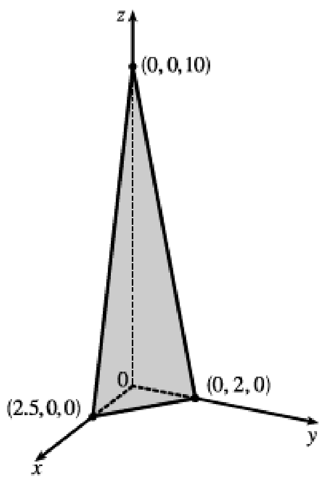
9. (a) $g(2,-1) = \cos(2+2(-1)) = \cos(0) = 1$

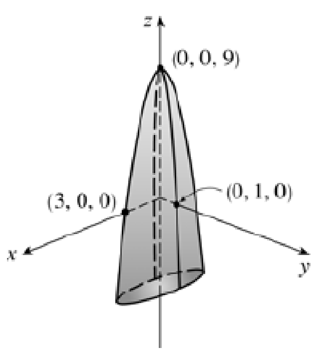
(b) $x + 2y$ is defined for all choices of values for x and y and the cosine function is defined for all input values, so the domain of g is $\mathbb{R}^2$.

(c) The range of the cosine function is $[-1, 1]$ and $x + 2y$ generates all possible input values for the cosine function, so the range of $\cos(x+2y)$ is $[-1, 1]$.

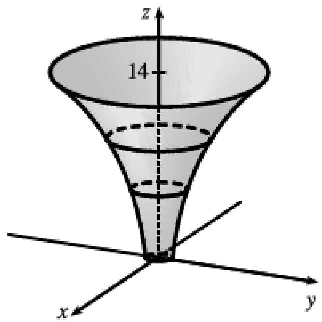
25. $z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with intercepts 2.5, 2, and 10.



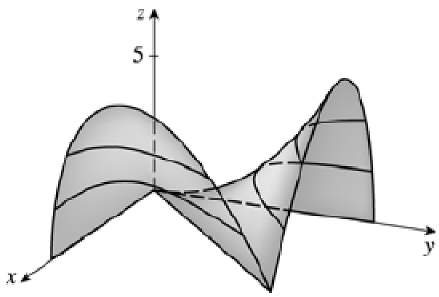
29. $z = 9 - x^2 - 9y^2$, an elliptic paraboloid opening downward with vertex at $(0, 0, 9)$.



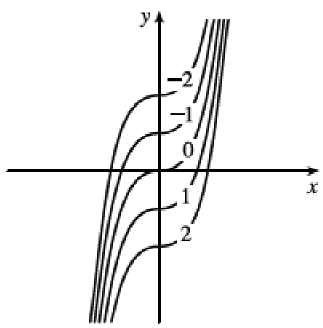
39.



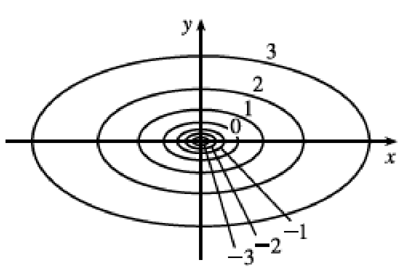
41.



44. The level curves are $x^3 - y = k$ or $y = x^3 - k$, a family of cubic curves.



46. The level curves are $\ln(x^2 + 4y^2) = k$ or $x^2 + 4y^2 = e^k$, a family of ellipses.



66. $k = x^2 + 3y^2 + 5z^2$ is a family of ellipsoids for $k > 0$ and the origin for $k = 0$.

68. Equations for the level surfaces are $x^2 - y^2 - z^2 = k$. For $k = 0$, the equation becomes $y^2 + z^2 = x^2$ and the surface is a right circular cone with vertex the origin and axis the x-axis. For $k > 0$, we have a family of hyperboloids of two sheets with axis the x-axis, and for $k < 0$, we have a family of hyperboloids of one sheet with axis the x-axis.

5. $f(x, y) = 5x^3 - x^2y^2$ is a polynomial, and hence continuous, so $\lim_{(x,y)\to(1,2)} f(x, y) = f(1, 2) = 5(1)^3 - (1)^2(2)^2 = 1$.

7. $f(x, y) = \dfrac{4 - xy}{x^2 + 3y^2}$ is a rational function and hence continuous on its domain.

$(2, 1)$ is in the domain of f, so f is continuous there and $\lim_{(x,y)\to(2,1)} f(x, y) = f(2, 1) = \dfrac{4 - (2)(1)}{(2)^2 + 3(1)^2} = \dfrac{2}{7}$.

13. $f(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$. We can see that the limit along any line through $(0, 0)$ is 0, as well as along other paths through $(0, 0)$ such as $x = y^2$ and $y = x^2$. So we suspect that the limit exists and equals 0; we use the Squeeze Theorem to prove our assertion. $0 \leq \left|\dfrac{xy}{\sqrt{x^2 + y^2}}\right| \leq |x|$ since $|y| \leq \sqrt{x^2 + y^2}$, and $|x| \to 0$ as $(x, y) \to (0, 0)$. So $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

15. $f(x, y) = y^5 - 3xy \quad \Rightarrow \quad f_x(x, y) = 0 - 3y = -3y$, $f_y(x, y) = 5y^4 - 3x$

17. $f(x, t) = e^{-t}\cos \pi x \quad \Rightarrow \quad f_x(x, t) = e^{-t}\,(-\sin \pi x)\,(\pi) = -\pi e^{-t}\sin \pi x$, $\; f_t(x, t) = e^{-t}(-1)\cos \pi x = -e^{-t}\cos \pi x$

32. $f(x, y, z) = x\sin(y - z) \quad \Rightarrow \quad f_x(x, y, z) = \sin(y - z)$, $\quad f_y(x, y, z) = x\cos(y - z)$,

$f_z(x, y, z) = x\cos(y - z)(-1) = -x\cos(y - z)$

33. $w = \ln(x + 2y + 3z) \quad \Rightarrow \quad \dfrac{\partial w}{\partial x} = \dfrac{1}{x + 2y + 3z}$, $\dfrac{\partial w}{\partial y} = \dfrac{2}{x + 2y + 3z}$, $\dfrac{\partial w}{\partial z} = \dfrac{3}{x + 2y + 3z}$

51. (a) $z = f(x) + g(y) \quad \Rightarrow \quad \dfrac{\partial z}{\partial x} = f'(x)$, $\quad \dfrac{\partial z}{\partial y} = g'(y)$

(b) $z = f(x + y)$. Let $u = x + y$. Then $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du}\,(1) = f'(u) = f'(x + y)$,

$\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du}\,(1) = f'(u) = f'(x + y)$.

52. (a) $z = f(x)g(y) \quad \Rightarrow \quad \dfrac{\partial z}{\partial x} = f'(x)g(y), \quad \dfrac{\partial z}{\partial y} = f(x)g'(y)$

(b) $z = f(xy)$. Let $u = xy$. Then $\dfrac{\partial u}{\partial x} = y$ and $\dfrac{\partial u}{\partial y} = x$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du} \cdot y = yf'(u) = yf'(xy)$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du} \cdot x = xf'(u) = xf'(xy)$.

(c) $z = f\left(\dfrac{x}{y}\right)$. Let $u = \dfrac{x}{y}$. Then $\dfrac{\partial u}{\partial x} = \dfrac{1}{y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{x}{y^2}$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = f'(u)\dfrac{1}{y} = \dfrac{f'(x/y)}{y}$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = f'(u)\left(-\dfrac{x}{y^2}\right) = -\dfrac{xf'(x/y)}{y^2}$.

60. $u = e^{xy}\sin y \quad \Rightarrow \quad u_x = ye^{xy}\sin y,\ u_{xy} = ye^{xy}\cos y + (\sin y)(y \cdot xe^{xy} + e^{xy} \cdot 1) = e^{xy}(y\cos y + xy\sin y + \sin y)$,

$u_y = e^{xy}\cos y + (\sin y)(xe^{xy}) = e^{xy}(\cos y + x\sin y)$,

$u_{yx} = e^{xy} \cdot \sin y + (\cos y + x\sin y) \cdot ye^{xy} = e^{xy}(\sin y + y\cos y + xy\sin y)$. Thus $u_{xy} = u_{yx}$.

11.

$f(x, y) = 1 + x\ln(xy - 5)$. The partial derivatives are $f_x(x, y) = x \cdot \dfrac{1}{xy-5}(y) + \ln(xy-5) \cdot 1 = \dfrac{xy}{xy-5} + \ln(xy-5)$

and $f_y(x, y) = x \cdot \dfrac{1}{xy-5}(x) = \dfrac{x^2}{xy-5}$, so $f_x(2, 3) = 6$ and $f_y(2, 3) = 4$. Both f_x and f_y are continuous functions for $xy > 5$, so by Theorem 8, f is differentiable at $(2, 3)$. By Equation 3, the linearization of f at $(2, 3)$ is given by

$L(x, y) = f(2, 3) + f_x(2, 3)(x - 2) + f_y(2, 3)(y - 3) = 1 + 6(x - 2) + 4(y - 3) = 6x + 4y - 23$.

13. $f(x, y) = \dfrac{x}{x+y}$. The partial derivatives are $f_x(x, y) = \dfrac{1(x+y) - x(1)}{(x+y)^2} = y/(x+y)^2$ and

$f_y(x, y) = x(-1)(x+y)^{-2} \cdot 1 = -x/(x+y)^2$, so $f_x(2, 1) = \frac{1}{9}$ and $f_y(2, 1) = -\frac{2}{9}$. Both f_x and f_y are continuous functions for $y \neq -x$, so f is differentiable at $(2, 1)$ by Theorem 8. The linearization of f at $(2, 1)$ is given by

$L(x, y) = f(2, 1) + f_x(2, 1)(x - 2) + f_y(2, 1)(y - 1) = \frac{2}{3} + \frac{1}{9}(x - 2) - \frac{2}{9}(y - 1) = \frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$.

19. We can estimate $f(2.2, 4.9)$ using a linear approximation of f at $(2, 5)$, given by

$f(x, y) \approx f(2, 5) + f_x(2, 5)(x - 2) + f_y(2, 5)(y - 5) = 6 + 1(x - 2) + (-1)(y - 5) = x - y + 9$. Thus

$f(2.2, 4.9) \approx 2.2 - 4.9 + 9 = 6.3$.

21. $f(x,y,z) = \sqrt{x^2+y^2+z^2} \quad \Rightarrow \quad f_x(x,y,z) = \dfrac{x}{\sqrt{x^2+y^2+z^2}},\ f_y(x,y,z) = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$, and

$f_z(x,y,z) = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$, so $f_x(3,2,6) = \frac{3}{7},\ f_y(3,2,6) = \frac{2}{7},\ f_z(3,2,6) = \frac{6}{7}$. Then the linear approximation of f at $(3,2,6)$ is given by

$$\begin{aligned} f(x,y,z) &\approx f(3,2,6) + f_x(3,2,6)(x-3) + f_y(3,2,6)(y-2) + f_z(3,2,6)(z-6) \\ &= 7 + \tfrac{3}{7}(x-3) + \tfrac{2}{7}(y-2) + \tfrac{6}{7}(z-6) = \tfrac{3}{7}x + \tfrac{2}{7}y + \tfrac{6}{7}z \end{aligned}$$

Thus $\sqrt{(3.02)^2+(1.97)^2+(5.99)^2} = f(3.02, 1.97, 5.99) \approx \frac{3}{7}(3.02) + \frac{2}{7}(1.97) + \frac{6}{7}(5.99) \approx 6.9914$.

7. $z = x^2y^3,\ x = s\cos t,\ y = s\sin t \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = 2xy^3\cos t + 3x^2y^2\sin t$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (2xy^3)(-s\sin t) + (3x^2y^2)(s\cos t) = -2sxy^3\sin t + 3sx^2y^2\cos t$$

9. $z = \sin\theta\cos\phi,\quad \theta = st^2,\quad \phi = s^2t \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial \phi}\frac{\partial \phi}{\partial s} = (\cos\theta\ \cos\phi)(t^2) + (-\sin\theta\ \sin\phi)(2st) = t^2\cos\theta\ \cos\phi - 2st\sin\theta\ \sin\phi$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial \phi}\frac{\partial \phi}{\partial t} = (\cos\theta\ \cos\phi)(2st) + (-\sin\theta\ \sin\phi)(s^2) = 2st\cos\theta\ \cos\phi - s^2\sin\theta\ \sin\phi$$

10. $z = e^{x+2y},\quad x = s/t,\quad y = t/s \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (e^{x+2y})(1/t) + (2e^{x+2y})(-ts^{-2}) = e^{x+2y}\left(\frac{1}{t} - \frac{2t}{s^2}\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (e^{x+2y})(-st^{-2}) + (2e^{x+2y})(1/s) = e^{x+2y}\left(\frac{2}{s} - \frac{s}{t^2}\right)$$

14. By the Chain Rule (3), $\dfrac{\partial W}{\partial s} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial s} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial s}$. Then

$$\begin{aligned} W_s(1,0) &= F_u(u(1,0),v(1,0))\,u_s(1,0) + F_v(u(1,0),v(1,0))\,v_s(1,0) = F_u(2,3)u_s(1,0) + F_v(2,3)v_s(1,0) \\ &= (-1)(-2) + (10)(5) = 52 \end{aligned}$$

Similarly, $\dfrac{\partial W}{\partial t} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial t} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial t} \quad \Rightarrow$

$$\begin{aligned} W_t(1,0) &= F_u(u(1,0),v(1,0))\,u_t(1,0) + F_v(u(1,0),v(1,0))\,v_t(1,0) = F_u(2,3)u_t(1,0) + F_v(2,3)v_t(1,0) \\ &= (-1)(6) + (10)(4) = 34 \end{aligned}$$

31. $x^2 + 2y^2 + 3z^2 = 1$, so let $F(x, y, z) = x^2 + 2y^2 + 3z^2 - 1 = 0$. Then by Equations 7

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x}{6z} = -\frac{x}{3z} \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{4y}{6z} = -\frac{2y}{3z}.$$

33. $e^z = xyz$, so let $F(x, y, z) = e^z - xyz = 0$. Then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{-yz}{e^z - xy} = \dfrac{yz}{e^z - xy}$ and

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-xz}{e^z - xy} = \frac{xz}{e^z - xy}.$$