Linear Algebra - 2

Quiz 2 :

L1 : $3x - 2y + 6 = 0$

L2 : $9x - 6y - 18 = 0$

Parallel lines $\longrightarrow$ equal slopes

$$3x - 2y + 6 = 0$$

$$2y = 3x + 6$$

$$y = \left(\frac{3}{2}\right) x + \left(6/2\right)$$

$$\boxed{y = \left(3/2\right) x + 3}$$

$$y = mx + c$$

3/2

3

* **Recap :**

features / independent var

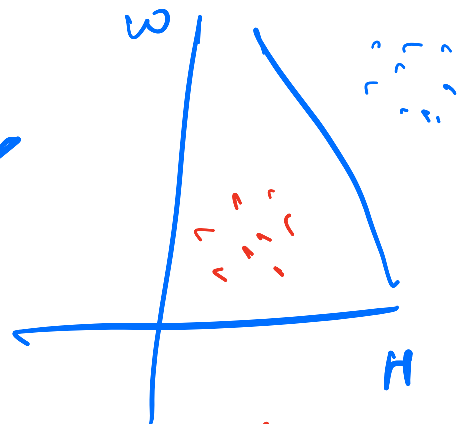
x_1	x_2	y

dependent var

rows / data points

height	weight	Target

$$w_1x_1 + w_2x_2 + w_0 = 0$$



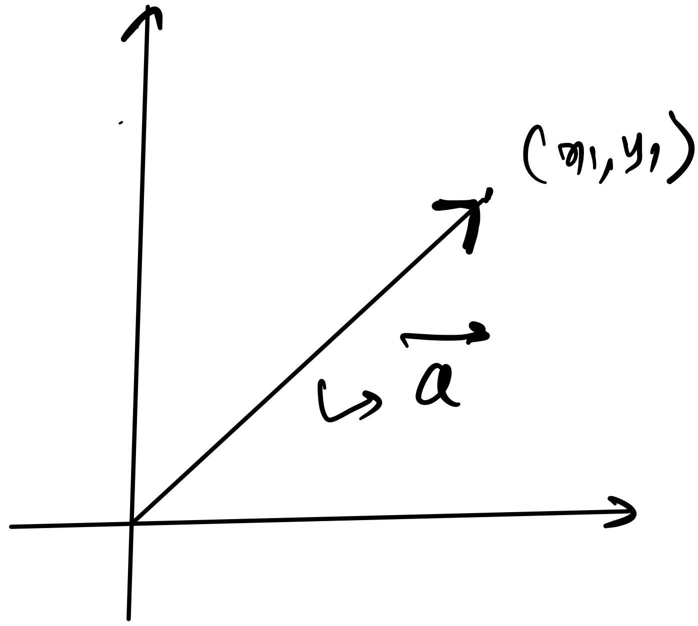
Weight	Height	Width

3D Hyperplane

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

* **Vectors**

→ Both magnitude and direction



→ Collection of numbers.

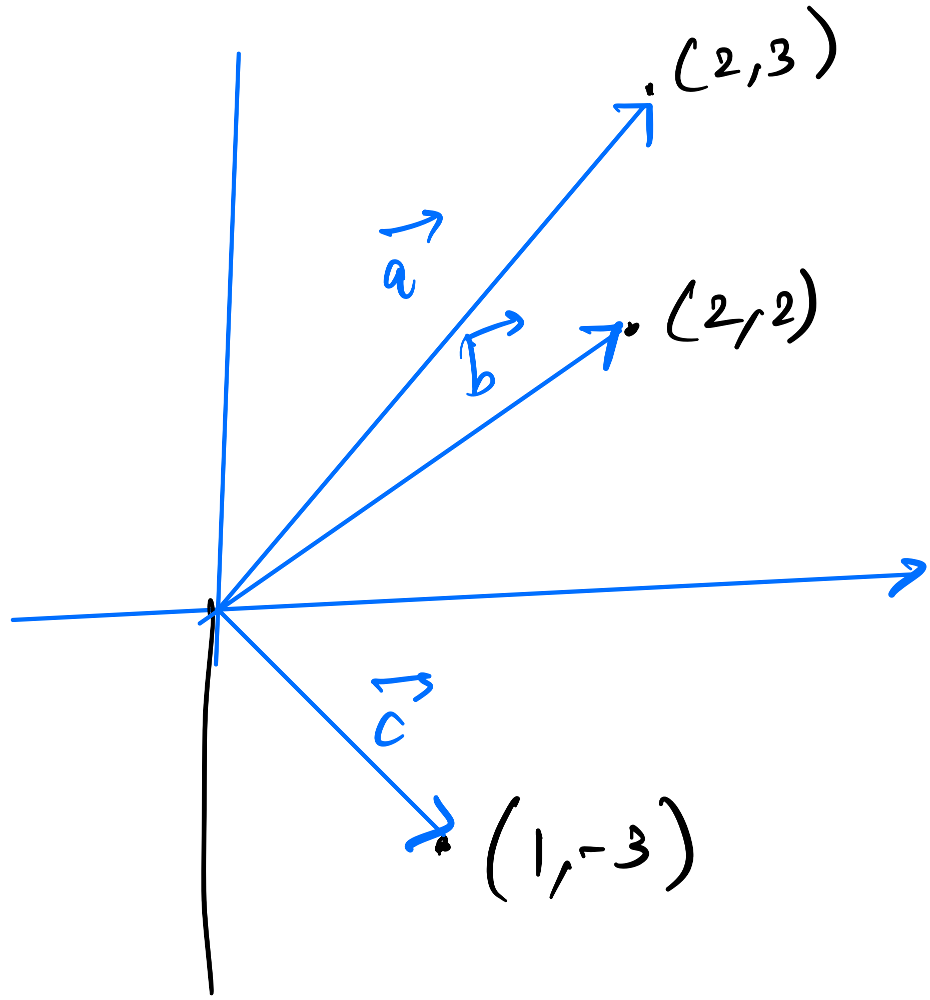
How do we represent a vector?

$$\vec{x} = [1, 2, 3, 5]$$

By-default :- Column vector

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\vec{x}^{T} = \begin{bmatrix} 1 & 2 & 3 & 5 \end{bmatrix}$$



$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \vec{c} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

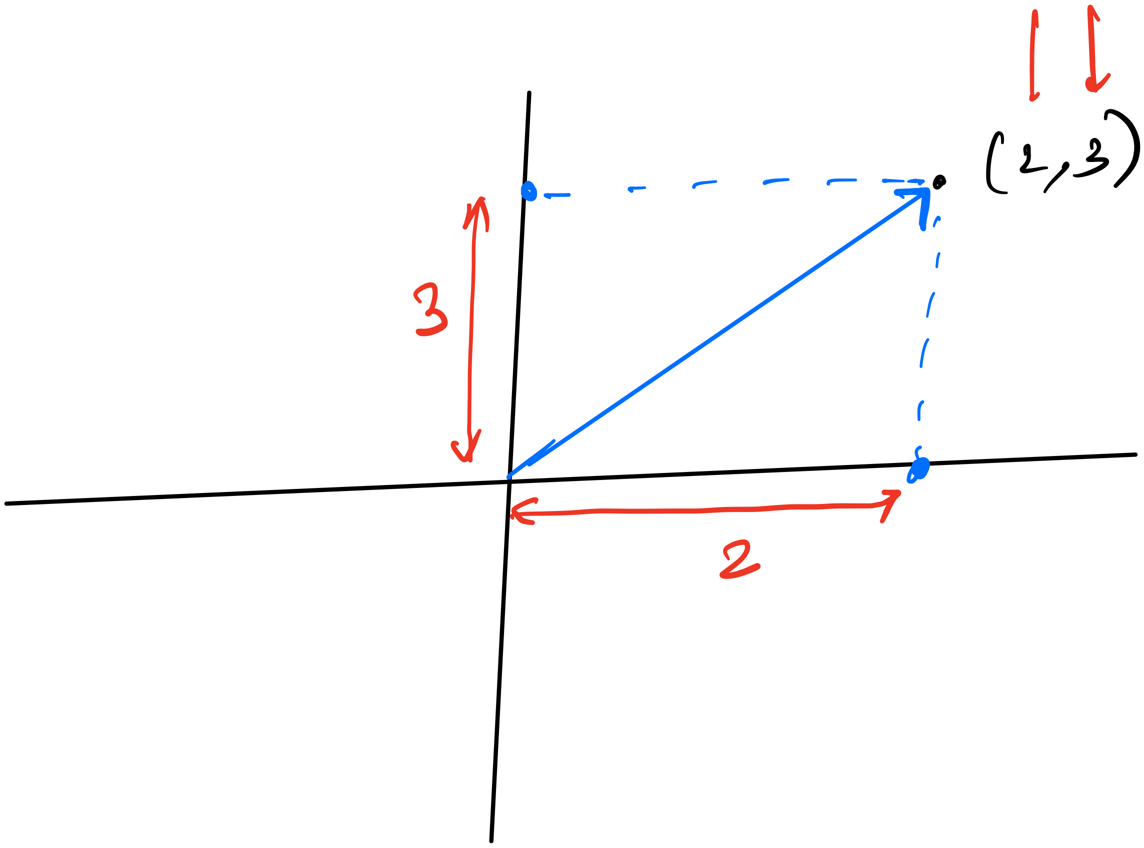
$$\vec{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

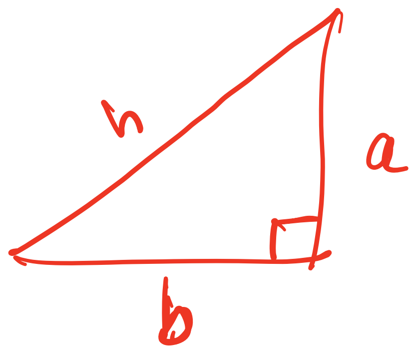
(Q) What is the magnitude of a vector?

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

for this vector, magnitude is

$$\boxed{\|\vec{x}\| = \sqrt{x_1^2 + x_2^2}}$$





By pythagoras Thm

$$h^2 = a^2 + b^2$$

$$\boxed{h = \sqrt{a^2 + b^2}}$$

* what will be the magnitude for a d-dimension vector?

2D: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\text{magnitude}(\vec{x}) = \|\vec{x}\| = \sqrt{x_1^2 + x_2^2}$$

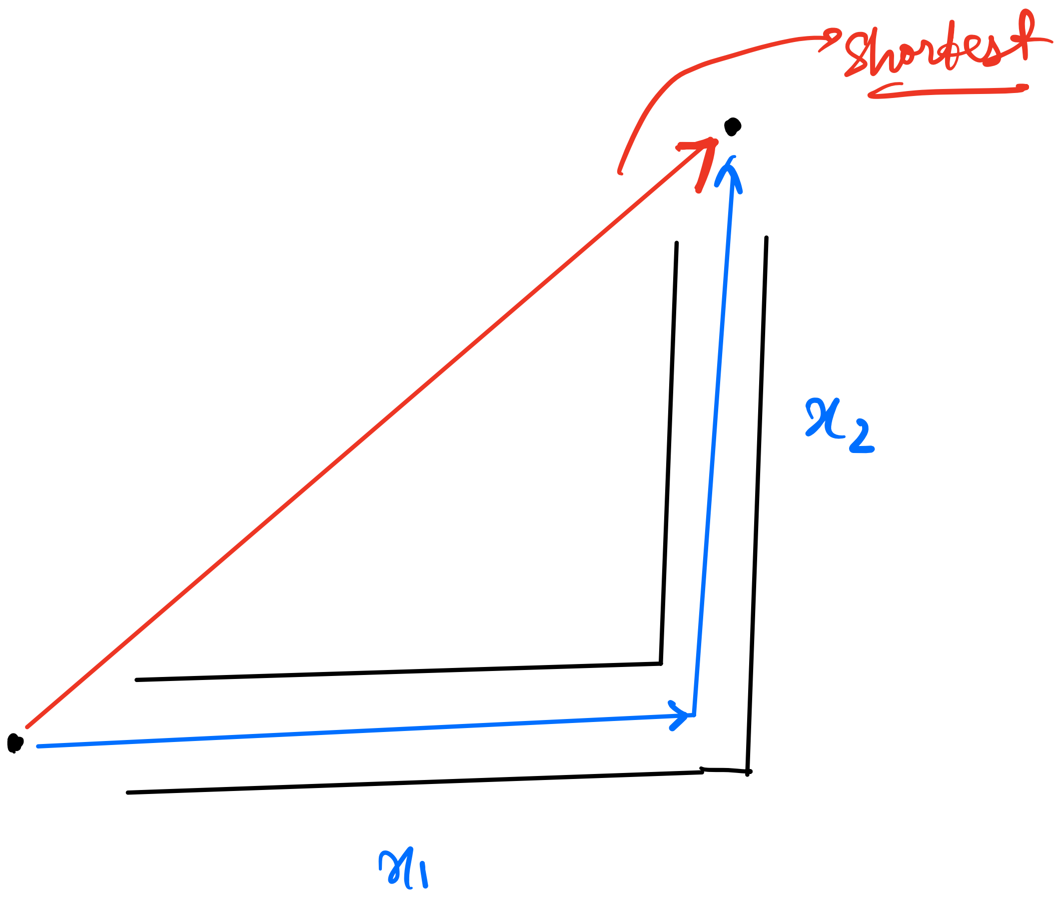
3D: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad \|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

.
.
.
.

nD: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \Rightarrow \|\vec{x}\| = \sqrt{x_1^2 + x_2^2 \cdots + x_n^2}$

* **Norm** of a vector → its length
→ its magnitude.

2 types of Distances



$|3| = 3$

$|-3| = 3$

$$d_1 = |x_1| + |x_2|$$

→ Manhattan Distance.

$d_2 = \sqrt{x_1^2 + x_2^2}$

$\longrightarrow$ Eucledian Distance

Terminologies :-

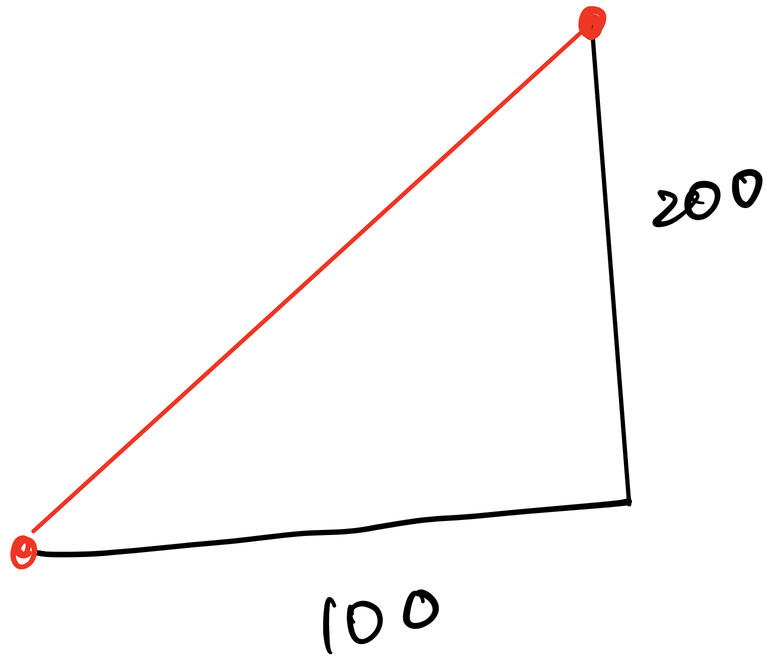
① L_1 Norm $\rightarrow$ $\|\vec{x}\|_1 = |x_1| + |x_2|$

(Manhattan Dist)

② L_2 Norm $\rightarrow$ $\|\vec{x}\|_2 = \sqrt{x_1^2 + x_2^2}$

(Eucledian Dist)

$\|\vec{x}\| \rightarrow \|\vec{x}\|_2$



L1 Norm → Manhattan Dist

$= (|x_1| + |x_2|)$

$= 100 + 200$

$= \underline{\underline{300}}$

L2 Norm →

Eucledian Dist

$= \sqrt{(x_1^2 + x_2^2)}$

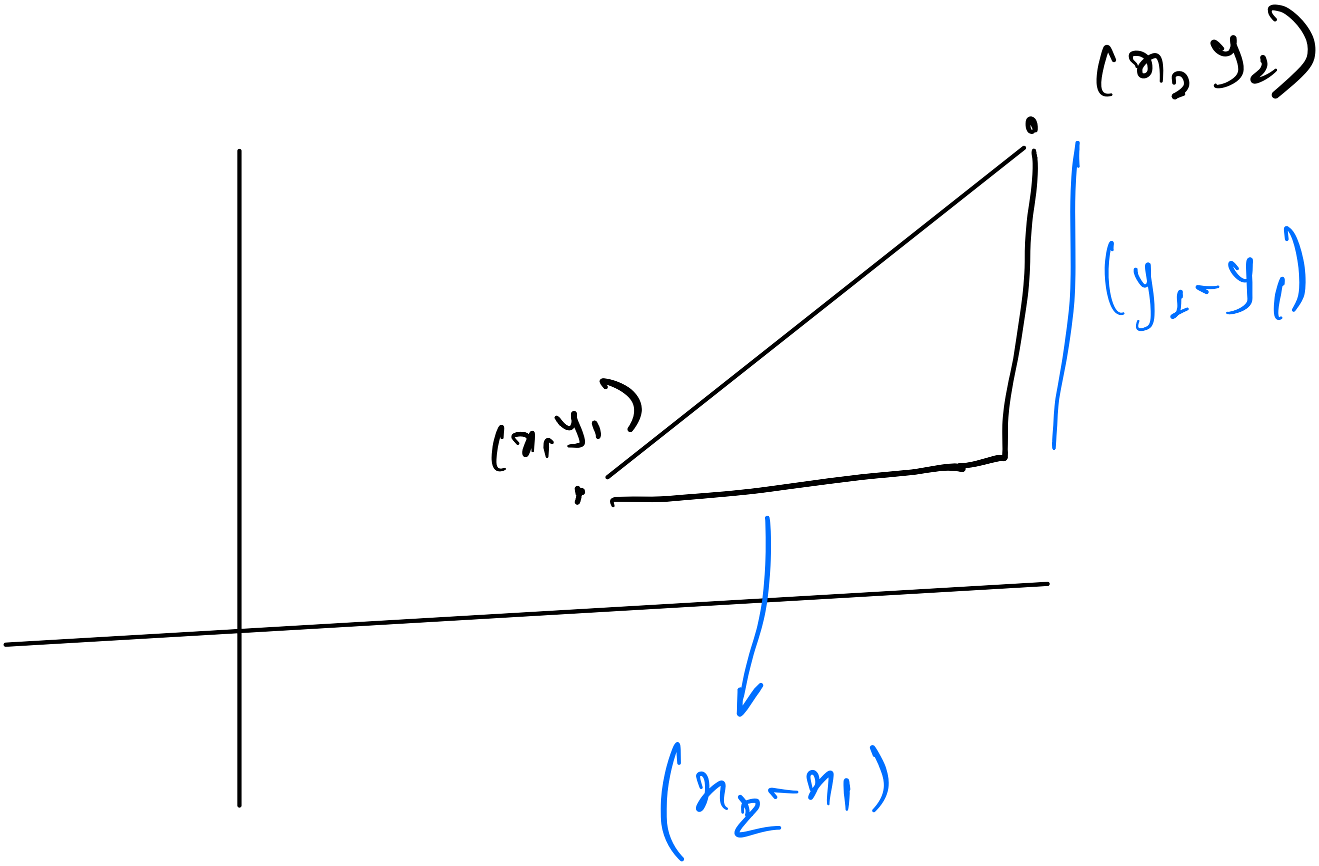
$$= \sqrt{(100)^2 + (200)^2}$$

$$= \sqrt{10000 + 40000}$$

$$= \sqrt{50000}$$

$$= \boxed{223.6}$$

* Generalised form:



Euclidean = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Manhattan = $|x_2 - x_1| + |y_2 - y_1|$

* Matrix multiplication

$$A_{m*n} * B_{p*q} = C_{m \times q}$$

$C_{m \times q} \downarrow$ Resultant

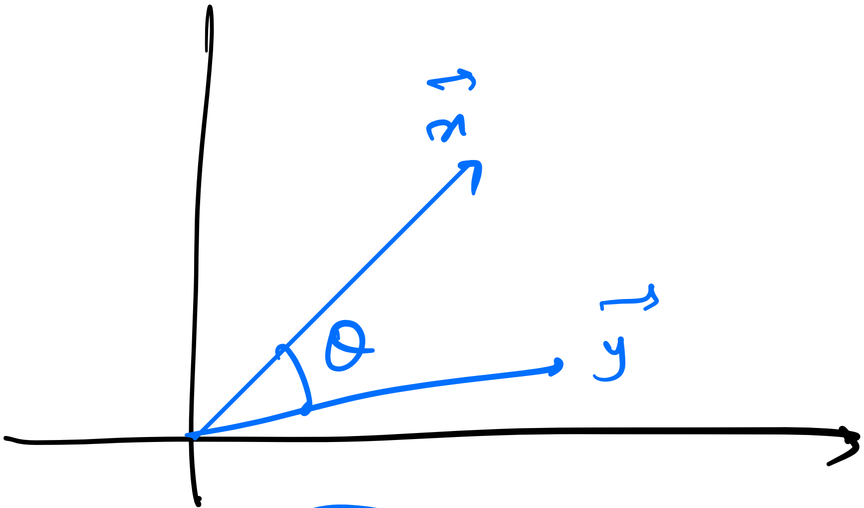
Condition : (1) $n = p$

eg

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} * \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2 \times 2}$$

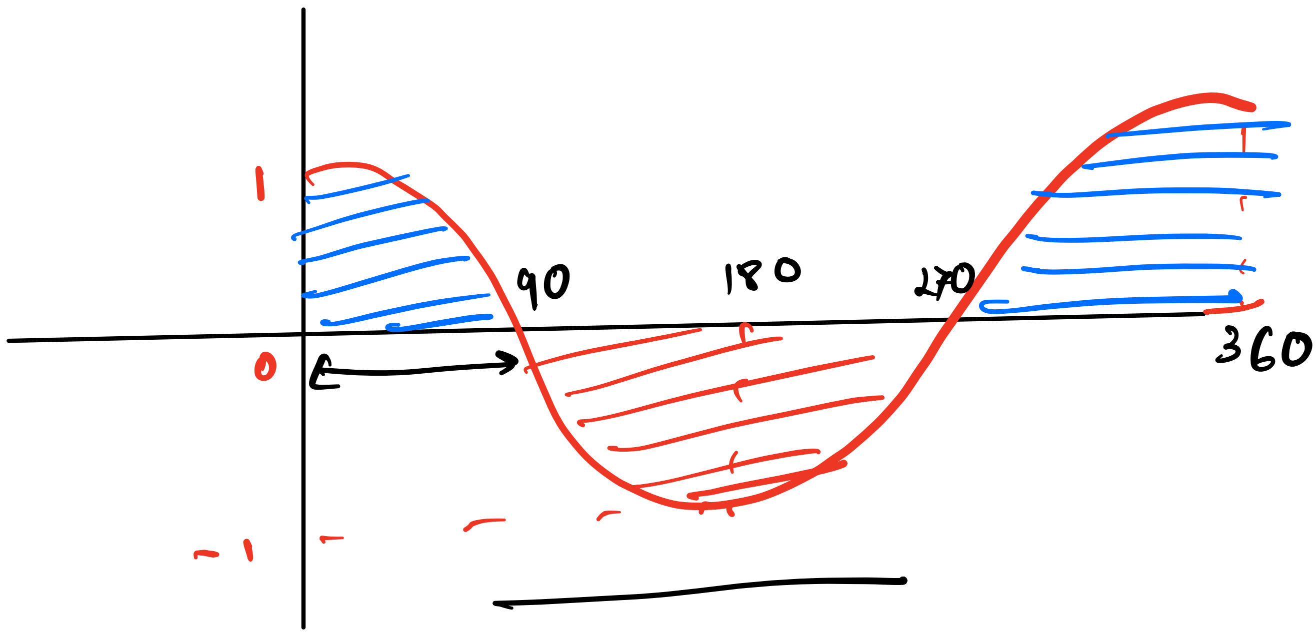
$$= \begin{bmatrix} 1 \times 5 + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ 3 \times 5 + 4 \times 7 & 3 \times 6 + 4 \times 8 \end{bmatrix}_{2 \times 2}$$

* Angle between 2 vectors



$$\boxed{\cos(\theta) = \frac{\vec{x}^{T} * \vec{y}}{||x|| * ||y||}}$$

Dot product



$\underline{\underline{\cos\theta > 0}}$ $\longrightarrow$ $0 < \theta < 90$

or

$270 < \theta < 360$

$\cos\theta < 0$ $\longrightarrow$ $\boxed{90 < \theta < 270}$

$\vec{a} = [1, 2]$

$\vec{b} = [3, 4]$

$\vec{a}^T * \vec{b}$

a

a[0] * b[0]

+

a[1] * b[1]

* $w_1x_1 + w_2x_2 + w_3x_3 \ldots + w_nx_n + w_0 = 0$

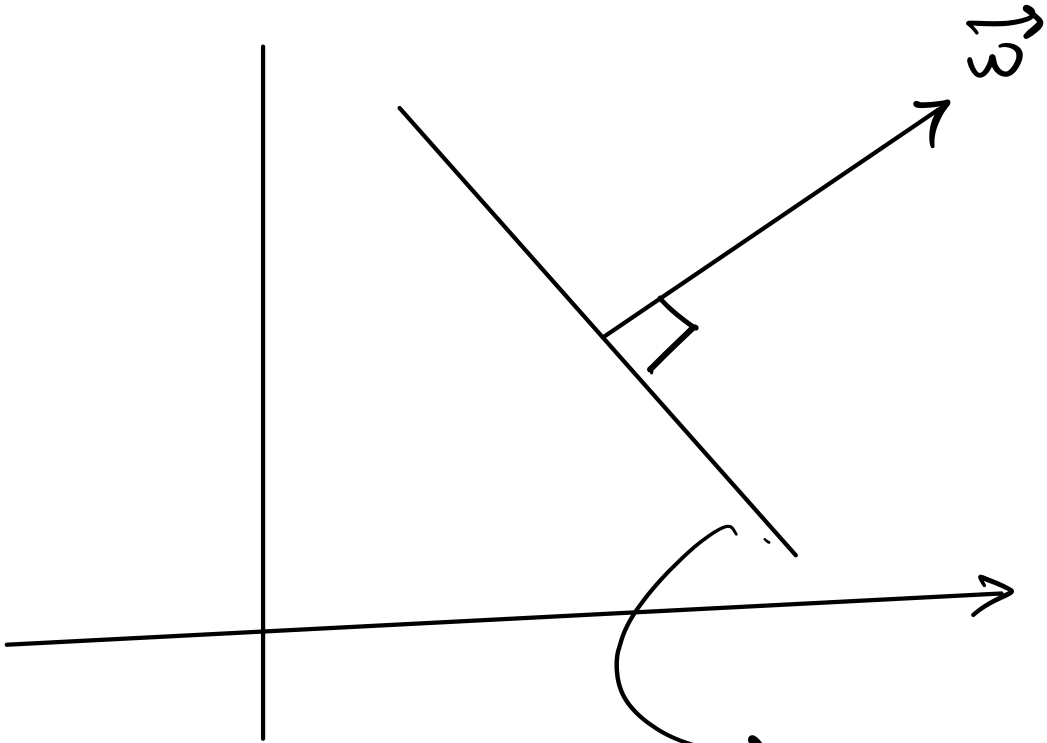
$$\vec{W} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

weight vector

feature vector

$$\boxed{\vec{W}^T * \vec{x} + w_0 = 0}$$

Bias



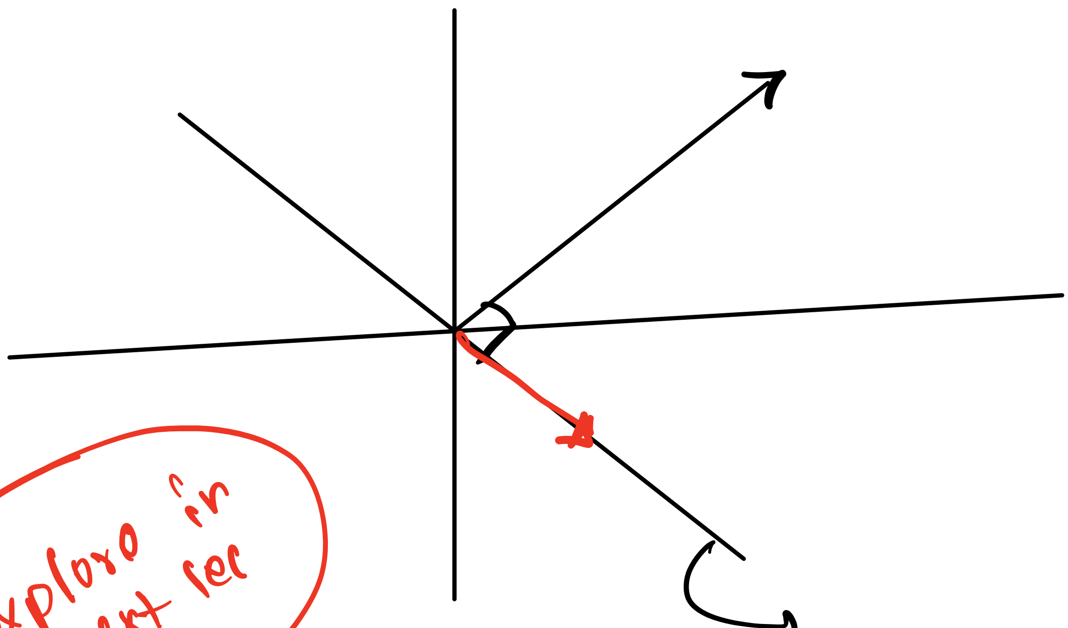
$$x_1 + x_2 - 98 = 0$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{\underline{w_0 = -98}}$$

* [The weight vector will always be perpendicular to the hyperplane.]



$x_1 + x_2 = 0$

will explore in next lec

$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad w_0 = 0$

$\vec{w}^T * \vec{x} \Rightarrow x_1 + x_2 \Rightarrow 0$

$$\cos(\theta) = \frac{\vec{w}^T * \vec{x}}{||w|| * ||\vec{x}||} \rightarrow 0$$

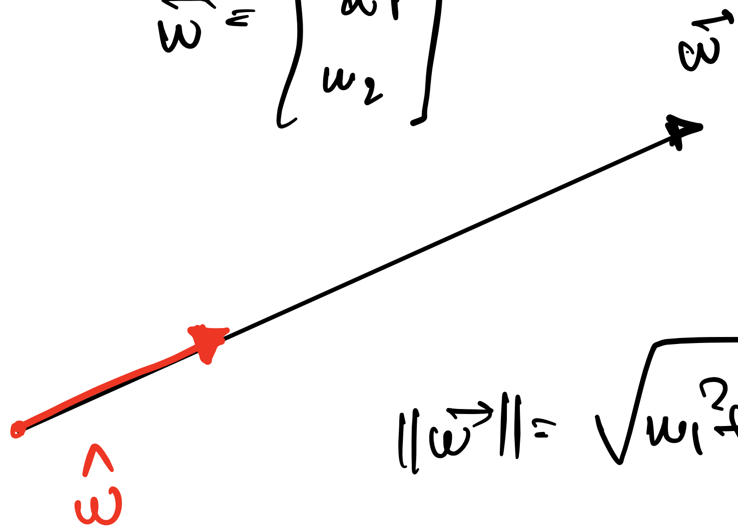
$$\cos\theta = 0$$

$$\boxed{\theta = 90^\circ}$$

* Unit Vector

magnitude = 1

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



$$\|\vec{w}\| = \sqrt{w_1^2 + w_2^2}$$

$$\boxed{\hat{w} = \frac{\vec{w}}{\|\vec{w}\|}}$$

$$\hat{w} = \begin{bmatrix} \dfrac{w_1}{\|\vec{w}\|} \\[2ex] \dfrac{w_2}{\|\vec{w}\|} \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix}$$

H.W

$$\boxed{\|\hat{w}\| = \sqrt{a^2 + b^2}}$$

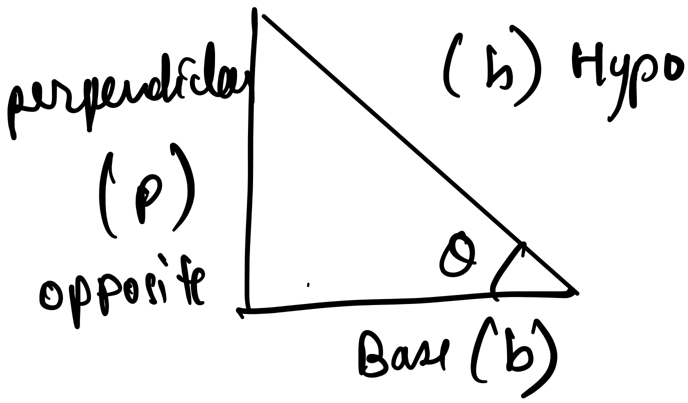
$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= \sqrt{\left(\frac{w_1}{\|w\|}\right)^2 + \left(\frac{w_2}{\|w\|}\right)^2}$$

$$= \sqrt{\frac{w_1^2 + w_2^2}{(\|w\|)^2}}$$

$$\|\hat{w}\| = 1$$

* Basic Trignometric Results

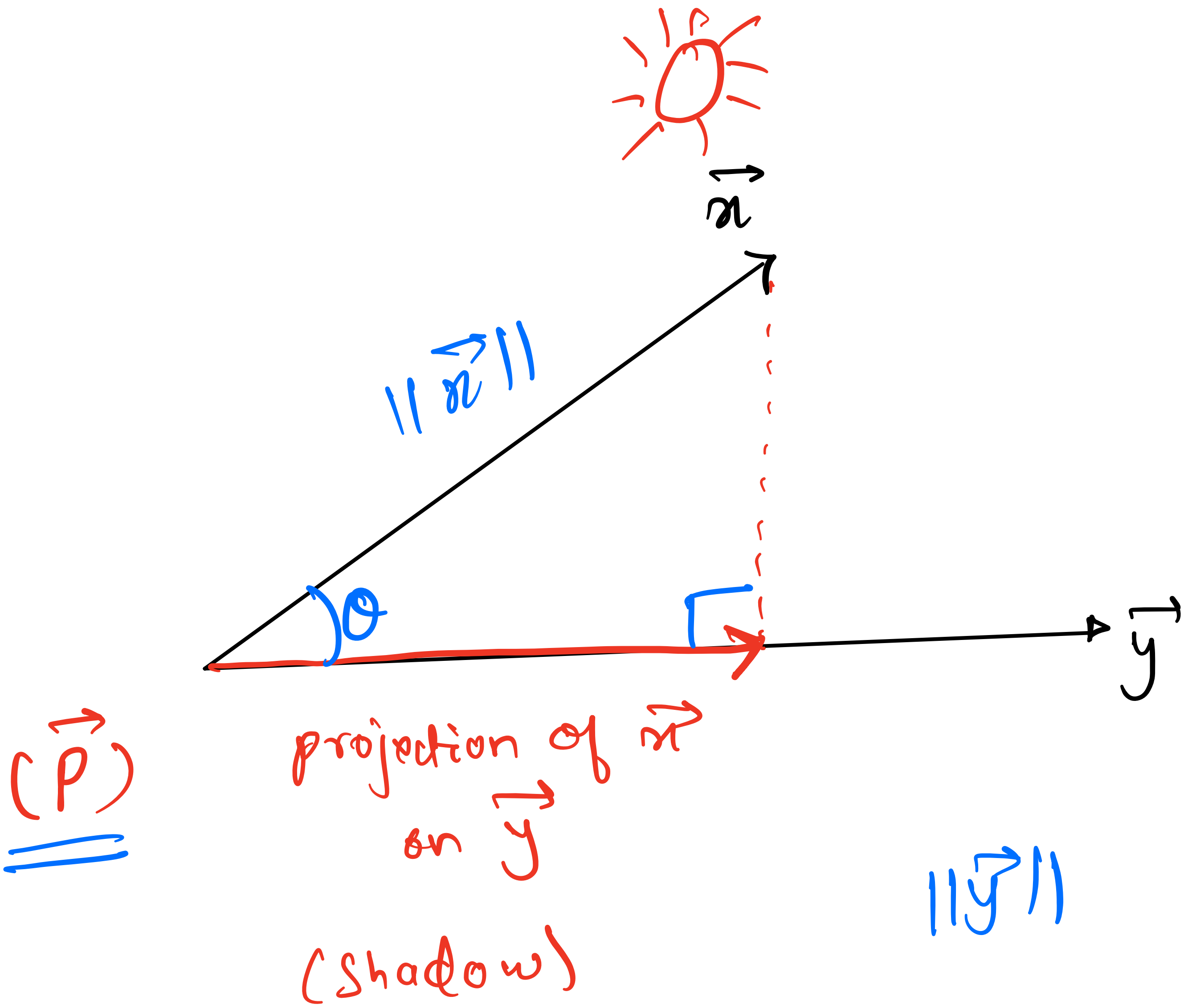


$$\boxed{\sin(\theta) = \frac{p}{h}}$$

$$\boxed{\tan\theta = \frac{p}{b}}$$

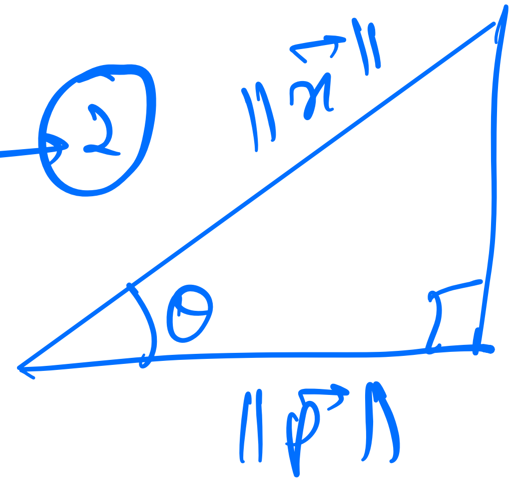
$$\boxed{\cos(\theta) = \frac{b}{h}}$$

* Projection of a Vector



$$\cos(\theta) = \frac{\vec{x}^T * \vec{y}}{\|\vec{x}\| * \|\vec{y}\|} \longrightarrow (1)$$

$$\cos\theta = \frac{\|\vec{p}\|}{\|\vec{x}\|} \rightarrow (2)$$



from (1) & (2)

$$\frac{\vec{x} * \vec{y}}{\cancel{\|\vec{x}\|} * \|\vec{y}\|} = \frac{\|\vec{p}\|}{\cancel{\|\vec{x}\|}}$$

$$\|\vec{p}\| = \frac{\vec{x}^T * \vec{y}}{\|\vec{y}\|}$$

$$\|\vec{p}\| = \vec{x}^T * \boxed{\left(\frac{\vec{y}}{\|\vec{y}\|}\right)}$$

$$\boxed{\|\vec{p}\| = \vec{x}^T * \hat{y}}$$ ✓

$\vec{\omega} = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$\|\vec{\omega}\| = \sqrt{2}$