Calc III: Workshop 7, Fall 2017

Problem 1.

(a) Find the appropriate description of the triangular region R in polar coordinates, where R has vertices $(0,0)$, $(1,0)$ and $(1,1)$. (There will be variable limits somewhere!)

(b) Compute the integral

$$\iint_R \frac{y}{\sqrt{x^2+y^2}}\,dA$$

using polar coordinates. (You can check your answer by computing the integral in cartesian coordinates as well.)

Problem 2. Find the volume of the solid region bounded by the paraboloid $z = x^2 + y^2$ and the cone $z^2 = 4(x^2 + y^2)$.

Problem 3. Compute the volume of a sphere of radius R using *cylindrical* (instead of spherical) coordinates (r, θ, z).

Problem 4. Find the volume inside the elliptic cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $0 \le z \le 1$, by using the change of variables $x = ar\cos\theta$, $y = br\sin\theta$, $z = z$.

Problem 5. Find the volume inside the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, using a change of variables $x = ax'$, $y = by'$, $z = cz'$. (You may use the fact, proved in class, that the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.)