Functional Analysis, Fall 2016

- **Instructor**: Professor Chris Kottke
- **Webpage**: `http://ckottke.ncf.edu/functional/` and/or Canvas
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- **Office Hours**: TBD
- **Lectures**: MWF 9:00-9:50, HNS 106
- **Textbook**: Introductory Functional Analysis with Applications, by Erwin Kreyszig.

Course Description: This will be an introduction to various topics in Functional Analysis, focusing on Banach and Hilbert spaces and associated linear operators. We will cover some of the classical theorems (and applications thereof) in the subject, including the Hahn-Banach, Alaoglu, Open Mapping, Closed Graph, and Banach Fixed Point theorems, as well as varoius aspects of the spectral theory of self-adjoint operators.

Reading Assignments: A reading assignment for each class will be posted on the course webpage and in the Canvas course prior to each lecture. This reading should be completed prior to the lecture, as I aim to expound on, rather than merely transcribe, the material in the book.

Homework: Problem sets will be assigned and graded on a roughly biweekly basis. These are perhaps the most important component of the course, and should be started early.

Midterm: There will be one mid-term exam at the end of Module I, with precise date and format (i.e., in-class vs. take-home) to be determined.

Final projects: In lieu of a final exam, you will complete a final project, which will be an expository paper on a topic or theorem which extends or goes beyond the material covered in lecture, of between 10 and 20 pages in length. This will give you an excellent opportunity to develop your skills in mathematical writing.

Assessment: Your course performance (Sat/Unsat) will be evaluated based on homework, exam and final projects, with equal weight given to each. Class participation will be reflected in the narrative evaluation.

Policies:

- Collaboration on homework is expected and in fact encouraged. However any work you submit should be your own; verbatim copying of solutions will be considered as a violation of academic honesty policies.
- A student claiming a need for special accommodations because of a disability must work with the Counseling and Wellness Center, which will establish the need for specific accommodations and communicate them to the instructor.
- No student shall be compelled to attend class or sit for an examination at a day or time when he or she would normally be engaged in a religious observance or on a day or time prohibited by his or her religious belief. Students are expected to notify their instructors if they intend to be absent for a class or announced examination, in accordance with this policy, prior to the scheduled meeting.

Topics: Below is a tentative list topics, which may be subject to revision as the course is underway.

- Metric spaces, completeness and completion.
- Normed vector spaces of finite and infinite dimension, subspaces, compactness.
- Banach and Hilbert spaces.
- Linear operators and functionals, boundedness, duality.
- A *brief* treatment of Lebesgue measure theory and L^p spaces.
- Hahn-Banach Theorem.
- Weak and weak* topologies, Banach-Alaoglu Theorem.
- Open Mapping and Closed Graph Theorems.
- Banach Fixed Point theorem and applications to ODE, Inverse Function Theorem.
- Compact and Fredholm Operators.
- Spectral Theorem for compact self-adjoint operators, functional calculus.
- (If time) Spectral Theorem for bounded self-adjoint operators via Gelfand theory and C^*-algebras.