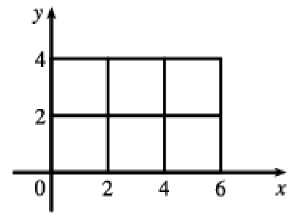
1. (a) The subrectangles are shown in the figure.



The surface is the graph of $f(x, y) = xy$ and $\Delta A = 4$, so we estimate

$$V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(x_i, y_j)\, \Delta A$$
$$= f(2,2)\,\Delta A + f(2,4)\,\Delta A + f(4,2)\,\Delta A + f(4,4)\,\Delta A + f(6,2)\,\Delta A + f(6,4)\,\Delta A$$
$$= 4(4) + 8(4) + 8(4) + 16(4) + 12(4) + 24(4) = 288$$

(b) $V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f\left(\overline{x}_i, \overline{y}_j\right) \Delta A = f(1,1)\,\Delta A + f(1,3)\,\Delta A + f(3,1)\,\Delta A + f(3,3)\,\Delta A + f(5,1)\,\Delta A + f(5,3)\,\Delta A$

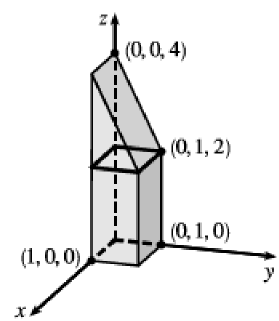
$$= 1(4) + 3(4) + 3(4) + 9(4) + 5(4) + 15(4) = 144$$

7. The values of $f(x, y) = \sqrt{52 - x^2 - y^2}$ get smaller as we move farther from the origin, so on any of the subrectangles in the problem, the function will have its largest value at the lower left corner of the subrectangle and its smallest value at the upper right corner, and any other value will lie between these two. So using these subrectangles we have $U < V < L$. (Note that this is true no matter how R is divided into subrectangles.)

11. $z = 3 > 0$, so we can interpret the integral as the volume of the solid S that lies below the plane $z = 3$ and above the rectangle $[-2, 2] \times [1, 6]$. S is a rectangular solid, thus $\iint_R 3\, dA = 4 \cdot 5 \cdot 3 = 60$.

13. $z = f(x, y) = 4 - 2y \geq 0$ for $0 \leq y \leq 1$. Thus the integral represents the volume of that part of the rectangular solid $[0, 1] \times [0, 1] \times [0, 4]$ which lies below the plane $z = 4 - 2y$. So

$$\iint_R (4 - 2y)\, dA = (1)(1)(2) + \tfrac{1}{2}(1)(1)(2) = 3$$



3. $\int_1^4 \int_0^2 (6x^2y - 2x)\, dy\, dx = \int_1^4 \left[3x^2y^2 - 2xy\right]_{y=0}^{y=2} dx = \int_1^4 (12x^2 - 4x)\, dx = \left[4x^3 - 2x^2\right]_1^4 = (256 - 32) - (4 - 2) = 222$

12. $\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2}\, dy\, dx = \int_0^1 x\left[\frac{1}{3}(x^2 + y^2)^{3/2}\right]_{y=0}^{y=1} dx = \frac{1}{3}\int_0^1 x[(x^2 + 1)^{3/2} - x^3]\, dx = \frac{1}{3}\int_0^1 [x(x^2 + 1)^{3/2} - x^4] dx$

$$= \frac{1}{3}\left[\frac{1}{5}(x^2 + 1)^{5/2} - \frac{1}{5}x^5\right]_0^1 = \frac{1}{15}\left[2^{5/2} - 1 - 1 + 0\right] = \frac{2}{15}\left(2\sqrt{2} - 1\right)$$

15. $\iint_R \sin(x - y)\, dA = \int_0^{\pi/2} \int_0^{\pi/2} \sin(x - y)\, dy\, dx = \int_0^{\pi/2} \left[\cos(x - y)\right]_{y=0}^{y=\pi/2} dx = \int_0^{\pi/2} \left[\cos(x - \frac{\pi}{2}) - \cos x\right] dx$

$$= \left[\sin(x - \tfrac{\pi}{2}) - \sin x\right]_0^{\pi/2} = \sin 0 - \sin\tfrac{\pi}{2} - \left[\sin(-\tfrac{\pi}{2}) - \sin 0\right]$$
$$= 0 - 1 - (-1 - 0) = 0$$

25. The solid lies under the plane $4x + 6y - 2z + 15 = 0$ or $z = 2x + 3y + \frac{15}{2}$ so

$$V = \iint_R (2x + 3y + \tfrac{15}{2})\, dA = \int_{-1}^{1} \int_{-1}^{2} (2x + 3y + \tfrac{15}{2})\, dx\, dy = \int_{-1}^{1} \left[x^2 + 3xy + \tfrac{15}{2}x\right]_{x=-1}^{x=2} dy$$

$$= \int_{-1}^{1} \left[(19 + 6y) - (-\tfrac{13}{2} - 3y)\right] dy = \int_{-1}^{1} (\tfrac{51}{2} + 9y)\, dy = \left[\tfrac{51}{2}y + \tfrac{9}{2}y^2\right]_{-1}^{1} = 30 - (-21) = 51$$

29. Here we need the volume of the solid lying under the surface $z = x\sec^2 y$ and above the rectangle $R = [0, 2] \times [0, \pi/4]$ in the xy-plane.

$$V = \int_0^2 \int_0^{\pi/4} x\sec^2 y\, dy\, dx = \int_0^2 x\, dx \int_0^{\pi/4} \sec^2 y\, dy = \left[\tfrac{1}{2}x^2\right]_0^2 \left[\tan y\right]_0^{\pi/4}$$

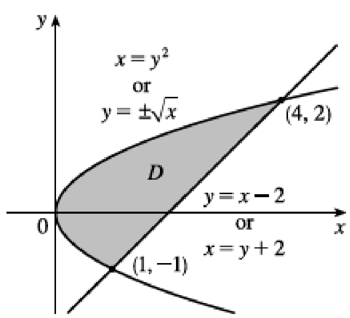
$$= (2 - 0)(\tan\tfrac{\pi}{4} - \tan 0) = 2(1 - 0) = 2$$

1. $\int_0^4 \int_0^{\sqrt{y}} xy^2\, dx\, dy = \int_0^4 \left[\frac{1}{2}x^2y^2\right]_{x=0}^{x=\sqrt{y}} dy = \int_0^4 \frac{1}{2}y^2[(\sqrt{y}\,)^2 - 0^2]dy = \frac{1}{2}\int_0^4 y^3\, dy = \frac{1}{2}\left[\frac{1}{4}y^4\right]_0^4 = \frac{1}{2}(64 - 0) = 32$

7. $\iint_D y^2\, dA = \int_{-1}^{1} \int_{-y-2}^{y} y^2\, dx\, dy = \int_{-1}^{1} \left[xy^2\right]_{x=-y-2}^{x=y} dy = \int_{-1}^{1} y^2\left[y - (-y - 2)\right] dy$

$$= \int_{-1}^{1} (2y^3 + 2y^2)dy = \left[\tfrac{1}{2}y^4 + \tfrac{2}{3}y^3\right]_{-1}^{1} = \tfrac{1}{2} + \tfrac{2}{3} - \tfrac{1}{2} + \tfrac{2}{3} = \tfrac{4}{3}$$

15.



The curves $y = x - 2$ or $x = y + 2$ and $x = y^2$ intersect when $y + 2 = y^2 \Leftrightarrow y^2 - y - 2 = 0 \Leftrightarrow (y - 2)(y + 1) = 0 \Leftrightarrow y = -1, y = 2$, so the points of intersection are $(1, -1)$ and $(4, 2)$. If we describe D as a type I region, the upper boundary curve is $y = \sqrt{x}$ but the lower boundary curve consists of two parts, $y = -\sqrt{x}$ for $0 \le x \le 1$ and $y = x - 2$ for $1 \le x \le 4$.

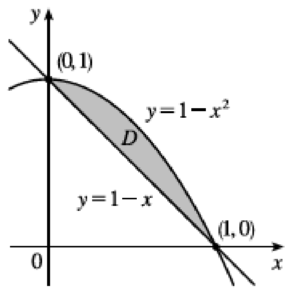
Thus $D = \{(x, y) \mid 0 \le x \le 1,\ -\sqrt{x} \le y \le \sqrt{x}\,\} \cup \{(x, y) \mid 1 \le x \le 4, x - 2 \le y \le \sqrt{x}\,\}$ and $\iint_D y\, dA = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y\, dy\, dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y\, dy\, dx$. If we describe D as a type II region, D is enclosed by the left boundary $x = y^2$ and the right boundary $x = y + 2$ for $-1 \le y \le 2$, so $D = \left\{(x, y) \mid -1 \le y \le 2, y^2 \le x \le y + 2\right\}$ and $\iint_D y\, dA = \int_{-1}^{2} \int_{y^2}^{y+2} y\, dx\, dy$. In either case, the resulting iterated integrals are not difficult to evaluate but the region D is more simply described as a type II region, giving one iterated integral rather than a sum of two, so we evaluate the latter integral:

$$\iint_D y\, dA = \int_{-1}^{2} \int_{y^2}^{y+2} y\, dx\, dy = \int_{-1}^{2} \left[xy\right]_{x=y^2}^{x=y+2} dy = \int_{-1}^{2} (y + 2 - y^2)y\, dy = \int_{-1}^{2} (y^2 + 2y - y^3)\, dy$$

$$= \left[\tfrac{1}{3}y^3 + y^2 - \tfrac{1}{4}y^4\right]_{-1}^{2} = \left(\tfrac{8}{3} + 4 - 4\right) - \left(-\tfrac{1}{3} + 1 - \tfrac{1}{4}\right) = \tfrac{9}{4}$$

17. $\int_0^1 \int_0^{x^2} x\cos y\, dy\, dx = \int_0^1 \left[x\sin y\right]_{y=0}^{y=x^2} dx = \int_0^1 x\sin x^2\, dx = -\frac{1}{2}\cos x^2\Big]_0^1 = \frac{1}{2}(1 - \cos 1)$

23.



$$V = \int_0^1 \int_{1-x}^{1-x^2} (1 - x + 2y)\,dy\,dx = \int_0^1 \left[y - xy + y^2\right]_{y=1-x}^{y=1-x^2}\,dx$$

$$= \int_0^1 \Big[\Big((1-x^2) - x(1-x^2) + (1-x^2)^2\Big) - \Big((1-x) - x(1-x) + (1-x)^2\Big)\Big]\,dx$$

$$= \int_0^1 \left[\left(x^4 + x^3 - 3x^2 - x + 2\right) - \left(2x^2 - 4x + 2\right)\right] dx$$

$$= \int_0^1 \left(x^4 + x^3 - 5x^2 + 3x\right) dx = \left[\tfrac{1}{5}x^5 + \tfrac{1}{4}x^4 - \tfrac{5}{3}x^3 + \tfrac{3}{2}x^2\right]_0^1$$

$$= \tfrac{1}{5} + \tfrac{1}{4} - \tfrac{5}{3} + \tfrac{3}{2} = \tfrac{17}{60}$$