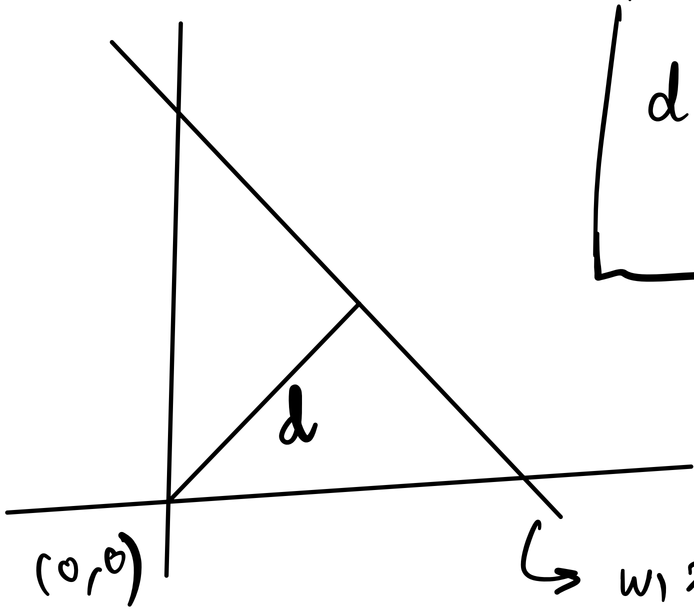
Linear Algebra - 4

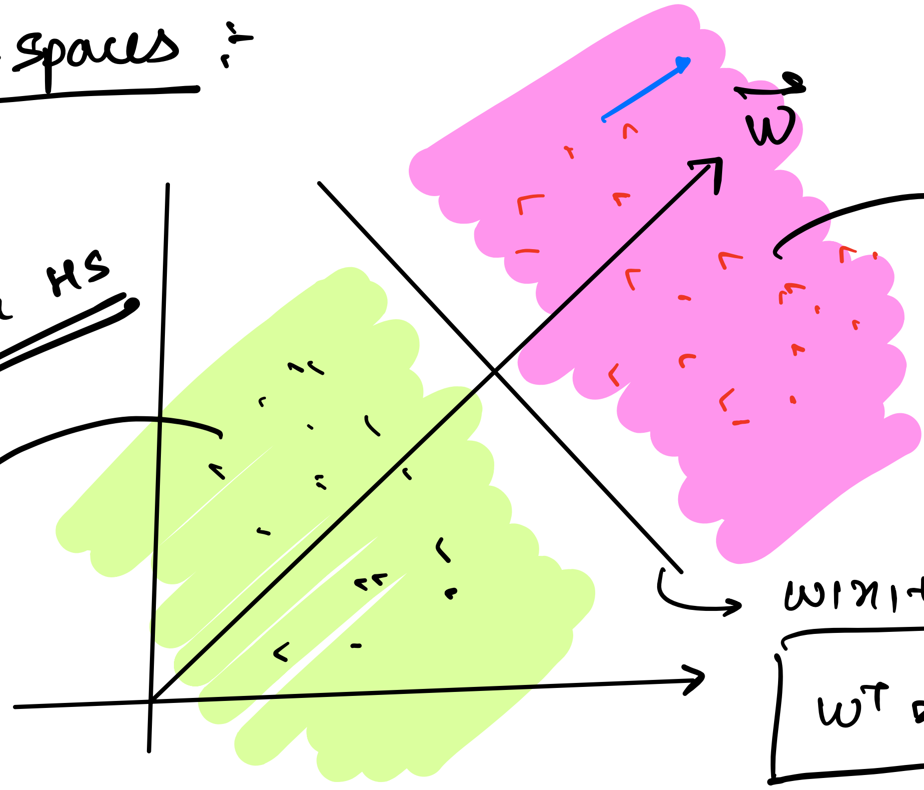
Recap:-



$w_1 x_1 + w_2 x_2 + w_0 = 0$

$$\boxed{d = \frac{-w_0}{\|\vec{w}\|}}$$

Half spaces :



$-ve$ HS

$w^T x + w_0 < 0$

$w^T x + w_0 > 0$

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$$\boxed{w^T x + w_0 = 0}$$

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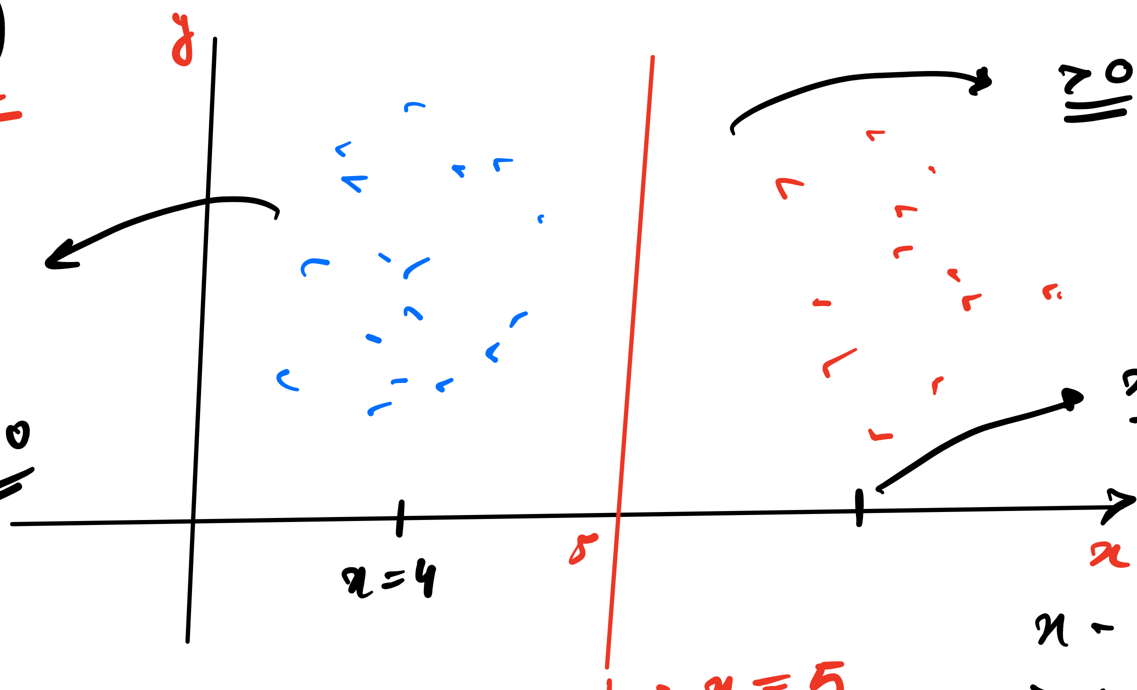
eg



$x - 5$
$4 - 5$
$-1 < 0$

≥ 0

$x = 6$

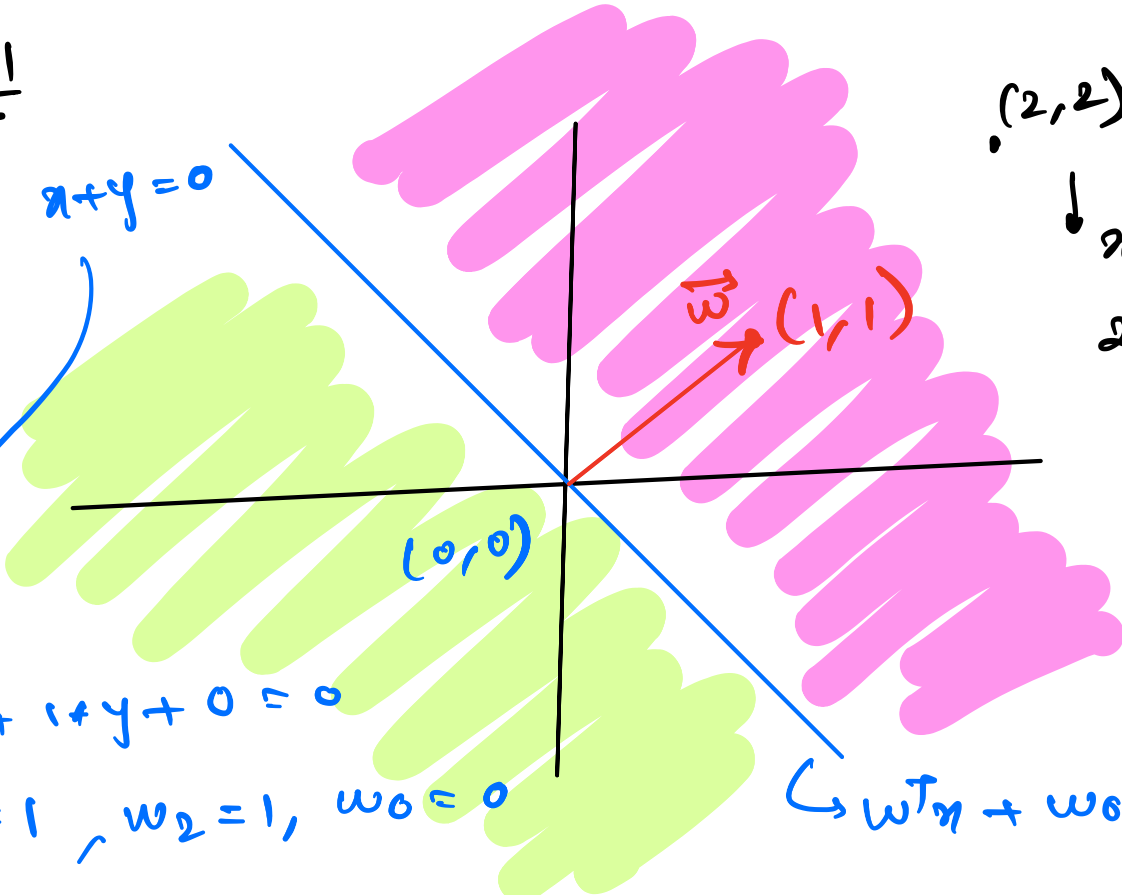
$x - 5$
$\Rightarrow 6 - 5$
$= 1 > 0$

$x = 5$

$x - 5 = 0$

eg1

$x+y=0$



$(2,2)$

$x+y$

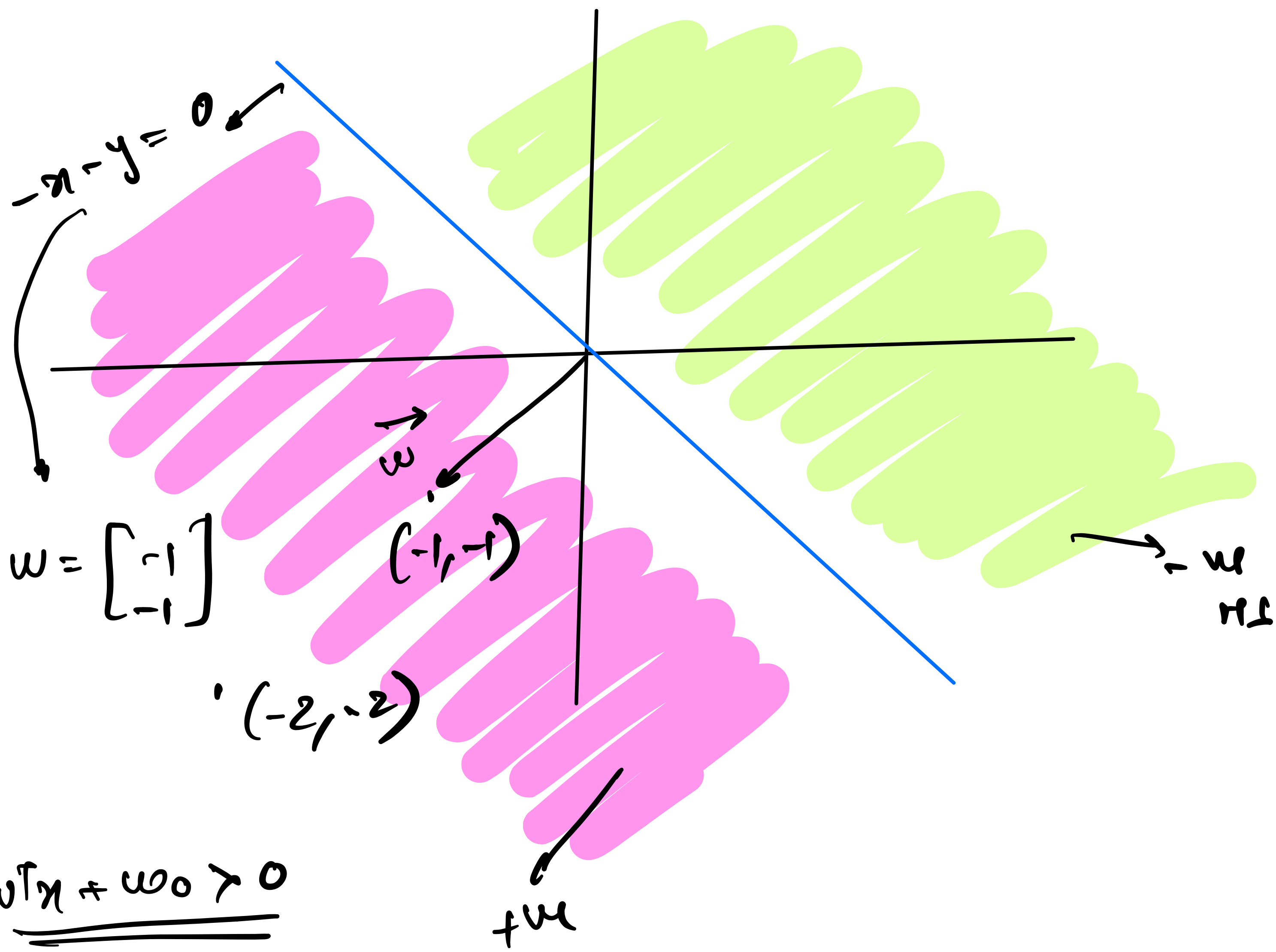
$2+2>0$

$1*x+1*y+0=0$

$\hookrightarrow w^T x + w_0 = 0$

$w_1=1,\ w_2=1,\ w_0=0$

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$\underline{\underline{w^T x + w_0 > 0}}$

$-x-y$

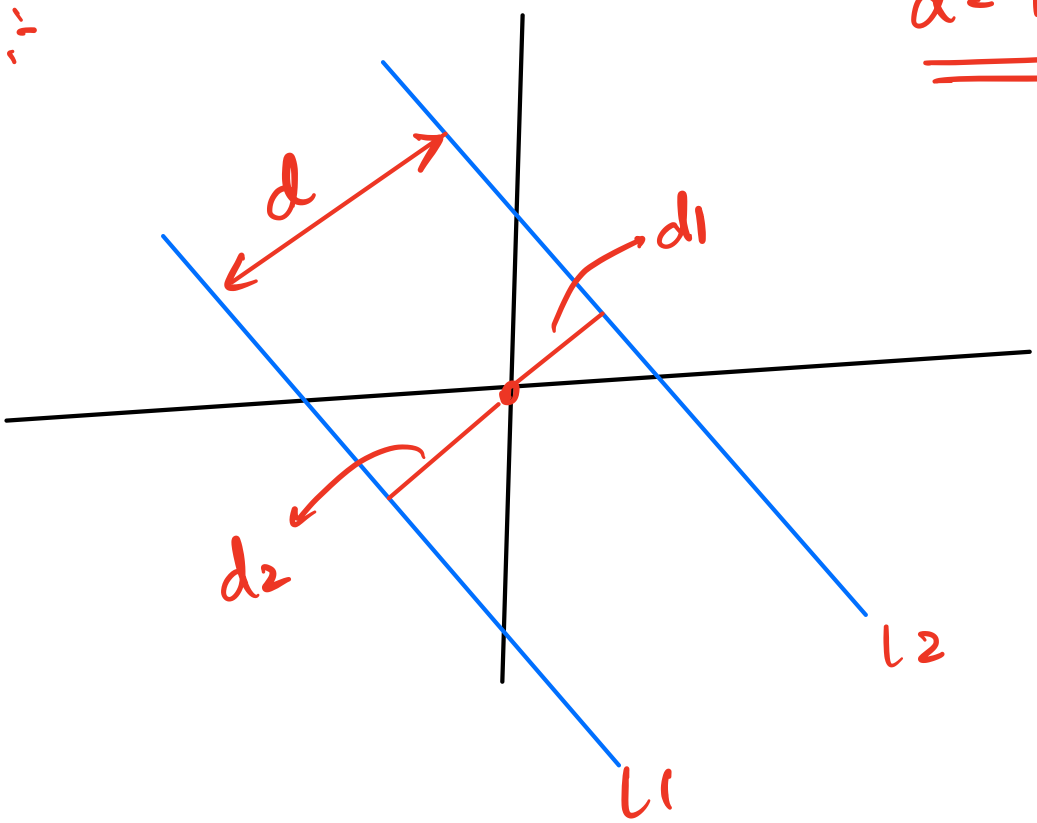
$-(-2)-(-2)$

$2+2 = \underline{\underline{4 > 0}}$

⊕ Distance between 2 Parallel lines

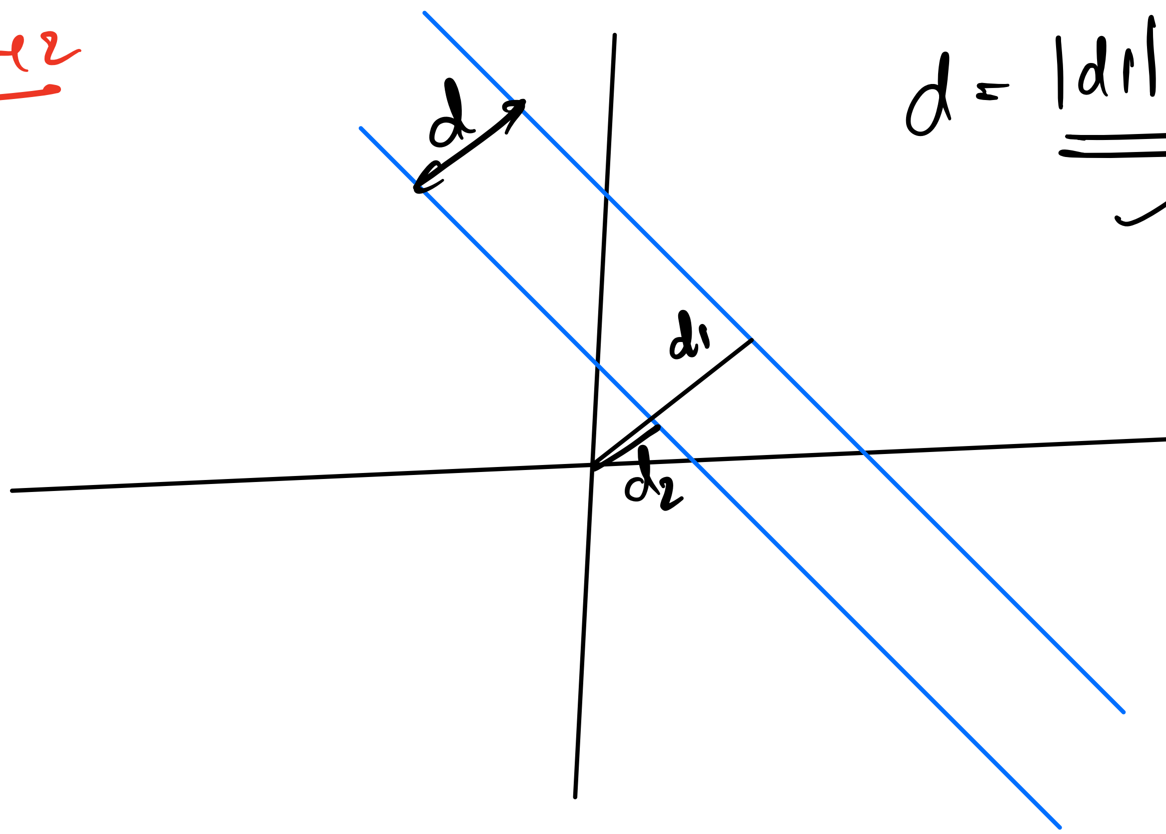
Case 1 :-

$$d = |d_1| + |d_2|$$



Case 2

$$d = |d_1| - |d_2|$$



eg :- L1: $w^1 = [4, 3]$, $w_0^1 = 3$

L2: $w^2 = [16, 12]$, $w_0^2 = 7$

$\boxed{d = ?}$

$$d_1 = \frac{-w_0^1}{\|w^1\|} = \frac{-3}{\sqrt{4^2+3^2}} = \frac{-3}{5}$$

$$d_2 = \frac{-w_0^2}{\|w^2\|} = \frac{-7}{\sqrt{16^2+12^2}} = \frac{-7}{20}$$

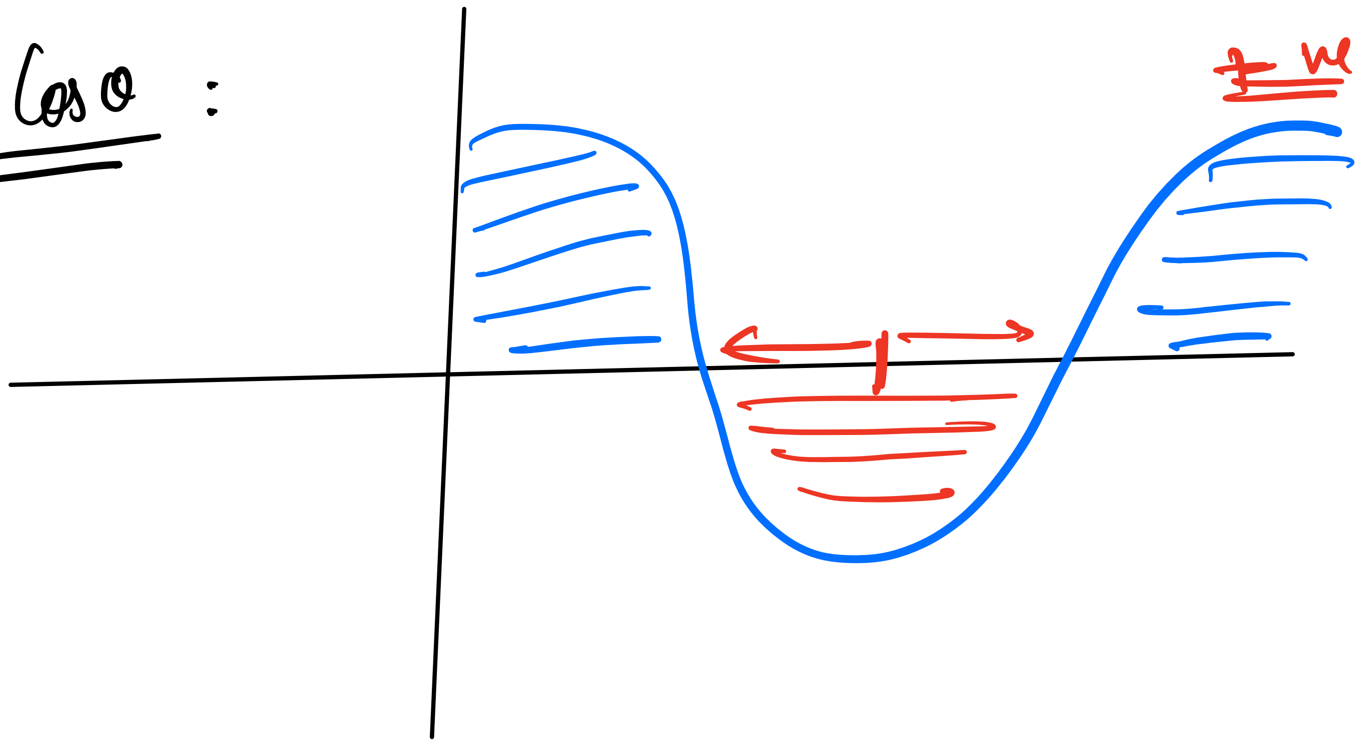
$$d = |d_1| - |d_2| \qquad (|d_1| > |d_2|)$$

$$= \frac{3}{5} - \frac{7}{20} = \frac{12-7}{20} = \frac{5}{20}$$

$$= 1/4 = \boxed{0.25}$$

method 2 :-

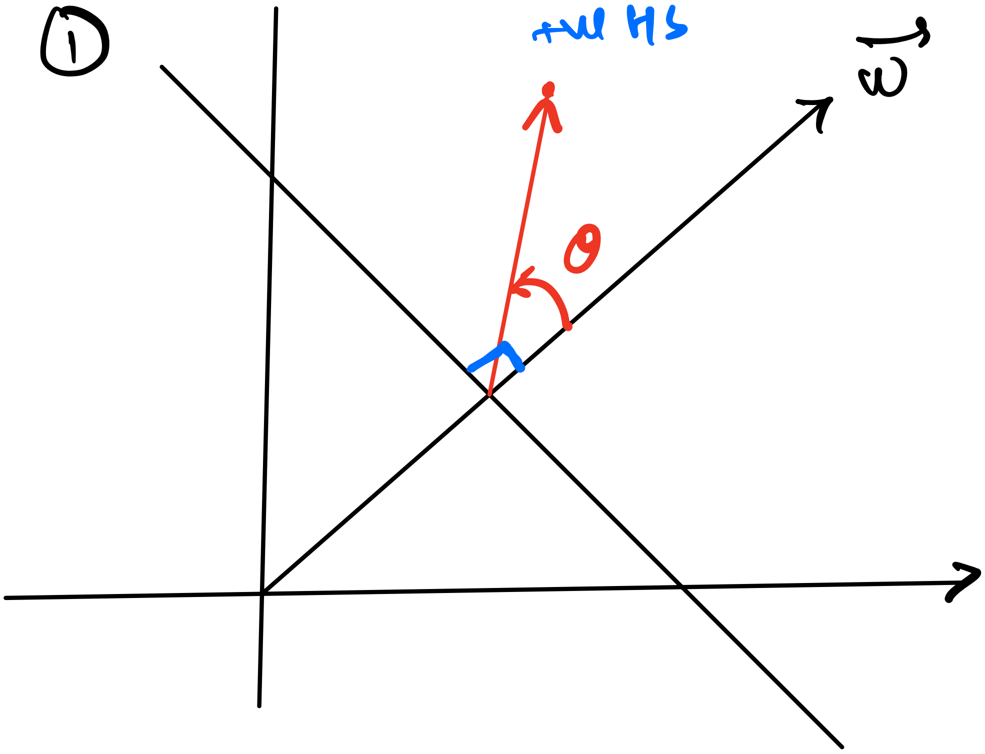
Cos θ :



Cos θ > 0 $\longrightarrow$ $\theta \Rightarrow$ 0 - 90, 270 - 360

Cos θ < 0 $\longrightarrow$ $\theta \Rightarrow$ $\boxed{90^\circ - 270^\circ}$

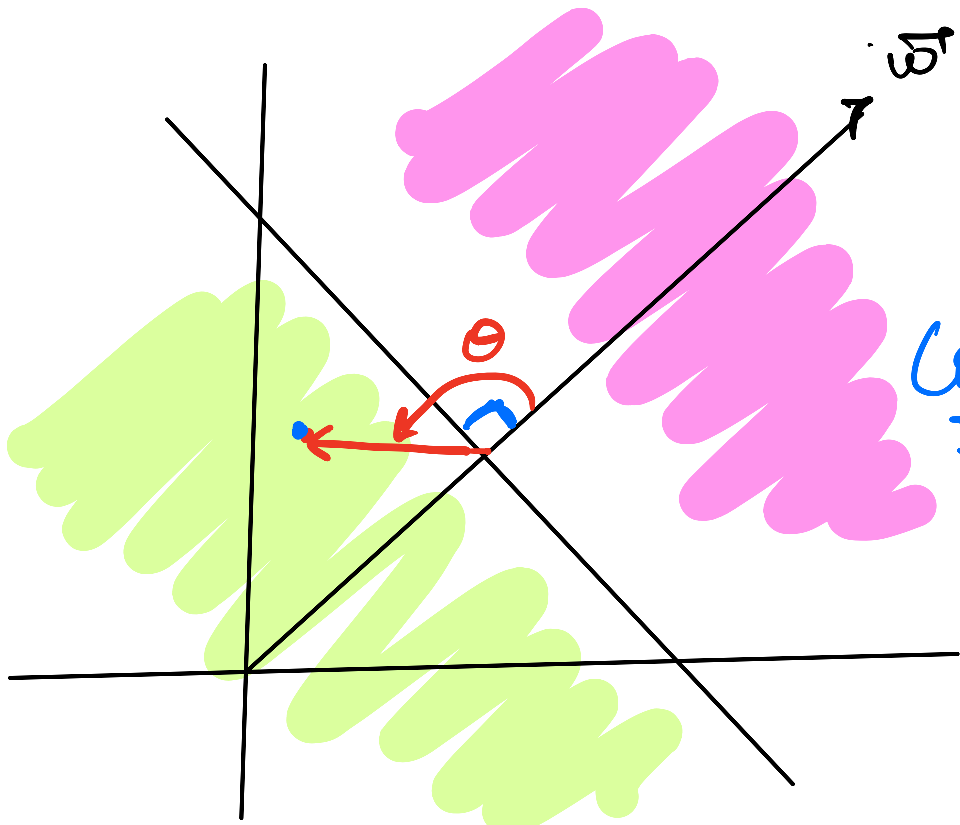
①



$\theta \rightarrow 0 - 90$

$\cos\theta > 0$

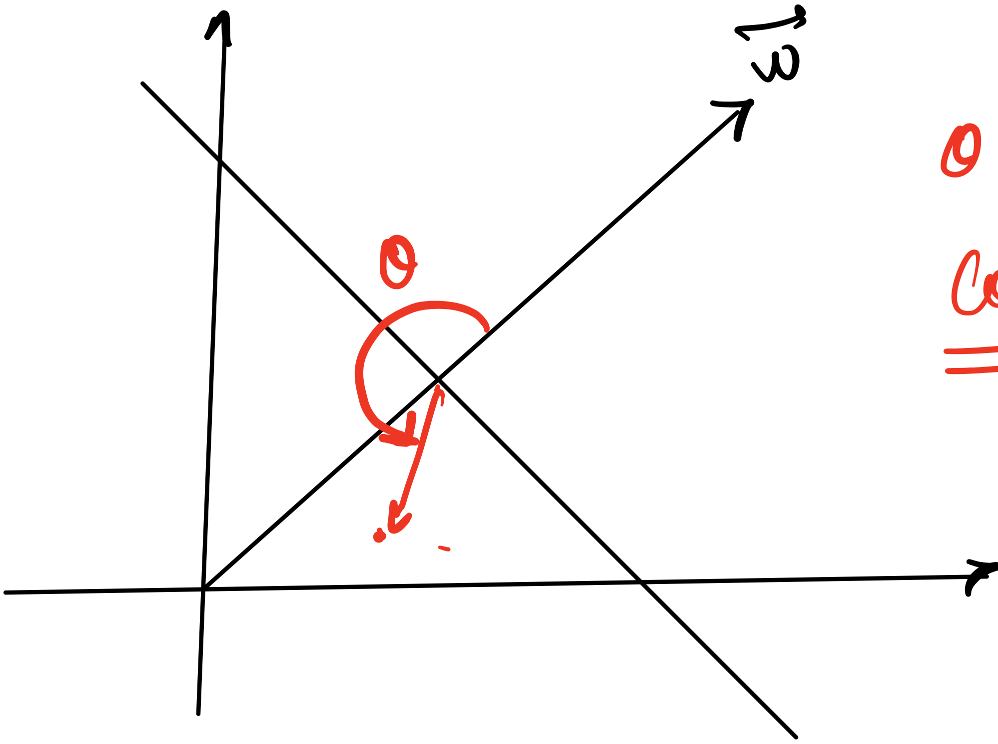
②



$\theta \rightarrow 90 - 180$

$\cos\theta < 0$

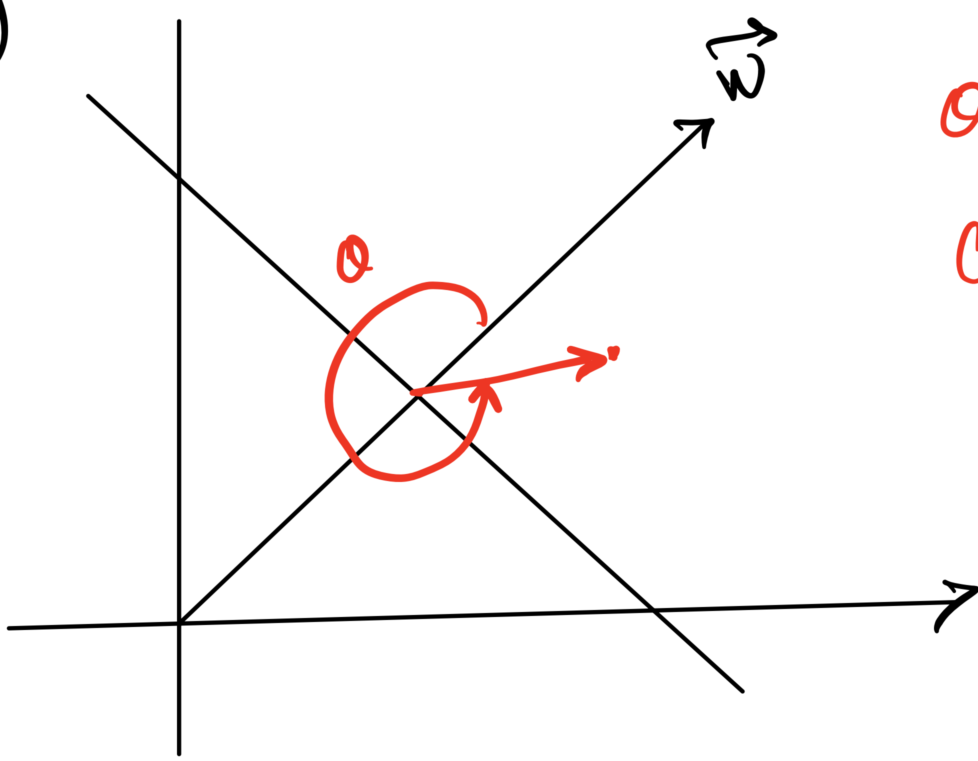
3



$\theta \Rightarrow 180 - 270$

$\cos\theta < 0$

4



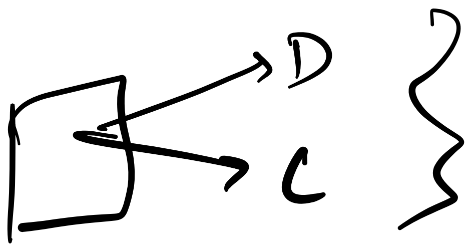
$\theta \rightarrow 270 - 360$

$\cos\theta > 0$

eg

dog $\longrightarrow$ +ve $\longrightarrow$ +1

Cat $\longrightarrow$ −ve $\longrightarrow$ −1



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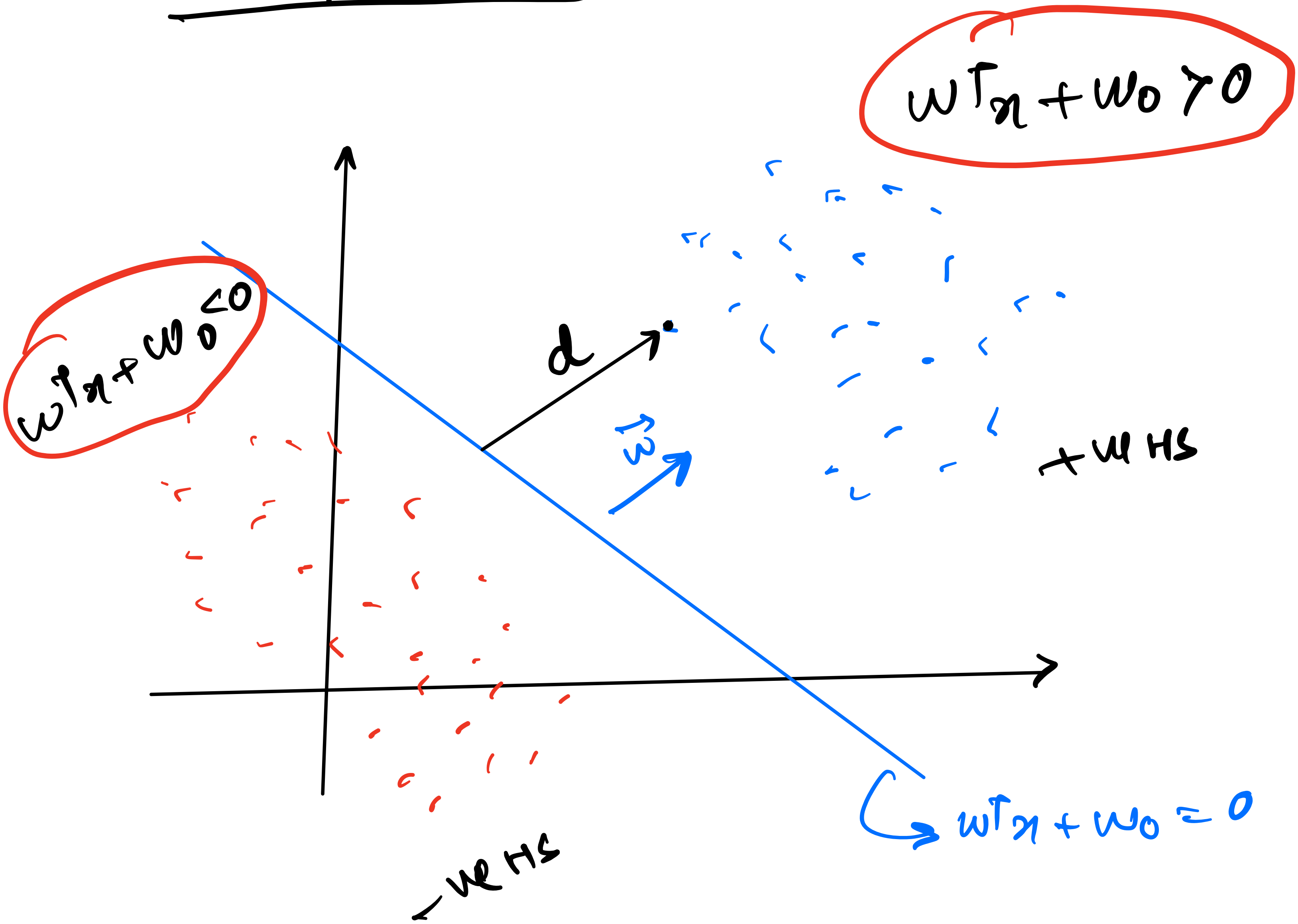
* Distance of a point from the hyperplane.

$$\boxed{d = \frac{w^T x_0 + w_0}{\|\vec{w}\|}}$$

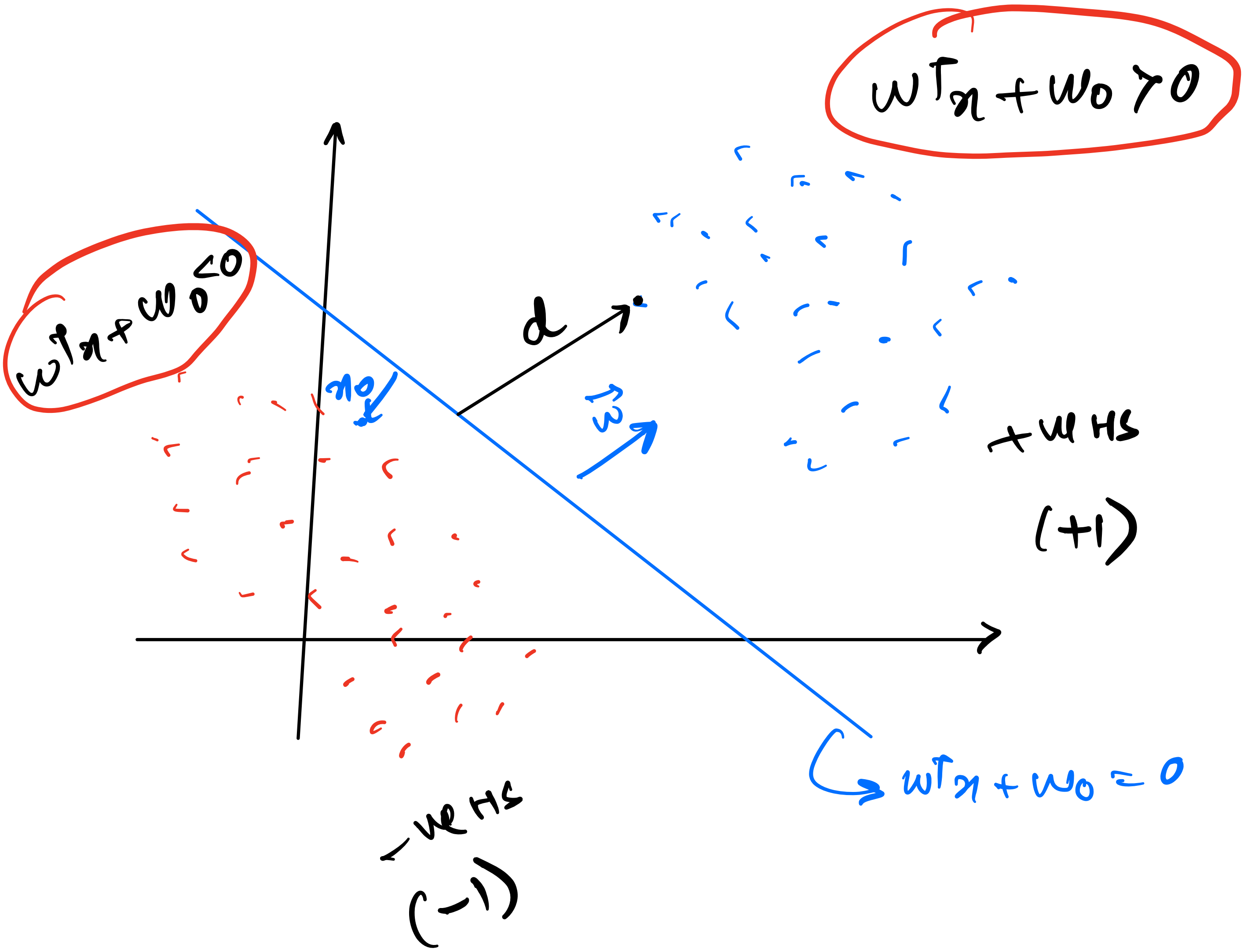
* Loss function :



$$d = \left(\frac{w^T x + w_0}{\|\vec{w}\|} \right)$$

+ve (+ve HS)

-ve (-ve HS)



$\vec{x}_0 \Rightarrow$ Actual $\longrightarrow$ +ve

Assigned $\longrightarrow$ -ve

Actual → +ve
Assigned → −ve

or

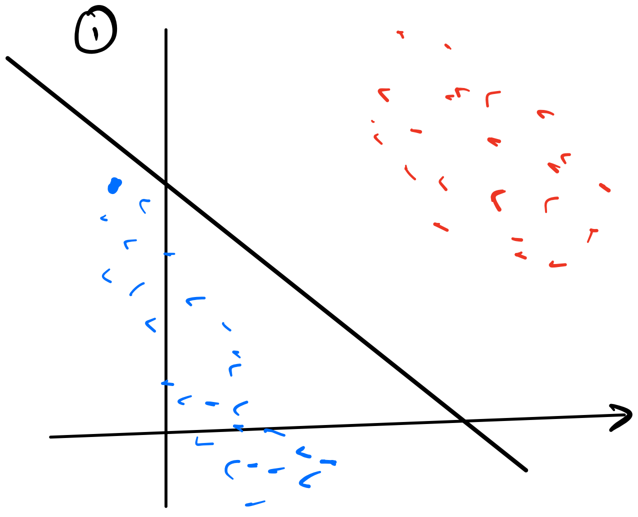
Actual → −ve
Assigned → +ve

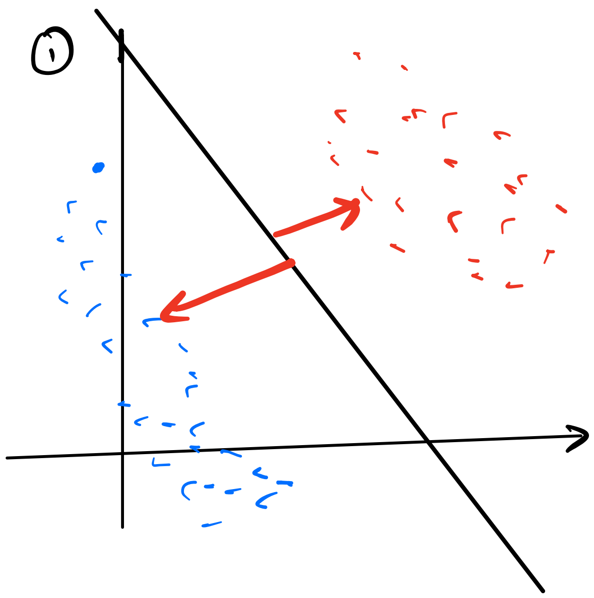
→ **Misclassified**

How to assign labels :

$$d = \frac{w^T x_0 + w_0}{\|\vec{w}\|}$$

$$\boxed{\begin{array}{l} d > 0 \longrightarrow \text{Assign } +1 \\ d < 0 \longrightarrow \text{Assign } -1 \end{array}}$$


i


i

Goal: we need to maximize these distances.

→ Mathematical Notation:

$$D = \{ \underline{x_i}, \underline{y_i} \}_{i=1}^{n}$$

x_i → data point for feature vec

y_i → Actual label.

+1 -1

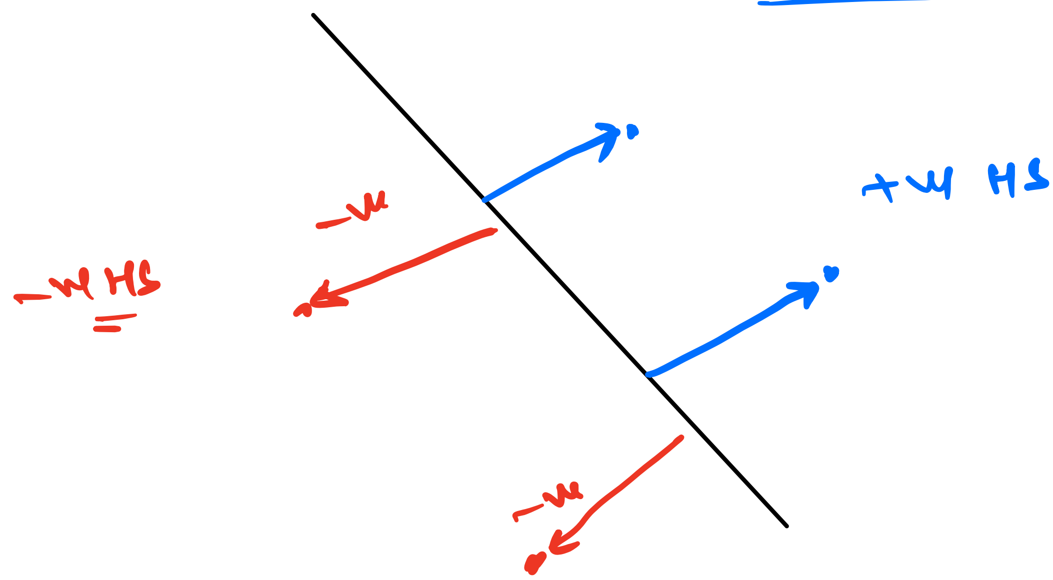
?

$$G(D, w, w_0) \underset{=}{\max} \sum_{i=1}^{n} \left(\frac{w^T x_i + w_0}{\|\vec{w}\|} \right)$$

+ve
+ve
or
-ve
-ve

→ Correct

$w^T x + w_0 \geq 0$



Soln :-

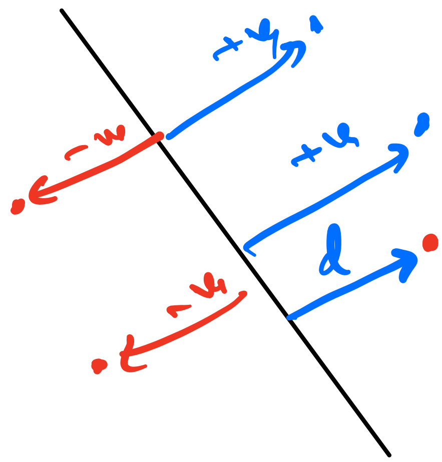
Actual point → +ve

d_1 → −ve

d_1 * sign (Actual point)

−ve * +ve → −ve

Actual Class	d	d * sign(Actual value) ↳ y_i
+ve	+ve	→ +ve ✓
+ve	−ve	→ −ve ✗
−ve	+ve	→ −ve ✗
−ve	−ve	→ +ve ✓



$$\boxed{d * \text{sign}(y_i)}$$

$$+w * -w$$

$$\Rightarrow -w$$

$$G(D, w, w_0) = \max \sum_{i=1}^{n} \left[\left(\frac{w^T x_i + w_0}{\|\vec{w}\|} \right) * y_i \right]$$

Gain $\longrightarrow$ $\max \sum d_i * l_i$

d_i: dist (assigned)

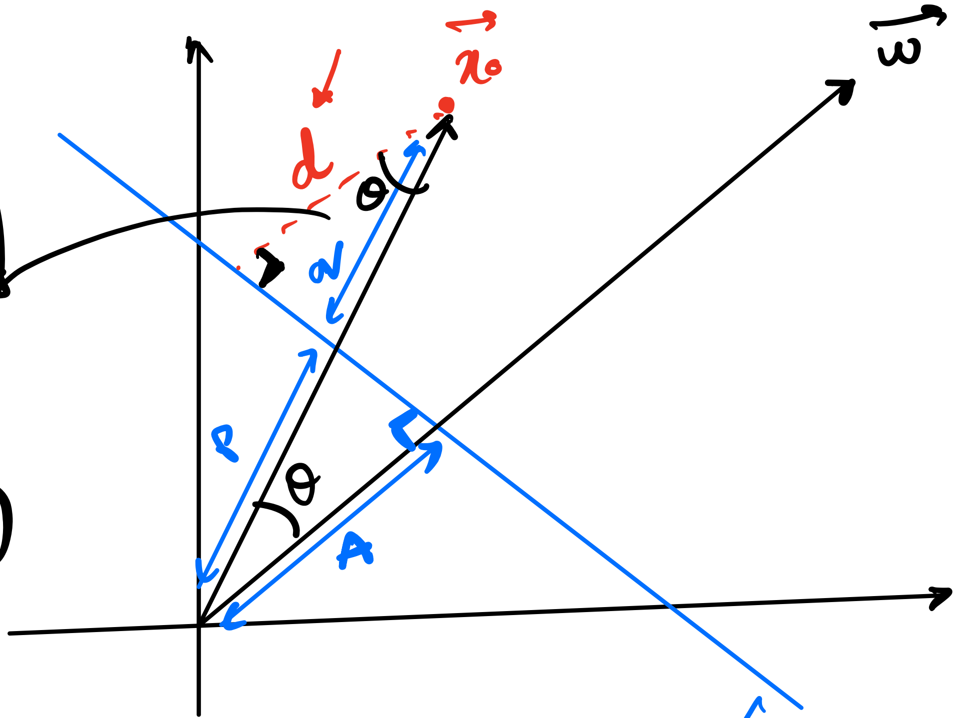
l_i: label (actual)

$$\boxed{\text{Loss} \longrightarrow \min \; -\sum d_i * l_i}$$

$$L(D, \vec{w}, w_0) = -G(D, \vec{w}, w_0)$$

(*) Distance between Point and line

$$\boxed{\cos\theta = \frac{d}{v}}$$ (3)



$\hookrightarrow w^T x + w_0 = 0$

$$\cos\theta = \frac{A}{P}$$

$$\boxed{A = \frac{-w_0}{\|\vec{w}\|}}$$ (2)

$$\boxed{P = \frac{A}{\cos\theta}}$$ (1)

$$P + q = \|\vec{x}_0\|$$

$$q = \|\vec{x}_0\| - P$$

Put P from eqn (1)

$$q = \|\vec{x}_0\| - \frac{A}{\cos\theta}$$

from (3) $\cos\theta = \frac{d}{q}$

$$\boxed{d = q * \cos\theta}$$

$$d = \left(\|\vec{x}_0\| - \frac{A}{\cos\theta}\right) * \cos\theta$$

$$d = \|\vec{x_0}\| * \cos\theta - A$$

from Angle between 2 vectors $\vec{\omega}, \vec{x_0}$

$$\boxed{\cos\theta = \frac{\vec{\omega}^T * x_0}{\|\vec{\omega}\| * \|\vec{x_0}\|}} \rightarrow 4$$

$$d = \cancel{\|\vec{x_0}\|} * \left(\frac{\omega^T * x_0}{\|\vec{\omega}\| * \cancel{\|\vec{x_0}\|}} \right) - A$$

$$d = \frac{\omega^T x_0}{\|\vec{\omega}\|} - A$$

from eqn 2

$$d = \frac{w^T x_0}{\|\vec{w}\|} - \left(\frac{-w_0}{\|\vec{w}\|} \right)$$

$$\boxed{d = \frac{w^T x_0 + w_0}{\|\vec{w}\|}}$$

x