Calculus III equation sheet for Exam 2

- Straight line segment from $\mathbf{p}_0$ to $\mathbf{p}_1$:

$$\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0), \quad 0 \le t \le 1.$$

- Change of variables formula for integration

$$dA = dx\,dy = \det \frac{\partial(x,y)}{\partial(u,v)}\,du\,dv, \qquad \frac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$$

$$dV = dx\,dy\,dz = \det \frac{\partial(x,y,z)}{\partial(u,v,w)}\,du\,dv\,dw \quad \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{pmatrix}$$

- Center of mass $(\overline{x}, \overline{y})$ or $(\overline{x}, \overline{y}, \overline{z})$ where:

$$\overline{x} = \frac{\iint_R x\delta(x,y)\,dA}{\iint_R \delta(x,y)\,dA}, \quad \overline{y} = \frac{\iint_R y\delta(x,y)\,dA}{\iint_R \delta(x,y)\,dA}$$

$$\overline{x} = \frac{\iiint_R x\delta(x,y,z)\,dV}{\iiint_R \delta(x,y,z)\,dV}, \quad \overline{y} = \frac{\iiint_R y\delta(x,y,z)\,dV}{\iiint_R \delta(x,y,z)\,dV}, \quad \overline{z} = \frac{\iiint_R z\delta(x,y,z)\,dV}{\iiint_R \delta(x,y,z)\,dV}.$$

- Polar coordinates:

$$(x,y) = (r\cos\theta, r\sin\theta),$$
$$dA = r\,dr\,d\theta$$

- Cylindrical coordinates:

$$(x,y,z) = (r\cos\theta, r\sin\theta, z),$$
$$dV = r\,dz\,dr\,d\theta$$

- Spherical coordinates:

$$(x,y,z) = (\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi),$$
$$dV = \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

- Arc length elements:

$$ds = |\mathbf{p}'(t)|\,dt,$$
$$\mathbf{T}\,ds = \mathbf{p}'(t)\,dt$$

- Fundamental Theorem for Line Integrals:

$$f(\mathbf{p}_1) - f(\mathbf{p}_0) = \int_C \nabla f \cdot \mathbf{T}\,ds$$

- Green's Theorem:

$$\oint_{\partial R} \big(P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}\big) \cdot \mathbf{T}\,ds = \iint_R (Q_x - P_y)\,dA$$

- Trig values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0