- **What is derivative**

  The rate of change of a function with respect to a variable is called the **derivative** of the function with respect to that variable.

- **Derivative** of a function $f(x)$ with respect to variable *x* is denoted as:

$$f'(x) = \frac{df(x)}{dx}$$

- **Differentiation using the first principles:**

The derivative of a function $f(x)$ at a point x = a is given as:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- **Common derivatives:**

$\frac{d}{dx} c = 0$	Constant Rule
$\frac{d}{dx} x^n = nx^{n-1}$	Power Rule
$\frac{d}{dx} \sin(x) = \cos(x)$ $\frac{d}{dx} \cos(x) = -\sin(x)$	Trigonometric Rules
$\frac{d}{dx} b^x = b^x \ln(b)$	Exponential Rule
$\frac{d}{dx} \ln(x) = \frac{1}{x}$	Logarithmic Rule

- **Rules of differentiation:**

1. **Sum\Difference rule:** $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

2. **Constant multiple rule:** $\frac{d}{dx}[k.f(x)] = k.\frac{d}{dx}f(x)$

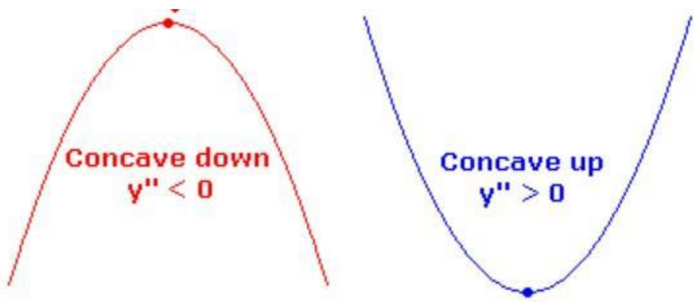
3. **Product rule:** $\frac{d}{dx}[f(x).g(x)] = f(x).\frac{d}{dx}g(x) + g(x).\frac{d}{dx}f(x)$

4. **Quotient rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).f'(x) - f(x).g'(x)}{[g(x)]^2}$

5. **Chain rule:** $\frac{d}{dx} f(g(x)) = f^{'}(g(x)).g^{'}(x)$

- The derivative of a function gives us the **slope** of the **tangent** line to the function at any point on the graph.
    - If the derivative/slope of the tangent at a certain point is **positive**, then the function is **increasing**.
    - If the derivative/slope of the tangent at a certain point is **negative**, then the function is **decreasing**.
    - If the slope of the tangent is **zero**, then the function is neither decreasing nor increasing at that point.

- The **Second derivative** of a function represents its concavity.
  If the second derivative is **positive**, then the function is **concave upwards**.
  If the second derivative is **negative**, then the function is **concave downwards**.



- **Steps to find the optima:**

1. Given a function ***f(x)***, firstly calculate its derivative. i.e. ***f'(x)***
2. Put ***f'(x) = 0*** to obtain the stationary points **x = c**.
3. calculate ***f''(x)*** at each stationary points x = c (i.e *f''(c)*)
4. We get the following situations:
   i). If ***f''(c)>0***, then f(x) has a **minimum value** at x=c. ii). **If *f''(c)<0***, then f(x) has a **maximum value** at x=c. iii). If ***f''(c)=0***, then f(x) may or may not have a maxima or minima at x = c.

Example: Let's find the optima of the function $f(x) = 2x^2 - 4x + 1$

Step 1: Calculate the first derivative.

$$f'(x) = 4x - 4$$

Step 2: Put $f'(x) = 0$ to obtain the stationary points.

$$4x - 4 = 0 \Rightarrow x = 1$$

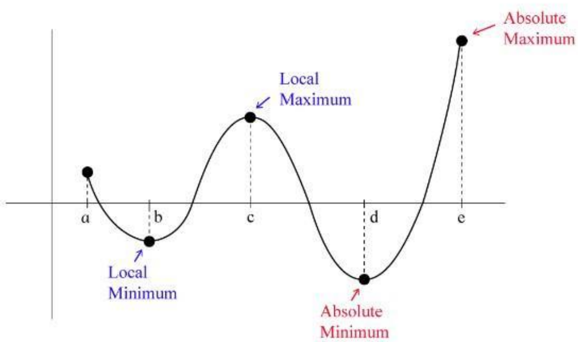
Step 3: Calculate $f''(1)$

$$f''(1) = 4 > 0$$

Step 4: Since, ***f”(1)>0*** therefore, there exists minima at x = 1.

- **Local minima/maxima** can be defined as a point where the function has minimum/maximum value with respect to its vicinity/surroundings. We have marked it as Lmin and Lmax in the image above.

Global minima/maxima can be defined as the minimum/maximum value across the whole domain. It is also called **absolute maxima/minima**.



- A function f(x) is said to be differentiable if it satisfies the following conditions:
  1. f(x) should be **smooth** in its domain.
  2. f(x) is **continuous** in its domain and
  3. **f’(x)** is **continuous**.

Example: f(x) = |x| is not differentiable at x = 0 as it has a sharp point (not smooth) at this point.