Calculus III Workshop questions: 9/14/16

Problem 1 (14.3, #67)**.** Let $u = e^{r\theta} \sin \theta$. Compute the partial derivative $\frac{\partial^3 u}{\partial r^2 \partial \theta}$.

Problem 2 (14.3, #76)**.** Determine whether each of the following functions is a solution to *Laplace's equation* $u_{xx} + u_{yy} = 0$.
(a) $u = x^2 + y^2$
(b) $u = x^2 - y^2$
(c) $u = x^3 + 3xy^2$
(d) $u = \ln \sqrt{x^2 + y^2}$

Problem 3 (14.3, #93)**.** Is it possible that a function $f(x, y)$ has partial derivatives $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$?

Problem 4 (14.4, #1)**.** Find the tangent plane at $(2, -1, -3)$ to the surface

$$z = 3y^2 - 2x^2 + x$$

Problem 5 (14.4, #34)**.** Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

Problem 6 (14.4, #42)**.** Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \left\langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \right\rangle, \quad \mathbf{r}_2(u) = \left\langle 1 + u^2, 2u^3 - 1, 2u + 1 \right\rangle$$

both lie on S. Find an equation for the tangent plane at P.

Problem 7 (14.5, #20)**.** Use a tree diagram to write out the chain rule for the function

$$t = f(u, v, w), \quad u = u(p, q, r, s), \quad v = v(p, q, r, s), \quad w = w(p, q, r, s).$$

Problem 8 (14.5, #39)**.** The length ℓ, width w and height h of a box change with time. At a certain instant the dimensions are $\ell = 1$ m and $w = h = 2$ m, and ℓ and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that instant find the rates at which the following quantities are changing:
(a) The volume
(b) The surface area
(c) The length of a diagonal