

Calculus III Workshop questions: 9/28/16

Problem 1 (15.2, #31). Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y - 2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$.

Problem 2 (15.2, #35). Find the average value of $f(x, y) = x^2y$ over the rectangle with vertices $(-1, 0)$, $(-1, 5)$, $(1, 5)$, and $(1, 0)$.

Problem 3 (15.2, #37, 38). Use symmetry to evaluate the double integrals

$$\iint_R \frac{xy}{1+x^4} dx dy, \quad R = \{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 1\},$$

and

$$\iint_R (1 + x^2 \sin y + y^2 \sin x) dA, \quad R = [-\pi, \pi] \times [-\pi, \pi].$$

Problem 4 (15.3, #11, 12). Draw a region that is

- (a) type I but not type II
- (b) type II but not type I
- (c) both type I and type II
- (d) neither type I nor type II

Problem 5 (15.3, #20). Evaluate $\iint_D xy^2 dA$, where D is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$.

Problem 6 (15.3, #36). Find the volume of the solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$ and $z = 2 + y$, by subtracting two volumes.

Problem 7 (15.3, #44, 46). Sketch the region of integration and change the order of integration in the following integrals:

$$\int_0^2 \int_{x^2}^4 f(x, y) dy dx,$$

and

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} f(x, y) dx dy.$$