

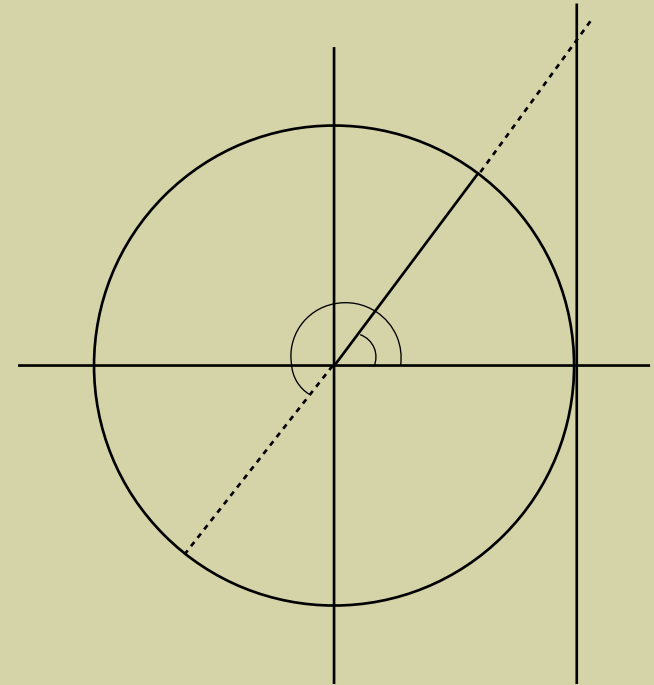
# Άλγεβρα Β' Λυκείου

## Τριγωνομετρικές Εξισώσεις ΙΙΙ

## Η εξίσωση $\varepsilon\varphi x = \alpha$

Έστω η εξίσωση  $\varepsilon\varphi x = \sqrt{3}$

οι λύσεις της εξίσωσης  $\varepsilon\varphi x = \sqrt{3}$  είναι:  $x = \kappa\pi + \frac{\pi}{3}, \quad \kappa \in \mathbb{Z}$



Γενικότερα, αν  $\theta$  είναι μια λύση της εξίσωσης  $\varepsilon\varphi x = \alpha$ , αν δηλαδή ισχύει  $\varepsilon\varphi x = \varepsilon\varphi \theta$ , τότε οι λύσεις της εξίσωσης αυτής είναι:

$$x = \kappa\pi + \theta, \quad \kappa \in \mathbb{Z}$$

Ο ίδιος τύπος λύσεων ισχύει και για την εξίσωση  $\sigma\varphi x = \alpha$ .

## ΠΑΡΑΔΕΙΓΜΑΤΑ

**1<sup>ο</sup>** Να λυθεί η εξίσωση  $\varepsilon\varphi x = -1$

**ΛΥΣΗ**

Επειδή  $\varepsilon\varphi \frac{\pi}{4} = 1$ , ισχύει  $\varepsilon\varphi \left(-\frac{\pi}{4}\right) = -1$ . Έχουμε επομένως  $\varepsilon\varphi x = \varepsilon\varphi \left(-\frac{\pi}{4}\right)$ ,  
οπότε

$$x = \kappa\pi - \frac{\pi}{4}, \quad \kappa \in \mathbb{Z}$$

**2<sup>ο</sup>** Να λυθεί η εξίσωση  $\sigma\varphi x = \sqrt{3}$

**ΛΥΣΗ**

Επειδή  $\sigma\varphi \frac{\pi}{6} = \sqrt{3}$ , έχουμε  $\sigma\varphi x = \sigma\varphi \frac{\pi}{6}$ , οπότε οι λύσεις της εξίσωσης είναι

$$x = \kappa\pi + \frac{\pi}{6}, \quad \kappa \in \mathbb{Z}$$

3. Να λύσετε τις εξισώσεις

i)  $\epsilon\varphi x = 0$       ii)  $\epsilon\varphi x = \frac{\sqrt{3}}{3}$       iii)  $\sigma\varphi x = 1$       iv)  $\sigma\varphi x = \sqrt{3}$

i)  $\epsilon\varphi x = 0 \Leftrightarrow \epsilon\varphi x = \epsilon\varphi 0 \Leftrightarrow x = k\pi, \quad k \in \mathbb{Z}$

ii)  $\epsilon\varphi x = \frac{\sqrt{3}}{3} \Leftrightarrow \epsilon\varphi x = \epsilon\varphi \frac{\pi}{6} \Leftrightarrow x = k\pi + \frac{\pi}{6}, \quad k \in \mathbb{Z}$

iii)  $\sigma\varphi x = 1 \Leftrightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{4} \Leftrightarrow x = k\pi + \frac{\pi}{4}, \quad k \in \mathbb{Z}$

iv)  $\sigma\varphi x = \sqrt{3} \Leftrightarrow \sigma\varphi x = \sigma\varphi \left(\frac{\pi}{6}\right) \Leftrightarrow x = k\pi + \frac{\pi}{6}, \quad k \in \mathbb{Z}$

4. Να λύσετε τις εξισώσεις

$$\text{i) } \varepsilon\varphi x = -\frac{\sqrt{3}}{3}$$

$$\text{ii) } \sigma\varphi x = -\frac{\sqrt{3}}{3}$$

$$\text{i) } \varepsilon\varphi x = -\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi\left(-\frac{\pi}{6}\right) \Leftrightarrow x = k\pi - \frac{\pi}{6}, \quad k \in \mathbb{Z}$$

$$\text{ii) } \sigma\varphi x = -\frac{\sqrt{3}}{3} \Leftrightarrow \sigma\varphi x = \sigma\varphi\left(-\frac{\pi}{3}\right) \Leftrightarrow x = k\pi - \frac{\pi}{3}, \quad k \in \mathbb{Z}$$

6. Να λύσετε τις εξισώσεις

i)  $(\sqrt{3} + \epsilon\phi x)(1 - \epsilon\phi x) = 0$

ii)  $(2\sigma\upsilon\nu x + 1)(\epsilon\phi^2 x - 3)\sigma\phi x = 0$

i)  $(\sqrt{3} + \epsilon\phi x)(1 - \epsilon\phi x) = 0$

$\begin{cases} \sqrt{3} + \epsilon\phi x = 0 \Leftrightarrow \epsilon\phi x = -\sqrt{3} \Leftrightarrow \epsilon\phi x = \epsilon\phi\left(-\frac{\pi}{3}\right) \Leftrightarrow x = k\pi - \frac{\pi}{3} \\ 1 - \epsilon\phi x = 0 \Leftrightarrow \epsilon\phi x = 1 \Leftrightarrow \epsilon\phi x = \epsilon\phi\frac{\pi}{4} \Leftrightarrow x = k\pi + \frac{\pi}{4} \end{cases} \quad k \in \mathbb{Z}$

ii)  $(2\sigma\upsilon\nu x + 1)(\epsilon\phi^2 x - 3)\sigma\phi x = 0$  επειδή είναι γινόμενο ίσο με μηδέν, θα είναι κάποιος παράγοντας μηδέν.

$$2\sigma\upsilon\nu x + 1 = 0 \Leftrightarrow \sigma\upsilon\nu x = -\frac{1}{2} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{2\pi}{3} \Leftrightarrow$$

$$x = 2k\pi + \frac{2\pi}{3}, \quad k \in \mathbb{Z}$$

$$x = 2k\pi - \frac{2\pi}{3}, \quad k \in \mathbb{Z}$$

6. Να λύσετε τις εξισώσεις

i)  $(\sqrt{3} + \epsilon\phi x)(1 - \epsilon\phi x) = 0$

ii)  $(2\sigma\upsilon\nu x + 1)(\epsilon\phi^2 x - 3)\sigma\phi x = 0$

$$\cdot \epsilon\phi^2 x - 3 = 0 \Leftrightarrow \epsilon\phi^2 x = 3 \Leftrightarrow \epsilon\phi x = \sqrt{3} \quad \text{ή} \quad \epsilon\phi x = -\sqrt{3}$$

$$\text{δηλαδή} \quad \epsilon\phi x = \epsilon\phi \frac{\pi}{3} \Leftrightarrow x = k\pi + \frac{\pi}{3}, \quad k \in \mathbb{Z}, \quad \text{ή} \quad \epsilon\phi x = \epsilon\phi\left(-\frac{\pi}{3}\right) \Leftrightarrow$$

$$x = k\pi - \frac{\pi}{3}, \quad k \in \mathbb{Z}$$

$$\cdot \sigma\phi x = 0 \Leftrightarrow \sigma\phi x = \sigma\phi \frac{\pi}{2} \Leftrightarrow x = k\pi + \frac{\pi}{2}, \quad k \in \mathbb{Z}.$$



8. Να λύσετε τις εξισώσεις

i)  $2 \cdot \eta\mu 3x = \sqrt{3}$

ii)  $\sigma\upsilon\nu \frac{x}{5} + 1 = 0$

iii)  $3\varepsilon\varphi \frac{2x}{7} - \sqrt{3} = 0$

i)  $2\eta\mu 3x = \sqrt{3} \Leftrightarrow \eta\mu 3x = \frac{\sqrt{3}}{2} \Rightarrow \eta\mu 3x = \eta\mu \frac{\pi}{3} \quad \text{άρα}$

$$3x = 2k\pi + \frac{\pi}{3} \quad \eta \quad 3x = 2k\pi + \pi - \frac{\pi}{3} \quad k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{2k\pi + \frac{\pi}{3}}{3} = \frac{2k\pi}{3} + \frac{\pi}{9} \quad \eta \quad x = \frac{2k\pi}{3} + \frac{2\pi}{9}, k \in \mathbb{Z}$$

ii)  $\sigma\upsilon\nu \frac{x}{5} + 1 = 0 \Leftrightarrow \sigma\upsilon\nu \frac{x}{5} = -1 \Rightarrow \sigma\upsilon\nu \frac{x}{5} = \sigma\upsilon\nu \pi \Rightarrow \frac{x}{5} = 2k\pi + \pi \quad \eta$

$$\frac{x}{5} = 2k\pi - \pi, k \in \mathbb{Z}. \quad \text{δηλαδή} \quad x = 10k\pi + 5\pi \quad \eta \quad x = 10k\pi - 5\pi$$

iii)  $3\varepsilon\varphi \frac{2x}{7} - \sqrt{3} = 0 \Leftrightarrow 3\varepsilon\varphi \frac{2x}{7} = \sqrt{3} \Leftrightarrow \varepsilon\varphi \frac{2x}{7} = \frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi \frac{2x}{7} = \varepsilon\varphi \frac{\pi}{6}$

$$\frac{2x}{7} = k\pi + \frac{\pi}{6}, k \in \mathbb{Z}, \quad 2x = 7k\pi + \frac{7\pi}{6} \Leftrightarrow x = 7k\frac{\pi}{2} + \frac{7\pi}{12} \quad k \in \mathbb{Z}$$

9. Να λύσετε τις εξισώσεις

i)  $\eta\mu\left(x + \frac{\pi}{3}\right) = -1$     ii)  $2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1$     iii)  $\epsilon\phi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3}$

i)  $\eta\mu\left(x + \frac{\pi}{3}\right) = \eta\mu\left(-\frac{\pi}{2}\right)$  άρα  $x + \frac{\pi}{3} = 2k\pi - \frac{\pi}{2}$  ή  $x + \frac{\pi}{3} = 2k\pi + \pi - \left(-\frac{\pi}{2}\right)$   
 $\Leftrightarrow x = 2k\pi - \frac{2\pi}{2} - \frac{3\pi}{3} = 2k\pi - \frac{2\pi+3\pi}{6}$   $x = 2k\pi + \frac{3\pi}{2} - \frac{3\pi}{3} = 2k\pi + \frac{6\pi-3\pi}{6}$   
 $\Leftrightarrow x = 2k\pi - \frac{5\pi}{6}$  ή  $x = 2k\pi + \frac{3\pi}{6} = 2k\pi + \frac{\pi}{2}$

ii)  $2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1 \Leftrightarrow \sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow \sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = \sigma\upsilon\nu\left(\frac{\pi}{3}\right) \Leftrightarrow$   
 $\Leftrightarrow 3x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{3}$  ή  $3x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{3}$   
 $\Leftrightarrow 3x = 2k\pi + \frac{3\pi}{3} + \frac{\pi}{4}$  ή  $3x = 2k\pi + \frac{3\pi}{4} - \frac{\pi}{3}$   
 $\Leftrightarrow 3x = 2k\pi + \frac{7\pi}{12}$  ή  $3x = 2k\pi - \frac{\pi}{12}$   
 $\Leftrightarrow x = 2k\frac{\pi}{3} + \frac{7\pi}{36}$  ή  $x = 2k\frac{\pi}{3} - \frac{\pi}{36} \quad k \in \mathbb{Z}$

9. Να λύσετε τις εξισώσεις

$$\text{i) } \eta\mu\left(x + \frac{\pi}{3}\right) = -1 \quad \text{ii) } 2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1 \quad \text{iii) } \epsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3}$$

$$\text{iii) } \epsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3} \Leftrightarrow \epsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \epsilon\varphi\frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{4} - 5x = k\pi + \frac{\pi}{3} \Leftrightarrow -5x = k\pi + \frac{\pi}{3} - \frac{\pi}{4}$$

$$\Leftrightarrow -5x = k\pi + \frac{\pi}{12}$$

$$\Leftrightarrow x = -\frac{k\pi}{5} - \frac{\pi}{60} \quad k \in \mathbb{Z}$$

10. Να λύσετε τις εξισώσεις

i)  $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

ii)  $2\sigma\upsilon\nu^2x + 3\sigma\upsilon\nu x - 2 = 0$     iii)  $3\varepsilon\varphi^2t = 3 + 2\sqrt{3}\varepsilon\varphi t$

i)  $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$     Θέτω  $\eta\mu\omega = x$     οπότε:

$$2x^2 + x - 1 = 0 \quad \Delta = 1^2 - 4 \cdot 2 \cdot (-1) = 1 + 8 = 9$$

$$x_{1,2} = \frac{-1 \pm 3}{4} = \begin{cases} x_1 = \frac{2}{4} = \frac{1}{2} \\ x_2 = \frac{-4}{4} = -1 \end{cases}$$

$$\bullet \quad \eta\mu\omega = \frac{1}{2} \Leftrightarrow \eta\mu\omega = \eta\mu\frac{\pi}{6} \Leftrightarrow \omega = 2k\pi + \frac{\pi}{6} \quad \text{ή} \quad \omega = 2k\pi + \pi - \frac{\pi}{6}$$

$$\Leftrightarrow \omega = 2k\pi + \frac{\pi}{6} \quad \text{ή} \quad \omega = 2k\pi + \frac{5\pi}{6}$$

$$\bullet \quad \eta\mu\omega = -1 \Leftrightarrow \eta\mu\omega = \eta\mu\left(-\frac{\pi}{2}\right) \Leftrightarrow \omega = 2k\pi - \frac{\pi}{2} \quad \text{ή} \quad \omega = 2k\pi + \pi - \left(-\frac{\pi}{2}\right)$$

$$\omega = 2k\pi - \frac{\pi}{2} \quad \text{ή} \quad \omega = 2k\pi + \frac{3\pi}{2}$$

$$\text{συνολικά} \quad \omega = 2k\pi - \frac{\pi}{2}, \quad k \in \mathbb{Z}.$$

10. Να λύσετε τις εξισώσεις

i)  $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

ii)  $2\sigma\upsilon\nu^2 x + 3\sigma\upsilon\nu x - 2 = 0$     iii)  $3\varepsilon\varphi^2 t = 3 + 2\sqrt{3}\varepsilon\varphi t$

ii)  $2\sigma\upsilon\nu^2 x + 3\sigma\upsilon\nu x - 2 = 0$     Θέτω  $\sigma\upsilon\nu x = \omega$     οπότε :

$$2\omega^2 + 3\omega - 2 = 0 \quad \Delta = 9 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25 \quad \text{οπότε :}$$

$$\omega_{1,2} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4} \quad \begin{cases} \frac{2}{4} = \frac{1}{2} \\ -\frac{8}{4} = -2 \end{cases}$$

$$\text{άρα } \sigma\upsilon\nu x = \frac{1}{2} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu\left(\frac{\pi}{3}\right)$$

$$x = 2k\pi + \frac{\pi}{3} \quad \eta \quad x = 2k\pi - \frac{\pi}{3}$$

•  $\sigma\upsilon\nu x = -2$  απορρίνεται γιατί δε γίνεται το  $\sigma\upsilon\nu$  να είναι  $-2$ .

(είναι από  $-1, \dots, 1$ )

10. Να λύσετε τις εξισώσεις

i)  $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

ii)  $2\sigma\upsilon\nu^2x + 3\sigma\upsilon\nu x - 2 = 0$     iii)  $3\varepsilon\varphi^2t = 3 + 2\sqrt{3}\varepsilon\varphi t$

iii)  $3\varepsilon\varphi^2t = 3 + 2\sqrt{3}\varepsilon\varphi t$     Θέλω  $\varepsilon\varphi t = \omega$

$$3\omega^2 = 3 + 2\sqrt{3}\omega \Rightarrow 3\omega^2 - 2\sqrt{3}\omega - 3 = 0$$

$$\Delta = 4 \cdot 3 - 4 \cdot 3 \cdot (-3) = 12 + 36 = 48$$

$$\text{άρα } \omega = \frac{2\sqrt{3} \pm \sqrt{48}}{6} = \frac{2\sqrt{3} \pm \sqrt{3 \cdot 16}}{6} = \frac{2\sqrt{3} \pm \sqrt{3} \cdot \sqrt{16}}{6} = \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} \begin{cases} \frac{6\sqrt{3}}{6} = \sqrt{3} \\ \frac{-2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3} \end{cases}$$

$$\text{οπότε } \bullet \omega = \sqrt{3} \Leftrightarrow \varepsilon\varphi t = \sqrt{3} \Leftrightarrow \varepsilon\varphi t = \varepsilon\varphi \frac{\pi}{3} \Leftrightarrow t = k\pi + \frac{\pi}{3}, \quad k \in \mathbb{Z}$$

$$\bullet \omega = -\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi t = -\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi t = \varepsilon\varphi\left(-\frac{\pi}{6}\right) \Leftrightarrow t = k\pi + \left(-\frac{\pi}{6}\right)$$

$$\Leftrightarrow t = k\pi - \frac{\pi}{6}, \quad k \in \mathbb{Z}$$

# 11. Να λύσετε τις εξισώσεις

i)  $\eta\mu^2 x + 5\sigma\upsilon\nu^2 x = 4$

ii)  $\epsilon\phi x \cdot \sigma\phi 2x = 1$

i)  $\eta\mu^2 x + 5\sigma\upsilon\nu^2 x = 4$  . Θυμάμαι την ταυτότητα όπου  $\eta\mu^2 x + \sigma\upsilon\nu^2 x = 1$

$\Leftrightarrow \eta\mu^2 x = 1 - \sigma\upsilon\nu^2 x$  και αντικαθιστώ στην εξίσωση

$\eta\mu^2 x$  +  $5\sigma\upsilon\nu^2 x = 4 \Leftrightarrow 1 - \sigma\upsilon\nu^2 x + 5\sigma\upsilon\nu^2 x = 4 \Leftrightarrow 4\sigma\upsilon\nu^2 x = 3 \Leftrightarrow \sigma\upsilon\nu^2 x = \frac{3}{4}$

$\Leftrightarrow \sigma\upsilon\nu x = \pm \frac{\sqrt{3}}{2}$

•  $\sigma\upsilon\nu x = \frac{\sqrt{3}}{2} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{6} \Leftrightarrow x = 2k\pi - \frac{\pi}{6}$   
 $x = 2k\pi - \frac{\pi}{6}$  }  $k \in \mathbb{Z}$

•  $\sigma\upsilon\nu x = -\frac{\sqrt{3}}{2} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \left(\pi - \frac{\pi}{6}\right) \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{5\pi}{6}$

$\Leftrightarrow x = 2k\pi + \frac{5\pi}{6}$  ή

$x = 2k\pi - \frac{5\pi}{6} \quad k \in \mathbb{Z}$