# Calc III: Workshop 10, Fall 2017

**Problem 1.** Let $R = \{(x, y) : x^2 + y^2 \leq a^2, y \geq 0\}$ be the upper half disk of radius $a$, with mass density $\delta(x, y) = x^2 + y^2$.
(a) Compute the mass of $R$.
(b) Compute the center of mass $(\overline{x}, \overline{y})$ of $R$.

**Problem 2.** Use Green's Theorem to compute the line integral $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y) = (x^2+y^2)\mathbf{i}+(2xy+x)\mathbf{j}$ and $C$ is the closed triangular path consisting of straight line segments from $(0, 0)$ to $(1, 1)$, then to $(1, 0)$ and back to $(0, 0)$.

**Problem 3.** Compute the volume of the solid region between the surfaces $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$

**Problem 4.** Set up, but do not evaluate, the triple integral $\iiint_E xy\, dV$, where $E$ is the region bounded below by the cone $z = \sqrt{3(x^2 + y^2)}$ and above by the sphere $x^2 + y^2 + z^2 = 4$, using:
(a) Cartesian coordinates $(x, y, z)$,
(b) Cylindrical coordinates $(z, r, \theta)$, and
(c) Spherical coordinates $(\rho, \varphi, \theta)$.

**Problem 5.**
(a) Verify that the vector field $\mathbf{F}(x, y) = (2xy + ye^x)\mathbf{i} + (x^2 + e^x)\mathbf{j}$ is conservative, and find a potential function $f(x, y)$.
(b) Compute the line integral $\int_C \mathbf{F}(x, y) \cdot \mathbf{T}\, ds$, where $C$ is the curve $y = 1 + x^2$ from $(0, 1)$ to $(1, 2)$.