Math 2321 Fall 2015: Quiz 2 Solutions

Problem 1. Consider the function

$$f(x, y, z) = x^2 + (y - 1)^2 e^z.$$

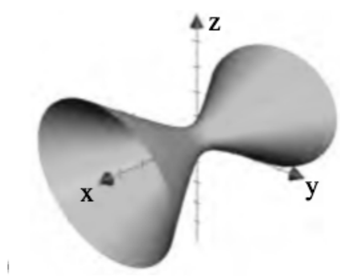
Determine whether f is a) continuous, and/or b) linear. Describe the level set where $f = 0$.

Solution. The function is continuous, since the functions $g_1(x) = x^2$, $g_2(y) = (y-1)^2$ and $g_3(z) = e^z$ are continuous 1-variable functions, and $f(x, y, z) = g_1(x) + g_2(y)g_3(z)$.

The function is *not* linear, since it is not of the form $f(x, y, z) = ax + by + cz$ for constants a, b and c.

The level set for $f = 0$ consists of the set of points $(0, 1, z)$ for any $z \in \mathbb{R}$ (which is a straight line in $\mathbb{R}^3$). Indeed, the terms x^2 and $(y-1)^2$ are non-negative, while e^z is strictly positive, so for $f = 0$ we must have $x^2 = 0$ and $(y-1)^2 = 0$, which implies $x = 0$ and $y = 1$. Then it does not matter what value z takes, and any (x, y, z) such that $x = 0$ and $y = 1$ is a solution. □

Problem 2. Determine the correct equation for the depicted surface:



a) $x^2 = 3z^2 + 2y^2$, b) $1 = 3z^2 + 2y^2$, c) $x^2 - 1 = 3z^2 + 2y^2$, d) $x^2 + 1 = 3z^2 + 2y^2$, e) $x^2 - 1 = 3z^2 - 2y^2$.

Solution. The surface is a 1-sheeted hyperboloid centered on the x-axis. The correct equation is $x^2 + 1 = 3x^2 + 2y^2$. Note that for $x = 0$ we have an ellipse $3z^2 + 2y^2 = 1$, while for $y = 0$ or $z = 0$ we have hyperbolas $x^2 + 1 = 3z^2$ or $x^2 + 1 = 2y^2$. □

Problem 3. Determine the gradient function $\nabla v(z, s, t)$ for the function

$$v(z, s, t) = z \ln s - z^2 \ln t.$$

Solution. The gradient is

$$\nabla v(z, s, t) = (v_z, v_s, v_t) = \left(\ln s - 2z \ln t,\ \tfrac{z}{s},\ \tfrac{z^2}{t}\right). \qquad \square$$