Calc III: Workshop 9, Fall 2017

Problem 1. Let

$$\mathbf{F}(x,y,z) = (2xy+1)z\mathbf{i} + x^2z\mathbf{j} + (x^2y + x + 2z)\mathbf{k}.$$

Compute the line integral $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ where C is the line segment from $(0,0,0)$ to $(1,2,3)$.

Problem 2. In fact the vector field of problem 1 is conservative. Find a potential function (i.e., $f(x,y,z)$ such that $\mathbf{F}(x,y,z) = \nabla f(x,y,z)$) and re-evaluate the line integral using the FTCLI.

Problem 3. Let

$$\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j} = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$$

be a vector field defined on $\mathbb{R}^2 \setminus \{(0,0)\}$, i.e., the whole plane minus the origin, where $\mathbf{F}(x,y)$ is undefined. Let C be the unit circle, oriented counterclockwise and compute

$$\oint_C \mathbf{F}(x,y)\cdot\mathbf{T}\,ds$$

Problem 4. Show that the vector field from the previous problem satisfies $Q_x - P_y = 0$ on $\mathbb{R}^2 \setminus \{(0,0)\}$. Is $\mathbf{F}$ conservative? Is there a potential function?