Math 2321 Fall 2015: Quiz 1 Solutions

Problem 1. Find the points of intersection, if any, of the plane defined by $x+2y+5z-8=0$ and the line parameterized by $(x,y,z)=(3,0,7)+t(-1,1,1)$.

Solution. We plug in the parameterization for the line into the equation for the plane and solve for t:

$$\begin{aligned} 0 &= (3-t)+2(t)+5(7+t)-8 \\ &= 6t+30 \\ \iff\ t &= -5. \end{aligned}$$

Plugging this back into the parameterization gives the point

$$(x,y,z)=(3,0,7)-5(-1,1,1)=(8,-5,2).$$

□

Problem 2. Find the area of the triangle with vertices

$$(0,0,0),\quad (-2,-1,-5),\quad (1,1,1)$$

and write a standard form equation for the plane through these 3 points.

Solution. The area of the triangle is one half of the area of the parallelogram defined by the displacement vectors $\mathbf{v}_1=(-2,-1,-5)-(0,0,0)=(-2,-1,-5)$ and $\mathbf{v}_2=(1,1,1)-(0,0,0)=(1,1,1)$; the area of the parallelogram is given by $|\mathbf{v}_1\times\mathbf{v}_2|$. We compute

$$\begin{aligned} \mathbf{v}_1\times\mathbf{v}_2 &= \det\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -1 & -5 \\ 1 & 1 & 1 \end{pmatrix} \\ &= \mathbf{i}(-1+5)-\mathbf{j}(-2+5)+\mathbf{k}(-2+1) \\ &= (4,-3,-1). \end{aligned}$$

This has magnitude

$$|\mathbf{v}_1\times\mathbf{v}_2|=|(4,-3,-1)|=\sqrt{16+9+1}=\sqrt{26},$$

so the area of the triangle is $\frac{\sqrt{26}}{2}$.

The plane through the three points has tangent vectors $\mathbf{v}_1$ and $\mathbf{v}_2$, but since we want an equation in standard form, we will use the normal vector $\mathbf{n}=\mathbf{v}_1\times\mathbf{v}_2=(4,-3,-1)$. Since the plane passes through $(0,0,0)$, it is defined by the standard form equation

$$0=\mathbf{n}\cdot\big((x,y,z)-(0,0,0)\big)=4x-3y-z.$$

□

Problem 3. Find a parameterization of the tangent line to the curve $\mathbf{p}(t)=(5t,2,e^t)$ at $t_0=0$. Use the variable s as a parameter.

Solution. The derivative/velocity of the curve is

$$\mathbf{p}'(t)=(5,0,e^t).$$

Then $\mathbf{p}'(0)=(5,0,1)$ is a tangent vector to the curve at $t_0=0$, which is the point $\mathbf{p}(0)=(0,2,1)$. The tangent line is parameterized by

$$(x,y,z)=(0,2,1)+s(5,0,1),\quad s\in\mathbb{R}.$$

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