Calc III: Quiz 3 Solutions, Fall 2017

Problem 1. Compute the partial derivatives $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y^2}$ for the function $f(x,y) = \sqrt{x^2+y+4}$.

Solution. First we compute

$$f_x = \frac{x}{\sqrt{x^2+y+4}}, \quad f_y = \frac{1}{2\sqrt{x^2+y+4}},$$

and then

$$f_{xx} = \frac{1}{\sqrt{x^2+y+4}} - \frac{x^2}{(x^2+y+4)^{3/2}} = \frac{y+4}{(x^2+y+4)^{3/2}}$$
$$f_{yy} = -\frac{1}{4(x^2+y+4)^{3/2}}$$
$$f_{xy} = -\frac{x}{2(x^2+y+4)^{3/2}}$$

□

Problem 2. Find the tangent plane to the surface $z = x^2 + y^3$ at the point $P = (1,1,2)$.

Solution. The surface is in the form $z = f(x,y) = x^2+y^3$. We compute the partial derivatives of f at $(1,1)$ to be

$$f_x(1,1) = 2x\big|_{(x,y)=(1,1)} = 2, \quad f_y(1,1) = 3y^2\big|_{(x,y)=(1,1)} = 3.$$

Then the tangent plane is given by

$$2(x-1) + 3(y-1) - z + 2 = 0, \quad \text{or} \quad 2x + 3y - z - 3 = 0.$$

□

Problem 3. Find the directional derivative of the function $f(x,y,z) = \sin(xyz)$ at the point $P = (1,1,1)$ in the direction $\mathbf{v} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

Solution. The gradient of f is

$$\nabla f(x,y,z) = \big(yz\cos(xyz), xz\cos(xyz), xy\cos(xyz)\big).$$

The directional derivative is given by

$$D_{\mathbf{v}} f(1,1,1) = \nabla f(1,1,1) \cdot \mathbf{v} = \big(\cos(1), \cos(1), \cos(1)\big) \cdot (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = \frac{3\cos(1)}{\sqrt{3}}.$$

□