9. $\mathbf{v} = \langle 3-(-8), -2-1, 4-4\rangle = \langle 11, -3, 0\rangle$, and letting $P_0 = (-8, 1, 4)$, parametric equations are $x = -8 + 11t$, $y = 1 - 3t$, $z = 4 + 0t = 4$, while symmetric equations are $\dfrac{x+8}{11} = \dfrac{y-1}{-3}$, $z = 4$. Notice here that the direction number $c = 0$, so rather than writing $\dfrac{z-4}{0}$ in the symmetric equation we must write the equation $z = 4$ separately.

10. $\mathbf{v} = (\mathbf{i}+\mathbf{j}) \times (\mathbf{j}+\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ is the direction of the line perpendicular to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$.

With $P_0 = (2, 1, 0)$, parametric equations are $x = 2 + t$, $y = 1 - t$, $z = t$ and symmetric equations are $x - 2 = \dfrac{y-1}{-1} = z$ or $x - 2 = 1 - y = z$.

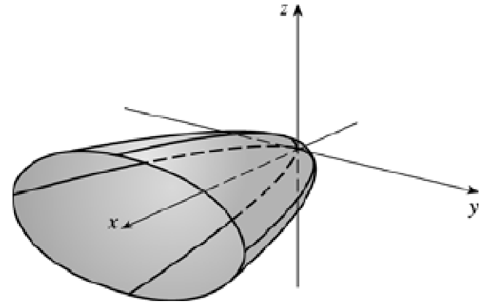
13. Direction vectors of the lines are $\mathbf{v}_1 = \langle -2-(-4), 0-(-6), -3-1\rangle = \langle 2, 6, -4\rangle$ and $\mathbf{v}_2 = \langle 5-10, 3-18, 14-4\rangle = \langle -5, -15, 10\rangle$, and since $\mathbf{v}_2 = -\frac{5}{2}\mathbf{v}_1$, the direction vectors and thus the lines are parallel.

19.
Since the direction vectors $\langle 2, -1, 3\rangle$ and $\langle 4, -2, 5\rangle$ are not scalar multiples of each other, the lines aren't parallel. For the lines to intersect, we must be able to find one value of t and one value of s that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations: $3 + 2t = 1 + 4s$, $4 - t = 3 - 2s$, $1 + 3t = 4 + 5s$. Solving the last two equations we get $t = 1$, $s = 0$ and checking, we see that these values don't satisfy the first equation. Thus the lines aren't parallel and don't intersect, so they must be skew lines.

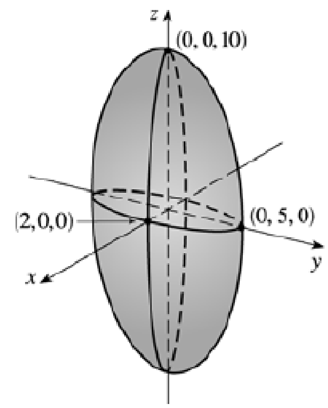
23. Since the plane is perpendicular to the vector $\langle 1, -2, 5\rangle$, we can take $\langle 1, -2, 5\rangle$ as a normal vector to the plane. $(0, 0, 0)$ is a point on the plane, so setting $a = 1$, $b = -2$, $c = 5$ and $x_0 = 0$, $y_0 = 0$, $z_0 = 0$ in Equation 7 gives $1(x-0) + (-2)(y-0) + 5(z-0) = 0$ or $x - 2y + 5z = 0$ as an equation of the plane.

33. Here the vectors $\mathbf{a} = \langle 8-3, 2-(-1), 4-2\rangle = \langle 5, 3, 2\rangle$ and $\mathbf{b} = \langle -1-3, -2-(-1), -3-2\rangle = \langle -4, -1, -5\rangle$ lie in the plane, so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -15+2, -8+25, -5+12\rangle = \langle -13, 17, 7\rangle$ and an equation of the plane is $-13(x-3) + 17[y-(-1)] + 7(z-2) = 0$ or $-13x + 17y + 7z = -42$.

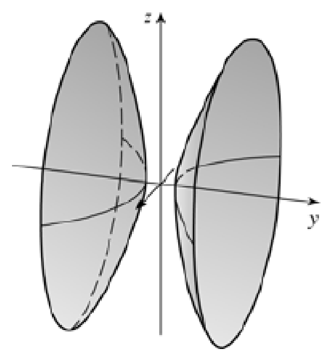
11. For $x = y^2 + 4z^2$, the traces in $x = k$ are $y^2 + 4z^2 = k$. When $k > 0$ we have a family of ellipses. When $k = 0$ we have just a point at the origin, and the trace is empty for $k < 0$. The traces in $y = k$ are $x = 4z^2 + k^2$, a family of parabolas opening in the positive x-direction. Similarly, the traces in $z = k$ are $x = y^2 + 4k^2$, a family of parabolas opening in the positive x-direction. We recognize the graph as an elliptic paraboloid with axis the x-axis and vertex the origin.



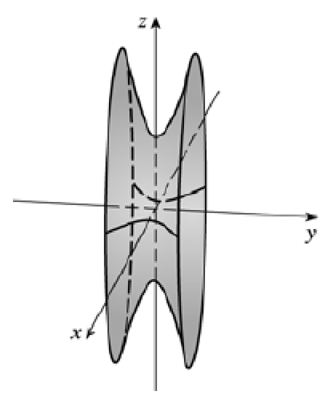
14. $25x^2 + 4y^2 + z^2 = 100$. The traces in $x = k$ are $4y^2 + z^2 = 100 - 25k^2$, a family of ellipses for $|k| < 2$. (The traces are a single point for $|k| = 2$ and are empty for $|k| > 2$.) Similarly, the traces in $y = k$ are the ellipses $25x^2 + z^2 = 100 - 4k^2$, $|k| < 5$, and the traces in $z = k$ are the ellipses $25x^2 + 4y^2 = 100 - k^2$, $|k| < 10$. The graph is an ellipsoid centered at the origin with intercepts $x = \pm 2$, $y = \pm 5$, $z = \pm 10$.



15. $-x^2 + 4y^2 - z^2 = 4$. The traces in $x = k$ are the hyperbolas $4y^2 - z^2 = 4 + k^2$. The traces in $y = k$ are $x^2 + z^2 = 4k^2 - 4$, a family of circles for $|k| > 1$, and the traces in $z = k$ are $4y^2 - x^2 = 4 + k^2$, a family of hyperbolas. Thus the surface is a hyperboloid of two sheets with axis the y-axis.



18. $4x^2 - 16y^2 + z^2 = 16$. The traces in $x = k$ are $z^2 - 16y^2 = 16 - 4k^2$, a family of hyperbolas for $|k| \neq 2$ and two intersecting lines when $|k| = 2$. (Note that the hyperbolas are oriented differently for $|k| < 2$ than for $|k| > 2$.) The traces in $y = k$ are $4x^2 + z^2 = 16(1 + k^2)$, a family of ellipses, and the traces in $z = k$ are $4x^2 - 16y^2 = 16 - k^2$, two intersecting lines when $|k| = 4$ and a family of hyperbolas when $|k| \neq 4$ (oriented differently for $|k| < 4$ than for $|k| > 4$). We recognize the graph as a hyperboloid of one sheet with axis the y-axis.



21. This is the equation of an ellipsoid: $x^2 + 4y^2 + 9z^2 = x^2 + \dfrac{y^2}{(1/2)^2} + \dfrac{z^2}{(1/3)^2} = 1$, with x-intercepts ± 1, y-intercepts $\pm\frac{1}{2}$ and z-intercepts $\pm\frac{1}{3}$. So the major axis is the x-axis and the only possible graph is VII.

22. This is the equation of an ellipsoid: $9x^2 + 4y^2 + z^2 = \dfrac{x^2}{(1/3)^2} + \dfrac{y^2}{(1/2)^2} + z^2 = 1$, with x-intercepts $\pm\frac{1}{3}$, y-intercepts $\pm\frac{1}{2}$ and z-intercepts ± 1. So the major axis is the z-axis and the only possible graph is IV.

23.
This is the equation of a hyperboloid of one sheet, with $a = b = c = 1$. Since the coefficient of y^2 is negative, the axis of the hyperboloid is the y-axis, hence the correct graph is II.

24. This is a hyperboloid of two sheets, with $a = b = c = 1$. This surface does not intersect the xz-plane at all, so the axis of the hyperboloid is the y-axis and the graph is III.

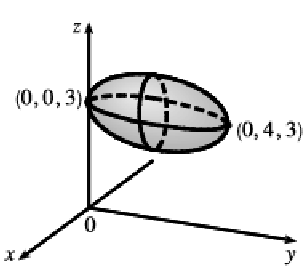
25. There are no real values of x and z that satisfy this equation for $y < 0$, so this surface does not extend to the left of the xz-plane. The surface intersects the plane $y = k > 0$ in an ellipse. Notice that y occurs to the first power whereas x and z occur to the second power. So the surface is an elliptic paraboloid with axis the y-axis. Its graph is VI.

26. This is the equation of a cone with axis the y-axis, so the graph is I.

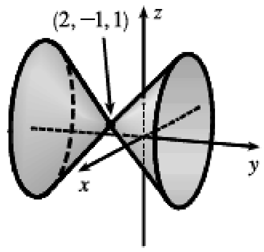
27.
This surface is a cylinder because the variable y is missing from the equation. The intersection of the surface and the xz-plane is an ellipse. So the graph is VIII.

28. This is the equation of a hyperbolic paraboloid. The trace in the xy-plane is the parabola $y = x^2$. So the correct graph is V.

33. Completing squares in y and z gives
$4x^2 + (y-2)^2 + 4(z-3)^2 = 4$ or
$x^2 + \dfrac{(y-2)^2}{4} + (z-3)^2 = 1$, an ellipsoid with
center $(0, 2, 3)$.



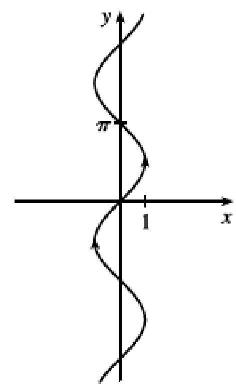
35. Completing squares in all three variables gives
$(x-2)^2 - (y+1)^2 + (z-1)^2 = 0$ or
$(y+1)^2 = (x-2)^2 + (z-1)^2$, a circular cone with
center $(2,-1,1)$ and axis the horizontal line $x = 2$,
$z = 1$.



7. The corresponding parametric equations for this curve are $x = \sin t$, $y = t$.

We can make a table of values, or we can eliminate the parameter: $t = y \quad \Rightarrow$

$x = \sin y$, with $y \in \mathbb{R}$. By comparing different values of t, we find the direction in which t increases as indicated in the graph.

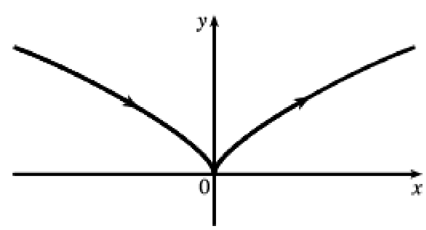


8. The corresponding parametric equations for this curve are $x = t^3$, $y = t^2$.

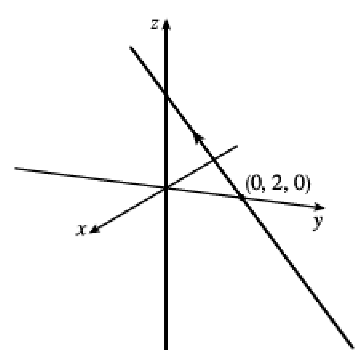
We can make a table of values, or we can eliminate the parameter:

$x = t^3 \quad \Rightarrow \quad t = \sqrt[3]{x} \quad \Rightarrow \quad y = t^2 = (\sqrt[3]{x})^2 = x^{2/3}$,

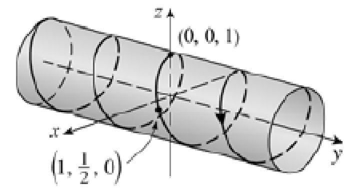
with $t \in \mathbb{R} \quad \Rightarrow \quad x \in \mathbb{R}$. By comparing different values of t, we find the direction in which t increases as indicated in the graph.



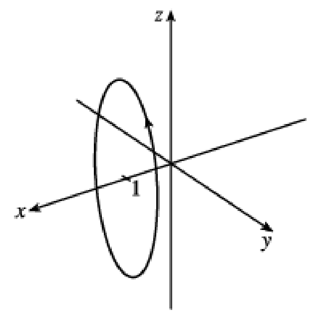
9. The corresponding parametric equations are $x = t$, $y = 2 - t$, $z = 2t$, which are parametric equations of a line through the point $(0, 2, 0)$ and with direction vector $\langle 1, -1, 2 \rangle$.



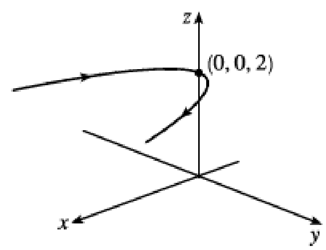
10. The corresponding parametric equations are $x = \sin \pi t$, $y = t$, $z = \cos \pi t$. Note that $x^2 + z^2 = \sin^2 \pi t + \cos^2 \pi t = 1$, so the curve lies on the circular cylinder $x^2 + z^2 = 1$. A point (x, y, z) on the curve lies directly to the left or right of the point $(x, 0, z)$ which moves clockwise (when viewed from the left) along the circle $x^2 + z^2 = 1$ in the xz-plane as t increases. Since $y = t$, the curve is a helix that spirals toward the right around the cylinder.



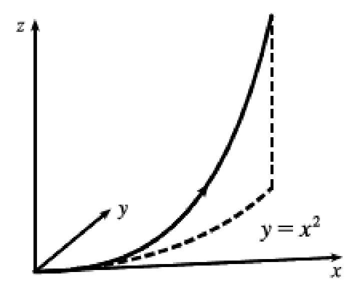
11. The corresponding parametric equations are $x = 1$, $y = \cos t$, $z = 2\sin t$. Eliminating the parameter in y and z gives $y^2 + (z/2)^2 = \cos^2 t + \sin^2 t = 1$ or $y^2 + z^2/4 = 1$. Since $x = 1$, the curve is an ellipse centered at $(1, 0, 0)$ in the plane $x = 1$.



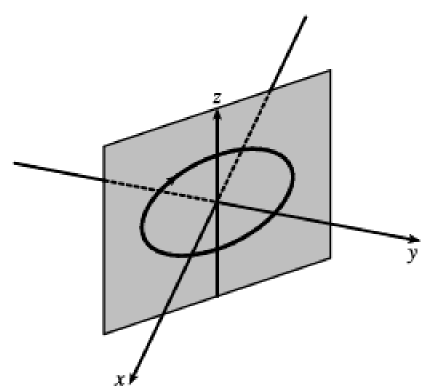
12. The parametric equations are $x = t^2$, $y = t$, $z = 2$, so we have $x = y^2$ with $z = 2$. Thus the curve is a parabola in the plane $z = 2$ with vertex $(0, 0, 2)$.



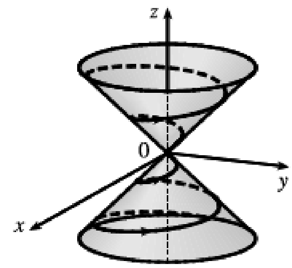
13. The parametric equations are $x = t^2$, $y = t^4$, $z = t^6$. These are positive for $t \neq 0$ and 0 when $t = 0$. So the curve lies entirely in the first octant. The projection of the graph onto the xy-plane is $y = x^2$, $y > 0$, a half parabola. Onto the xz-plane $z = x^3$, $z > 0$, a half cubic, and the yz-plane, $y^3 = z^2$.



14. If $x = \cos t$, $y = -\cos t$, $z = \sin t$, then $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$, so the curve is contained in the intersection of circular cylinders along the x- and y-axes. Furthermore, $y = -x$, so the curve is an ellipse in the plane $y = -x$, centered at the origin.

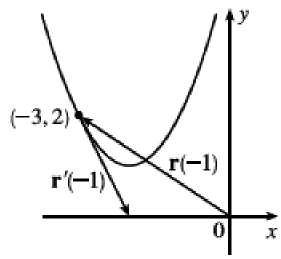


27. If $x = t\cos t,\ y = t\sin t,\ z = t$, then $x^2 + y^2 = t^2\cos^2 t + t^2\sin^2 t = t^2 = z^2$, so the curve lies on the cone $z^2 = x^2 + y^2$. Since $z = t$, the curve is a spiral on this cone.



3. Since $(x+2)^2 = t^2 = y - 1 \quad\Rightarrow$ $y = (x+2)^2 + 1$, the curve is a parabola.

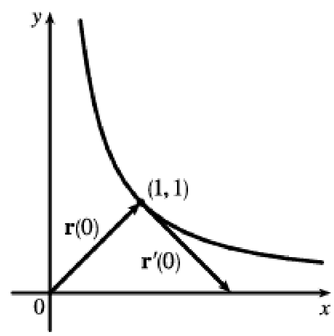
(a), (c)



(b) $\mathbf{r}'(t) = \langle 1, 2t\rangle$,
$\mathbf{r}'(-1) = \langle 1, -2\rangle$

6. Since $y = e^{-t} = \dfrac{1}{e^t} = \dfrac{1}{x}$ the curve is part of the hyperbola $y = \dfrac{1}{x}$. Note that $x > 0$, $y > 0$.

(a), (c)



(b) $\mathbf{r}'(t) = e^t\,\mathbf{i} - e^{-t}\,\mathbf{j}$,
$\mathbf{r}'(0) = \mathbf{i} - \mathbf{j}$

9. $\mathbf{r}'(t) = \left\langle \dfrac{d}{dt}[t\sin t], \dfrac{d}{dt}[t^2], \dfrac{d}{dt}[t\cos 2t] \right\rangle = \langle t\cos t + \sin t, 2t, t(-\sin 2t)\cdot 2 + \cos 2t\rangle$

$= \langle t\cos t + \sin t, 2t, \cos 2t - 2t\sin 2t\rangle$

18. $\mathbf{r}'(t) = \langle 3t^2 + 3, 2t, 3\rangle \quad\Rightarrow\quad \mathbf{r}'(1) = \langle 6, 2, 3\rangle$. Thus

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{6^2 + 2^2 + 3^2}}\langle 6, 2, 3\rangle = \tfrac{1}{7}\langle 6, 2, 3\rangle = \left\langle \tfrac{6}{7}, \tfrac{2}{7}, \tfrac{3}{7}\right\rangle.$$

19. $\mathbf{r}'(t) = -\sin t\,\mathbf{i} + 3\,\mathbf{j} + 4\cos 2t\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(0) = 3\,\mathbf{j} + 4\,\mathbf{k}$. Thus

$$\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{1}{\sqrt{0^2 + 3^2 + 4^2}}(3\,\mathbf{j} + 4\,\mathbf{k}) = \tfrac{1}{5}(3\,\mathbf{j} + 4\,\mathbf{k}) = \tfrac{3}{5}\,\mathbf{j} + \tfrac{4}{5}\,\mathbf{k}.$$

23. The vector equation for the curve is $\mathbf{r}(t) = \left\langle 1 + 2\sqrt{t}, t^3 - t, t^3 + t\right\rangle$, so $\mathbf{r}'(t) = \left\langle 1/\sqrt{t}, 3t^2 - 1, 3t^2 + 1\right\rangle$. The point $(3, 0, 2)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle 1, 2, 4\rangle$. Thus, the tangent line goes through the point $(3, 0, 2)$ and is parallel to the vector $\langle 1, 2, 4\rangle$. Parametric equations are $x = 3 + t$, $y = 2t$, $z = 2 + 4t$.

24. The vector equation for the curve is $\mathbf{r}(t) = \left\langle e^t, te^t, te^{t^2} \right\rangle$, so $\mathbf{r}'(t) = \left\langle e^t, te^t + e^t, 2t^2e^{t^2} + e^{t^2} \right\rangle$. The point $(1, \mathbf{0}, 0)$ corresponds to $t = 0$, so the tangent vector there is $\mathbf{r}'(0) = \langle 1, 1, 1 \rangle$. Thus, the tangent line is parallel to the vector $\langle 1, 1, 1 \rangle$ and includes the point $(1, 0, 0)$. Parametric equations are $x = 1 + 1 \cdot t = 1 + t$, $y = 0 + 1 \cdot t = t$, $z = 0 + 1 \cdot t = t$.