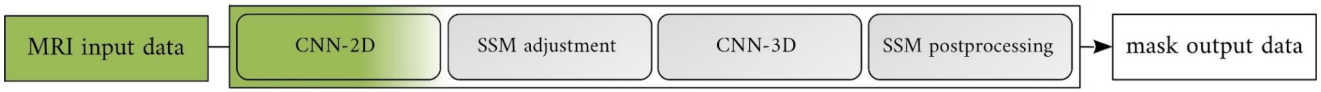
# Segmentation of bones: 2D U-Net



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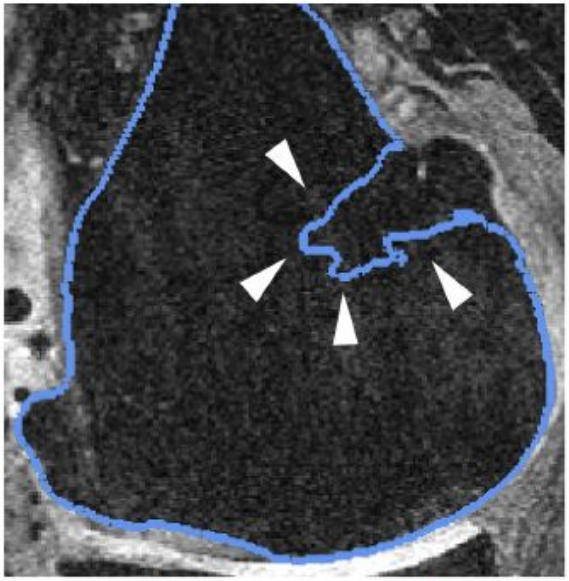
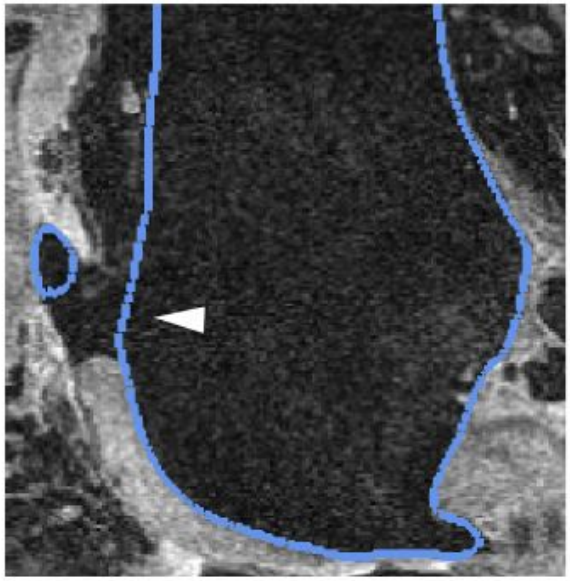
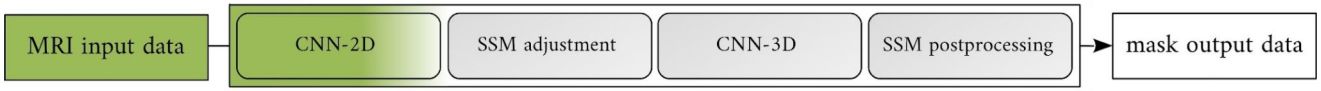
# CNN-2D: Segmentation problems



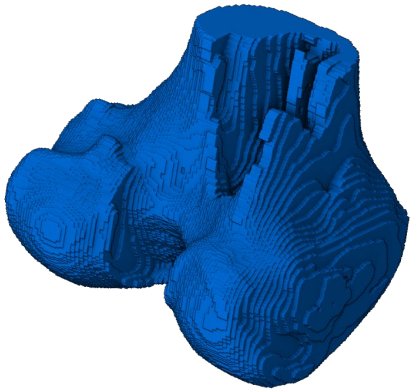


Dashed: Correct segmentation

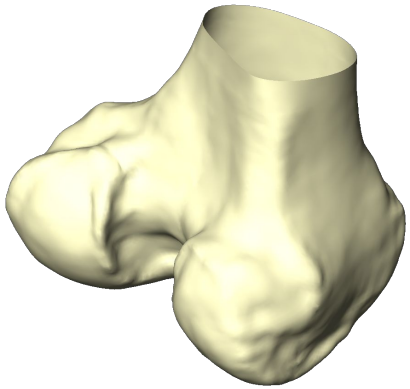


# Segmentation

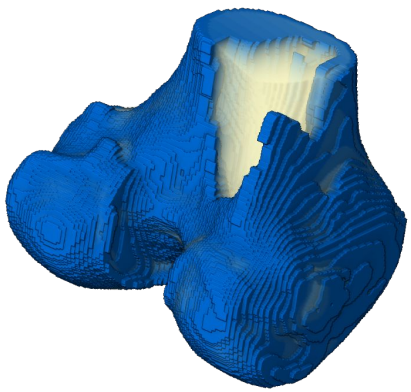
## 3D PDM for regularization of bone masks



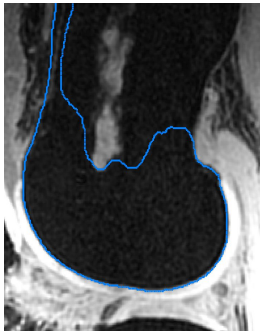
CNN-2D mask

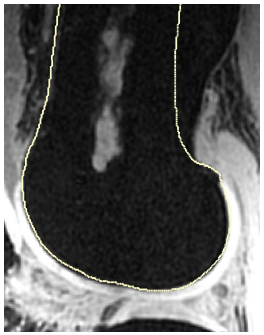


Adjusted PDM



Comparison





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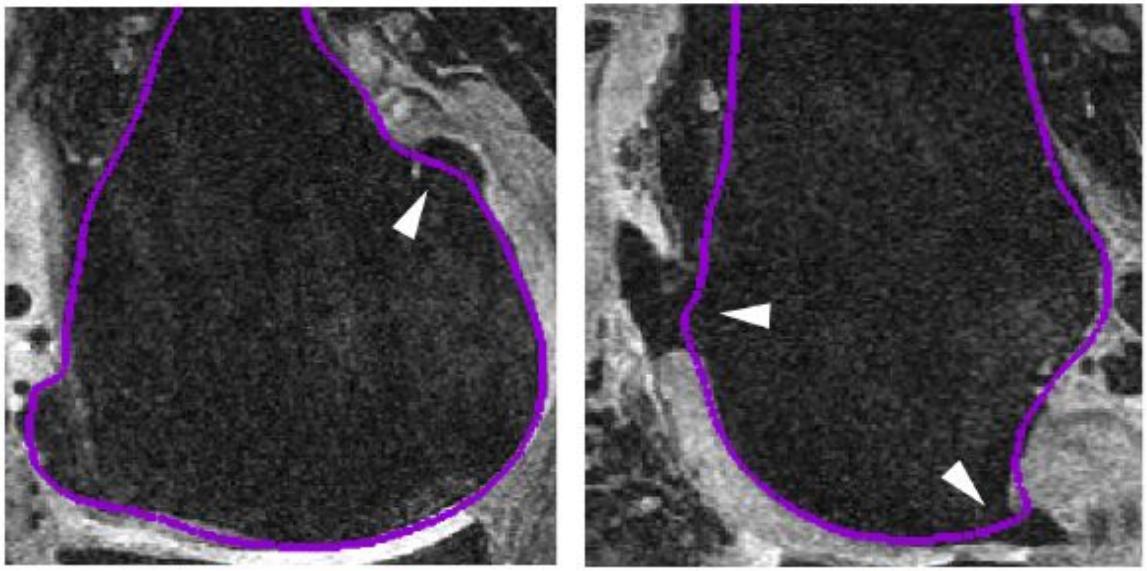
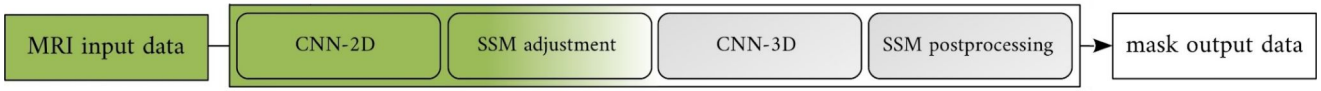
# PDM: Regularization problems



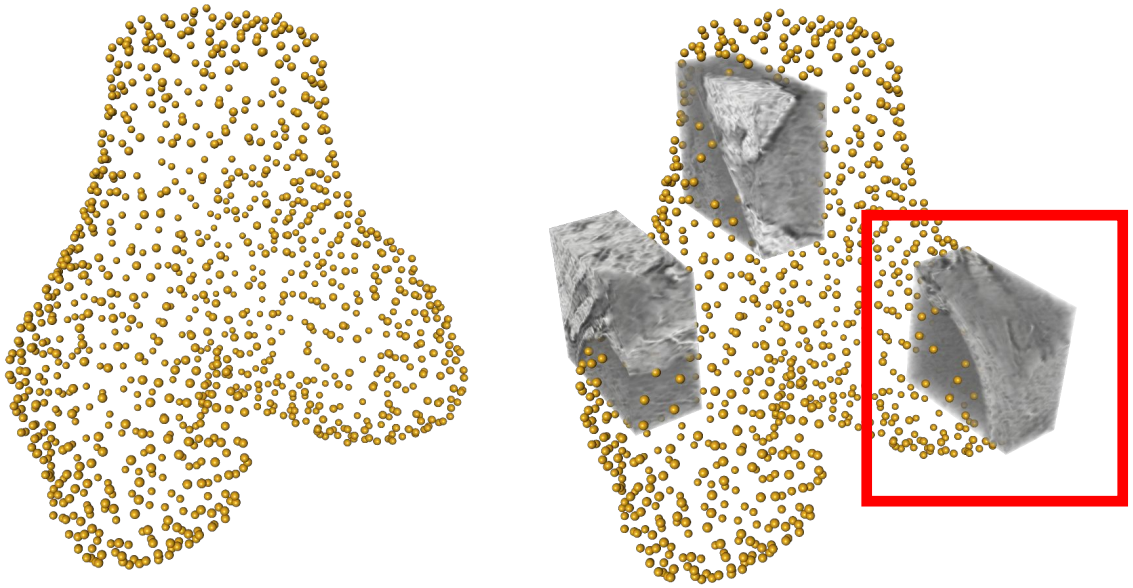
Dashed: Correct segmentation

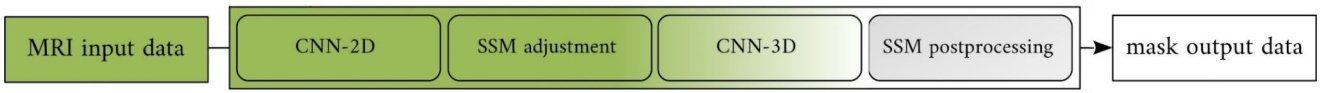


# CNN-3D Bone: Sparse sampling

≈100.000 points → ≈1.000 points

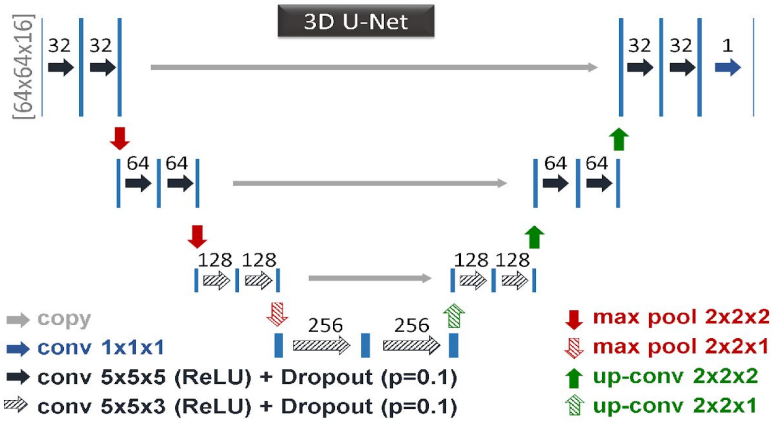
# CNN-3D Bone: Sparse sampling

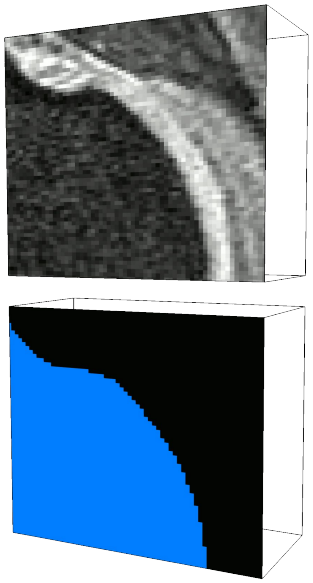


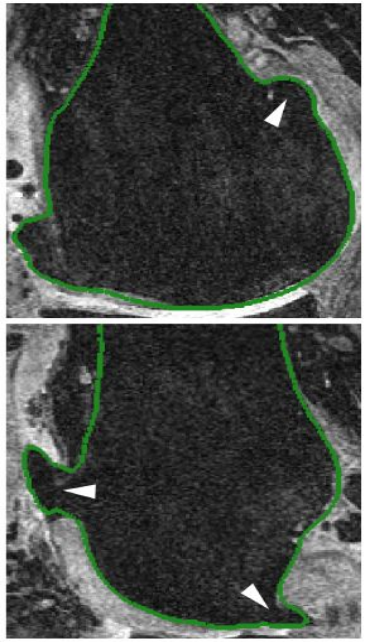


# Segmentation

## Bone: CNN-3D





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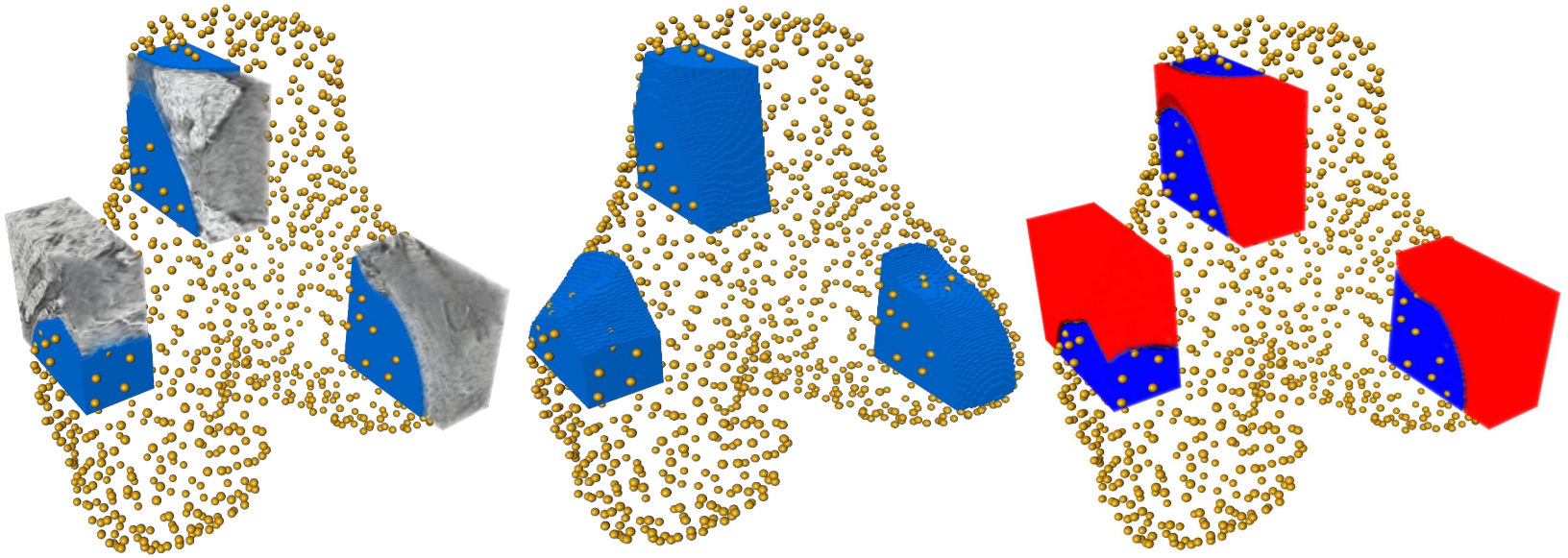
# Segmentation

## CNN-3D Bone: Segmentation of subvolumes







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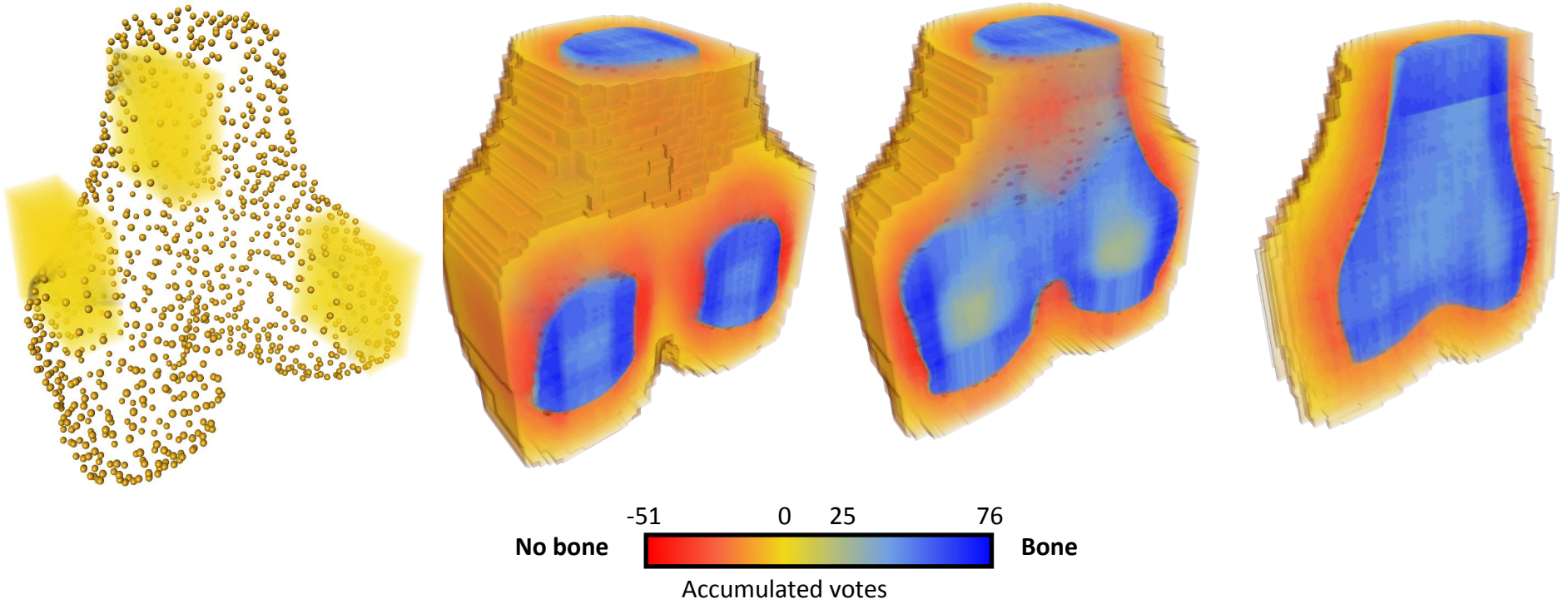
# CNN-3D Bone: Merging of subvolumes







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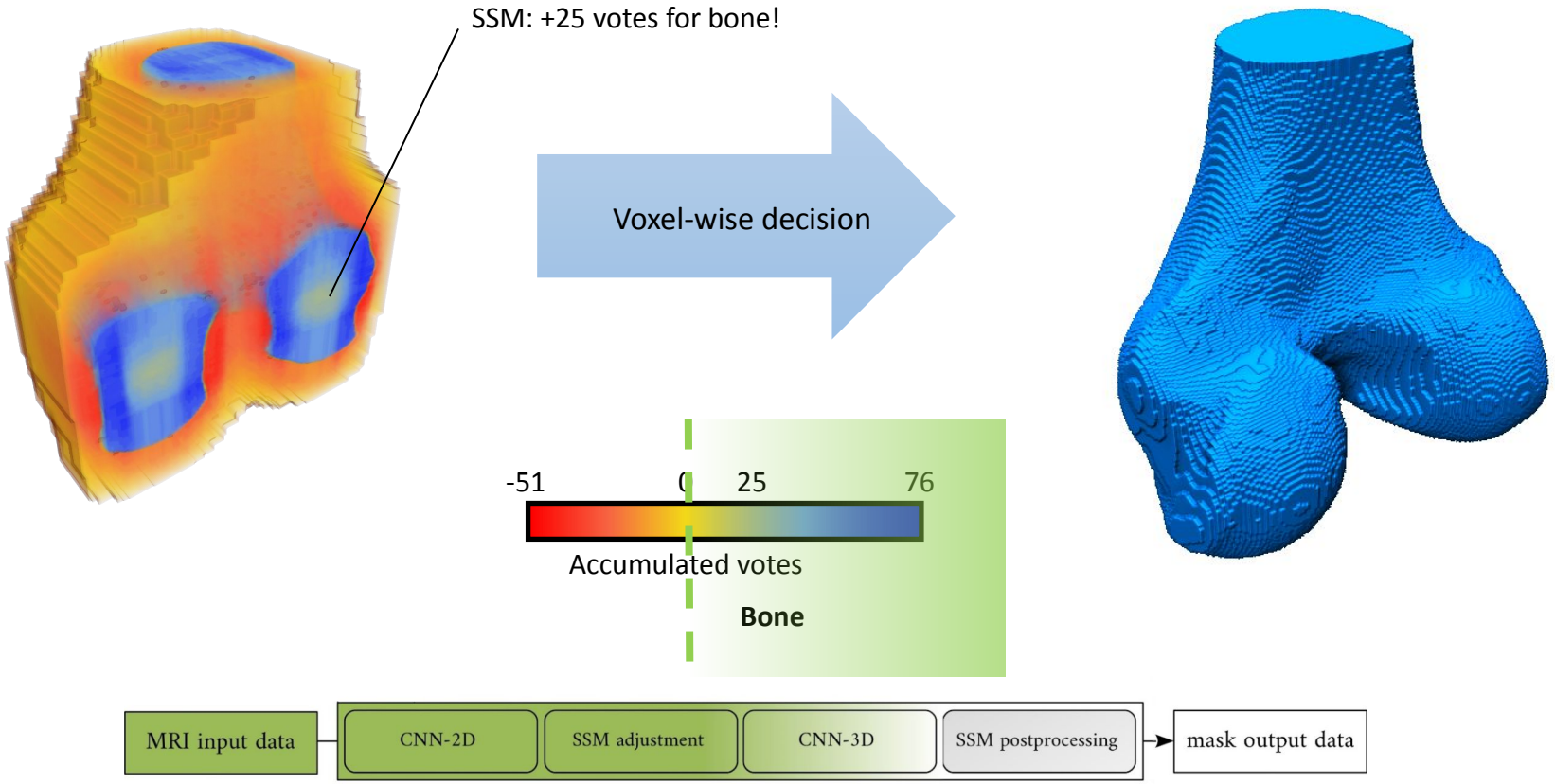
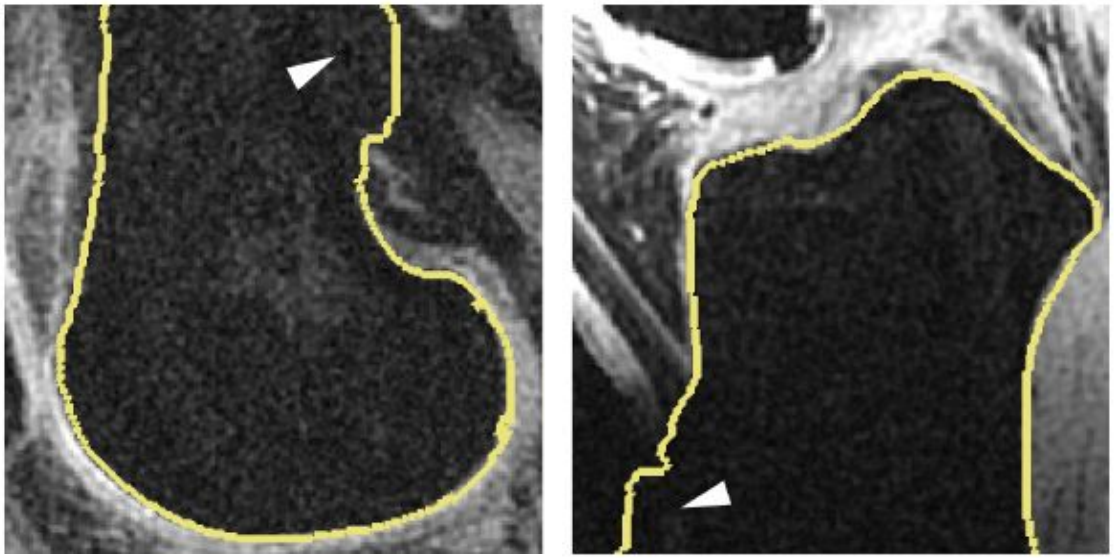
# CNN-3D Bone: Merging of subvolumes



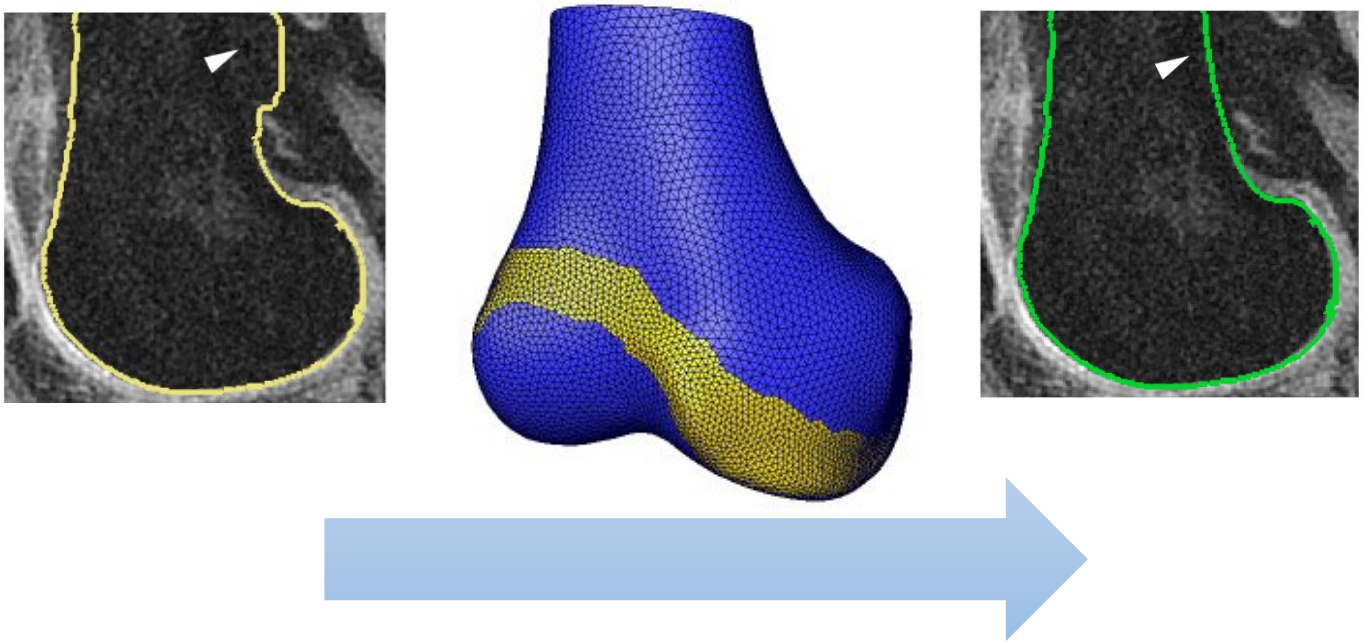
# CNN-3D Bone: Merging of subvolumes



Dashed: Correct segmentation

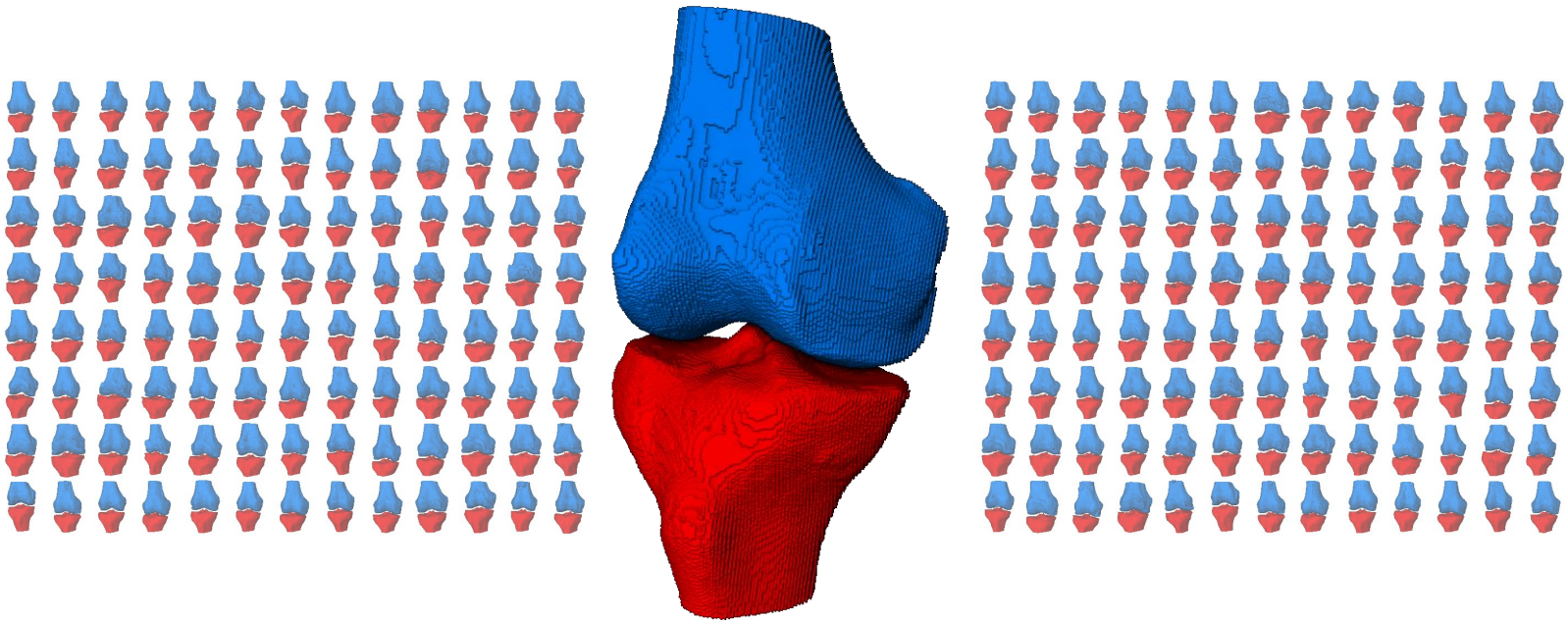
# PDM Postprocessing

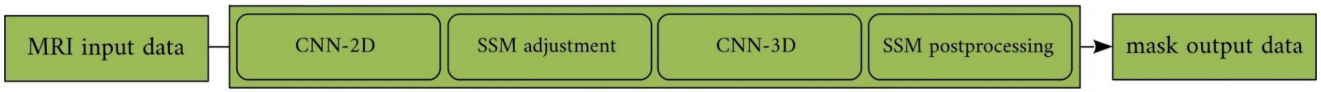
Femur PDM



# Segmentation

## Segmentation of many bones







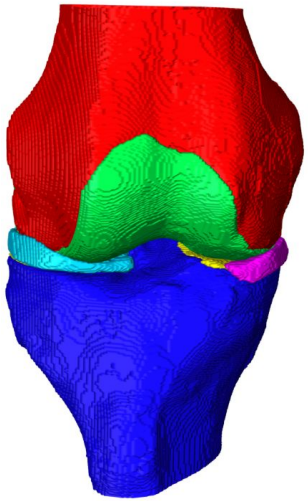

[Ambellan et al. 2019, Med Image Anal 52]

# Segmentation - Further Material

507 manual segmentations and 47k automatic segmentations of Dual Echo Steady State - MRI (DESS) from the OAI database* are

publicly available at

*pubdata.zib.de*





[Tack et al. 2021, PloS one] *nda.nih.gov/oai

# Segmentation - Further Material

If interested in U-Net segmentation see e.g.

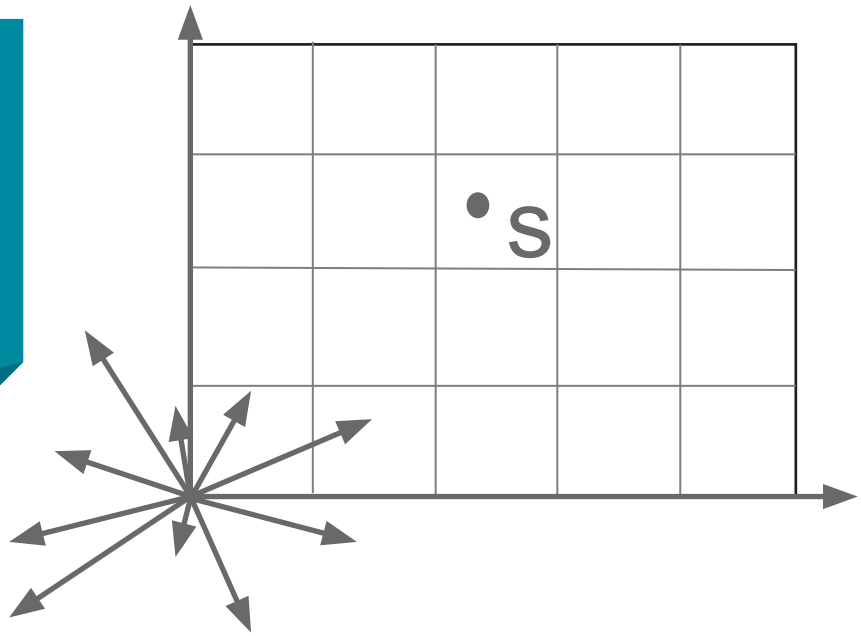
*https://www.kaggle.com/vbookshelf/simple-cell-segmentation-with-keras-and-u-net*

# Riemannian Shape Spaces

# Shape Distance

Configuration Space

Pseudo-Metric
- Invariant under similarity transforms
- Favor natural deformations (Physically-based)






[von Tycowicz et al. 2018, Med Image Anal 42]

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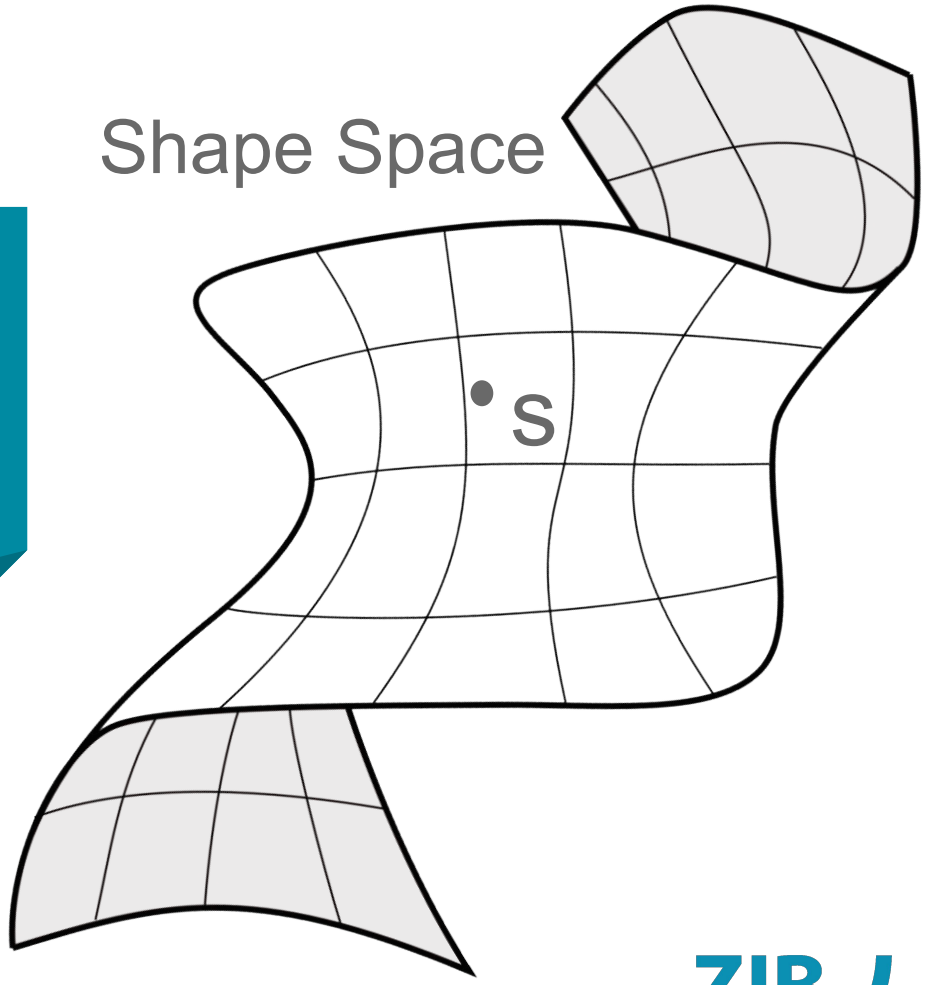
# Shape Space

Pseudo-Metric
- Invariant under similarity transforms
- Favor natural deformations (Physically-based)

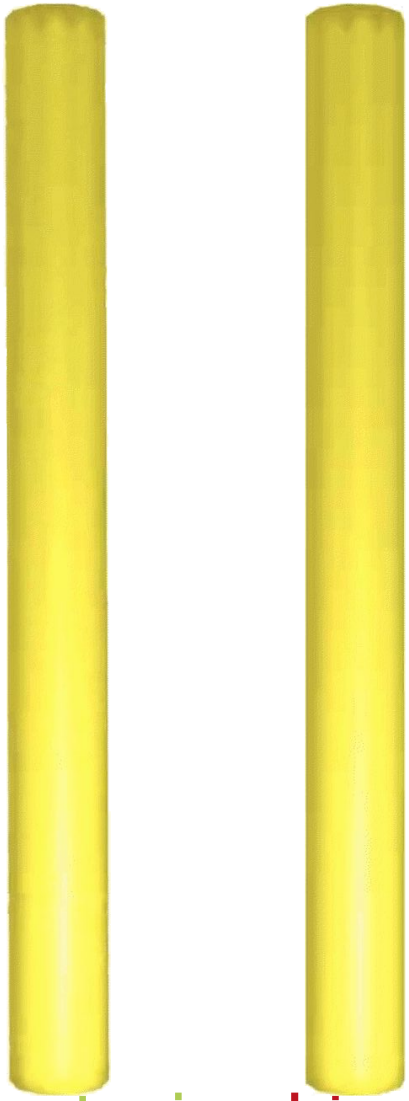
Manifold





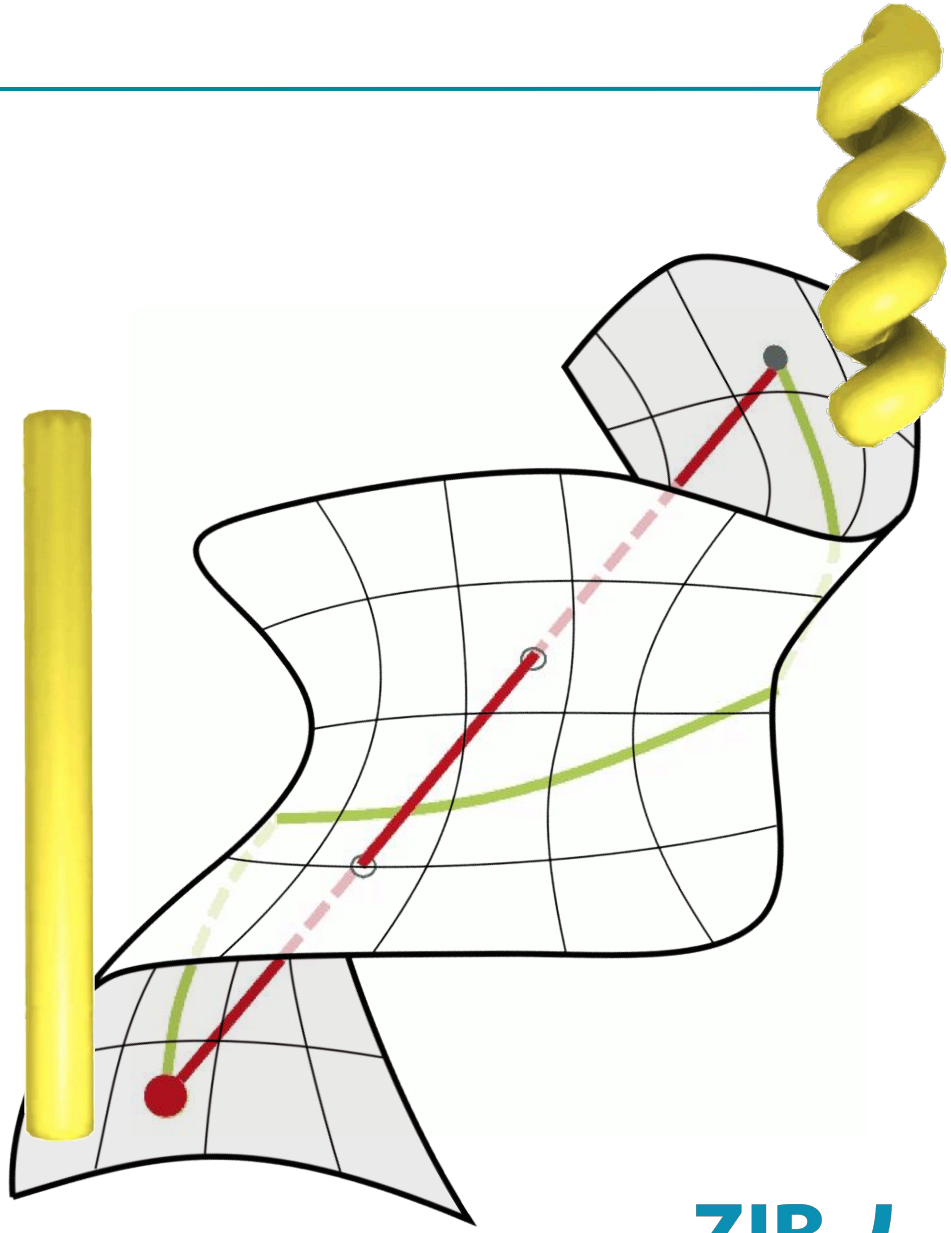

[von Tycowicz et al. 2018, Med Image Anal 42]

# Shortest Path



Geodesic Line

[Ambellan et al. 2019, MICCAI]





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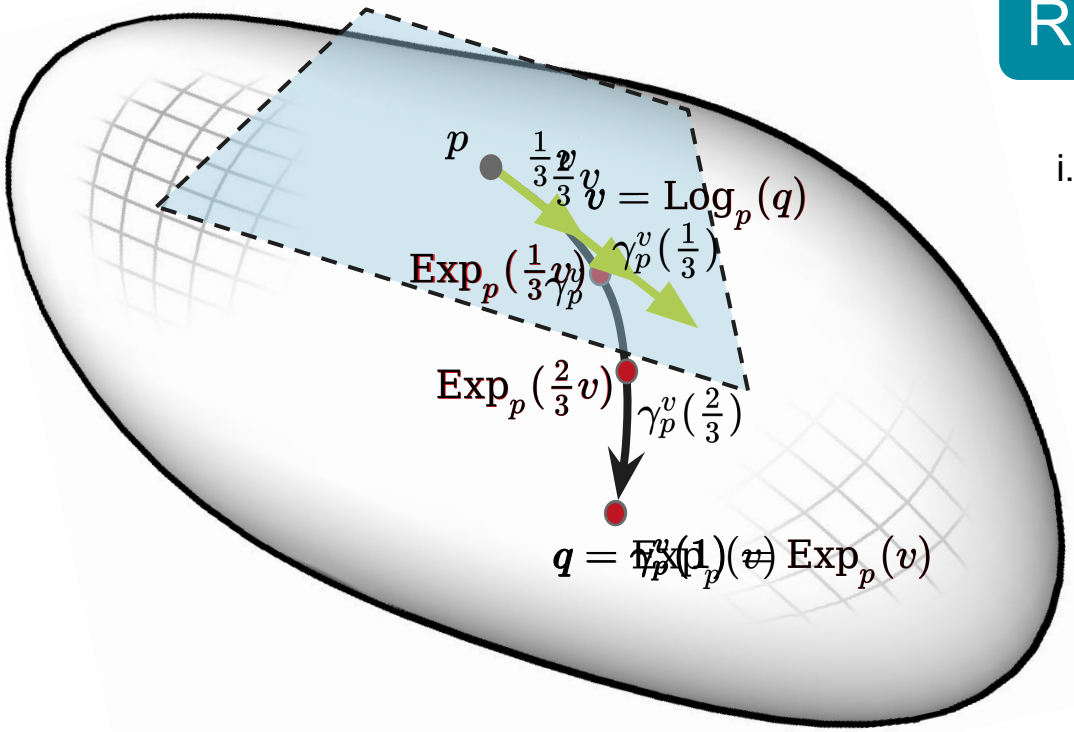
# Geodesics and Interpolation

Local characterization of Geodesics

- As-straight-as-possible paths




Riem. exponential

i.e. $\mathrm{Log}_p = \mathrm{Exp}_p^{-1}$

Riem. logarithm



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# Challenge

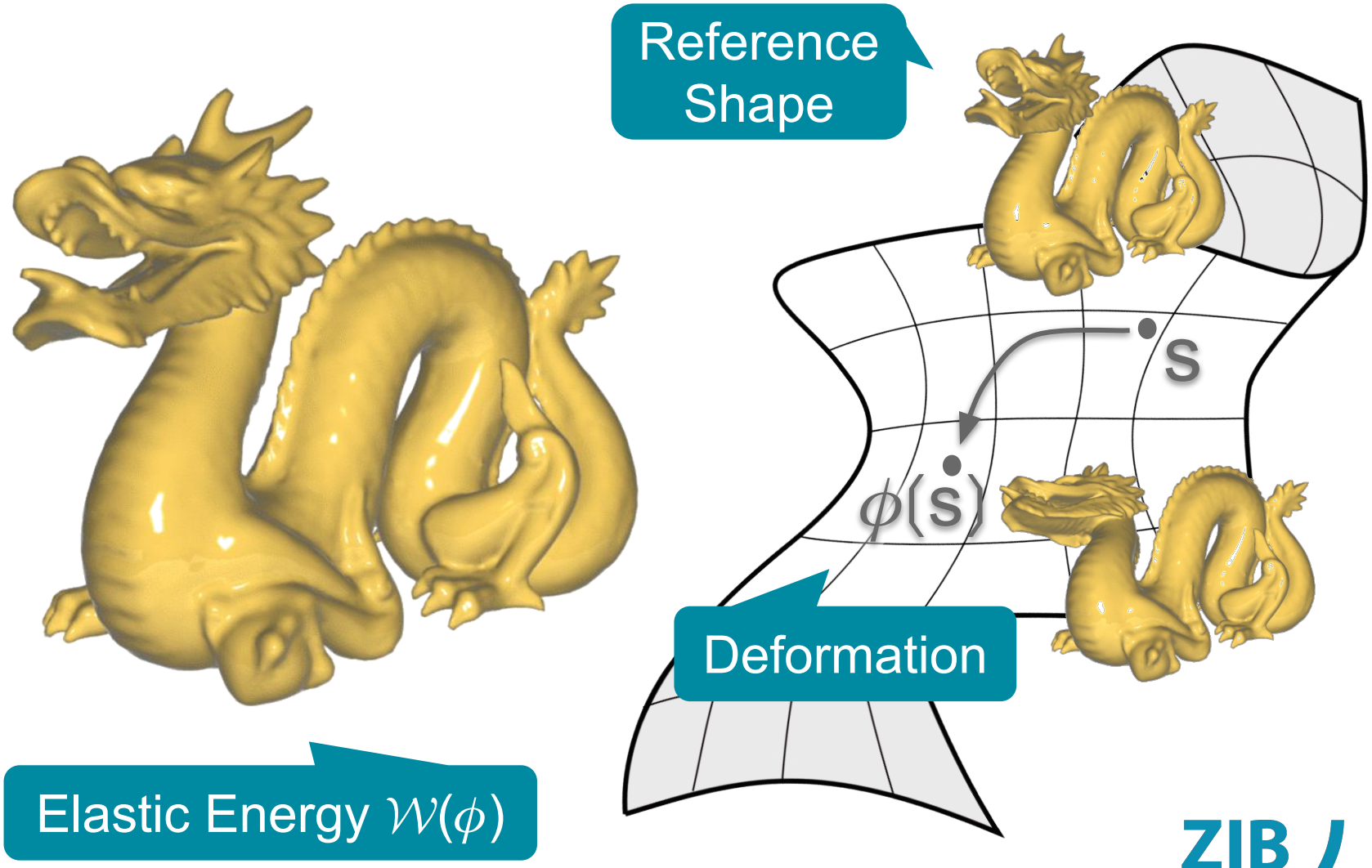
Provide efficient analysis of shape data suitable for large shape populations

# Idea

Change of Variables

Fundamental Coordinates

# Deformation-based Morphometry



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# Shell Space

Membrane Term

Bending Term

$$\mathcal{W}(\phi) = \mathcal{W}_{\text{mem}}(\mathrm{I}_\phi) + \mathcal{W}_{\text{bend}}(\mathrm{II}_\phi)$$

Fundamental Forms

Surface-Theory:
Fundamental Forms determine $\phi(s)$ uniquely up to rigid motion



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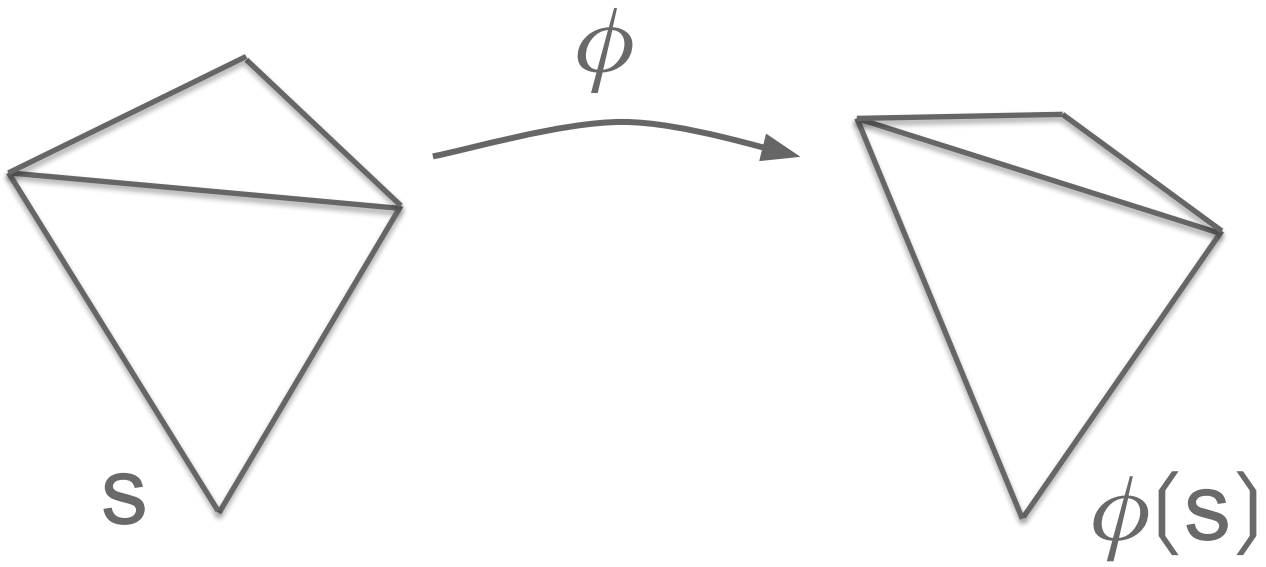
# Simplicial Map

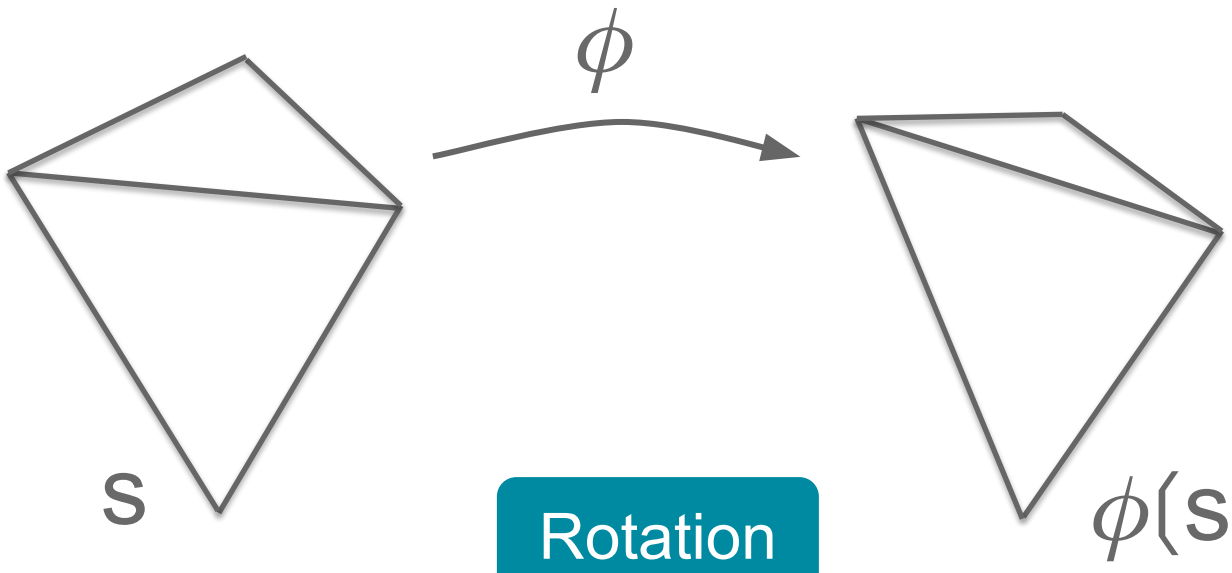



- $\phi$ affine on each triangle $T$
- Gradient $\nabla\phi|_T$ const. 3-by-3 matrix



[von Tycowicz et al. 2018, Med Image Anal 42]

# 1st Fundamental Form




Rotation

$$\nabla\phi|_T = RU$$

Stretch

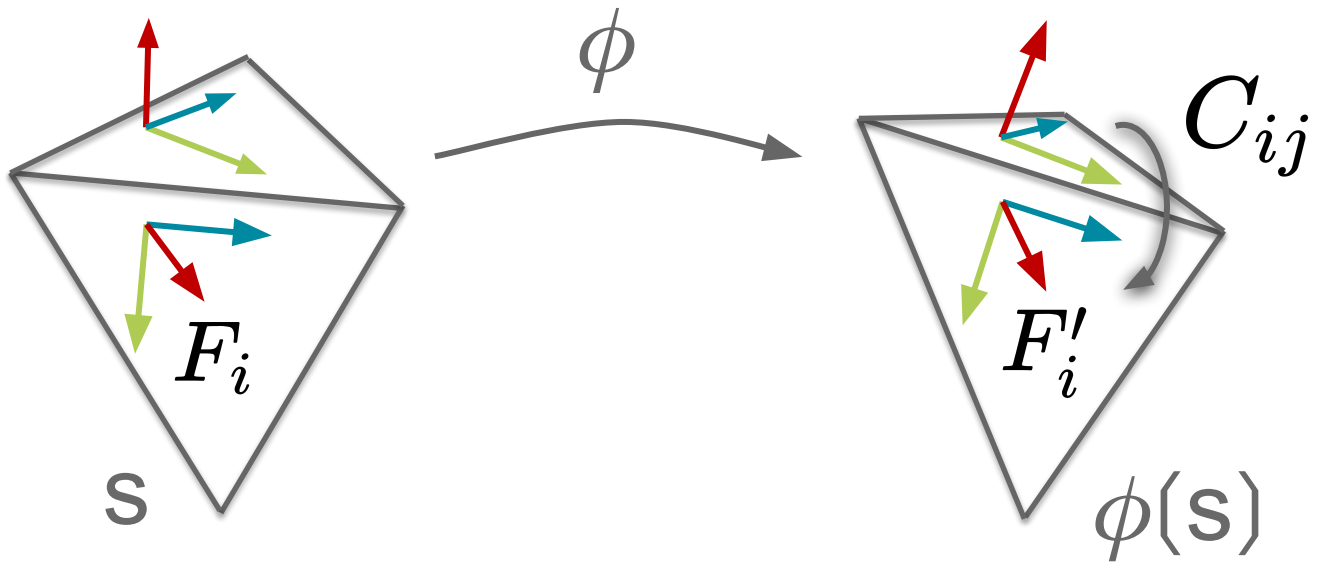
Polar Decomposition

$U$ restricted to tangent plane $\rightarrow$ $I$



[Ambellan et al. 2021, Med Image Anal 73]

# 2nd Fundamental Form




$$F'_i = R_i F_i$$

$$F'_i C_{ij} = F'_j$$

Push Frames Forward

Transition Rotations



[Ambellan et al. 2021, Med Image Anal 73]

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# Fundamental Coordinates

Shape Representation

- Tangential stretches (one per triangle)
- Transition rotations (one per inner edge)

+ Invariant under Euclidean motion
+ Bi-invariant Lie group structure
+ Efficient Riemannian calculus

Inverse Problem (representation → surface)

- Local/global solver allowing interactive rates

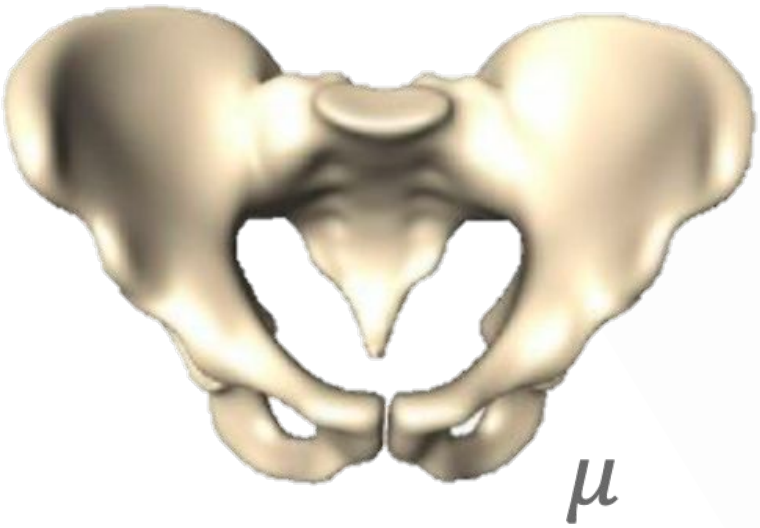


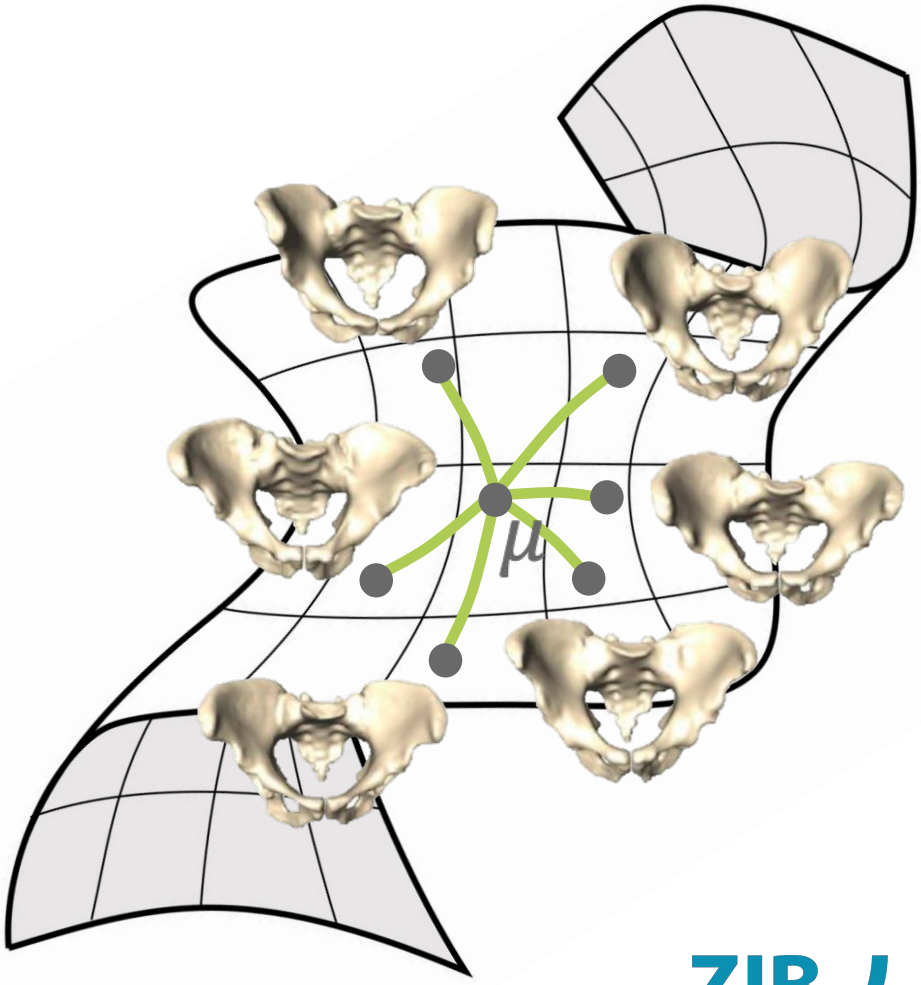
[Ambellan et al. 2021, Med Image Anal 73]

# Riemannian Shape Analysis

# Mean Shape

Riemannian Center of Mass



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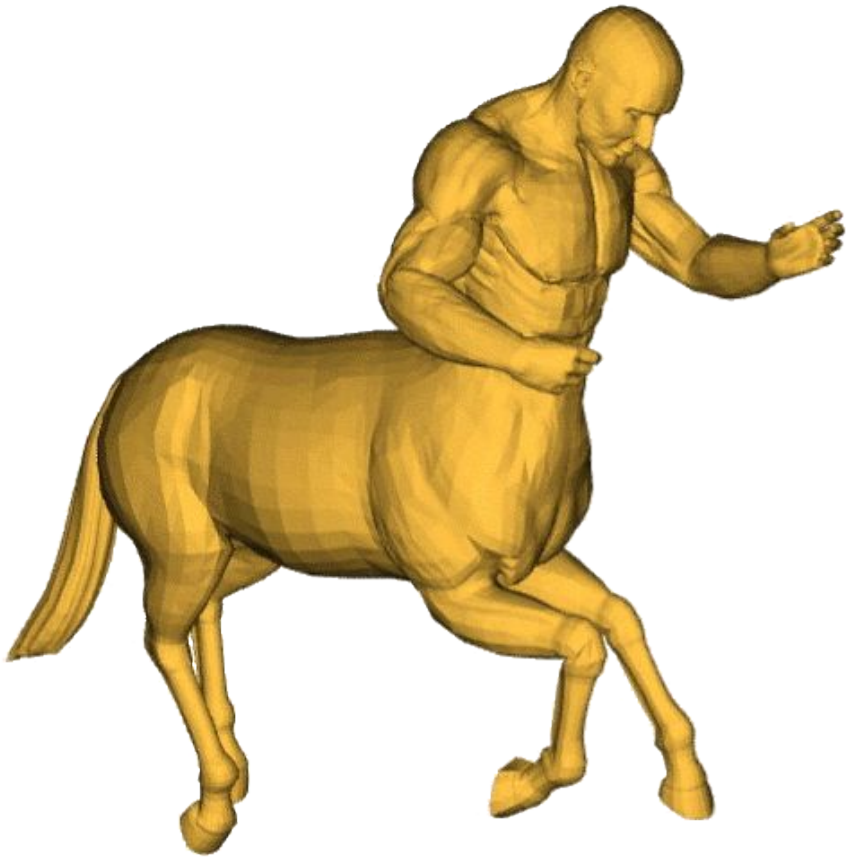
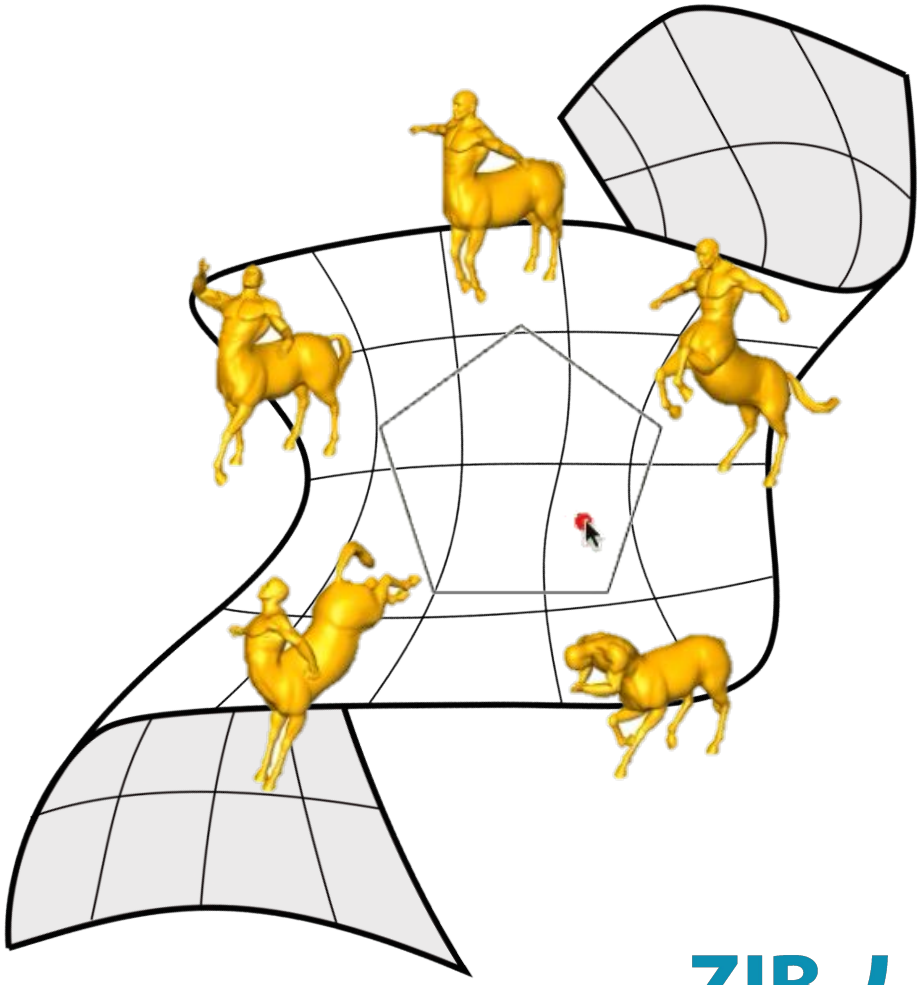
# Exploration of Example Manifold







[von Tycowicz et al. 2015, ACM Trans Graph 34(3)]

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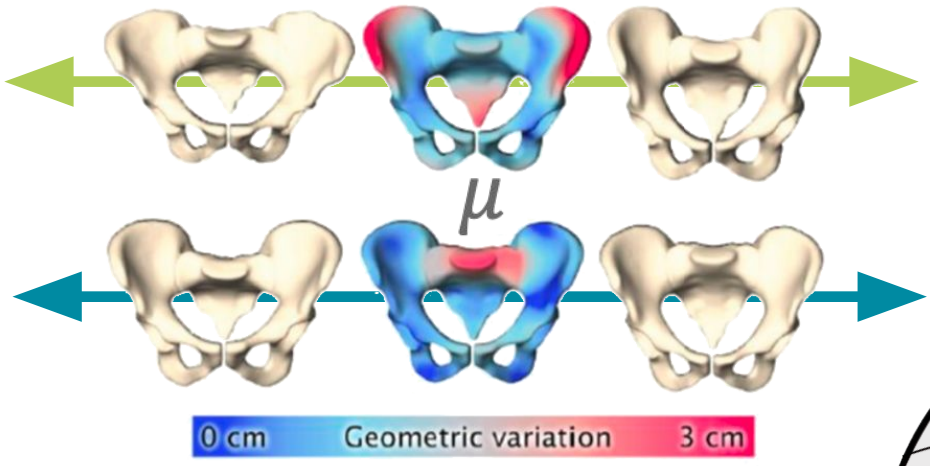
# Principal Geodesic Analysis

Principal Modes of Variation







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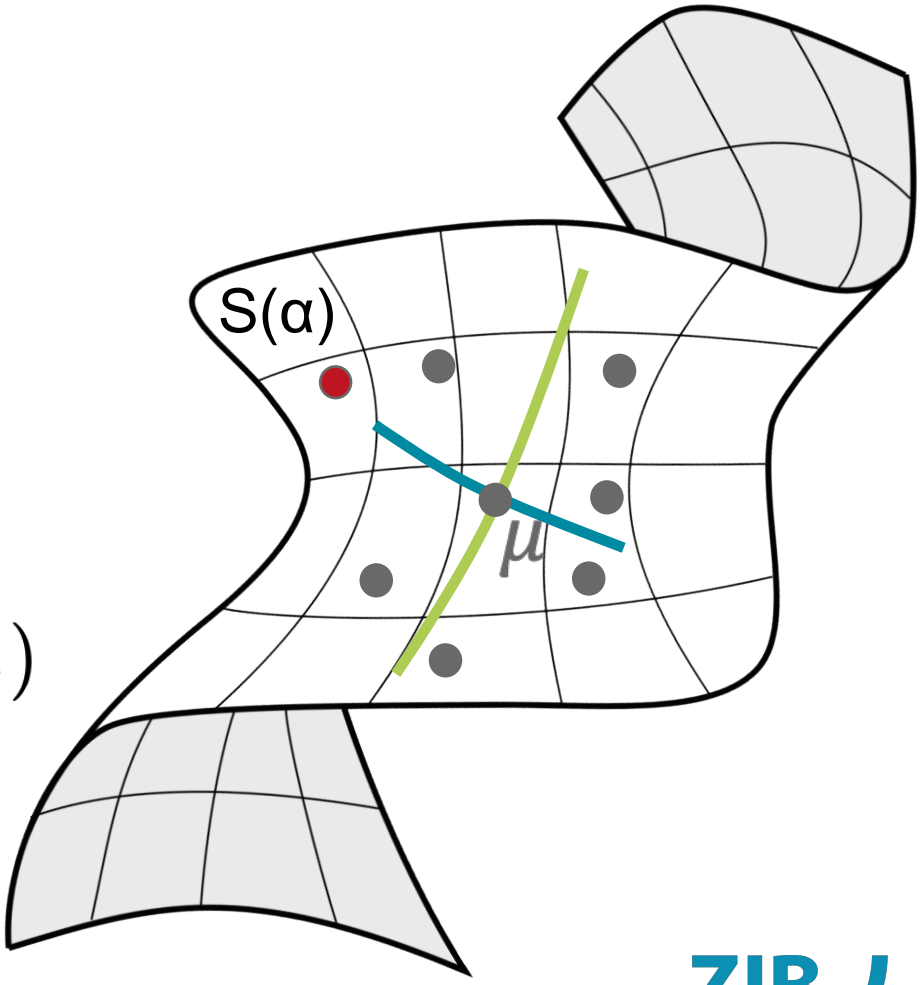
# Generation of Shapes

$S(\alpha) = \mu + \alpha_1 + \alpha_2$

Shape Weights

$$S(\alpha) = \mathrm{Exp}_\mu\left(\sum_k \alpha_k \cdot \vartheta_k\right)$$



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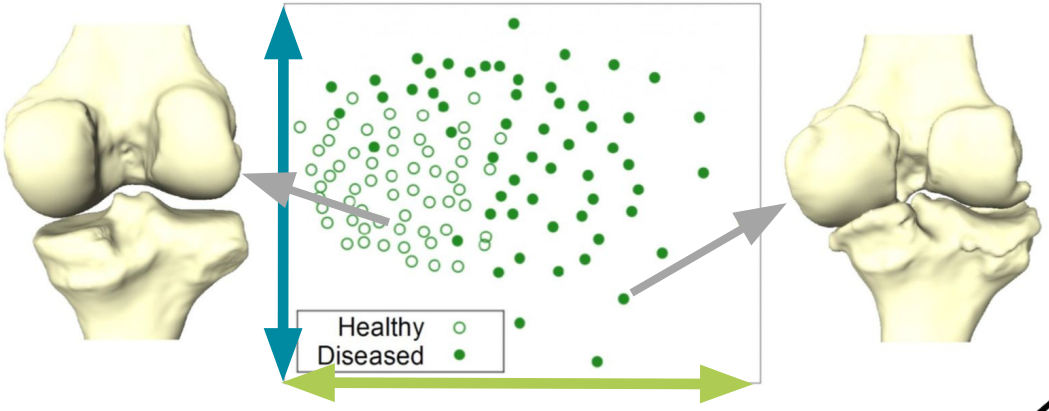
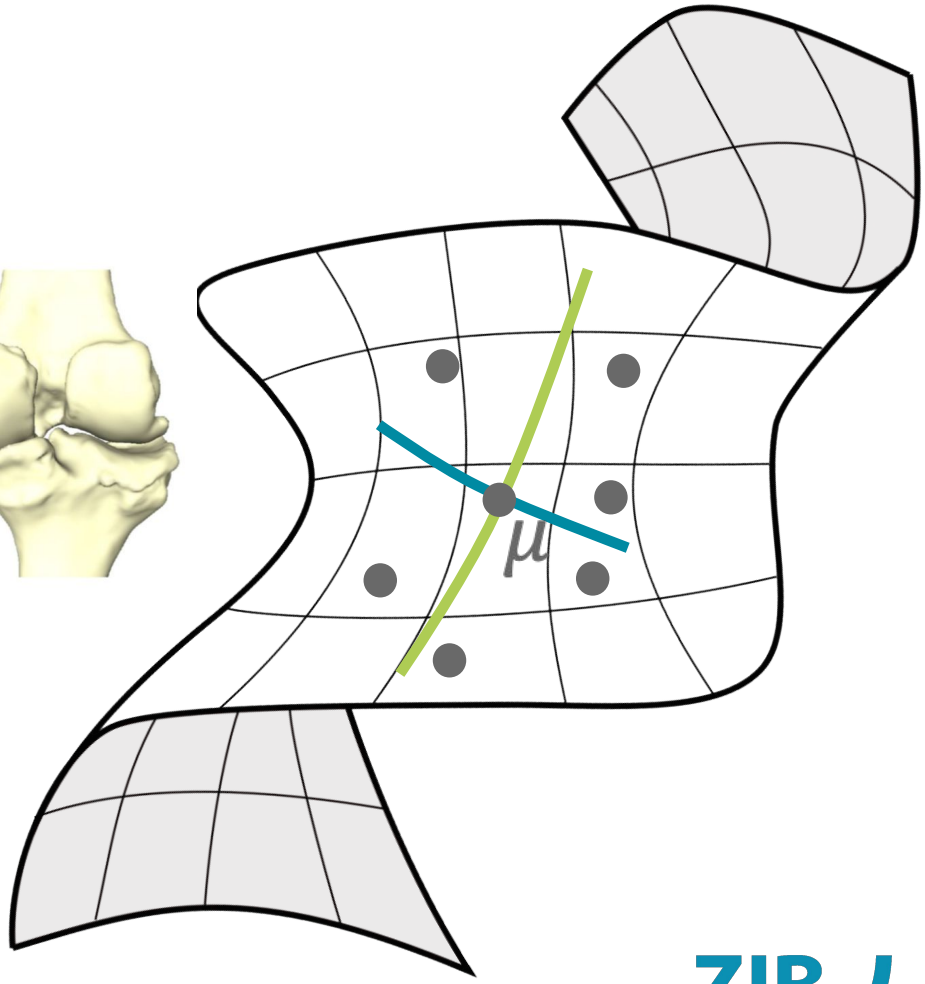
# SSM-based Machine Learning

## Dimension Reduction




- Clustering
- Classification
- …



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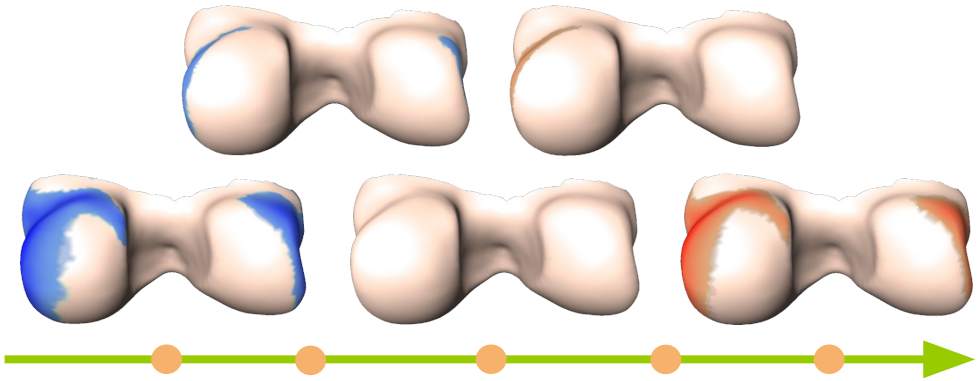
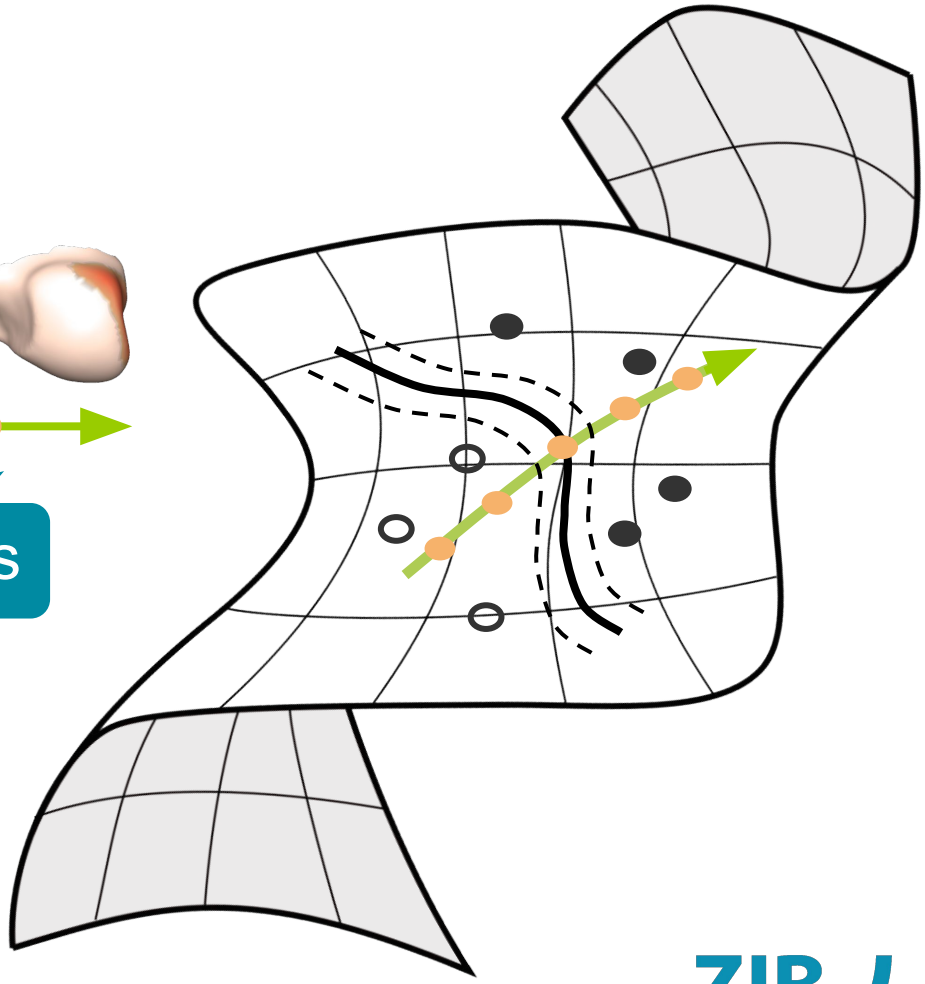
# Support Vector Machine



Osteoarthritis





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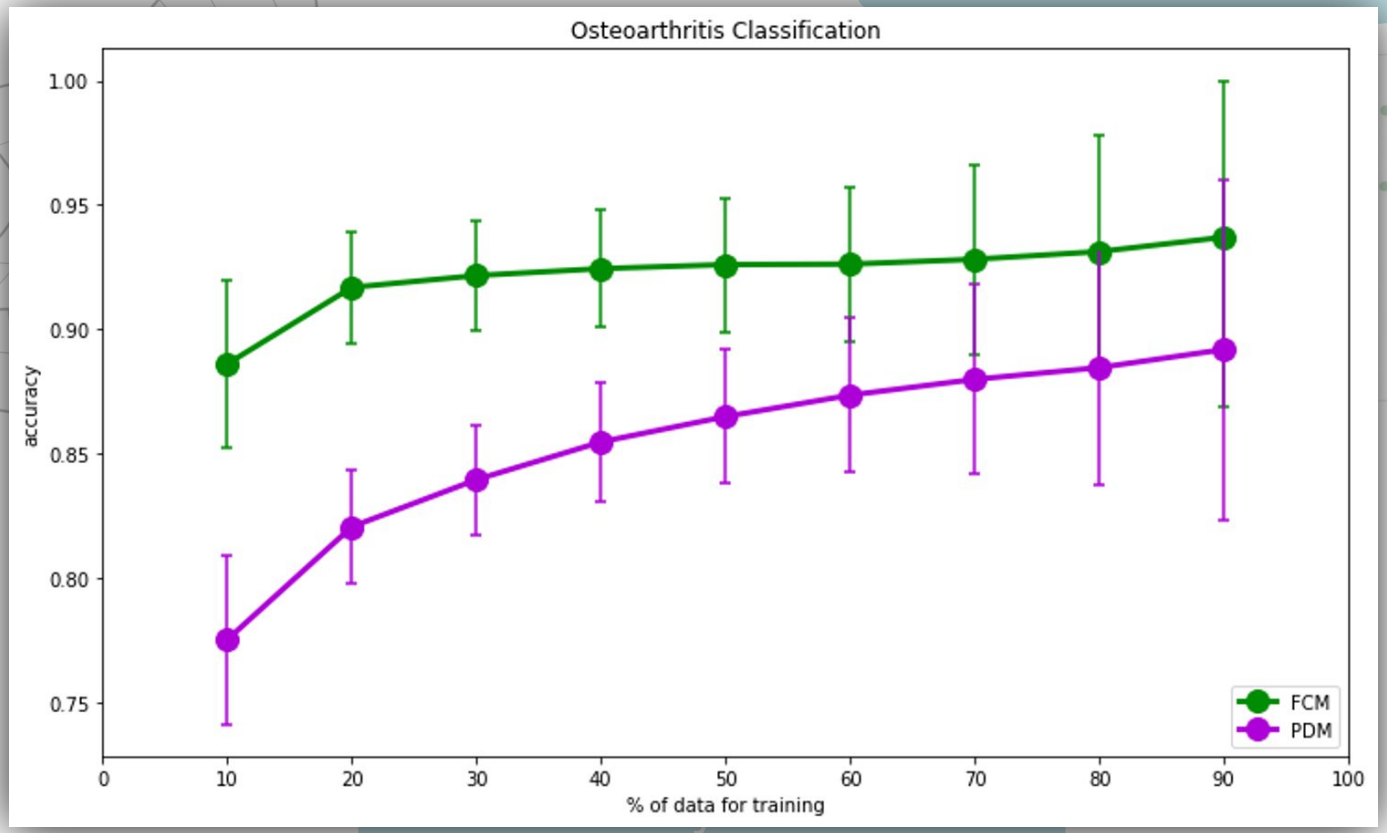
# Knee Osteoarthritis Classification

Support Vector Machine






[Ambellan et al. 2019, MICCAI]

# Open Source Software

The notebooks in the practical part are built upon the python library *'Morphomatics: Geometric morphometrics in non-Euclidean shape spaces'**

available at

*github.com/morphomatics/morphomatics*



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*morphomatics.github.io

# Practical Part

README.md

## TES_21_22_Tutorials

Tutorials for the Thematic Einstein Semester on Mathematics of Imaging in Real-World Challenges

The idea of the tutorials is to run them via Jupyter notebooks. The notebooks are designed to require little computational power and can be be run by using binder:


The link above will open the notebooks directly in your browser and you do not need not install any additional software. If you would like to run to code on your own computer, please follow the installation instructions below. If you want to do this, then please **try it well in advance to the tutorial**, because it will take some time.

### Installation

#### Prerequisites

You will need to have git and anaconda installed on your computer. The notebooks do not require GPU support.



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