**Calc III: Quiz 4 Solutions, Fall 2017**

**Problem 1.** Compute the double integral

$$\int_0^2 \int_1^4 xy\,dx\,dy.$$

*Solution.* We have

$$\begin{aligned}\int_0^2 \int_1^4 xy\,dx\,dy &= \int_0^2 \tfrac{1}{2}x^2y\big|_{x=1}^4\,dy \\ &= \int_0^2 8y - \tfrac{1}{2}y\,dy = \int_0^2 \tfrac{15}{2}y\,dy \\ &= \tfrac{15}{4}y^2\big|_{y=0}^2 \\ &= 15.\end{aligned}$$

□

**Problem 2.** Compute the double integral

$$\int_0^{\pi/2} \int_0^y \cos x\,\sin y\,dx\,dy.$$

*Solution.* We have

$$\begin{aligned}\int_0^{\pi/2} \int_0^y \cos x\,\sin y\,dx\,dy &= \int_0^{\pi/2} \sin x \sin y\big|_{x=0}^y dy \\ &= \int_0^{\pi/2} \sin^2 y\,dy \\ &= \frac{1}{2}\int_0^{\pi/2} 1 - \cos 2y\,dy \\ &= \frac{1}{2}y - \frac{1}{4}\sin 2y\big|_{y=0}^{\pi/2} \\ &= \frac{\pi}{4}.\end{aligned}$$

□

**Problem 3.** Find the volume $V$ of the solid bounded by the three coordinate planes and the plane $x + y + z = 1$.

*Solution.* Solving for $z$, we see that we are interested in the double integral of $f(x, y) = 1 - x - y$, over the region in the $xy$-plane bounded by the coordinate axes and the line where

the plane $x + y + z = 1$ intersects the plane $z = 0$, i.e., $x + y = 1$. This is the integral

$$\begin{aligned} V = \int_0^1 \int_0^{1-x} 1 - x - y \, dy \, dx &= \int_0^1 y - xy - \tfrac{1}{2}y^2 \big|_{y=0}^{1-x} \, dx \\ &= \int_0^1 1 - x - x(1-x) - \tfrac{1}{2}(1-x)^2 \, dx \\ &= \int_0^1 \tfrac{1}{2} - x + \tfrac{1}{2}x^2 \, dx \\ &= \tfrac{1}{2}x - \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 \big|_{x=0}^{1} \\ &= \frac{1}{6}. \end{aligned}$$

□