Calculus III Workshop questions: 10/26/16

Problem 1 (15.9, #23)**.** Evaluate $\iiint_E (x^2 + y^2)\, dV$ where E lies in between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.

Problem 2 (15.9, #30)**.** Find the volume of the solid that lies within the sphere $x^2+y^2+z^2 = 4$, above the x-y plane, and below the cone $z = \sqrt{x^2 + y^2}$.

Problem 3 (15.9, #36)**.** Use either spherical or cylindrical coordinates, as appropriate, to find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$.

Problem 4 (15.9, #39, #40)**.** Evaluate the integrals by changing to spherical coordinates:

(a) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx$

(b) $\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 z + y^2 z + z^3)\, dz\, dx\, dy$

Problem 5 (16.1, #29–32)**.** Match the functions f with the plots of their gradient vector fields labeled I–IV. Give reasons for your answers.

(29) $f(x, y) = x^2 + y^2$ (30) $f(x, y) = x(x + y)$

(31) $f(x, y) = (x + y)^2$ (32) $f(x, y) = \sin \sqrt{x^2 + y^2}$