Calculus III Workshop (Exam 2 review) questions: 11/2/16

Problem 1 (15.3 #20). Evaluate $\iint_D xy^2 dA$, where D is enclosed by x = 0 and $x = \sqrt{1-y^2}$.

Problem 2 (15.3 #19). Evaluate $\iint_D y^2, dA$, where D is the triangle with vertices (0,1), (1,2) and (4,1).

Problem 3 (15.4 #8). Evaluate the integral $\iint_R (2x - y) dA$ in polar coordinates, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$ and the lines x = 0 and y = x.

Problem 4 (15.4 #29). Evaluate the following integral by converting to polar coordinates

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin(x^2 + y^2) \, dy \, dx$$

Problem 5 (15.5, #8). Find the mass and center of mass of the lamina D which is bounded by $y = x^2$ and y = x + 2 with density $\rho(x, y) = kx$.

Problem 6 (15.5 #11). A lamina occupies the part of the disk $x^2 + y^2 \le 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis.

Problem 7 (15.7, #14). Evaluate $\iiint_E xy \, dV$, where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes z = 0 and z = x + y.

Problem 8 (15.7, #41). Find the mass and center of mass of the cube E given by $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le a$, with density $\rho(x, y, z) = x^2 + y^2 + z^2$.

Problem 9 (15.8, #19). Evaluate $\iiint_E (x+y+z) dV$, where E is the solid in the first octant that lies under the paraboloid $z=4-x^2-y^2$.

Problem 10 (15.8, #24). Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

Problem 11 (15.9, #23). Evaluate $\iiint_E (x^2 + y^2) dV$, where *E* lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.

Problem 12 (15.9, #34). Find the mass and center of mass of a solid hemisphere of radius a if the density at any point is proportional to its distance from the base.

Problem 13 (16.2, #4). Evaluate the line integral $\int_C x \sin y \, ds$, where C is the line segment from (0,3) to (4,6).

Problem 14 (16.2, #20). Evaluate the line integral $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$, where

$$\mathbf{F}(x, y, z) = (x + y)\mathbf{i} + (y - z)\mathbf{j} + z^2\mathbf{k},$$

and C is parameterized by $\mathbf{r}(t) = t^2 \mathbf{i} + t^3 \mathbf{j} + t^2 \mathbf{k}$.