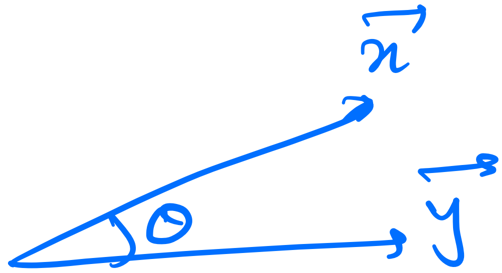
Linear Algebra - 3

Agenda :-

1. Recap
2. Questions
3. Shifting 2D lines
4. Proof : $\vec{w} \perp$ hyperplane
5. Distance between Origin and a line
6. $\boxed{\text{Distance between Point and a line.}}$

* **Recap** :



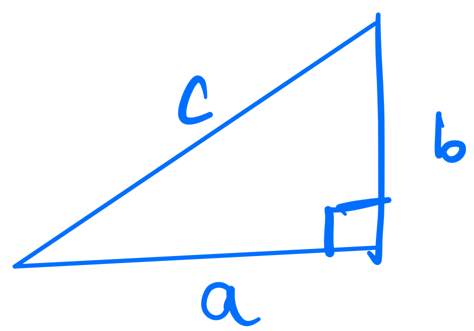
①

$$\boxed{\cos\theta = \frac{\vec{x}^T * \vec{y}}{\|\vec{x}\| * \|\vec{y}\|}}$$

②

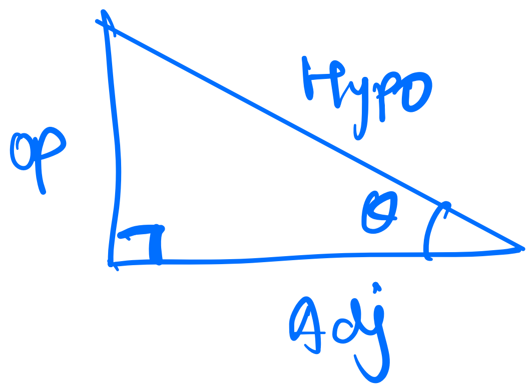
$$\hat{y} = \frac{\vec{y}}{\|\vec{y}\|}$$

* Trignometry basics :



$\Rightarrow$ Pythagoras Thm

$$a^2 + b^2 = c^2$$



$\Rightarrow$ $\cos\theta = \dfrac{\text{Adj}}{\text{Hypo}}$

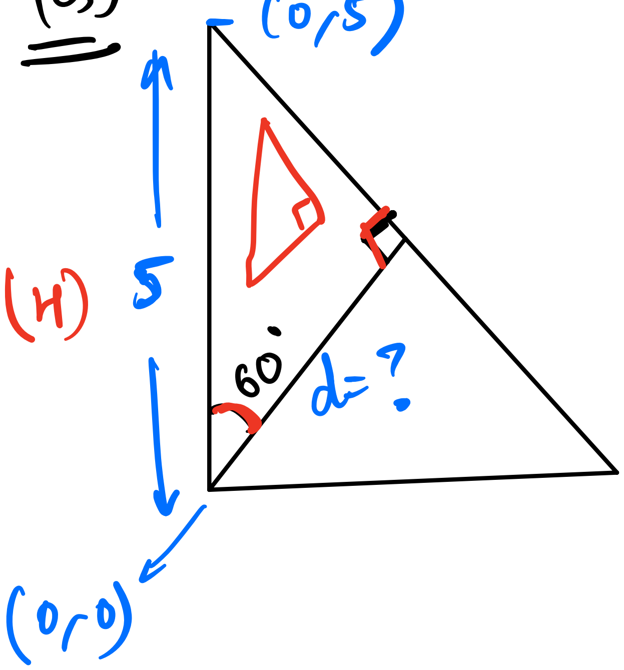
$$\sin\theta = \frac{\text{op}}{\text{Hypo}}$$

$$\tan\theta = \frac{\text{op}}{\text{Adj}}$$

Cos θ :

	0°	30°	45°	60°	90°
Cos θ :	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\boxed{0}$

(Q)



$H = 5$

$\cos 60^\circ = \frac{d}{H}$

$d = H * \cos 60$

$d = 5 * \frac{1}{2}$

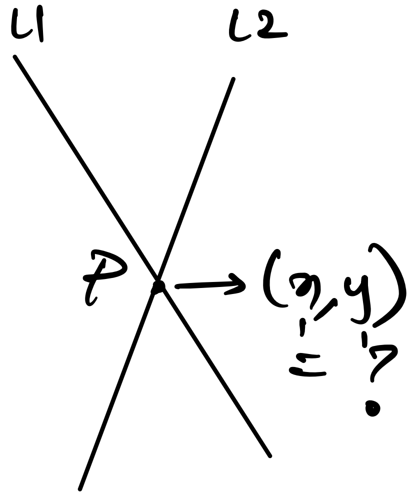
$d = 5/2 = 2.5$

(Q2)

L1: $3x - y + 7 = 0$

L2: $2x + 2y = 0$

Point of intersection = ?



Assume $P \rightarrow (x_1, y_1)$

L1: $3x_1 - y_1 + 7 = 0$ —— (1)

L2: $2x_1 + 2y_1 = 0$

$x_1 = -y_1$ —— (2)

using (2) in (1)

$3(-y_1) - y_1 + 7 = 0$

$4y_1 = 7$

$\boxed{y_1 = \frac{7}{4}}$ —— (3)

$$\boxed{x_1 = -y_1 \Rightarrow -\frac{7}{4}}$$

$$P \to \left(-7/4,\ 7/4\right)$$

$\star$ $L: \ ax + by + c = 0$

y - intersect ?

$$y = mx + c$$

$$ax + by + c = 0$$

$$by = -ax - c$$

$$\boxed{y = \left(\frac{-a}{b}\right)x + \left(\frac{-c}{b}\right)}$$

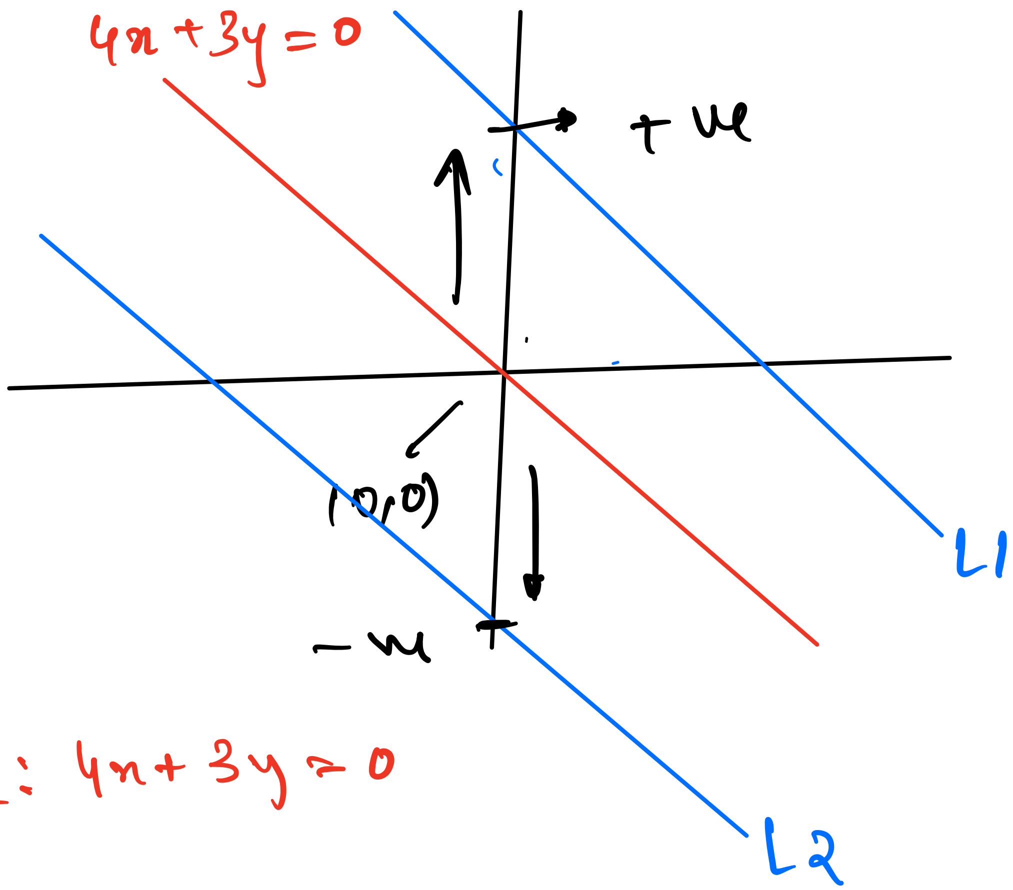
m
(Slope)

y intercept

$$w_1 x + w_2 y + w_0 = 0$$

$$\boxed{y = \left(\frac{-w_1}{w_2}\right) x + \left(\frac{-w_0}{w_2}\right)}$$

18)



L: $4x + 3y = 0$

① $4x + 3y - 5 = 0$

② $4x + 3y + 2 = 0$

L1: $4x + 3y - 5 = 0$

$$3y = -4x + 5$$

$$y = \underline{\underline{\left(-\tfrac{4}{3}\right)}}x + \left(\tfrac{5}{3}\right)$$

$\rightarrow$ +ve (L1)

L2: $4x + 3y + 2 = 0$

$$3y = -4x - 2$$

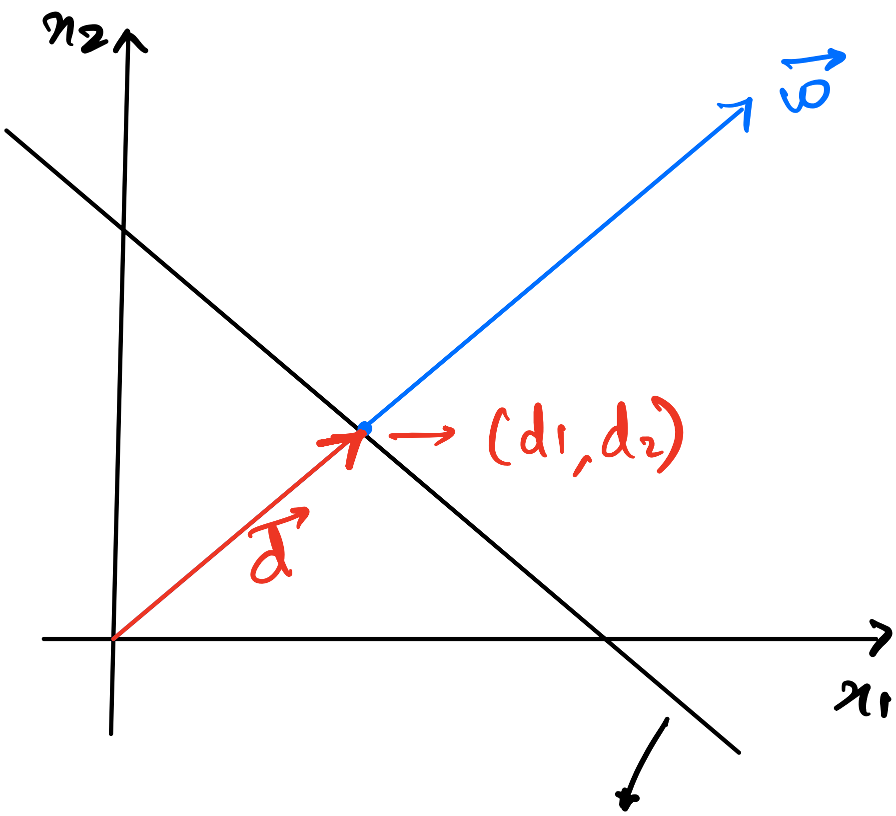
$$y = \underline{\underline{\left(-\tfrac{4}{3}\right)}}x + \underline{\underline{\left(-\tfrac{2}{3}\right)}}$$

$\rightarrow$ -ve

$\downarrow$

L2

* Proof of weight vector is $\perp$ to the hyperplane.



$$w_1x_1 + w_2x_2 + w_0 = 0$$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$\|\vec{w}\| = \sqrt{w_1^2 + w_2^2} \longrightarrow ①$$

$$L: w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\underline{\underline{(d_1, d_2)}}$$

$$w_1 d_1 + w_2 d_2 + w_0 = 0 \longrightarrow ②$$

$$\hat{w} = \frac{\vec{w}}{\|\vec{w}\|}$$

$$\boxed{\vec{d} = \underset{\text{const}}{K} * \underline{\underline{\hat{w}}}}$$

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \longrightarrow \begin{bmatrix} K * \hat{w_1} \\ K * \hat{w}_2 \end{bmatrix}$$

$$\hat{w}_1 = \frac{w_1}{\|\vec{w}\|} \quad , \quad \hat{w}_2 = \frac{w_2}{\|\vec{w}\|}$$

$$d_1 = K * \frac{w_1}{\|\vec{w}\|} \quad , \quad d_2 = K * \frac{w_2}{\|\vec{w}\|}$$

use this in eq (2)

$$w_1 d_1 + w_2 d_2 + w_0 = 0$$

$$w_1 \left(K * \frac{w_1}{\|\vec{w}\|} \right) + w_2 * \left(\frac{K * w_2}{\|\vec{w}\|} \right) + w_0 = 0$$

$$K * \left(\frac{w_1^2 + w_2^2}{\|\vec{w}\|} \right) + w_0 = 0$$

$$K = \frac{-w_0}{\left(\frac{w_1^2 + w_2^2}{\|\vec{w}\|}\right)}$$

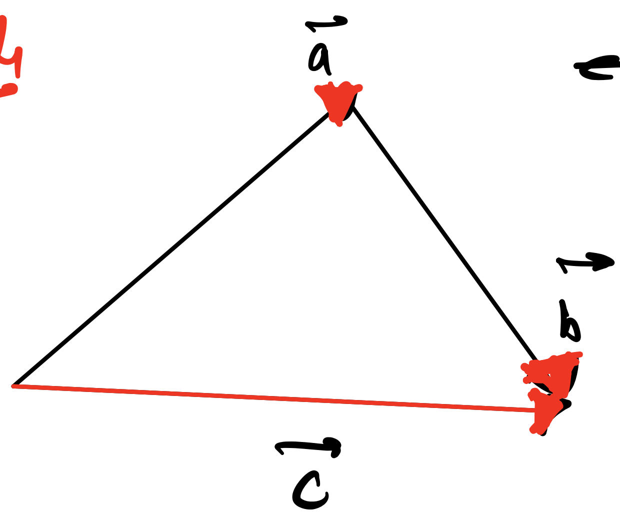
$$\vec{d} = \begin{bmatrix} K * \frac{w_1}{\|\vec{w}\|} \\ K * \frac{w_2}{\|\vec{w}\|} \end{bmatrix}$$

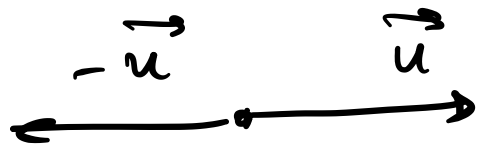
$$\boxed{\frac{a}{b/c} \Rightarrow \frac{a*c}{b}}$$

$$\vec{d} = \begin{bmatrix} \frac{-w_0 * \cancel{\|\vec{w}\|}}{w_1^2 + w_2^2} * \frac{w_1}{\cancel{\|\vec{w}\|}} \\ \\ \frac{-w_0 * \cancel{\|\vec{w}\|}}{w_1^2 + w_2^2} * \frac{w_2}{\cancel{\|\vec{w}\|}} \end{bmatrix}$$

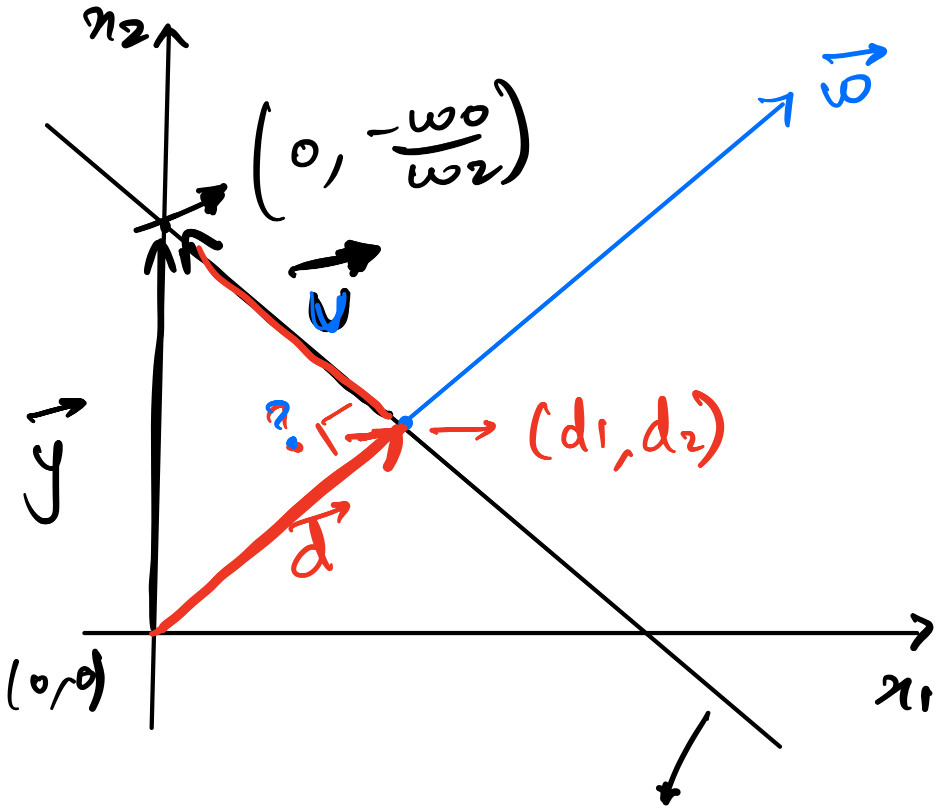
$$\vec{d} = \begin{bmatrix} \dfrac{-w_0 w_1}{w_1^2 + w_2^2} \\[2em] \dfrac{-w_0 w_2}{w_1^2 + w_2^2} \end{bmatrix}$$

Addition Rule





$$\boxed{\vec{C} = \vec{a} + \vec{b}}$$



$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$w_2 x_2 = -w_1 x_1 - w_0$$

$$x_2 = \left(\frac{-w_1}{w_2}\right) x + \left(\frac{-w_0}{w_2}\right)$$

$$\vec{y} = \vec{d} + \vec{u}$$

$$\boxed{\vec{u} = \vec{y} - \vec{d}}$$

$$\vec{y}-\vec{d} = \begin{bmatrix} 0 - \left(-\frac{w_0 w_1}{w_1^2+w_2^2}\right) \\ \frac{-w_0}{w_2} - \left(\frac{-w_0 w_2}{w_1^2+w_2^2}\right) \end{bmatrix}$$

$$\vec{d}^{T} * (\vec{y}-\vec{d}) = 0$$

$$\vec{d}^{T} \qquad (\vec{y}-\vec{d})$$

$$\cos\theta = \frac{\vec{d}^{T} * (\vec{y} - \vec{d})}{\|\vec{d}\| * \|\vec{y} - \vec{d}\|}$$

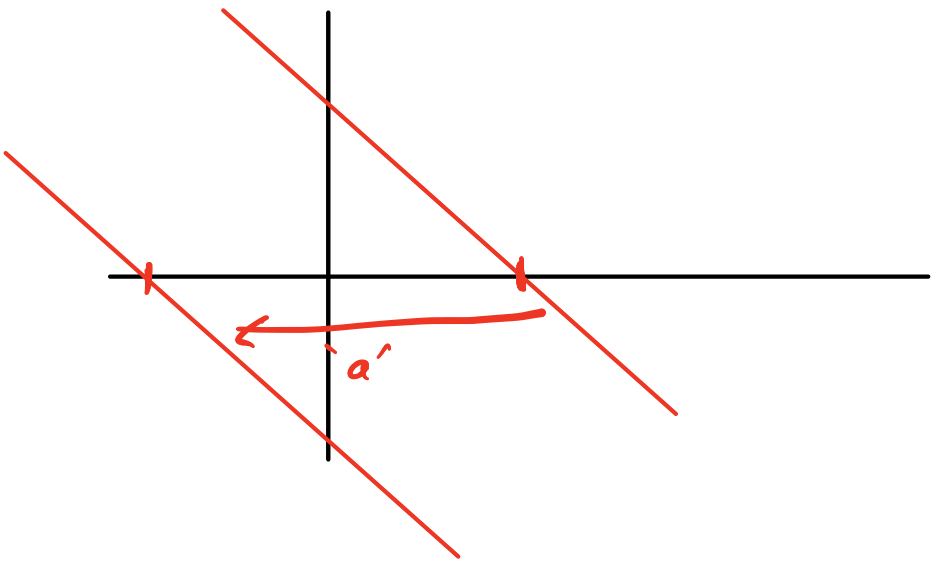
$$\cos\theta = 0$$

$$\boxed{\theta = 90^{\circ}}$$

* Shifting 2D lines : $\underline{\underline{H.W}}$

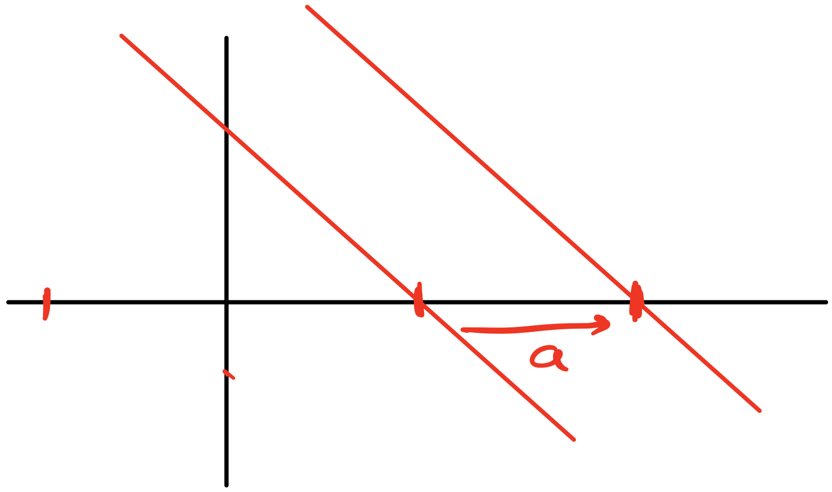
$\longrightarrow \quad w_1x_1 + w_2x_2 + w_0 = 0$

(i) 'a' units to the $\underline{\underline{\text{left}}}$



$$w_1\underline{\underline{(x_1+a)}} + w_2x_2 + w_0 = 0$$

② 'a' units to the right



$$w_1(x_1 - a) + w_2 x_2 + w_0 = 0$$

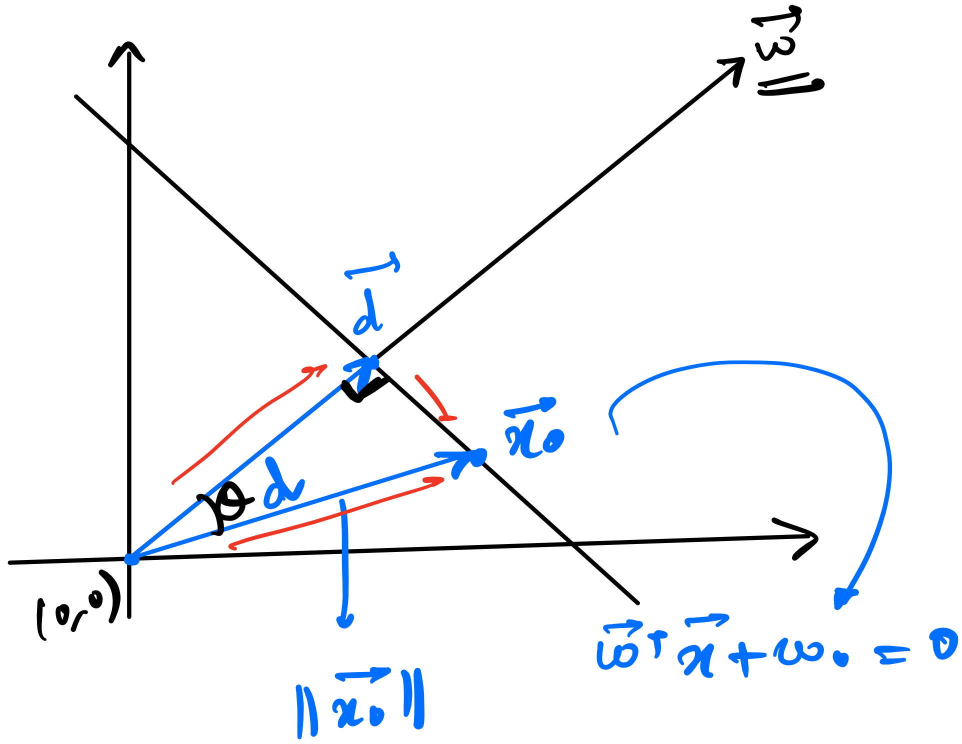
③ 'a' $\longrightarrow$ up

$$w_1 x_1 + w_2(x_2 - a) + w_0 = 0$$

④ 'a' $\longrightarrow$ down

$$w_1 x_1 + w_2(x_2 + a) + w_0 = 0$$

* Distance between origin and line



$$\vec{w}^T\vec{x} + w_0 = 0$$

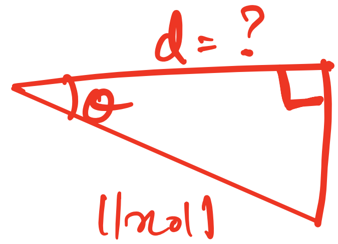
$$\vec{w}^T\vec{x}_0 + w_0 = 0$$

$$\boxed{\vec{w}^T * \vec{x}_0 = -w_0 \rightarrow (1)}$$

$$\boxed{\cos\theta = \frac{\vec{w}^T * \vec{x_0}}{\|\vec{w}\| * \|\vec{x_0}\|}} \quad (2)$$

angle between $\vec{w}$ & $\vec{x_0}$

from Trig



$$\cos\theta = \frac{d}{\|x_0\|} \quad (3)$$

Trigno

from (2) & (3)

$$\frac{\vec{w}^T * \vec{x_0}}{\|\vec{w}\| * \cancel{\|x_0\|}} = \frac{d}{\cancel{\|x_0\|}}$$

$$d = \frac{\vec{\omega}^T * \vec{x}_0}{\|\vec{\omega}\|} \longrightarrow (4)$$

using (1) in (4)

$$\boxed{d = \frac{-\omega_0}{\|\vec{\omega}\|}}$$