10. $\mathbf{r}(t) = \langle \tan t, \sec t, 1/t^2 \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle \sec^2 t, \sec t \tan t, -2/t^3 \rangle$

18. $\mathbf{r}'(t) = \langle 3t^2 + 3, 2t, 3 \rangle \quad \Rightarrow \quad \mathbf{r}'(1) = \langle 6, 2, 3 \rangle$. Thus

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{6^2 + 2^2 + 3^2}} \langle 6, 2, 3 \rangle = \tfrac{1}{7} \langle 6, 2, 3 \rangle = \left\langle \tfrac{6}{7}, \tfrac{2}{7}, \tfrac{3}{7} \right\rangle.$$

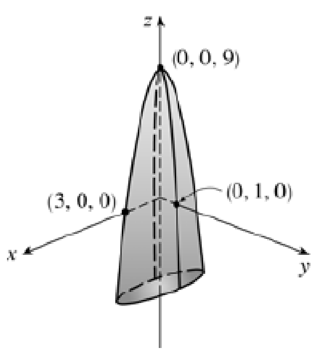
25. The vector equation for the curve is $\mathbf{r}(t) = \langle e^{-t} \cos t, e^{-t} \sin t, e^{-t} \rangle$, so

$$\begin{aligned} \mathbf{r}'(t) &= \langle e^{-t}(-\sin t) + (\cos t)(-e^{-t}), e^{-t} \cos t + (\sin t)(-e^{-t}), (-e^{-t}) \rangle \\ &= \langle -e^{-t}(\cos t + \sin t), e^{-t}(\cos t - \sin t), -e^{-t} \rangle \end{aligned}$$

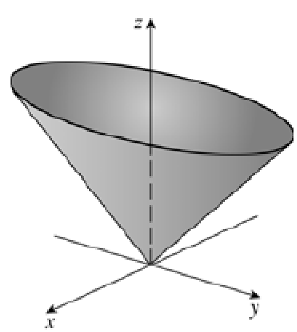
The point $(1, 0, 1)$ corresponds to $t = 0$, so the tangent vector there is

$\mathbf{r}'(0) = \langle -e^0(\cos 0 + \sin 0), e^0(\cos 0 - \sin 0), -e^0 \rangle = \langle -1, 1, -1 \rangle$. Thus, the tangent line is parallel to the vector $\langle -1, 1, -1 \rangle$ and parametric equations are $x = 1 + (-1)t = 1 - t$, $y = 0 + 1 \cdot t = t$, $z = 1 + (-1)t = 1 - t$.

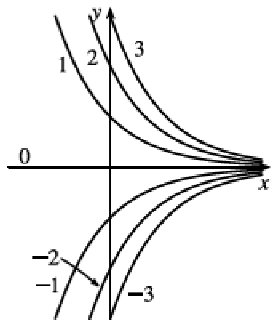
29. $z = 9 - x^2 - 9y^2$, an elliptic paraboloid opening downward with vertex at $(0, 0, 9)$.



30. $z = \sqrt{4x^2 + y^2}$ so $4x^2 + y^2 = z^2$ and $z \geq 0$, the top half of an elliptic cone.



47. The level curves are $ye^x = k$ or $y = ke^{-x}$, a family of exponential curves.



69. (a) The graph of g is the graph of f shifted upward 2 units.

(b) The graph of g is the graph of f stretched vertically by a factor of 2.

(c) The graph of g is the graph of f reflected about the xy-plane.

(d) The graph of $g(x, y) = -f(x, y) + 2$ is the graph of f reflected about the xy-plane and then shifted upward 2 units.

70. (a) The graph of g is the graph of f shifted 2 units in the positive x-direction.

(b) The graph of g is the graph of f shifted 2 units in the negative y-direction.

(c) The graph of g is the graph of f shifted 3 units in the negative x-direction and 4 units in the positive y-direction.

20. $z = \tan xy \quad \Rightarrow \quad \dfrac{\partial z}{\partial x} = (\sec^2 xy)(y) = y\sec^2 xy,\ \dfrac{\partial z}{\partial y} = (\sec^2 xy)(x) = x\sec^2 xy$

31. $f(x, y, z) = xz - 5x^2y^3z^4 \quad \Rightarrow \quad f_x(x, y, z) = z - 10xy^3z^4,\ f_y(x, y, z) = -15x^2y^2z^4,\ f_z(x, y, z) = x - 20x^2y^3z^3$

69. $w = \dfrac{x}{y+2z} = x(y+2z)^{-1} \quad \Rightarrow \quad \dfrac{\partial w}{\partial x} = (y+2z)^{-1},\ \dfrac{\partial^2 w}{\partial y\,\partial x} = -(y+2z)^{-2}(1) = -(y+2z)^{-2},$

$\dfrac{\partial^3 w}{\partial z\,\partial y\,\partial x} = -(-2)(y+2z)^{-3}(2) = 4(y+2z)^{-3} = \dfrac{4}{(y+2z)^3}$ and $\dfrac{\partial w}{\partial y} = x(-1)(y+2z)^{-2}(1) = -x(y+2z)^{-2},$

$\dfrac{\partial^2 w}{\partial x\,\partial y} = -(y+2z)^{-2},\ \dfrac{\partial^3 w}{\partial x^2\,\partial y} = 0.$

1. $z = f(x, y) = 3y^2 - 2x^2 + x \quad \Rightarrow \quad f_x(x, y) = -4x + 1,\ f_y(x, y) = 6y,$ so $f_x(2, -1) = -7,\ f_y(2, -1) = -6.$

By Equation 2, an equation of the tangent plane is $z - (-3) = f_x(2, -1)(x - 2) + f_y(2, -1)[y - (-1)] \quad \Rightarrow$

$z + 3 = -7(x - 2) - 6(y + 1)$ or $z = -7x - 6y + 5.$

5. $z = f(x, y) = x\sin(x+y) \quad \Rightarrow \quad f_x(x,y) = x \cdot \cos(x+y) + \sin(x+y) \cdot 1 = x\cos(x+y) + \sin(x+y)$,

$f_y(x,y) = x\cos(x+y)$, so $f_x(-1,1) = (-1)\cos 0 + \sin 0 = -1$, $f_y(-1,1) = (-1)\cos 0 = -1$ and an equation of the tangent plane is $z - 0 = (-1)(x+1) + (-1)(y-1)$ or $x + y + z = 0$.

17.

Let $f(x,y) = \dfrac{2x+3}{4y+1}$. Then $f_x(x,y) = \dfrac{2}{4y+1}$ and $f_y(x,y) = (2x+3)(-1)(4y+1)^{-2}(4) = \dfrac{-8x-12}{(4y+1)^2}$. Both f_x and f_y are continuous functions for $y \neq -\frac{1}{4}$, so by Theorem 8, f is differentiable at $(0,0)$. We have $f_x(0,0) = 2$, $f_y(0,0) = -12$ and the linear approximation of f at $(0,0)$ is $f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) = 3 + 2x - 12y$.

11. $z = e^r\cos\theta,\ r = st,\ \theta = \sqrt{s^2+t^2} \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s} = e^r\cos\theta \cdot t + e^r(-\sin\theta) \cdot \tfrac{1}{2}(s^2+t^2)^{-1/2}(2s) = te^r\cos\theta - e^r\sin\theta \cdot \frac{s}{\sqrt{s^2+t^2}}$$
$$= e^r\left(t\cos\theta - \frac{s}{\sqrt{s^2+t^2}}\sin\theta\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t} = e^r\cos\theta \cdot s + e^r(-\sin\theta) \cdot \tfrac{1}{2}(s^2+t^2)^{-1/2}(2t) = se^r\cos\theta - e^r\sin\theta \cdot \frac{t}{\sqrt{s^2+t^2}}$$
$$= e^r\left(s\cos\theta - \frac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$$

21. $z = x^4 + x^2y,\ x = s + 2t - u,\ y = stu^2 \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (4x^3 + 2xy)(1) + (x^2)(tu^2),$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (4x^3 + 2xy)(2) + (x^2)(su^2),$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = (4x^3 + 2xy)(-1) + (x^2)(2stu).$$

When $s = 4$, $t = 2$, and $u = 1$ we have $x = 7$ and $y = 8$,

so $\dfrac{\partial z}{\partial s} = (1484)(1) + (49)(2) = 1582$, $\dfrac{\partial z}{\partial t} = (1484)(2) + (49)(4) = 3164$, $\dfrac{\partial z}{\partial u} = (1484)(-1) + (49)(16) = -700$.

22. $T = v/(2u+v) = v(2u+v)^{-1}$, $u = pq\sqrt{r}$, $v = p\sqrt{q}\,r \quad \Rightarrow$

$$\frac{\partial T}{\partial p} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial p} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial p} = [-v(2u+v)^{-2}(2)](q\sqrt{r}) + \frac{(2u+v)(1) - v(1)}{(2u+v)^2}\,(\sqrt{q}\,r)$$

$$= \frac{-2v}{(2u+v)^2}\,(q\sqrt{r}) + \frac{2u}{(2u+v)^2}\,(\sqrt{q}\,r),$$

$$\frac{\partial T}{\partial q} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial q} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial q} = \frac{-2v}{(2u+v)^2}\,(p\sqrt{r}) + \frac{2u}{(2u+v)^2}\,\frac{pr}{2\sqrt{q}},$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial r} = \frac{-2v}{(2u+v)^2}\,\frac{pq}{2\sqrt{r}} + \frac{2u}{(2u+v)^2}\,(p\sqrt{q}).$$

When $p = 2$, $q = 1$, and $r = 4$ we have $u = 4$ and $v = 8$,

so $\dfrac{\partial T}{\partial p} = \left(-\frac{1}{16}\right)(2) + \left(\frac{1}{32}\right)(4) = 0$, $\dfrac{\partial T}{\partial q} = \left(-\frac{1}{16}\right)(4) + \left(\frac{1}{32}\right)(4) = -\frac{1}{8}$, $\dfrac{\partial T}{\partial r} = \left(-\frac{1}{16}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{32}\right)(2) = \frac{1}{32}$.

10. $f(x,y,z) = y^2 e^{xyz}$

(a) $\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z)\rangle = \langle y^2 e^{xyz}(yz), y^2 \cdot e^{xyz}(xz) + e^{xyz}\cdot 2y, y^2 e^{xyz}(xy)\rangle$

$$= \langle y^3 z e^{xyz}, (xy^2 z + 2y)e^{xyz}, xy^3 e^{xyz}\rangle$$

(b) $\nabla f(0,1,-1) = \langle -1, 2, 0\rangle$

(c) $D_{\mathbf{u}} f(0,1,-1) = \nabla f(0,1,-1)\cdot \mathbf{u} = \langle -1,2,0\rangle \cdot \left\langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right\rangle = -\frac{3}{13} + \frac{8}{13} + 0 = \frac{5}{13}$

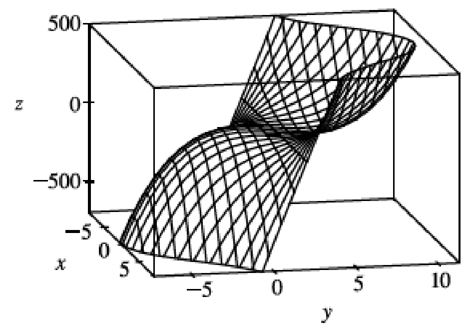
24. $f(x,y,z) = \dfrac{x+y}{z} \quad \Rightarrow \quad \nabla f(x,y,z) = \left\langle \dfrac{1}{z}, \dfrac{1}{z}, -\dfrac{x+y}{z^2}\right\rangle$, $\nabla f(1,1,-1) = \langle -1,-1,-2\rangle$. Thus the maximum rate of change is $|\nabla f(1,1,-1)| = \sqrt{1+1+4} = \sqrt{6}$ in the direction $\langle -1,-1,-2\rangle$.

42. Let $F(x,y,z) = x^2 - z^2 - y$. Then $y = x^2 - z^2 \quad \Leftrightarrow \quad x^2 - z^2 - y = 0$ is a level surface of F. $F_x(x,y,z) = 2x \quad \Rightarrow$ $F_x(4,7,3) = 8$, $F_y(x,y,z) = -1 \quad \Rightarrow \quad F_y(4,7,3) = -1$, and $F_z(x,y,z) = -2z \quad \Rightarrow \quad F_z(4,7,3) = -6$.

(a) An equation of the tangent plane at $(4,7,3)$ is $8(x-4) - 1(y-7) - 6(z-3) = 0$ or $8x - y - 6z = 7$.

(b) The normal line has symmetric equations $\dfrac{x-4}{8} = \dfrac{y-7}{-1} = \dfrac{z-3}{-6}$ and parametric equations $x = 4 + 8t$, $y = 7 - t$, $z = 3 - 6t$.

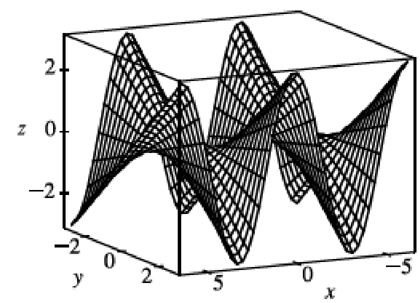
9. $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2 \quad \Rightarrow \quad f_x = 6xy - 12x,\ f_y = 3y^2 + 3x^2 - 12y,\ f_{xx} = 6y - 12,\ f_{xy} = 6x,$ $f_{yy} = 6y - 12$. Then $f_x = 0$ implies $6x(y-2) = 0$, so $x = 0$ or $y = 2$. If $x = 0$ then substitution into $f_y = 0$ gives $3y^2 - 12y = 0 \quad \Rightarrow \quad 3y(y-4) = 0 \quad \Rightarrow \quad y = 0$ or $y = 4$, so we have critical points $(0,0)$ and $(0,4)$. If $y = 2$, substitution into $f_y = 0$ gives $12 + 3x^2 - 24 = 0 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow$ $x = \pm 2$, so we have critical points $(\pm 2, 2)$.



$D(0,0) = (-12)(-12) - 0^2 = 144 > 0$ and $f_{xx}(0,0) = -12 < 0$, so $f(0,0) = 2$ is a local maximum. $D(0,4) = (12)(12) - 0^2 = 144 > 0$ and $f_{xx}(0,4) = 12 > 0$, so $f(0,4) = -30$ is a local minimum.

$D(\pm 2, 2) = (0)(0) - (\pm 12)^2 = -144 < 0$, so $(\pm 2, 2)$ are saddle points.

14. $f(x,y) = y \cos x \quad \Rightarrow \quad f_x = -y \sin x,\ f_y = \cos x,\ f_{xx} = -y \cos x,$ $f_{xy} = -\sin x,\ f_{yy} = 0$. Then $f_y = 0$ if and only if $x = \frac{\pi}{2} + n\pi$ for n an integer. But $\sin\left(\frac{\pi}{2} + n\pi\right) \neq 0$, so $f_x = 0 \quad \Rightarrow \quad y = 0$ and the critical points are $\left(\frac{\pi}{2} + n\pi, 0\right)$, n an integer.



$D\left(\frac{\pi}{2} + n\pi, 0\right) = (0)(0) - (\pm 1)^2 = -1 < 0$, so each critical point is a saddle point.