Calculus III Workshop 12, 11/30/16: Course Review

All problems are taken from the Exercises in the Review section of the corresponding chapter (not the concept check or true/false quiz sections) so 13.R #1 means Chapter 13, Review section, Exercise 1, etc.

Problem 1 (13.R #1). Sketch the curve with the vector function

$$\mathbf{r}(t) = t\mathbf{i} + \cos \pi t\mathbf{j} + \sin \pi t\mathbf{k}, \quad t \geq 0$$

and find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

Problem 2 (13.R #6.(a) and (b)). Let C be the curve with equations $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$. Find
(a) the point where C intersects the xz-plane, and
(b) parametric equations for the tangent line at $(1, 1, 0)$.

Problem 3 (14.R #16). Find the first partial derivatives of $G(x, y, z) = e^{xz} \sin(y/z)$.

Problem 4 (14.R #25.(a)). Find an equation of the tangent plane of the surface

$$z = 3x^2 - y^2 + 2x$$

at the point $(1, -2, 1)$.

Problem 5 (14.R #31). Find the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

Problem 6 (14.R #46). Find the directional derivative of $f(x, y, z) = x^2y + x\sqrt{1+z}$ at the point $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

Problem 7 (14.R #47). Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$. In which direction does it occur?

Problem 8 (14.R #53). Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = 3xy - x^2y - xy^2$$

Problem 9 (15.R #17). Calculate the multiple integral $\iint_D \frac{y}{1+x^2}\, dA$ where D is bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

Problem 10 (15.R #21). Compute $\iint_D (x^2 + y^2)^{3/2}\, dA$ where D is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

Problem 11 (15.R #23). Compute $\iiint_E xy\, dV$, where

$$E = \{(x, y, z) : 0 \leq x \leq 3,\ 0 \leq y \leq x,\ 0 \leq z \leq x + y\}.$$

Problem 12 (15.R #28). Compute $\iiint_H z^3\sqrt{x^2 + y^2 + z^2}\, dV$, where H is the solid hemisphere that lies above the xy-plane and has center the origin and radius 1.

Problem 13 (15.R #30). Find the volume of the solid under the surface $z = x^2y$ and above the triangle in the xy-plane with vertices $(1, 0)$, $(2, 1)$, and $(4, 0)$.

Problem 14 (15.R #34). Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and below the half cone $z = \sqrt{x^2 + y^2}$.

Problem 15 (16.R #2)**.** Evaluate the integral $\int_C x\,ds$, where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

Problem 16 (16.R #9)**.** Evaluate the line integral $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F}(x,y,z) = e^z\mathbf{i} + xz\mathbf{j} + (x+y)\mathbf{k}$ and C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} - t\mathbf{k}$, $0 \le t \le 1$.

Problem 17 (16.R #14)**.** Show that $\mathbf{F}(x,y,z) = e^y\mathbf{i} + (xe^y + e^z)\mathbf{j} + ye^z\mathbf{k}$ is conservative and use this fact to evaluate the line integral $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ where C is the line segment from $(0,2,0)$ to $(4,0,3)$.

Problem 18 (16.R #17)**.** Use Green's Theorem to evaluate $\int_C \langle x^2y, -xy^2\rangle \cdot \mathbf{T}\,ds$ where C is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,3)$.

Problem 19 (16.R #25)**.** Find the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0,0)$, $(1,0)$ and $(1,2)$.

Problem 20 (16.R #30)**.** Evaluate the surface/flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x,y,z) = \langle x^2, xy, z\rangle$ and S is the part of the parabolid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation

Problem 21 (16.R #32)**.** Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl}\mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x,y,z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + z^3e^{xy}\mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 5$ lying above the plane $z = 1$, oriented upward.

Problem 22 (16.R #33)**.** Use Stokes' Theorem to evaluate $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F}(x,y,z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$, and C is the triangle with vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$, oriented counterclockwise as viewed from above.

Problem 23 (16.R #34)**.** Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x,y,z) = \langle x^3, y^3, z^3\rangle$ and S is the boundary surface of the solid inside the cylinder $x^2 + y^2 = 1$ and between the planes $z = 0$ and $z = 2$.