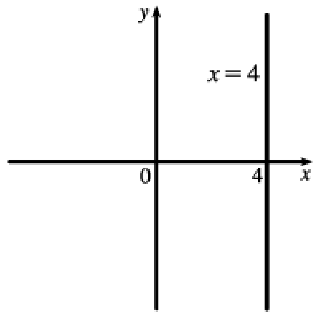
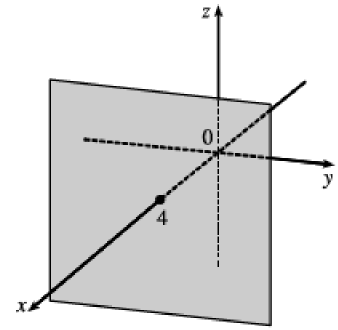
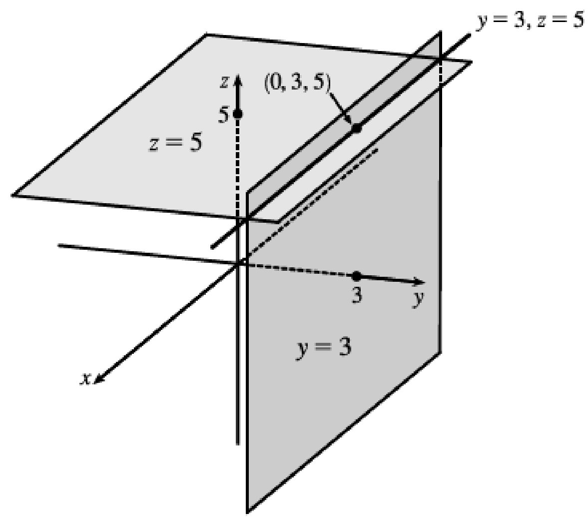
6. (a) In $\mathbb{R}^2$, the equation $x = 4$ represents a line parallel to the y-axis. In $\mathbb{R}^3$, the equation $x = 4$ represents the set $\{(x, y, z) \mid x = 4\}$, the set of all points whose x-coordinate is 4. This is the vertical plane that is parallel to the yz-plane and 4 units in front of it.





(b) In $\mathbb{R}^3$, the equation $y = 3$ represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation $z = 5$ represents a horizontal plane parallel to the xy-plane and 5 units above it. The pair of equations $y = 3$, $z = 5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y = 3$, $z = 5$. This line can also be described as the set $\{(x, 3, 5) \mid x \in \mathbb{R}\}$, which is the set of all points in $\mathbb{R}^3$ whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point $(0, 3, 5)$.



7. We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|PQ| = \sqrt{(7-3)^2 + [0-(-2)]^2 + [1-(-3)]^2} = \sqrt{16+4+16} = 6$$
$$|QR| = \sqrt{(1-7)^2 + (2-0)^2 + (1-1)^2} = \sqrt{36+4+0} = \sqrt{40} = 2\sqrt{10}$$
$$|RP| = \sqrt{(3-1)^2 + (-2-2)^2 + (-3-1)^2} = \sqrt{4+16+16} = 6$$

The longest side is QR, but the Pythagorean Theorem is not satisfied: $|PQ|^2 + |RP|^2 \neq |QR|^2$. Thus PQR is not a right triangle. PQR is isosceles, as two sides have the same length.

9. (a) First we find the distances between points:

$$|AB| = \sqrt{(3-2)^2 + (7-4)^2 + (-2-2)^2} = \sqrt{26}$$
$$|BC| = \sqrt{(1-3)^2 + (3-7)^2 + [3-(-2)]^2} = \sqrt{45} = 3\sqrt{5}$$
$$|AC| = \sqrt{(1-2)^2 + (3-4)^2 + (3-2)^2} = \sqrt{3}$$

In order for the points to lie on a straight line, the sum of the two shortest distances must be equal to the longest distance. Since $\sqrt{26} + \sqrt{3} \neq 3\sqrt{5}$, the three points do not lie on a straight line.

(b) First we find the distances between points:

$$|DE| = \sqrt{(1-0)^2 + [-2-(-5)]^2 + (4-5)^2} = \sqrt{11}$$
$$|EF| = \sqrt{(3-1)^2 + [4-(-2)]^2 + (2-4)^2} = \sqrt{44} = 2\sqrt{11}$$
$$|DF| = \sqrt{(3-0)^2 + [4-(-5)]^2 + (2-5)^2} = \sqrt{99} = 3\sqrt{11}$$

Since $|DE| + |EF| = |DF|$, the three points lie on a straight line.

12.

An equation of the sphere with center $(2, -6, 4)$ and radius 5 is $(x-2)^2 + [y-(-6)]^2 + (z-4)^2 = 5^2$ or $(x-2)^2 + (y+6)^2 + (z-4)^2 = 25$. The intersection of this sphere with the xy-plane is the set of points on the sphere whose z-coordinate is 0. Putting $z = 0$ into the equation, we have $(x-2)^2 + (y+6)^2 = 9, z = 0$ which represents a circle in the xy-plane with center $(2, -6, 0)$ and radius 3. To find the intersection with the xz-plane, we set $y = 0$: $(x-2)^2 + (z-4)^2 = -11$. Since no points satisfy this equation, the sphere does not intersect the xz-plane. (Also note that the distance from the center of the sphere to the xz-plane is greater than the radius of the sphere.) To find the intersection with the yz-plane, we set $x = 0$: $(y+6)^2 + (z-4)^2 = 21, x = 0$, a circle in the yz-plane with center $(0, -6, 4)$ and radius $\sqrt{21}$.

15.

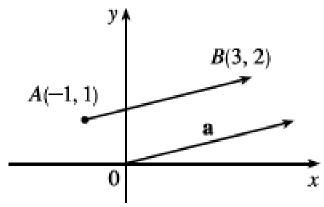
Completing squares in the equation $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$ gives

$(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 + 8z + 16) = 15 + 1 + 4 + 16 \quad \Rightarrow \quad (x-1)^2 + (y-2)^2 + (z+4)^2 = 36$, which we recognize as an equation of a sphere with center $(1, 2, -4)$ and radius 6.

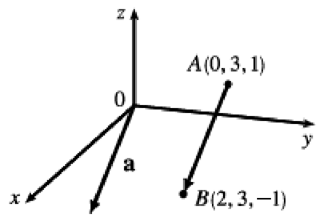
35. This describes all points whose x-coordinate is between 0 and 5, that is, $0 < x < 5$.

36. For any point on or above the disk in the xy-plane with center the origin and radius 2 we have $x^2 + y^2 \leq 4$. Also each point lies on or between the planes $z = 0$ and $z = 8$, so the region is described by $x^2 + y^2 \leq 4$, $0 \leq z \leq 8$.

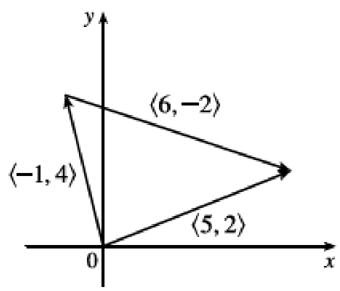
9. $\mathbf{a} = \langle 3 - (-1), 2 - 1 \rangle = \langle 4, 1 \rangle$



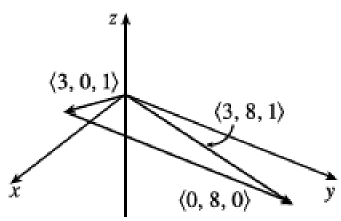
13. $\mathbf{a} = \langle 2 - 0, 3 - 3, -1 - 1 \rangle = \langle 2, 0, -2 \rangle$



15. $\langle -1, 4 \rangle + \langle 6, -2 \rangle = \langle -1 + 6, 4 + (-2) \rangle = \langle 5, 2 \rangle$



17. $\langle 3, 0, 1 \rangle + \langle 0, 8, 0 \rangle = \langle 3 + 0, 0 + 8, 1 + 0 \rangle$
$= \langle 3, 8, 1 \rangle$



25. The vector $8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ has length $|8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9$, so by Equation 4 the unit vector with the same direction is $\frac{1}{9}(8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) = \frac{8}{9}\,\mathbf{i} - \frac{1}{9}\,\mathbf{j} + \frac{4}{9}\,\mathbf{k}$.

26. $|\langle -2, 4, 2 \rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$, so a unit vector in the direction of $\langle -2, 4, 2 \rangle$ is $\mathbf{u} = \dfrac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle$.

A vector in the same direction but with length 6 is $6\mathbf{u} = 6 \cdot \dfrac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle = \left\langle -\dfrac{6}{\sqrt{6}}, \dfrac{12}{\sqrt{6}}, \dfrac{6}{\sqrt{6}} \right\rangle$ or $\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle$.

1. (a) $\mathbf{a} \cdot \mathbf{b}$ is a scalar, and the dot product is defined only for vectors, so $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ has no meaning.

(b) $(\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$ is a scalar multiple of a vector, so it does have meaning.

(c) Both $|\mathbf{a}|$ and $\mathbf{b} \cdot \mathbf{c}$ are scalars, so $|\mathbf{a}|\,(\mathbf{b} \cdot \mathbf{c})$ is an ordinary product of real numbers, and has meaning.

(d) Both $\mathbf{a}$ and $\mathbf{b} + \mathbf{c}$ are vectors, so the dot product $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ has meaning.

(e) $\mathbf{a} \cdot \mathbf{b}$ is a scalar, but $\mathbf{c}$ is a vector, and so the two quantities cannot be added and $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$ has no meaning.

(f) $|\mathbf{a}|$ is a scalar, and the dot product is defined only for vectors, so $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$ has no meaning.

2. $\mathbf{a} \cdot \mathbf{b} = \langle -2, 3 \rangle \cdot \langle 0.7, 1.2 \rangle = (-2)(0.7) + (3)(1.2) = 2.2$

5. $\mathbf{a} \cdot \mathbf{b} = \left\langle 4, 1, \frac{1}{4} \right\rangle \cdot \langle 6, -3, -8 \rangle = (4)(6) + (1)(-3) + \left(\frac{1}{4}\right)(-8) = 19$

6. $\mathbf{a} \cdot \mathbf{b} = \langle p, -p, 2p \rangle \cdot \langle 2q, q, -q \rangle = (p)(2q) + (-p)(q) + (2p)(-q) = 2pq - pq - 2pq = -pq$

15. $|\mathbf{a}| = \sqrt{4^2 + 3^2} = 5$, $|\mathbf{b}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$, and $\mathbf{a} \cdot \mathbf{b} = (4)(2) + (3)(-1) = 5$. From Corollary 6, we have

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{5}{5 \cdot \sqrt{5}} = \dfrac{1}{\sqrt{5}}$. So the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63^\circ$.

18. $|\mathbf{a}| = \sqrt{4^2 + 0^2 + 2^2} = \sqrt{20}$, $|\mathbf{b}| = \sqrt{2^2 + (-1)^2 + 0^2} = \sqrt{5}$, and $\mathbf{a} \cdot \mathbf{b} = (4)(2) + (0)(-1) + (2)(0) = 8$.

Then $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{8}{\sqrt{20} \cdot \sqrt{5}} = \dfrac{4}{5}$ and $\theta = \cos^{-1}\left(\frac{4}{5}\right) \approx 37^\circ$.

25.
$\overrightarrow{QP} = \langle -1, -3, 2 \rangle$, $\overrightarrow{QR} = \langle 4, -2, -1 \rangle$, and $\overrightarrow{QP} \cdot \overrightarrow{QR} = -4 + 6 - 2 = 0$. Thus $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are orthogonal, so the angle of the triangle at vertex Q is a right angle.

2. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 2 & 4 & 6 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 4 & 6 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -1 \\ 2 & 6 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} \mathbf{k}$

$= [6 - (-4)]\,\mathbf{i} - [6 - (-2)]\,\mathbf{j} + (4 - 2)\,\mathbf{k} = 10\,\mathbf{i} - 8\,\mathbf{j} + 2\,\mathbf{k}$

Now $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \langle 10, -8, 2 \rangle \cdot \langle 1, 1, -1 \rangle = 10 - 8 - 2 = 0$ and $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \langle 10, -8, 2 \rangle \cdot \langle 2, 4, 6 \rangle = 20 - 32 + 12 = 0$, so $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

3. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ -1 & 0 & 5 \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ 0 & 5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & -2 \\ -1 & 5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix}\mathbf{k}$

$= (15-0)\,\mathbf{i} - (5-2)\,\mathbf{j} + [0-(-3)]\,\mathbf{k} = 15\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k}$

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = (15\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k})\cdot(\mathbf{i} + 3\,\mathbf{j} - 2\,\mathbf{k}) = 15 - 9 - 6 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = (15\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k})\cdot(-\mathbf{i} + 5\,\mathbf{k}) = -15 + 0 + 15 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{b}$.

5. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 1 & \frac{1}{2} \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & -1 \\ \frac{1}{2} & 1 \end{vmatrix}\mathbf{k}$

$= \left[-\frac{1}{2} - (-1)\right]\mathbf{i} - \left[\frac{1}{2} - (-\frac{1}{2})\right]\mathbf{j} + \left[1 - (-\frac{1}{2})\right]\,\mathbf{k} = \frac{1}{2}\,\mathbf{i} - \mathbf{j} + \frac{3}{2}\,\mathbf{k}$

Now $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = \left(\frac{1}{2}\,\mathbf{i} - \mathbf{j} + \frac{3}{2}\,\mathbf{k}\right)\cdot(\mathbf{i} - \mathbf{j} - \mathbf{k}) = \frac{1}{2} + 1 - \frac{3}{2} = 0$ and

$(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = \left(\frac{1}{2}\,\mathbf{i} - \mathbf{j} + \frac{3}{2}\,\mathbf{k}\right)\cdot\left(\frac{1}{2}\,\mathbf{i} + \mathbf{j} + \frac{1}{2}\,\mathbf{k}\right) = \frac{1}{4} - 1 + \frac{3}{4} = 0$, so $\mathbf{a}\times\mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

13. (a) Since $\mathbf{b}\times\mathbf{c}$ is a vector, the dot product $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ is meaningful and is a scalar.

(b) $\mathbf{b}\cdot\mathbf{c}$ is a scalar, so $\mathbf{a}\times(\mathbf{b}\cdot\mathbf{c})$ is meaningless, as the cross product is defined only for two *vectors*.

(c) Since $\mathbf{b}\times\mathbf{c}$ is a vector, the cross product $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ is meaningful and results in another vector.

(d) $\mathbf{b}\cdot\mathbf{c}$ is a scalar, so the dot product $\mathbf{a}\cdot(\mathbf{b}\cdot\mathbf{c})$ is meaningless, as the dot product is defined only for two vectors.

(e) Since $(\mathbf{a}\cdot\mathbf{b})$ and $(\mathbf{c}\cdot\mathbf{d})$ are both scalars, the cross product $(\mathbf{a}\cdot\mathbf{b})\times(\mathbf{c}\cdot\mathbf{d})$ is meaningless.

(f) $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$ are both vectors, so the dot product $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d})$ is meaningful and is a scalar.

28. The parallelogram is determined by the vectors $\overrightarrow{KL} = \langle 0, 1, 3\rangle$ and $\overrightarrow{KN} = \langle 2, 5, 0\rangle$, so the area of parallelogram $KLMN$ is

$$\left|\overrightarrow{KL}\times\overrightarrow{KN}\right| = \left\|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 2 & 5 & 0 \end{matrix}\right\| = |(-15)\ \mathbf{i} - (-6)\,\mathbf{j} + (-2)\,\mathbf{k}| = |-15\,\mathbf{i} + 6\,\mathbf{j} - 2\,\mathbf{k}| = \sqrt{265} \approx 16.28$$

29.

(a) Because the plane through P, Q, and R contains the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, a vector orthogonal to both of these vectors (such as their cross product) is also orthogonal to the plane. Here $\overrightarrow{PQ} = \langle -3, 1, 2 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 4 \rangle$, so

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle (1)(4) - (2)(2), (2)(3) - (-3)(4), (-3)(2) - (1)(3) \rangle = \langle 0, 18, -9 \rangle$$

Therefore, $\langle 0, 18, -9 \rangle$ (or any nonzero scalar multiple thereof, such as $\langle 0, 2, -1 \rangle$) is orthogonal to the plane through P, Q, and R.

(b) Note that the area of the triangle determined by P, Q, and R is equal to half of the area of the parallelogram determined by the three points. From part (a), the area of the parallelogram is $\left| \overrightarrow{PQ} \times \overrightarrow{PR} \right| = |\langle 0, 18, -9 \rangle| = \sqrt{0 + 324 + 81} = \sqrt{405} = 9\sqrt{5}$, so the area of the triangle is $\frac{1}{2} \cdot 9\sqrt{5} = \frac{9}{2}\sqrt{5}$.