Distribution Theory: Spring 2019

- **Instructor**: Professor Chris Kottke
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- **Course Webpage**: http://ckottke.ncf.edu/distributions/, or Canvas
- **Lectures**: Mon/Thu 2:00-3:50, HNS 106
- **Office Hours**: Mon 3:30-4:30, Wed 1:30-2:30, Thu 11-12
- **Textbook**: *Distributions: Theory and Applications*, by Duistermaat and Kolk, Bikhauser Cornerstones series. This book is electronically available for free if you access the publisher's website via the preceding link from on campus. A hard copy has also been placed on reserve in the library.

Course Description: The desire to treat objects more general than functions arises quite naturally in physics and differential equations. For example, the so-called "Dirac delta function" vanishes everywhere except at 0 yet integrates to 1. Though no classical real valued function can have this property, it appears already in the work of Fourier and Cauchy in the early 19th century on partial differential equations. A rigorous theory of such objects–now called distributions–developed slowly over the course of the following century, culminating as recently as 1945 in the work of Laurent Schwartz.

Essentially, distributions are to functions what real numbers are to the rationals. Indeed, functions may be seen as embedded in this larger category of distributions in which many nice properties hold: for example, every distribution may be differentiated infininitely many times, and every constant coefficient PDE admits distributional solutions. Distributions also turn out to be the ideal framework in which to develop the theory of Fourier series and the Fourier transform, in which we write functions as an (infinite) linear combination of "basis elements" as in linear algebra, where the basis elements are "eigenvectors" of the partial derivative operators. By "diagonalizing" partial derivatives in this way, we turn PDE problems into algebraic ones.

Besides comprising a richly interesting mathematical theory, the theory of distributions and Fourier analysis thus have many important applications in physics, engineering and PDE, many of which will drive our developement of the subject.

Prerequisites:

- Linear Algebra
- Multidimensional Real Analysis (e.g. Real II, or permission of the instructor)
- Though not required, students with experience in PDE, complex analysis, or physics will enjoy connections to those subjects.

Reading Assignments: A reading assignment for each class will be posted on the course webpage and in the Canvas course prior to each lecture. This reading should be completed *before* the lecture. Unless otherwise specified, you will be responsible for all material in the reading assignment, even if it is not covered in lecture.

Though it won't be used explicitly, a second textbook, *Distributions, Partial Differential Equations, and Harmonic Analysis* by Mitrea is also on reserve in the library and freely downloadable from on campus. This is a more comprehensive treatment of the subject (and contains a staggering number of additional problems), for students desiring an additional resource.

Homework: Homework problems will be assigned and collected on a regular (approximately weekly) basis. You are encouraged to regard office hours as a kind of homework workshop and to attend them regularly.

Exams: There will be two exams, one at the end of each module, either oral (if class numbers permit) or written. Dates are yet to be determined.

Assessment: Your course performance (Sat/Unsat) will be evaluated based on homework and exams. Class participation and attendance will be reflected in the narrative evaluation.

Policies: Regular out-of-class work on problems is essential to learning this subject, and for this reason *no late assignments will be accepted* without prior instructor permission.

Students in need of academic accommodations for a disability may consult with the office of Students Disability Services (SDS) to arrange appropriate accommodations. Students are required to give reasonable notice prior to requesting an accommodation. Students may request an appointment with SDS in-person (HCL3), via phone at 941-487-4496, or via email at disabilityservices@ncf.edu.

No student shall be compelled to attend class or sit for an examination at a day or time when he or she would normally be engaged in a religious observance or on a day or time prohibited by his or her religious belief. Students are expected to notify their instructors if they intend to be absent for a class or announced examination, in accordance with this policy, well in advance of the scheduled meeting.