

- A **limit** is a value toward which an expression converges as one or more variables approach certain values. It is denoted as:

$$L = \lim_{x \rightarrow a} f(x)$$

- A **left-hand limit** means the limit of a function as it approaches from the left-hand side. It is denoted as:

$$LHL = \lim_{x \rightarrow a^-} f(x)$$

- On the other hand, a **right-hand limit** means the limit of a function as it approaches from the right-hand side.

$$RHL = \lim_{x \rightarrow a^+} f(x)$$

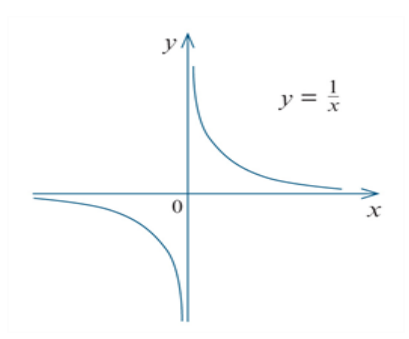
- A **Function** is a relationship between inputs and outputs where each input is related to exactly one output.

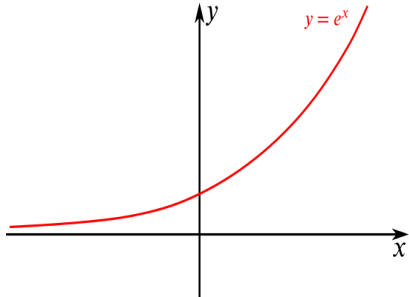
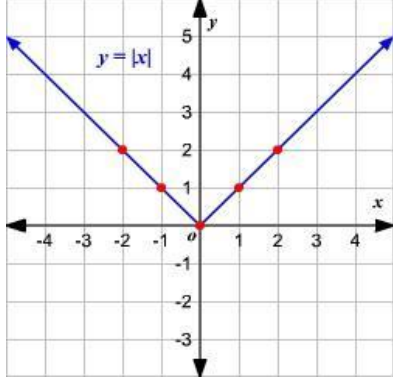
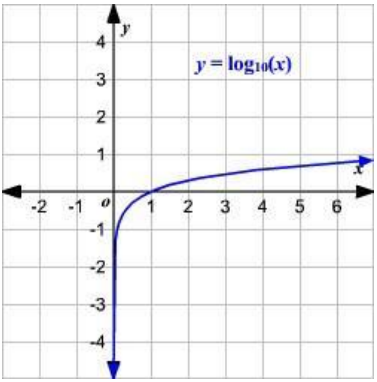
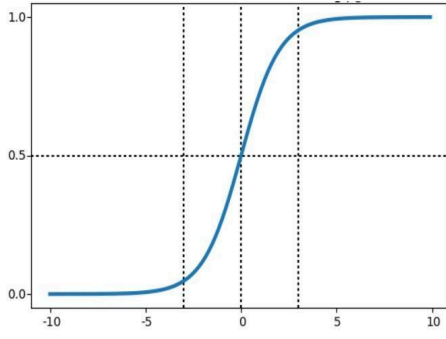
$$y = f(x)$$

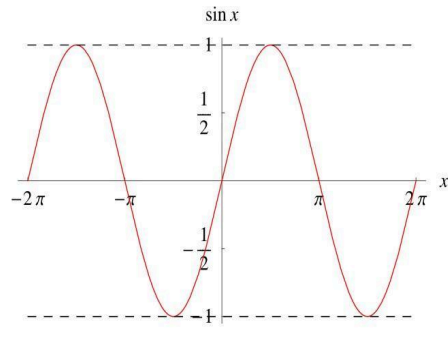
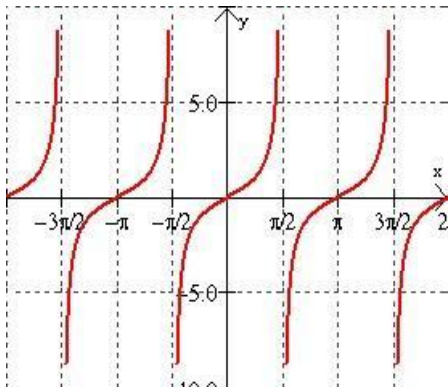
Output
Name of Function
Input

- The **domain** of a function is the set of input values for f, in which the function is real and defined.
- The set of all the outputs of a function is known as the **range** of the function.

- **Some important functions for Machine Learning:**

Function	Name	Domain	Range	Plot
$f(x) = \frac{1}{x}$	Hyperbola	R	R	

$f(x) = e^x$	Exponent	\mathbb{R}	\mathbb{R}_+	
$f(x) = x $	Modulus	\mathbb{R}	\mathbb{R}_+	
$f(x) = \log(x)$	Exponential	\mathbb{R}_+	\mathbb{R}	
$f(x) = \frac{1}{1 + e^{-x}}$	Sigmoid	\mathbb{R}	$(-1, 1)$	

$f(x) = \sin(x)$	sine	\mathbb{R}	$[-1, 1]$	
$f(x) = \tan(x)$	tangent	$\mathbb{R} - (2n + 1)\frac{\pi}{2}$	\mathbb{R}	

- $f(x)$ is **continuous** at a point $x = a$,

$$\text{if } \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a} f(x)$$

Example:

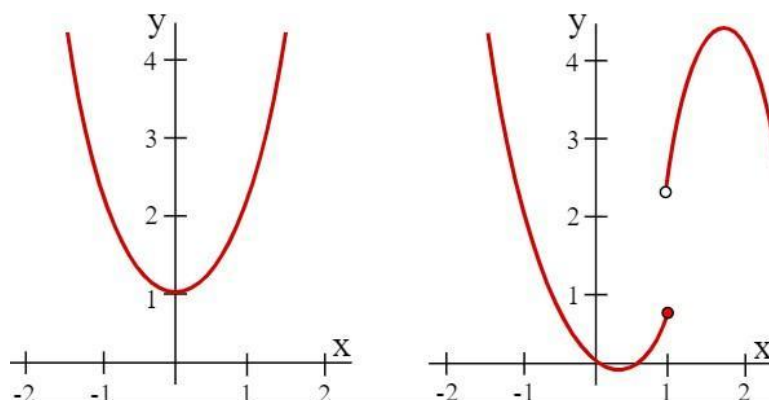


Figure 1

Figure 2

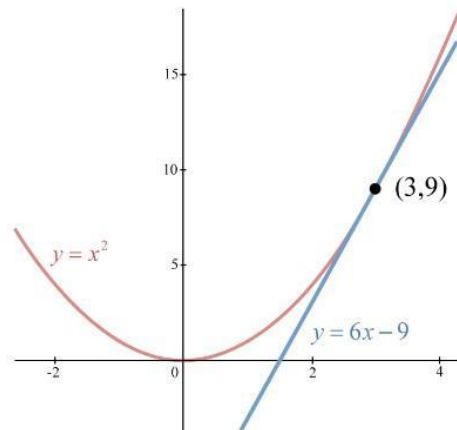
1. Figure 1 is continuous everywhere in its domain.

2. In Figure 2, at $x=1$, $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

Therefore, the function given in Figure 2 is not continuous at $x=1$.

- **A Tangent** is a straight line that touches a graph only at one point.

Example:



In the above-given graph, line $y = 6x - 9$ is a tangent line to the curve at the point (3,9).