1. Poisson problem

We wish to solve the equation $-\Delta\phi = f$ in a mixed formulation with suitable boundary conditions in a bounded domain Ω. The mixed formulation we worked with so far is

$$
\left\{
\begin{array}{lll}
(\boldsymbol{v}, \boldsymbol{u}) + (\operatorname{div}\, \boldsymbol{v}, \phi) & = 0 & \forall \boldsymbol{v} \in H(\operatorname{div}; \Omega) \\
(\varphi, \operatorname{div}\, \boldsymbol{u}) & = (\varphi, f) & \forall \varphi \in L^2(\Omega)
\end{array}
\right. \tag{1}
$$

Suppose we have a solution for the equation div
$bmu_f = f$, then we can write the solution as $\boldsymbol{u} = \boldsymbol{u}_f + \boldsymbol{u}_0$. If we fill this in (1) we obtain

$$
\left\{
\begin{array}{lll}
(\boldsymbol{v}, \boldsymbol{u}_0) + (\operatorname{div}\, \boldsymbol{v}, \phi) & = -(\boldsymbol{v}, \boldsymbol{u}_f) & \forall \boldsymbol{v} \in H(\operatorname{div}; \Omega) \\
(\varphi, \operatorname{div}\, \boldsymbol{u}_0) & = 0 & \forall \varphi \in L^2(\Omega)
\end{array}
\right. \tag{2}
$$

Now we have shifted the contribution from the right hand side function from the conservation law to the constitutive law.

Note that $\boldsymbol{u}_f$ is not unique, so a follow-up question could be: Does it make a difference which $\boldsymbol{u}_f$ we choose and if so, are there any indications which $\boldsymbol{u}_f$ is the best.

I expect that the rate of convergence for both methods is the same, but it might be that the error in the second case is lower because we insert more a priori knowledge about the solution. We can test all this for a simple square domain. Then we can look at a domain with a hole in the middle and, time permitting, we could insert an airfoil.

References

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