Linear Mixed effect model-day2

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Mixed effect models

possun Morphometic dataset.

```
possum <- read.csv("./data/possum.csv",header = T)
## Basic data data exploration
summary(possum)
```

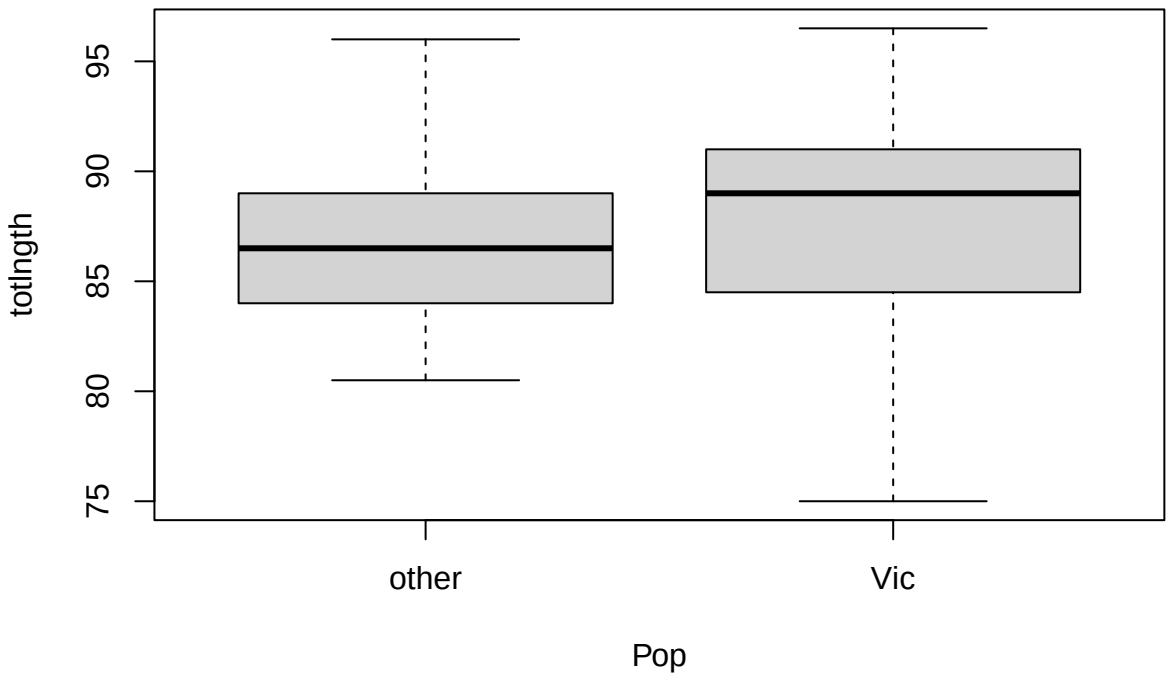
```
##       case             site           Pop                sex
##  Min.   :  1.00   Min.   :1.000   Length:104         Length:104
##  1st Qu.: 26.75   1st Qu.:1.000   Class :character   Class :character
##  Median : 52.50   Median :3.000   Mode  :character   Mode  :character
##  Mean   : 52.50   Mean   :3.625
##  3rd Qu.: 78.25   3rd Qu.:6.000
##  Max.   :104.00   Max.   :7.000
##
##       age           hdlngth           skullw         totlngth
##  Min.   :1.000   Min.   : 82.50   Min.   :50.00   Min.   :75.00
##  1st Qu.:2.250   1st Qu.: 90.67   1st Qu.:54.98   1st Qu.:84.00
##  Median :3.000   Median : 92.80   Median :56.35   Median :88.00
##  Mean   :3.833   Mean   : 92.60   Mean   :56.88   Mean   :87.09
##  3rd Qu.:5.000   3rd Qu.: 94.72   3rd Qu.:58.10   3rd Qu.:90.00
##  Max.   :9.000   Max.   :103.10   Max.   :68.60   Max.   :96.50
##  NA's   :2
##      taill          footlgth        earconch          eye            chest
##  Min.   :32.00   Min.   :60.30   Min.   :40.30   Min.   :12.80   Min.   :22.0
##  1st Qu.:35.88   1st Qu.:64.60   1st Qu.:44.80   1st Qu.:14.40   1st Qu.:25.5
##  Median :37.00   Median :68.00   Median :46.80   Median :14.90   Median :27.0
##  Mean   :37.01   Mean   :68.46   Mean   :48.13   Mean   :15.05   Mean   :27.0
##  3rd Qu.:38.00   3rd Qu.:72.50   3rd Qu.:52.00   3rd Qu.:15.72   3rd Qu.:28.0
##  Max.   :43.00   Max.   :77.90   Max.   :56.20   Max.   :17.80   Max.   :32.0
##                  NA's   :1
##      belly
##  Min.   :25.00
##  1st Qu.:31.00
##  Median :32.50
##  Mean   :32.59
##  3rd Qu.:34.12
##  Max.   :40.00
```

```
##
```

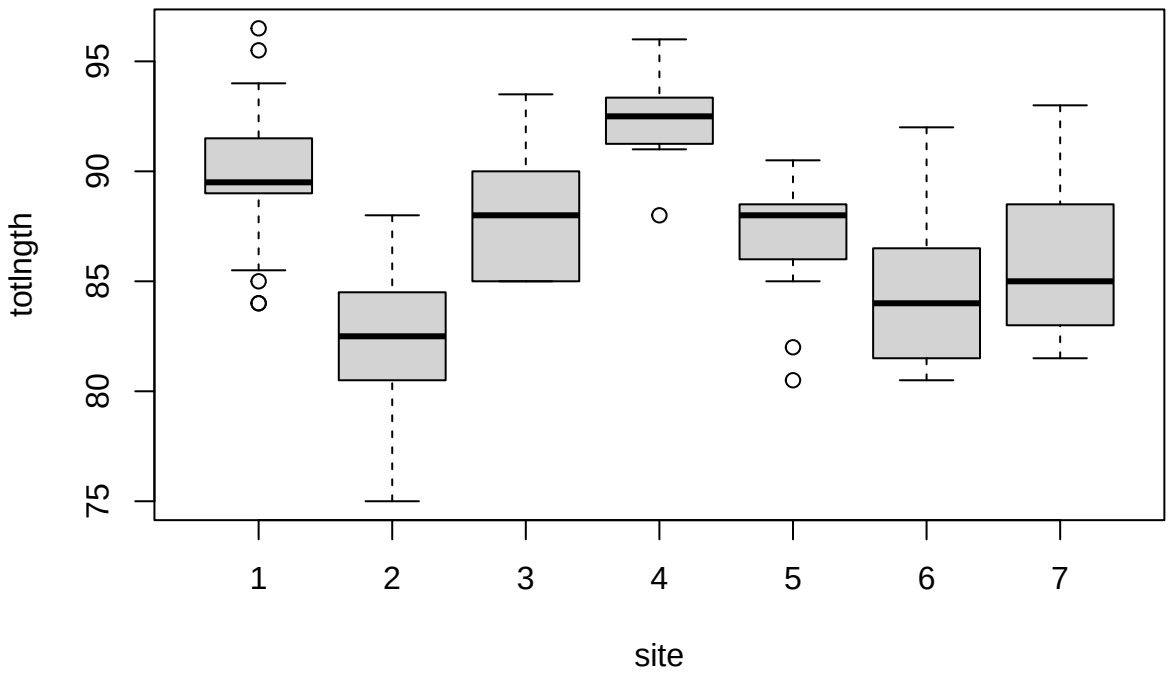
```
## convert the population,site and gender  into factors
possum$site <-as.factor(possum$site)
possum$sex <- as.factor(possum$sex)
possum$Pop <-as.factor(possum$Pop)
summary(possum)
```

```
##       case        site      Pop     sex         age            hdlngth
##  Min.   :  1.00   1:33   other:58   f:43   Min.   :1.000   Min.   : 82.50
##  1st Qu.: 26.75   2:13   Vic  :46   m:61   1st Qu.:2.250   1st Qu.: 90.67
##  Median : 52.50   3: 7                    Median :3.000   Median : 92.80
##  Mean   : 52.50   4: 7                    Mean   :3.833   Mean   : 92.60
##  3rd Qu.: 78.25   5:13                    3rd Qu.:5.000   3rd Qu.: 94.72
##  Max.   :104.00   6:13                    Max.   :9.000   Max.   :103.10
##                   7:18                    NA's   :2
##      skullw         totlngth         taill          footlgth
##  Min.   :50.00   Min.   :75.00   Min.   :32.00   Min.   :60.30
##  1st Qu.:54.98   1st Qu.:84.00   1st Qu.:35.88   1st Qu.:64.60
##  Median :56.35   Median :88.00   Median :37.00   Median :68.00
##  Mean   :56.88   Mean   :87.09   Mean   :37.01   Mean   :68.46
##  3rd Qu.:58.10   3rd Qu.:90.00   3rd Qu.:38.00   3rd Qu.:72.50
##  Max.   :68.60   Max.   :96.50   Max.   :43.00   Max.   :77.90
##                                                  NA's   :1
##     earconch          eye            chest          belly
##  Min.   :40.30   Min.   :12.80   Min.   :22.0   Min.   :25.00
##  1st Qu.:44.80   1st Qu.:14.40   1st Qu.:25.5   1st Qu.:31.00
##  Median :46.80   Median :14.90   Median :27.0   Median :32.50
##  Mean   :48.13   Mean   :15.05   Mean   :27.0   Mean   :32.59
##  3rd Qu.:52.00   3rd Qu.:15.72   3rd Qu.:28.0   3rd Qu.:34.12
##  Max.   :56.20   Max.   :17.80   Max.   :32.0   Max.   :40.00
##
```

```
## you can see the difference in the way population,site and gender columns
## box plot of lenght vs gender and pop
boxplot(totlngth~Pop, data = possum)
```




```
boxplot(totlngth~site, data = possum)
```




```
## pair plot to see if there is co - linearity
ggpairs(possum[,c(8,7,9,10)])
```

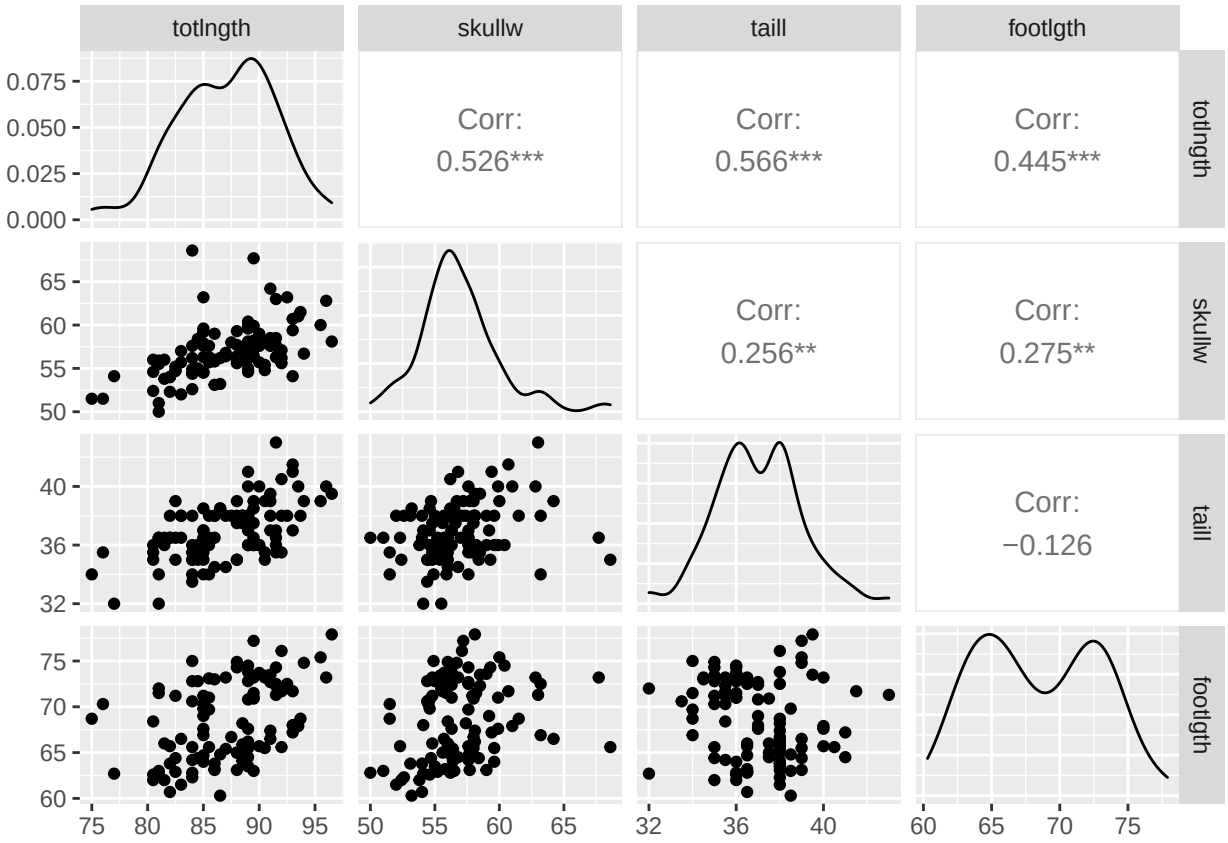
```
## Warning in ggally_statistic(data = data, mapping = mapping, na.rm = na.rm, :
## Removing 1 row that contained a missing value

## Warning in ggally_statistic(data = data, mapping = mapping, na.rm = na.rm, :
## Removing 1 row that contained a missing value

## Warning in ggally_statistic(data = data, mapping = mapping, na.rm = na.rm, :
## Removing 1 row that contained a missing value

## Warning: Removed 1 rows containing missing values (geom_point).
## Removed 1 rows containing missing values (geom_point).
## Removed 1 rows containing missing values (geom_point).

## Warning: Removed 1 rows containing non-finite values (stat_density).
```




```
#
```

Droping the co-linear variable.

There is small amount of correlation between tail and skull, foot len vs skull. However, since we want to know of skull width can predict the possum length.

- What you take out of this graphs on the population trend of the possum

Let fit simple linear model

$$Lenght = \alpha + \beta_1 * Skullwidth + \epsilon$$

Where

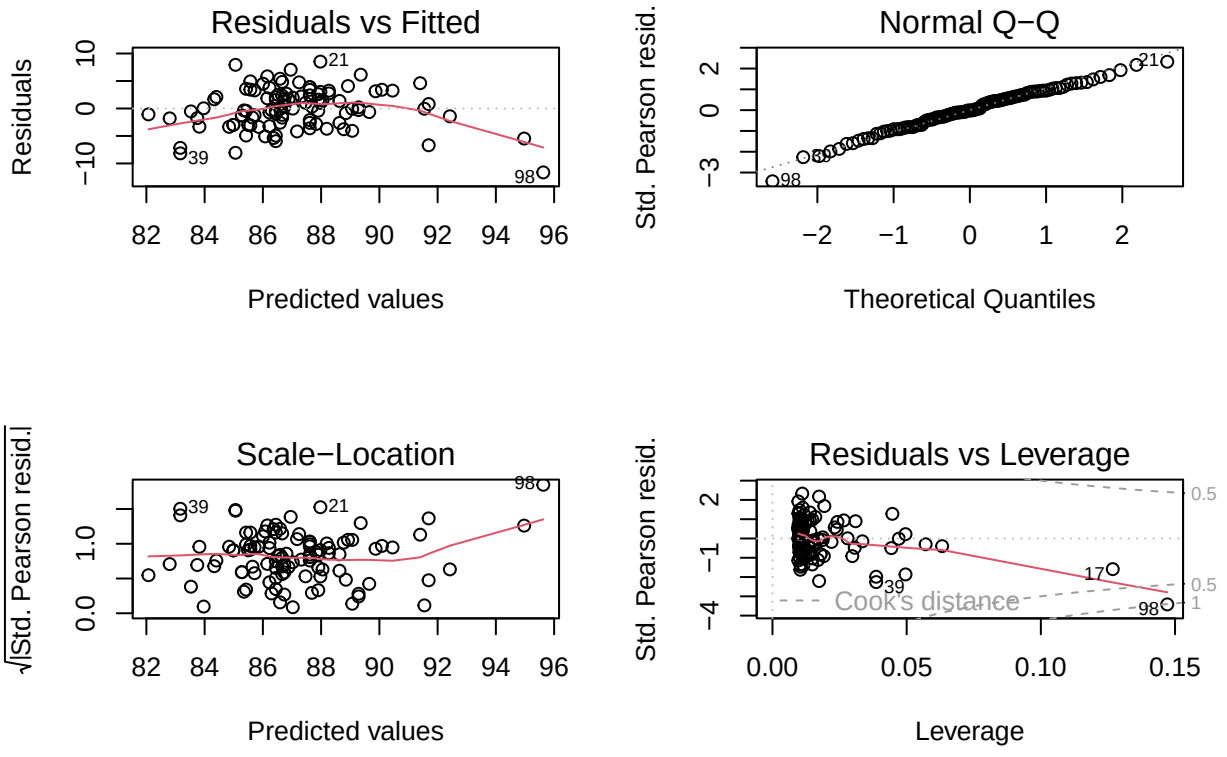
$$\epsilon \sim N(0, \sigma^2)$$

```
mod_lm <- glm(totlngth~skullw,data = possum,family = "gaussian")
summary(mod_lm)
```

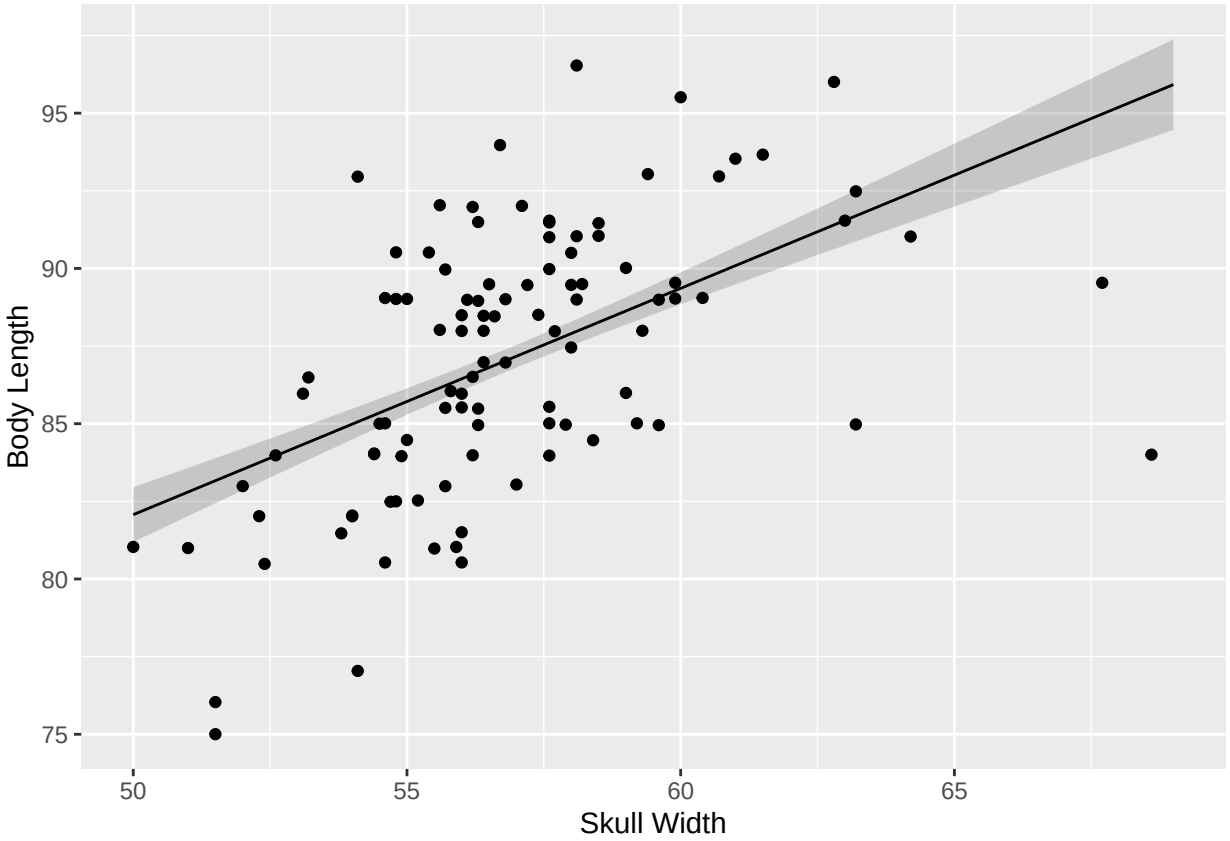
```
##
## Call:
## glm(formula = totlngth ~ skullw, family = "gaussian", data = possum)
```

```
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -11.6276   -2.6156   -0.0572    2.6212    8.5250
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  45.6305     6.6399   6.872 5.13e-10 ***
## skullw        0.7288     0.1166   6.253 9.50e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 13.56358)
##
##     Null deviance: 1913.8  on 103  degrees of freedom
## Residual deviance: 1383.5  on 102  degrees of freedom
## AIC: 570.29
##
## Number of Fisher Scoring iterations: 2
```

```
par(mfrow=c(2,2))
plot(mod_lm)
```




```
ggPredict(mod_lm,se = T,interactive = F)+labs(x="Skull Width", y= "Body Length")
```



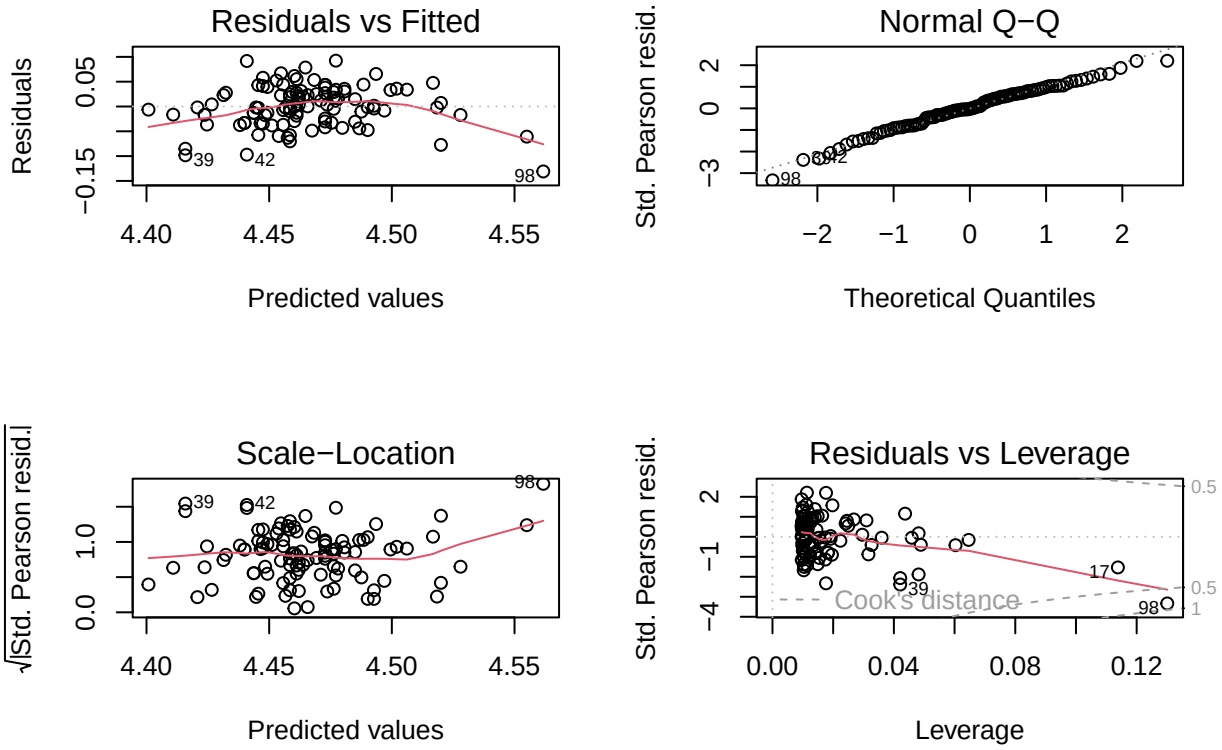
Clearly there the data is not homogeneous,

- We need to transform the variable to see if we can get out model to work

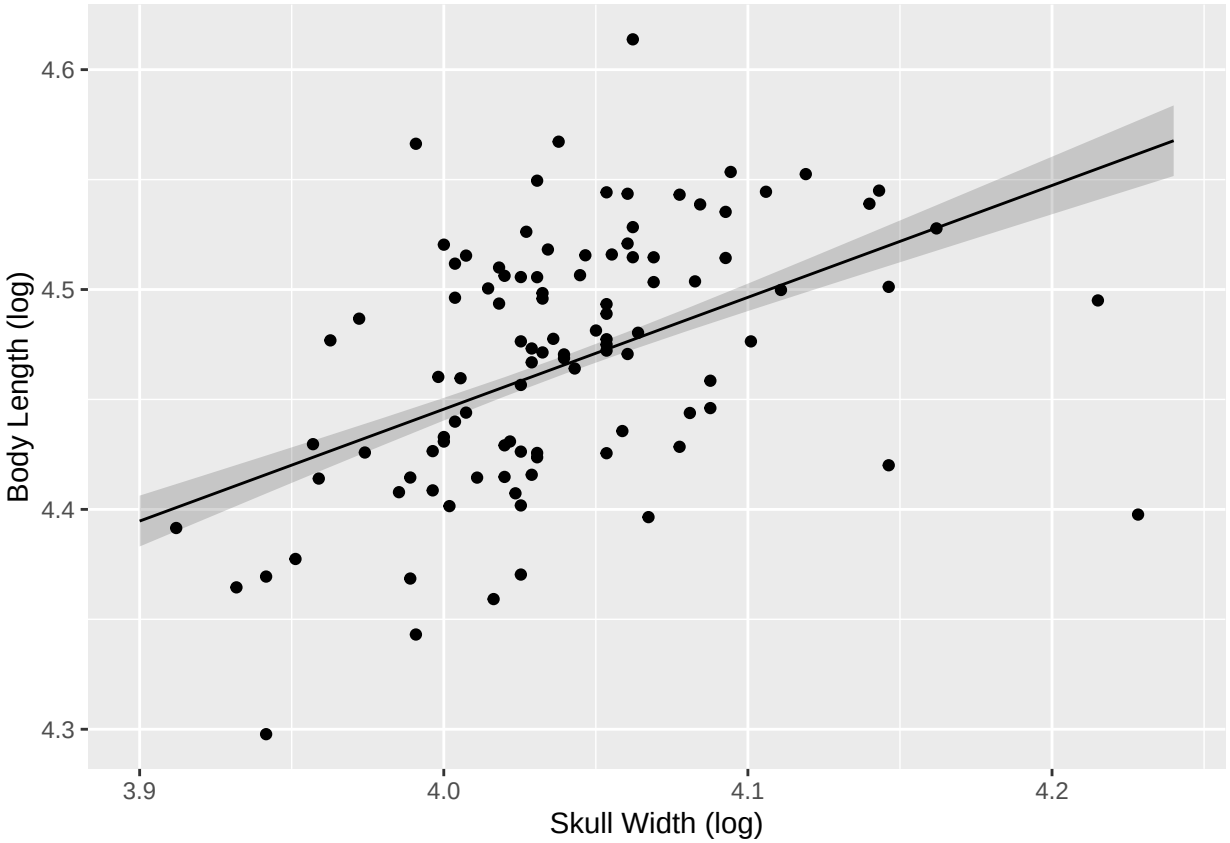
New model will be

$$\log(Length) = \alpha + \beta_1 * \log(Skullwidth) + \epsilon$$

```
possum <- possum%>%mutate(log_skullwid=log(skullw))
possum <- possum%>%mutate(log_len=log(totlngth))
mod_lm_log <- glm(log_len~log_skullwid,data = possum,family = "gaussian")
par(mfrow=c(2,2))
plot(mod_lm_log)
```




```
ggPredict(mod_lm_log,se = T,interactive = F)+labs(x="Skull Width (log)", y= "Body Length (log)")
```

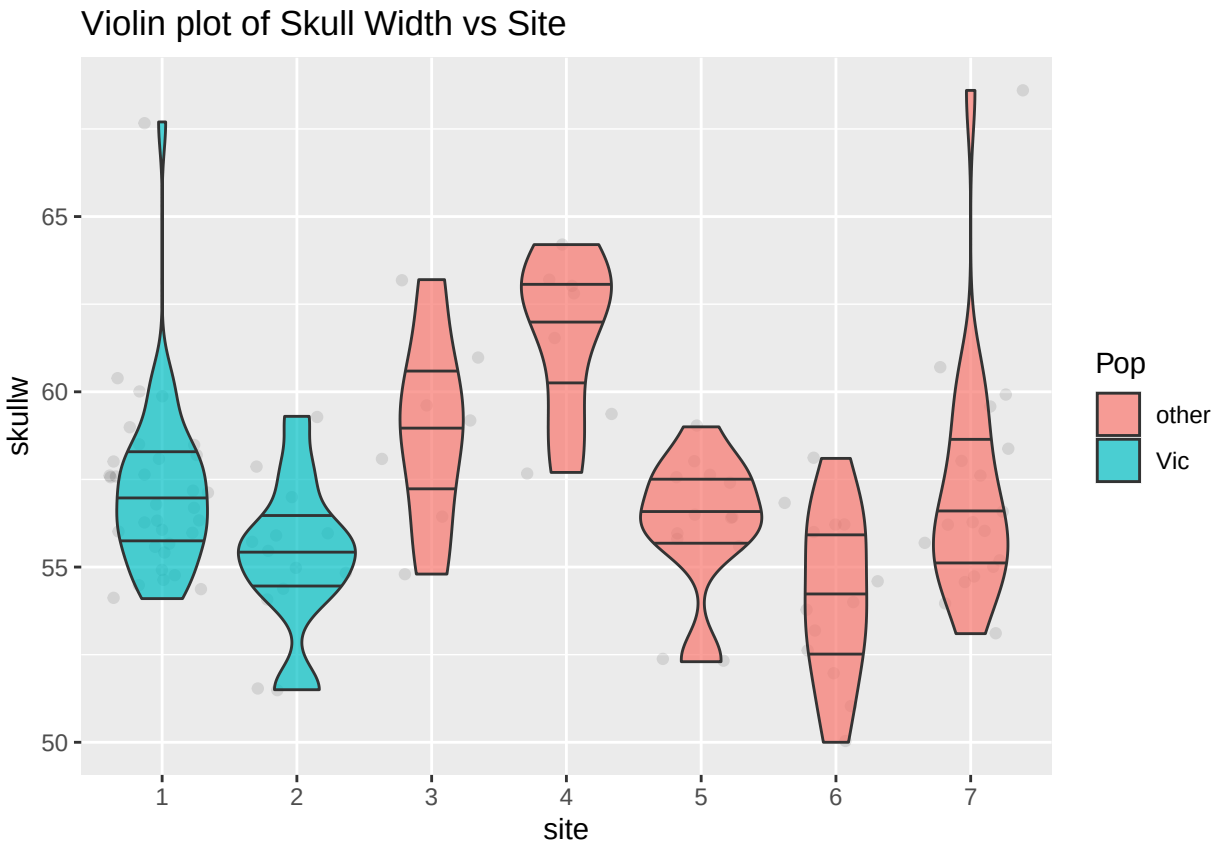


What is the assumption we are violating here for linear model of Gaussian Family ?

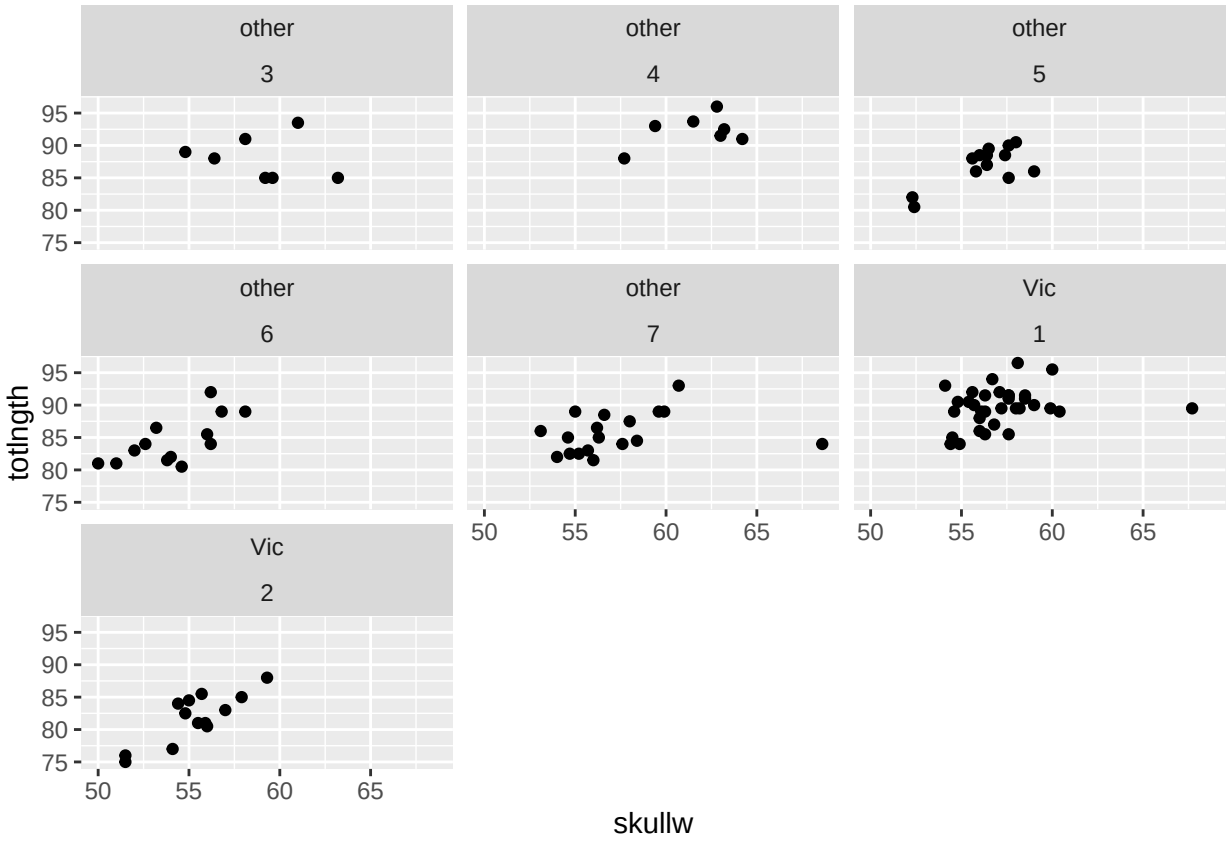
- Homoscedasticity: The variance of residual is the same for any value of X.
- what about "Independence: Observations are independent of each other."

Let us check

```
ggplot(possum, aes(x = site, y = skullw)) +
  geom_jitter(alpha = .1) +
  geom_violin(alpha = 0.7,aes(fill=Pop),draw_quantiles = c(0.25, 0.5, 0.75)) +
  ggtitle("Violin plot of Skull Width vs Site")
```




```
#scatter plot
scat_plot <- ggplot(data = possum, aes(x=skullw,y=totlngth))
scat_plot + geom_point()+facet_wrap(.~Pop+site)
```




```
# sample size
group_by(possum,site)%>%summarise(count=n())
```

```
## # A tibble: 7 x 2
##   site  count
##   <fct> <int>
## 1 1        33
## 2 2        13
## 3 3         7
## 4 4         7
## 5 5        13
## 6 6        13
## 7 7        18
```

- **the data is not independent**

As we can clearly see from the data collected the data is not independent but nested in the nature.

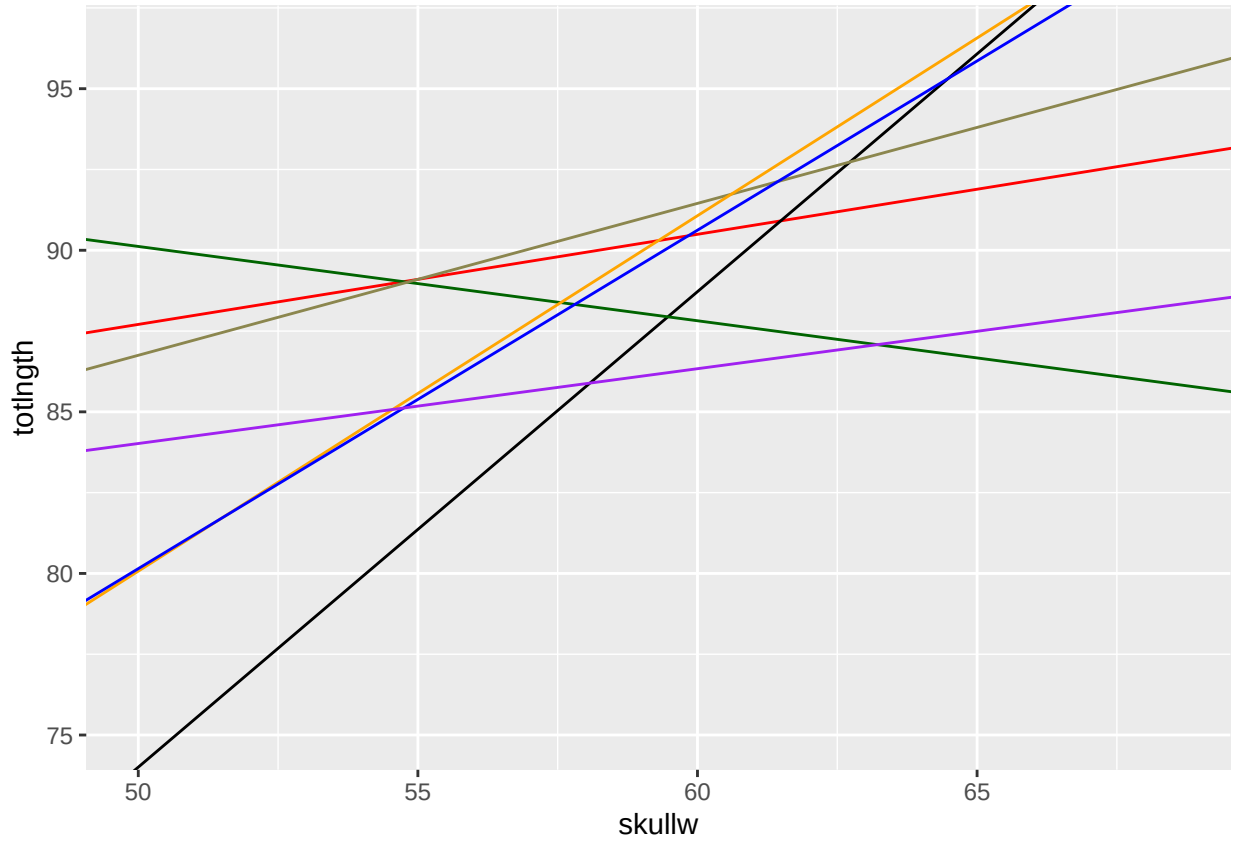
Moreover, the data is taken from 2 different population. In addition there are several site nested for each population.

This is called nested data.

One traditional way (wrong way)

Let make model of all the site individually.

```
site<-unique(possum$site)
Beta <- vector(length = length(site))
alpha <- vector(length = length(site))
for(k in 1:length(Beta)){
  flag_mod <- lm(totlngth~skullw,data = possum[possum$site==site[k],])
  alpha[k] <- as.numeric(flag_mod$coefficients[[1]])
  Beta[k] <- as.numeric(flag_mod$coefficients[[2]])
}
##let plot regression for each site.
ggplot(possum,aes(x=skullw,y=totlngth))+geom_blank()+
  geom_abline(slope = Beta[1],intercept = alpha[1],col="red")+
  geom_abline(slope = Beta[2],intercept = alpha[2],col="black")+
  geom_abline(slope = Beta[3],intercept = alpha[3],col="darkgreen")+
  geom_abline(slope = Beta[4],intercept = alpha[4],col="khaki4")+
  geom_abline(slope = Beta[5],intercept = alpha[5],col="orange")+
  geom_abline(slope = Beta[6],intercept = alpha[6],col="blue")+
  geom_abline(slope = Beta[7],intercept = alpha[7],col="purple")
```



Less traditional way (method in this case)

It to account for variation in site.

```
mod_account_site <- lm(totlngth~skullw + site ,data = possum)
AIC(mod_account_site,mod_lm)
```

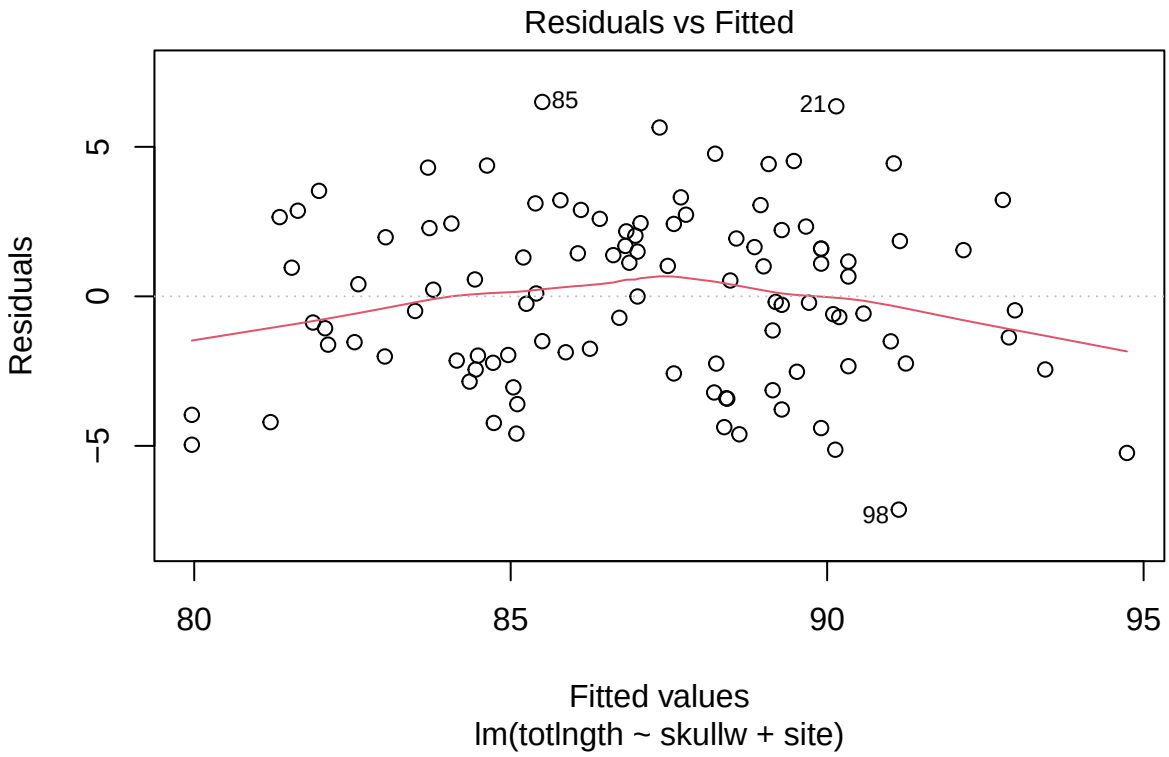
```
##                  df      AIC
```

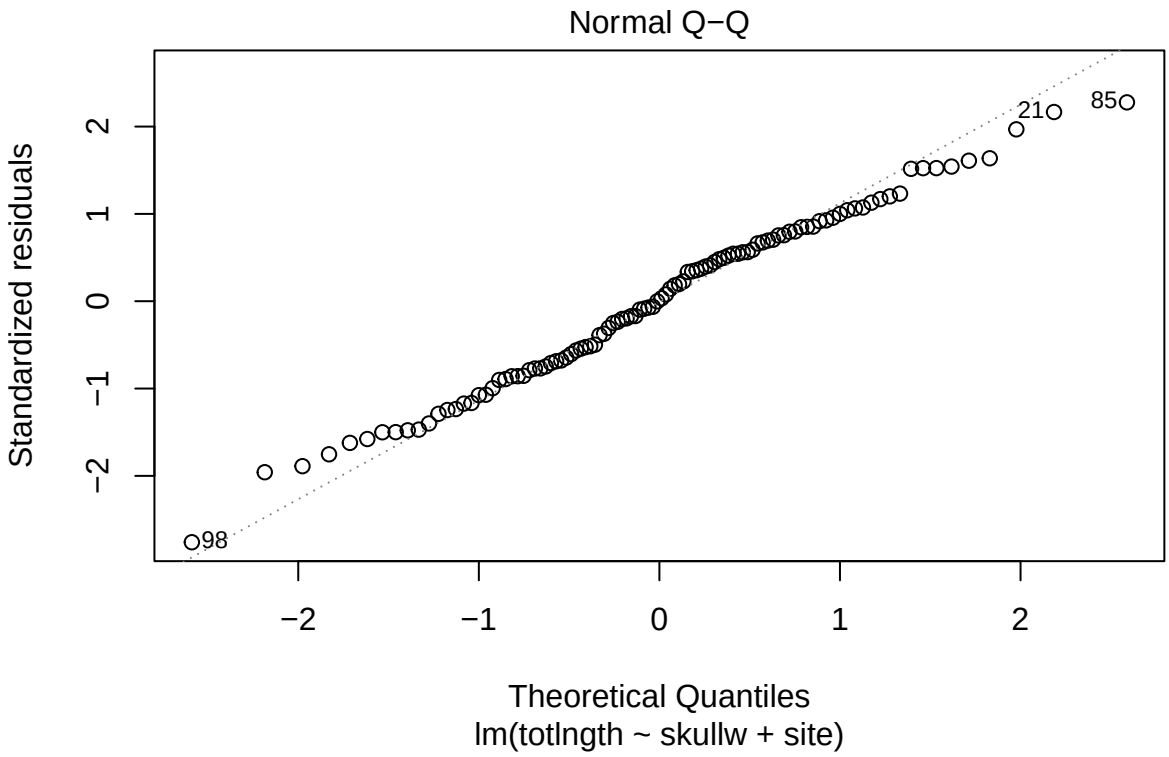
```
## mod_account_site  9 531.9851
## mod_lm            3 570.2881
```

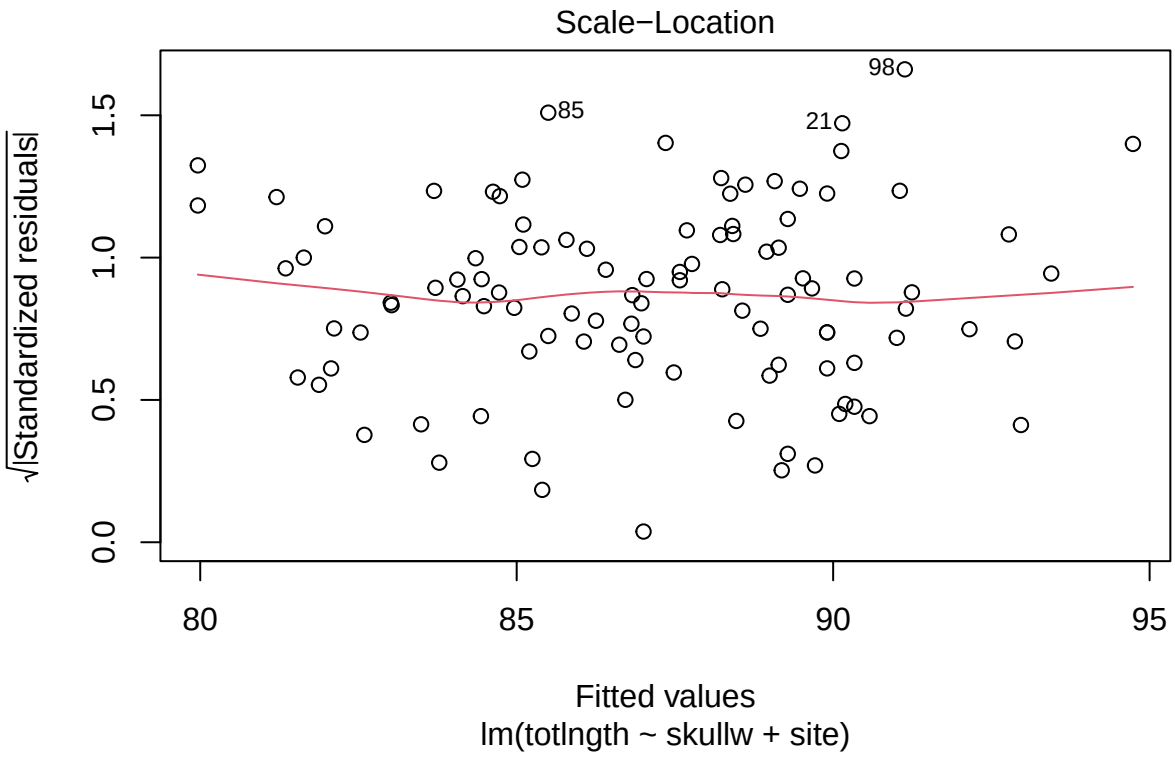
```
summary(mod_account_site)
```

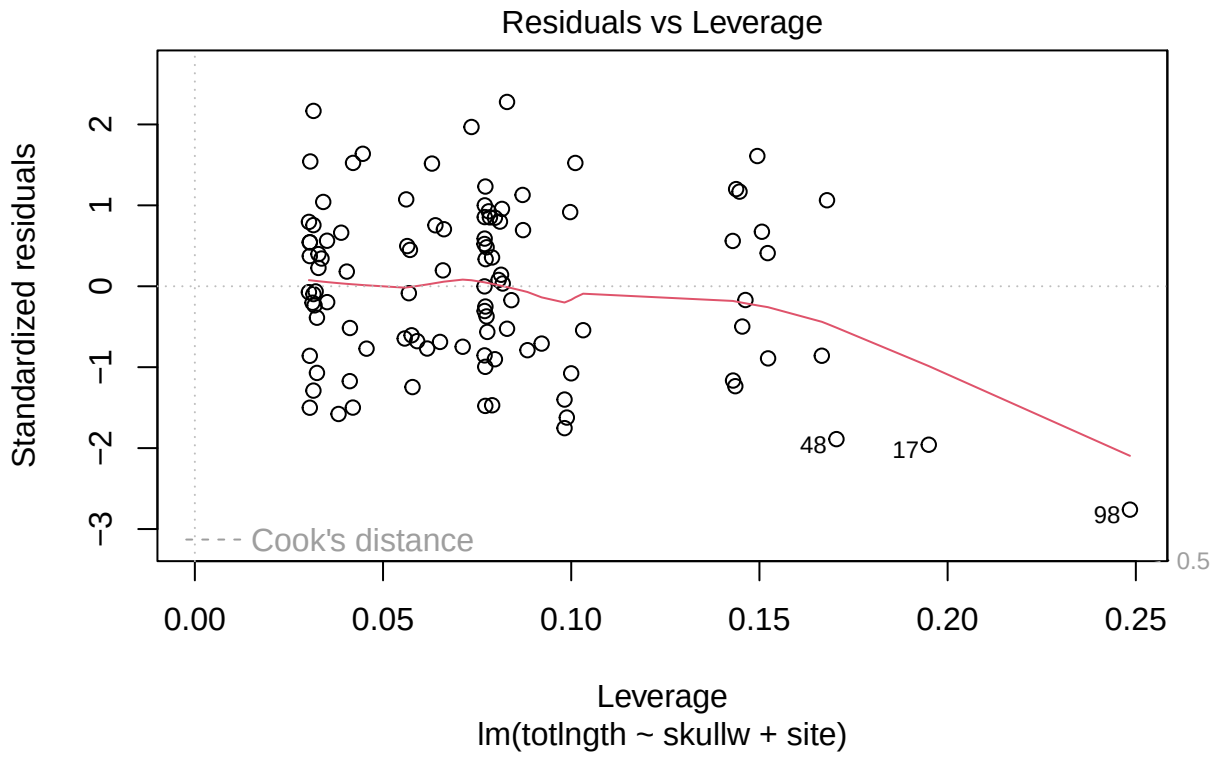
```
##
## Call:
## lm(formula = totlngth ~ skullw + site, data = possum)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.1323 -2.2274  0.0463  2.1842  6.5010
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  62.3488     6.6077   9.436 2.43e-15 ***
## skullw        0.4784     0.1152   4.154 7.09e-05 ***
## site2        -7.0243     1.0008  -7.019 3.17e-10 ***
## site3        -2.4554     1.2558  -1.955 0.053452 .
## site4         0.3833     1.3438   0.285 0.776076
## site5        -2.3268     0.9824  -2.369 0.019861 *
## site6        -3.7362     1.0356  -3.608 0.000492 ***
## site7        -4.0351     0.8734  -4.620 1.19e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.981 on 96 degrees of freedom
## Multiple R-squared:  0.5543, Adjusted R-squared:  0.5218
## F-statistic: 17.06 on 7 and 96 DF,  p-value: 1.779e-14
```

```
plot(mod_account_site)
```

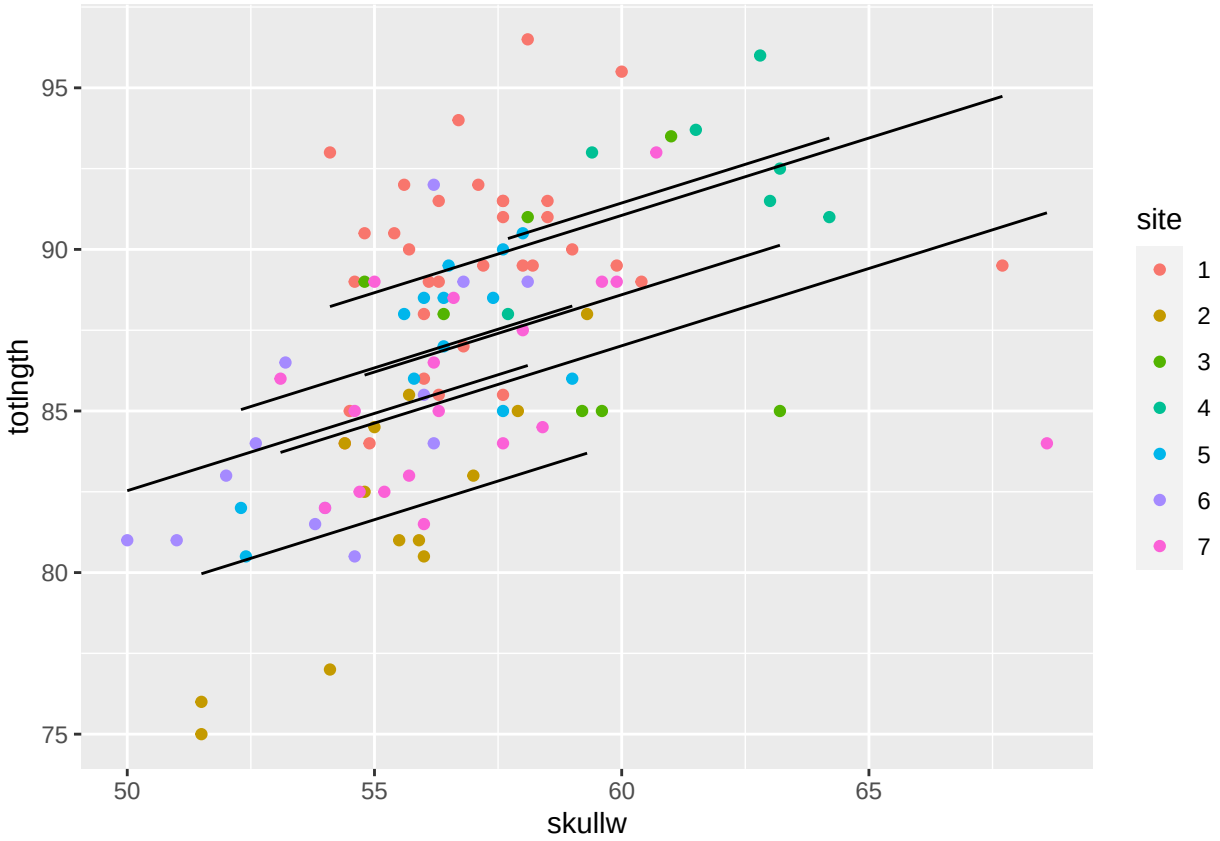

Residuals vs Fitted
85
21
98
Residuals
5
0
−5
80
85
90
95
Fitted values
lm(totlngth ~ skullw + site)


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(totlngth ~ skullw + site)
98
21
85
−2
−1
0
1
2


Scale−Location
98
85
21
√|Standardized residuals|
0.0
0.5
1.0
1.5
80
85
90
95
Fitted values
lm(totlngth ~ skullw + site)




```
#AIC value have decrease but What is interpretation of this model ?
pred_value <- as.data.frame(predict(mod_account_site,possum,se.fit = T))
pred_value <- cbind(pred_value,possum)
ggplot(data = pred_value,aes(x=skullw,y=totlngth))+
  geom_point(aes(col=site))+
  geom_line(aes(x=skullw,y = fit,group=site))
```



Now it is clear that for some random reason the behavior of data is dependent on site.

How to solve such problem

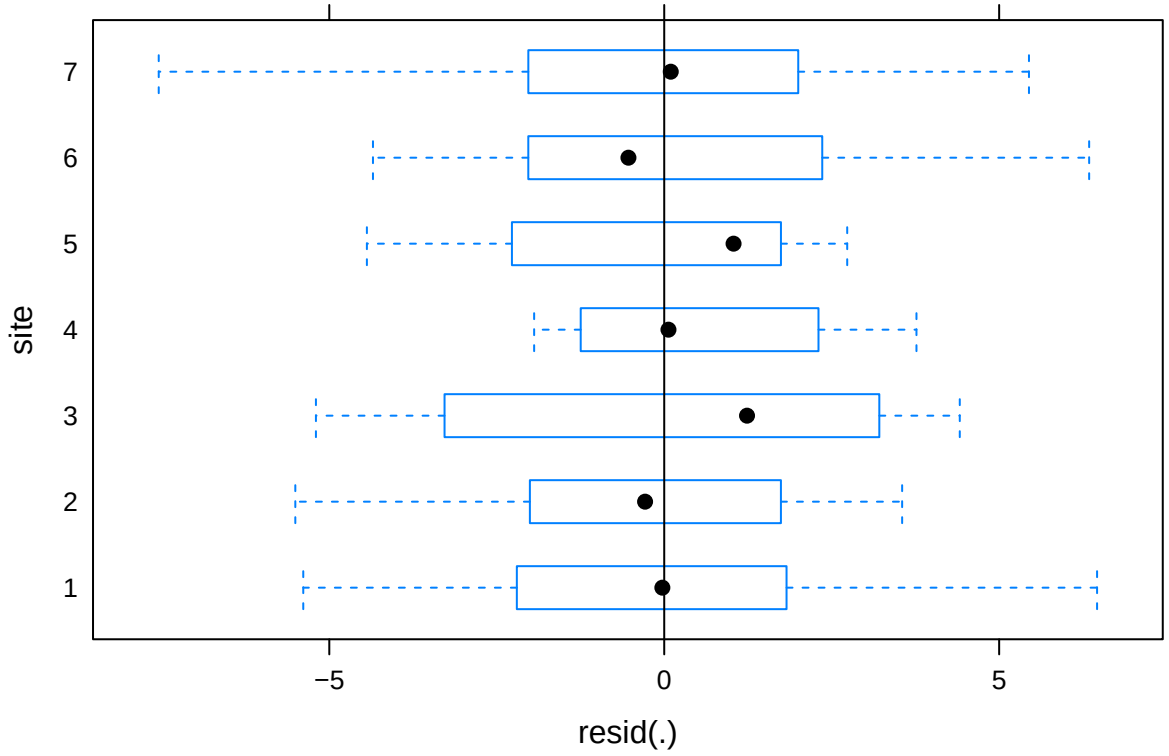
- One problem in using model $y = \alpha + \beta * X1 + \beta * X2$ is

```
model_LMM <- lmer(totlngth ~ skullw + (1|site) ,data = possum[is.na(possum$age)==F,])
summary(model_LMM)
```

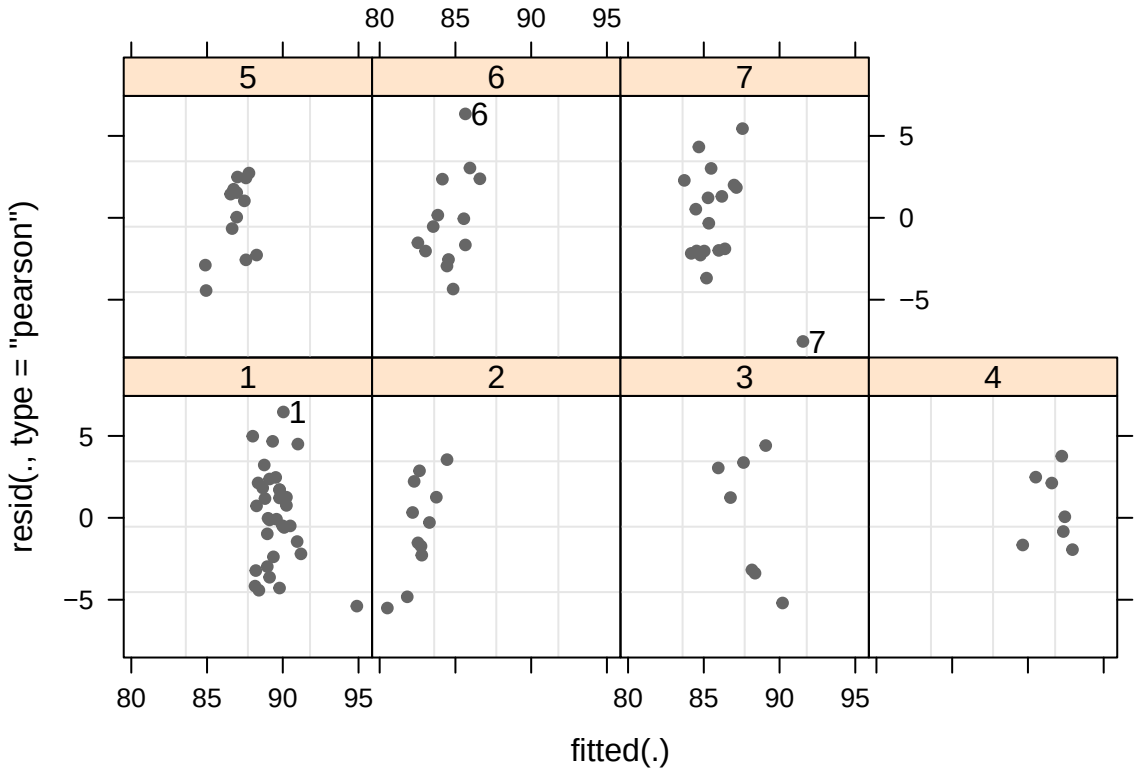
```
## Linear mixed model fit by REML ['lmerMod']
## Formula: totlngth ~ skullw + (1 | site)
##    Data: possum[is.na(possum$age) == F, ]
##
## REML criterion at convergence: 525.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.53879 -0.71822  0.00208  0.74030  2.17385
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  site     (Intercept) 5.360    2.315
##  Residual             8.836    2.972
## Number of obs: 102, groups:  site, 7
##
```

```
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  58.0801     6.4915   8.947
## skullw        0.5052     0.1122   4.501
##
## Correlation of Fixed Effects:
##        (Intr)
## skullw -0.990
```

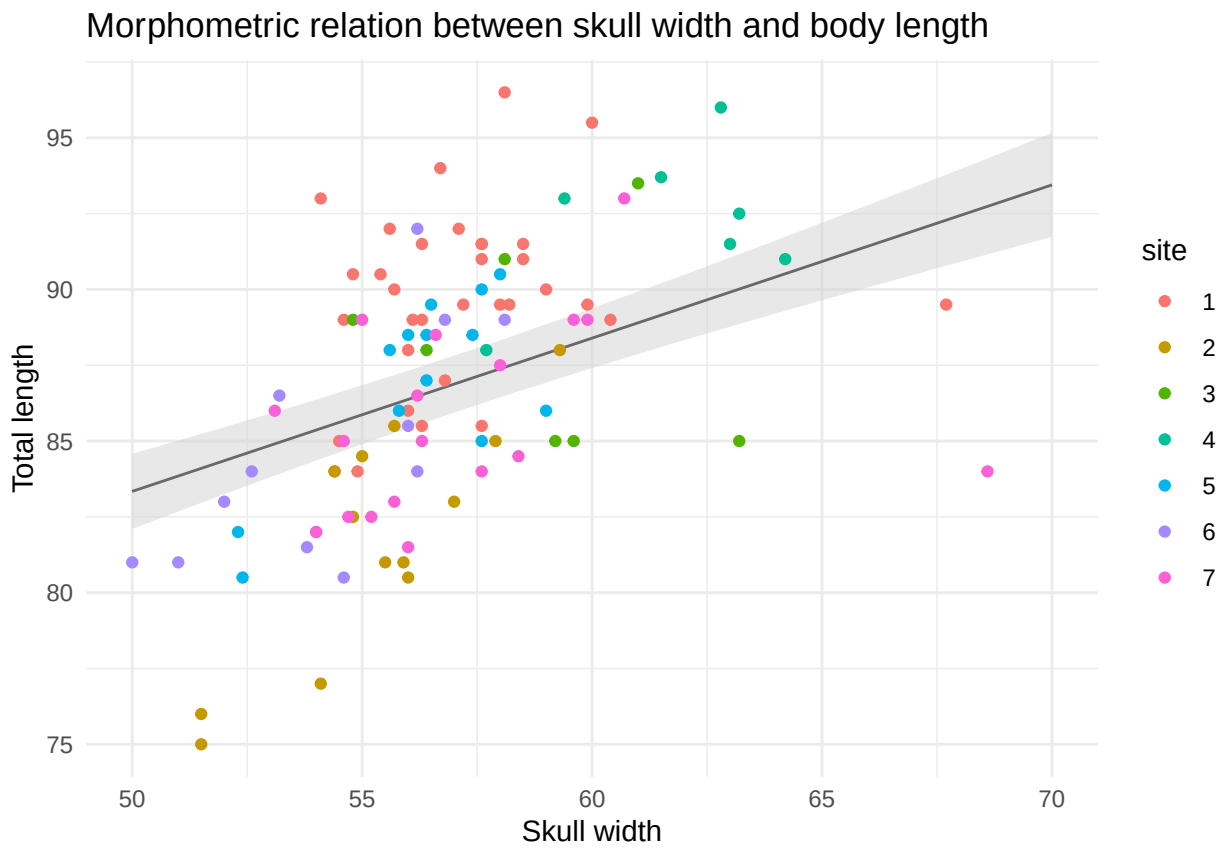
```
## ##getting SE and tabular format for fixed effect.
plot(model_LMM,site~ resid(.), abline = 0 )
```




```
plot(model_LMM, resid(., type = "pearson") ~ fitted(.) |site, id = 0.05,
     adj = -0.3, pch = 20, col = "gray40")
```




```
pred <- ggpredict(model_LMM,terms = "skullw")
ggplot(pred) +
   geom_line(aes(x = x, y = predicted)) +          # slope
   geom_ribbon(aes(x = x, ymin = predicted - std.error, ymax = predicted + std.error),
               fill = "lightgrey", alpha = 0.5) +  # error band
   geom_point(data = possum,                       # adding the raw data (scaled values)
              aes(x = skullw, y = totlngth, colour = site)) +
   labs(x = "Skull width", y = "Total length",
        title = "Morphometric relation between skull width and body length") +
   theme_minimal()
```


Morphometric relation between skull width and body length
Total length
95
90
85
80
75
50
55
60
65
70
Skull width
site
1
2
3
4
5
6
7