

45. Substitute the parametric equations of the line into the equation of the plane: $(3 - t) - (2 + t) + 2(5t) = 9 \Rightarrow 8t = 8 \Rightarrow t = 1$. Therefore, the point of intersection of the line and the plane is given by $x = 3 - 1 = 2$, $y = 2 + 1 = 3$, and $z = 5(1) = 5$, that is, the point $(2, 3, 5)$.

57.

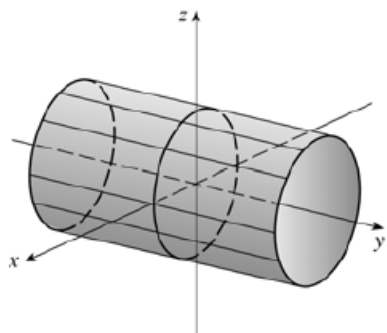
- (a) To find a point on the line of intersection, set one of the variables equal to a constant, say $z = 0$. (This will fail if the line of intersection does not cross the xy -plane; in that case, try setting x or y equal to 0.) The equations of the two planes reduce to $x + y = 1$ and $x + 2y = 1$. Solving these two equations gives $x = 1$, $y = 0$. Thus a point on the line is $(1, 0, 0)$.

A vector \mathbf{v} in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so we can take $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 2, 2 \rangle = \langle 2 - 2, 1 - 2, 2 - 1 \rangle = \langle 0, -1, 1 \rangle$. By Equations 2, parametric equations for the line are $x = 1$, $y = -t$, $z = t$.

- (b) The angle between the planes satisfies $\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| |\mathbf{n}_2|} = \frac{1 + 2 + 2}{\sqrt{3} \sqrt{9}} = \frac{5}{3\sqrt{3}}$. Therefore $\theta = \cos^{-1}\left(\frac{5}{3\sqrt{3}}\right) \approx 15.8^\circ$.

3. Since y is missing from the equation, the vertical traces

$x^2 + z^2 = 1$, $y = k$, are copies of the same circle in the plane $y = k$. Thus the surface $x^2 + z^2 = 1$ is a circular cylinder with rulings parallel to the y -axis.

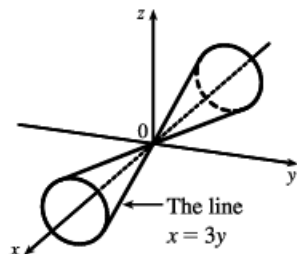


44. The surface is a right circular cone with vertex at $(0, 0, 0)$

and axis the x -axis. For $x = k \neq 0$, the trace is a circle

with center $(k, 0, 0)$ and radius $r = y = \frac{x}{3} = \frac{k}{3}$. Thus

the equation is $(x/3)^2 = y^2 + z^2$ or $x^2 = 9y^2 + 9z^2$.



27. First we parametrize the curve C of intersection. The projection of C onto the xy -plane is contained in the circle

$x^2 + y^2 = 25$, $z = 0$, so we can write $x = 5 \cos t$, $y = 5 \sin t$. C also lies on the cylinder $y^2 + z^2 = 20$, and $z \geq 0$

near the point $(3, 4, 2)$, so we can write $z = \sqrt{20 - y^2} = \sqrt{20 - 25 \sin^2 t}$. A vector equation then for C is

$$\mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, \sqrt{20 - 25 \sin^2 t} \rangle \Rightarrow \mathbf{r}'(t) = \left\langle -5 \sin t, 5 \cos t, \frac{1}{2}(20 - 25 \sin^2 t)^{-1/2}(-50 \sin t \cos t) \right\rangle.$$

The point $(3, 4, 2)$ corresponds to $t = \cos^{-1}(\frac{3}{5})$, so the tangent vector there is

$$\mathbf{r}'(\cos^{-1}(\frac{3}{5})) = \left\langle -5(\frac{4}{5}), 5(\frac{3}{5}), \frac{1}{2}(20 - 25(\frac{4}{5})^2)^{-1/2}(-50(\frac{4}{5})(\frac{3}{5})) \right\rangle = \langle -4, 3, -6 \rangle.$$

The tangent line is parallel to this vector and passes through $(3, 4, 2)$, so a vector equation for the line

is $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)\mathbf{k}$.