**67.** $u = e^{r\theta}\sin\theta \quad\Rightarrow\quad \dfrac{\partial u}{\partial\theta} = e^{r\theta}\cos\theta + \sin\theta\cdot e^{r\theta}\,(r) = e^{r\theta}\,(\cos\theta + r\sin\theta)$,

$$\frac{\partial^2 u}{\partial r\,\partial\theta} = e^{r\theta}\,(\sin\theta) + (\cos\theta + r\sin\theta)\,e^{r\theta}\,(\theta) = e^{r\theta}\,(\sin\theta + \theta\cos\theta + r\theta\sin\theta),$$

$$\frac{\partial^3 u}{\partial r^2\,\partial\theta} = e^{r\theta}\,(\theta\sin\theta) + (\sin\theta + \theta\cos\theta + r\theta\sin\theta)\cdot e^{r\theta}\;(\theta) = \theta e^{r\theta}\,(2\sin\theta + \theta\cos\theta + r\theta\sin\theta).$$

**76.** (a) $u = x^2 + y^2 \quad\Rightarrow\quad u_x = 2x,\ u_{xx} = 2;\quad u_y = 2y,\ u_{yy} = 2$. Thus $u_{xx} + u_{yy} \neq 0$ and $u = x^2 + y^2$ does not satisfy Laplace's Equation.

(b) $u = x^2 - y^2$ is a solution: $u_{xx} = 2,\ u_{yy} = -2$ so $u_{xx} + u_{yy} = 0$.

(c) $u = x^3 + 3xy^2$ is not a solution: $u_x = 3x^2 + 3y^2,\ u_{xx} = 6x;\quad u_y = 6xy,\ u_{yy} = 6x$.

(d) $u = \ln\sqrt{x^2 + y^2}$ is a solution: $u_x = \dfrac{1}{\sqrt{x^2 + y^2}}\left(\dfrac{1}{2}\right)(x^2 + y^2)^{-1/2}(2x) = \dfrac{x}{x^2 + y^2}$,

$u_{xx} = \dfrac{(x^2 + y^2) - x(2x)}{(x^2 + y^2)^2} = \dfrac{y^2 - x^2}{(x^2 + y^2)^2}$. By symmetry, $u_{yy} = \dfrac{x^2 - y^2}{(x^2 + y^2)^2}$, so $u_{xx} + u_{yy} = 0$.

(e) $u = \sin x\,\cosh y + \cos x\,\sinh y$ is a solution: $u_x = \cos x\,\cosh y - \sin x\,\sinh y$, $u_{xx} = -\sin x\,\cosh y - \cos x\,\sinh y$, and $u_y = \sin x\,\sinh y + \cos x\,\cosh y$, $u_{yy} = \sin x\,\cosh y + \cos x\,\sinh y$.

(f) $u = e^{-x}\cos y - e^{-y}\cos x$ is a solution: $u_x = -e^{-x}\cos y + e^{-y}\sin x,\quad u_{xx} = e^{-x}\cos y + e^{-y}\cos x$, and $u_y = -e^{-x}\sin y + e^{-y}\cos x,\quad u_{yy} = -e^{-x}\cos y - e^{-y}\cos x$.

**93.** $f_x(x,y) = x + 4y \quad\Rightarrow\quad f_{xy}(x,y) = 4$ and $f_y(x,y) = 3x - y \quad\Rightarrow\quad f_{yx}(x,y) = 3$. Since $f_{xy}$ and $f_{yx}$ are continuous everywhere but $f_{xy}(x,y) \neq f_{yx}(x,y)$, Clairaut's Theorem implies that such a function $f(x,y)$ does not exist.

**1.** $z = f(x,y) = 3y^2 - 2x^2 + x \quad\Rightarrow\quad f_x(x,y) = -4x + 1,\ f_y(x,y) = 6y,$ so $f_x(2,-1) = -7,\ f_y(2,-1) = -6$.

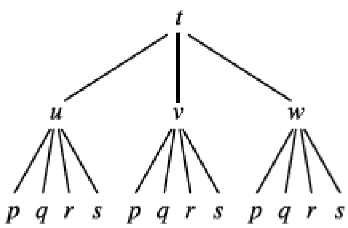
By Equation 2, an equation of the tangent plane is $z - (-3) = f_x(2,-1)(x-2) + f_y(2,-1)[y - (-1)] \quad\Rightarrow$

$z + 3 = -7(x-2) - 6(y+1)$ or $z = -7x - 6y + 5$.

**34.** Let $V$ be the volume. Then $V = \pi r^2 h$ and $\Delta V \approx dV = 2\pi rh\,dr + \pi r^2\,dh$ is an estimate of the amount of metal. With $dr = 0.05$ and $dh = 0.2$ we get $dV = 2\pi(2)(10)(0.05) + \pi(2)^2(0.2) = 2.80\pi \approx 8.8\ \text{cm}^3$.

**42.**
$\mathbf{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2 \rangle \quad \Rightarrow \quad \mathbf{r}_1'(t) = \langle 3, -2t, -4+2t \rangle, \quad \mathbf{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1 \rangle \quad \Rightarrow$
$\mathbf{r}_2'(u) = \langle 2u, 6u^2, 2 \rangle$. Both curves pass through $P$ since $\mathbf{r}_1(0) = \mathbf{r}_2(1) = \langle 2, 1, 3 \rangle$, so the tangent vectors $\mathbf{r}_1'(0) = \langle 3, 0, -4 \rangle$ and $\mathbf{r}_2'(1) = \langle 2, 6, 2 \rangle$ are both parallel to the tangent plane to $S$ at $P$. A normal vector for the tangent plane is $\mathbf{r}_1'(0) \times \mathbf{r}_2'(1) = \langle 3, 0, -4 \rangle \times \langle 2, 6, 2 \rangle = \langle 24, -14, 18 \rangle$, so an equation of the tangent plane is $24(x-2) - 14(y-1) + 18(z-3) = 0$ or $12x - 7y + 9z = 44$.

**20.**




$t = f(u, v, w), \quad u = u(p, q, r, s), \quad v = v(p, q, r, s), \quad w = w(p, q, r, s) \quad \Rightarrow$

$$\frac{\partial t}{\partial p} = \frac{\partial t}{\partial u}\frac{\partial u}{\partial p} + \frac{\partial t}{\partial v}\frac{\partial v}{\partial p} + \frac{\partial t}{\partial w}\frac{\partial w}{\partial p}, \quad \frac{\partial t}{\partial q} = \frac{\partial t}{\partial u}\frac{\partial u}{\partial q} + \frac{\partial t}{\partial v}\frac{\partial v}{\partial q} + \frac{\partial t}{\partial w}\frac{\partial w}{\partial q},$$

$$\frac{\partial t}{\partial r} = \frac{\partial t}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial t}{\partial v}\frac{\partial v}{\partial r} + \frac{\partial t}{\partial w}\frac{\partial w}{\partial r}, \quad \frac{\partial t}{\partial s} = \frac{\partial t}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial t}{\partial v}\frac{\partial v}{\partial s} + \frac{\partial t}{\partial w}\frac{\partial w}{\partial s}$$

**39.** (a) $V = \ell w h$, so by the Chain Rule,

$$\frac{dV}{dt} = \frac{\partial V}{\partial \ell}\frac{d\ell}{dt} + \frac{\partial V}{\partial w}\frac{dw}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} = wh\,\frac{d\ell}{dt} + \ell h\,\frac{dw}{dt} + \ell w\,\frac{dh}{dt} = 2 \cdot 2 \cdot 2 + 1 \cdot 2 \cdot 2 + 1 \cdot 2 \cdot (-3) = 6 \text{ m}^3/\text{s}.$$

(b) $S = 2(\ell w + \ell h + wh)$, so by the Chain Rule,

$$\begin{aligned}\frac{dS}{dt} &= \frac{\partial S}{\partial \ell}\frac{d\ell}{dt} + \frac{\partial S}{\partial w}\frac{dw}{dt} + \frac{\partial S}{\partial h}\frac{dh}{dt} = 2(w+h)\,\frac{d\ell}{dt} + 2(\ell+h)\,\frac{dw}{dt} + 2(\ell+w)\,\frac{dh}{dt} \\ &= 2(2+2)2 + 2(1+2)2 + 2(1+2)(-3) = 10 \text{ m}^2/\text{s}\end{aligned}$$

(c) $L^2 = \ell^2 + w^2 + h^2 \quad \Rightarrow \quad 2L\,\frac{dL}{dt} = 2\ell\,\frac{d\ell}{dt} + 2w\,\frac{dw}{dt} + 2h\,\frac{dh}{dt} = 2(1)(2) + 2(2)(2) + 2(2)(-3) = 0 \quad \Rightarrow$

$dL/dt = 0$ m/s.