$\lambda_{n,n-k+1}$

$$\cancel{\lambda^{(\alpha)}_{n,n-k+1}} = \nu + 2^{2/3} a_k \nu^{1/3} + \tfrac{1}{5} 2^{4/3} a_k^2 \nu^{-1/3} + \left(\frac{11}{35} - \alpha^2 - \frac{12}{175} a_k^3\right)\nu^{-1} + \left(\frac{16}{1575} a_k + \frac{92}{7875} a_k^4\right) 2^{2/3} \nu^{-5/3} - \left(\frac{15152}{3031875} a_k^5 + \frac{1088}{121275} a_k^2\right) 2^{1/3} \nu^{-7/3} + O(1/n^3)$$

where $a_k$ is the (with the negative sign) $k^{th}$ root of $Ai$,

and $\nu = 4n + 2\alpha + 2$

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$$\lambda^{(\alpha)}_{n,m} = \frac{j_{0,m}^2}{\nu}\left[1 + \left(\frac{j_{0,m}^2 + 2(\alpha^2 - 1)}{3\nu^2}\right)\right] + O(1/n^5)$$

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