Calc III: Quiz 1 Solutions, Fall 2017

Problem 1. For the points $P = (1, -1, 1)$, $Q = (2, -2, 2)$, $R = (2, 0, 1)$, and $S = (3, -1, 2)$, does $\overrightarrow{PQ} = \overrightarrow{RS}$? Justify your answer.

Solution. We have

$$\overrightarrow{PQ} = (2-1, (-2)-(-1), 1-2) = (1, -1, 1), \quad \text{and} \quad \overrightarrow{RS} = (3-2, -1-0, 2-1) = (1, -1, 1),$$

which are the same vector. □

Problem 2. Find the angle θ between the vectors $\mathbf{v} = (2, 1, 4)$ and $\mathbf{w} = (1, -2, 0)$.

Solution. The general formula is $\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$. We compute

$$\mathbf{v} \cdot \mathbf{w} = (2)(1) + (1)(-2) + (4)(0) = 0,$$

so in fact $\mathbf{v}$ and $\mathbf{w}$ are orthogonal, i.e., $\theta = \pi/2$. □

Problem 3. Find the area of the parallelogram with vertices $P = (2, 1, 3)$, $Q = (1, 4, 5)$, $R = (2, 5, 3)$, and $S = (3, 2, 1)$.

Solution. Let $\mathbf{v} = \overrightarrow{PQ} = (-1, 3, 2)$ and $\mathbf{w} = \overrightarrow{PS} = (1, 1, -2)$. These form two non-parallel sides of the parallelogram, whose area can be computed as by the cross product of $\mathbf{v} \times \mathbf{w}$:

$$\mathbf{v} \times \mathbf{w} = \big((3)(-2) - (1)(2), (1)(2) - (-1)(-2), (-1)(1) - (1)(3)\big) = (-8, 0, -4)$$

$$\text{Area} = \|\mathbf{v} \times \mathbf{w}\| = \sqrt{8^2 + 0^2 + 4^2} = \sqrt{80}.$$

□