Calc III: Workshop 8, Fall 2017

Problem 1. Use spherical coordinates to evaluate $\iiint_E y\,dV$, where E is the solid hemisphere inside $x^2+y^2+z^2=9$ where $y\geq 0$.

Problem 2. Suppose a hemisphere H has constant density. Find its center of mass.

Problem 3. Use cylindrical coordinates to evaluate $\iiint_E z\,dV$, where E is enclosed by the paraboloid $z=x^2+y^2$ and the plane $z=4$.

Problem 4. Find the center of mass of the solid S bounded by the parabolid $z=4x^2+4y^2$ and the plane $z=a$ (where $a>0$) if S has constant density.

Problem 5. Evaluate $\int_C (2+xy^2)\,ds$, where C is the upper half of the unit circle $x^2+y^2=1$.

Problem 6. Let C be a curve in the plane, parameterized by $\big(x(t),y(t)\big)$ for $a\leq t\leq b$.

(a) Note that $\big(x(a+b-t),y(a+b-t)\big)$, where $a\leq t\leq b$ parameterizes the same curve, but in the opposite direction. Denote this "opposite direction curve" by $-C$.

(b) Show that $\int_C f(x,y)\,ds=\int_{-C} f(x,y)\,ds$.