Delft University of Technology

Unsteady aerodynamics of thin airfoils

Rotor wake aerodynamics assignment 3

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1. Introduction

With unsteady flow imposed on an airfoil, the unsteady aerodynamics need to be considered and is modelled in this report. A flat plate is modelled together with a sinusoidal behaviour of its angle of attack at varying reduced frequency. The resultant loading and velocity fields are analysed and compared against the steady flow counterpart.

2. Validation of code

2.1. Code implementation

First, how the code has been implemented is shown in Figure 1 as a flowchart whereby the steps taking in the code can be observed.

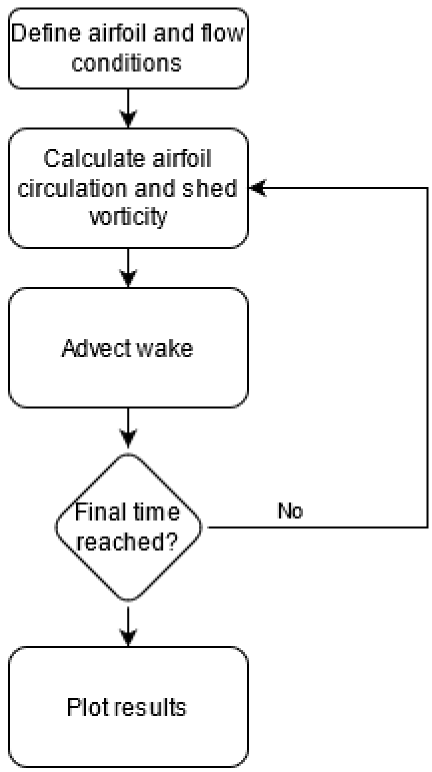

Fig. 1 Flowchart of how the code is implemented

2.2. Steady state lift coefficient

The lift coefficient of a flat plate was calculated at 5 degrees angle of attack. The calculated value was compared to the analytical result of flat plate lifting theory. It was found that the error stays constant with panel count, with less than 10^-4 of error magnitude.

2.3. Impulse response

To test the vorticity shedding module, the lift response to a step change in angle of attack was computed and compared to the analytical Wagner solution. This can be seen in Figure 2.

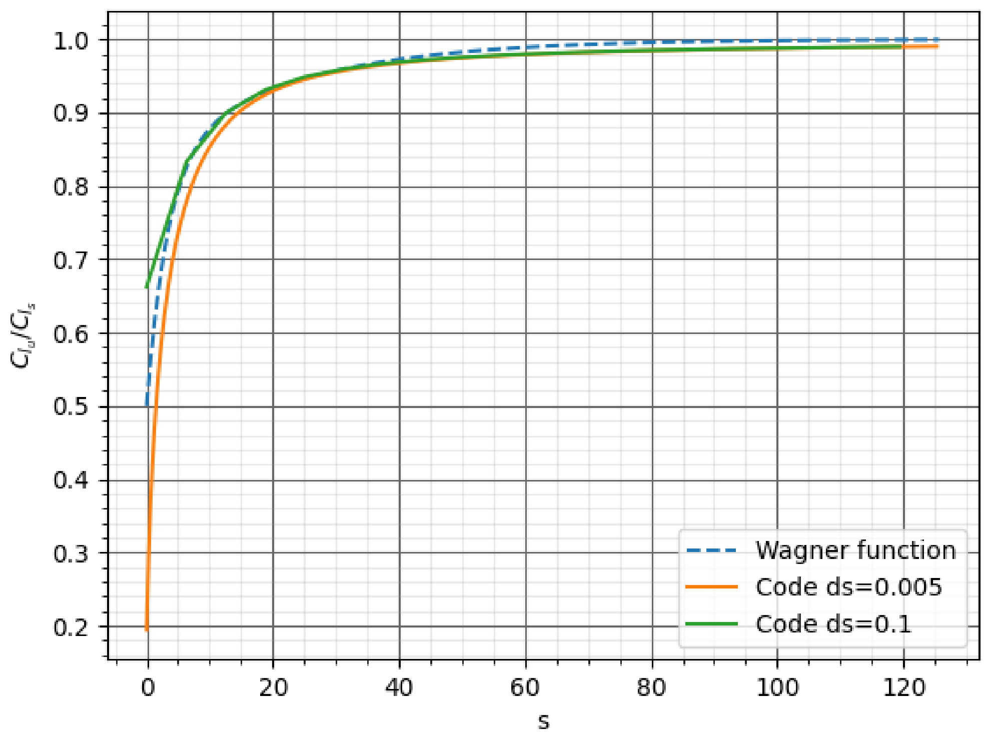

Fig. 2 Comparison between code and Wagner's function

it can be seen that the code follows the Wagners fucntion quite well. Furthermore a better fit with decreasing stepsize can be observed.

3. Modelling cases 1 and 2

Two given flow cases will be studied in this section which are namely: Steady flow at different angles of attack and Pitching airfoil at different reduced frequencies.

3.1. First flow case: Steady flow at different angles of attack

As for this steady flow case, a flat plate is modelled using a vortex distribution over the panel for different values of angles of attack. The lift polar is presented in Figure 3 in which the slope of steady case is presented alongside the ellipses of the unsteady cases at three different values of reduced frequency k. It can be seen that the flat plate has zero lift coefficient at zero angle of attack as it was expected.

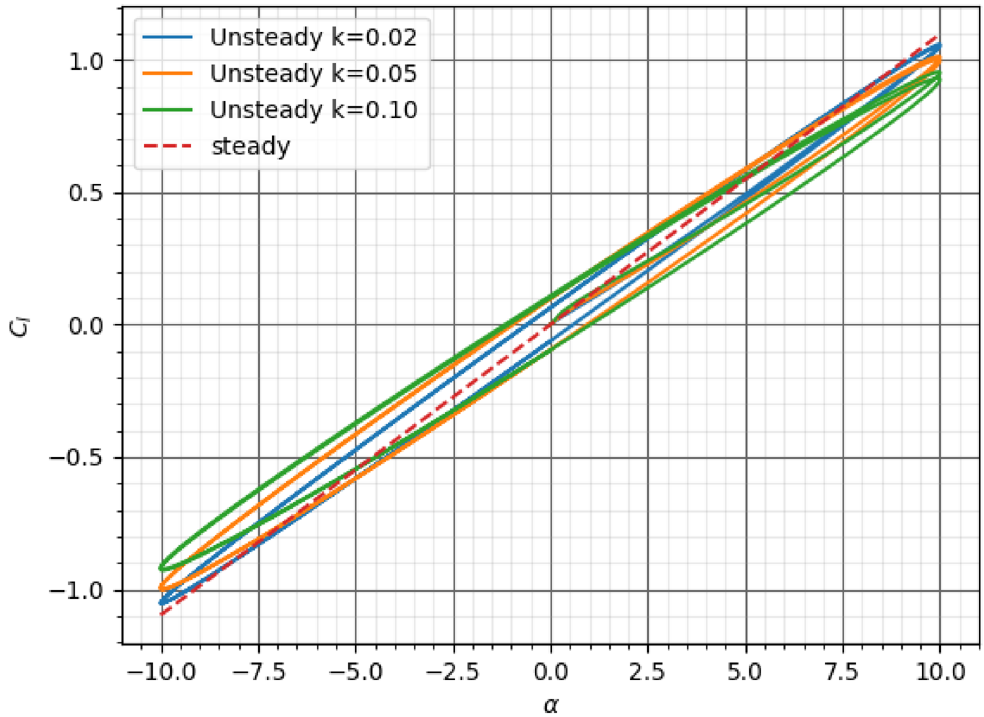

Fig. 3 Plot of lift coefficient C_l against angle of attack α

The velocity field around the flat plate is also plotted for three different values of angle of attack: -10, 0 and 10 degrees as shown in Figure 4, Figure 5 and Figure 6 respectively.

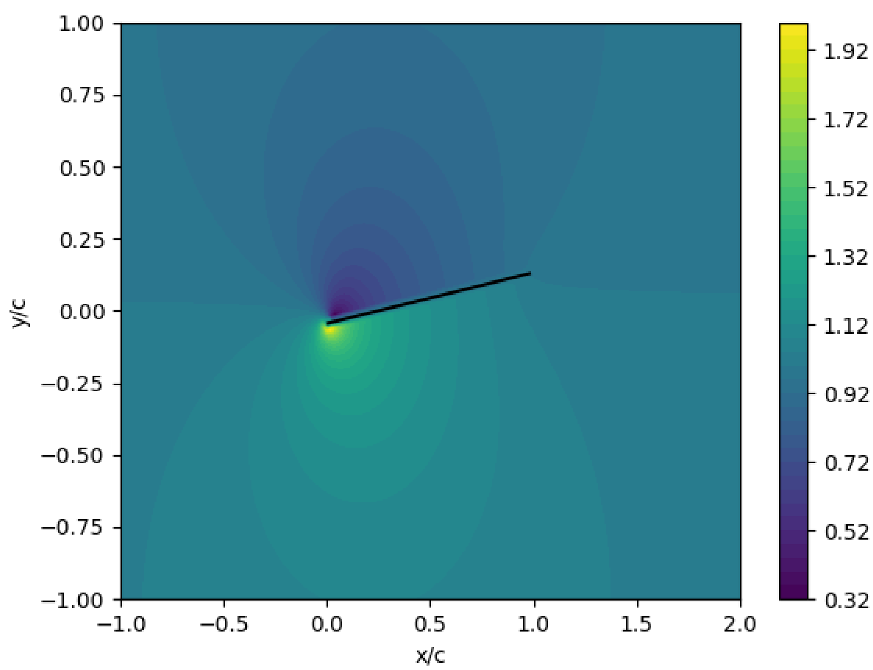

Fig. 4 Velocity field around the flat plate at $\alpha = -10°$

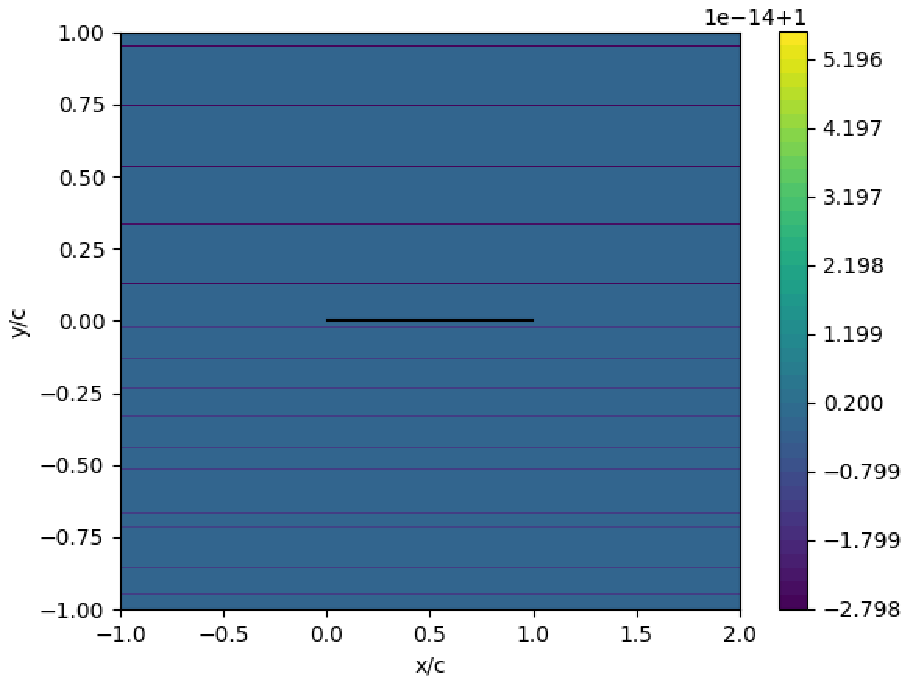

Fig. 5 Velocity field around the flat plate at $\alpha = 0°$

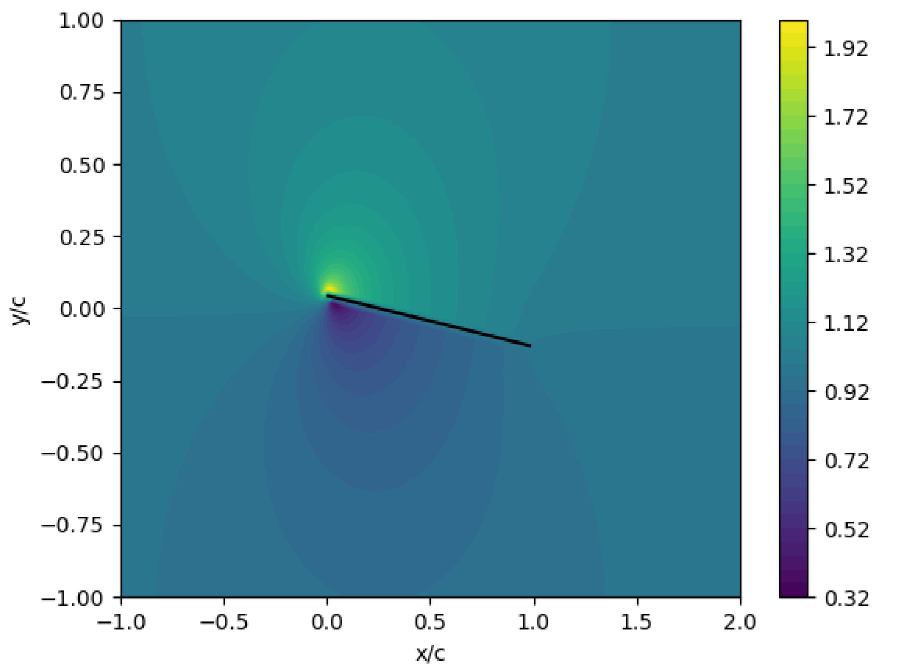

Fig. 6 Velocity field around the flat plate at $\alpha = 10°$

It can be observed that the regions of high and low velocities for the two extreme α cases are opposite of each other whereby the high velocity region is on the lower side of the leading edge for $\alpha = -10$ degrees and it is on the upper side of the leading edge for $\alpha = 10$ degrees. This is due to the circulation that is induced at the leading edge. The velocity magnitudes then gradually decreases as the coordinate gets further from those points of high velocity and thus the presence of contours as shown in the plots. Moreover, since this is a steady flow case, a symmetric circulation in the wake was not expected and is also not present in the results obtained. For the case of $\alpha = 0$ degree, horizontal lines among the uniform velocity field can be observed and they are due to the presence of numerical inaccuracies. Furthermore, circulation is not observed for this case.

Next, the pressure field around the flat plate is presented for the same set of angles of attack in Figure 7, Figure 8 and Figure 9.

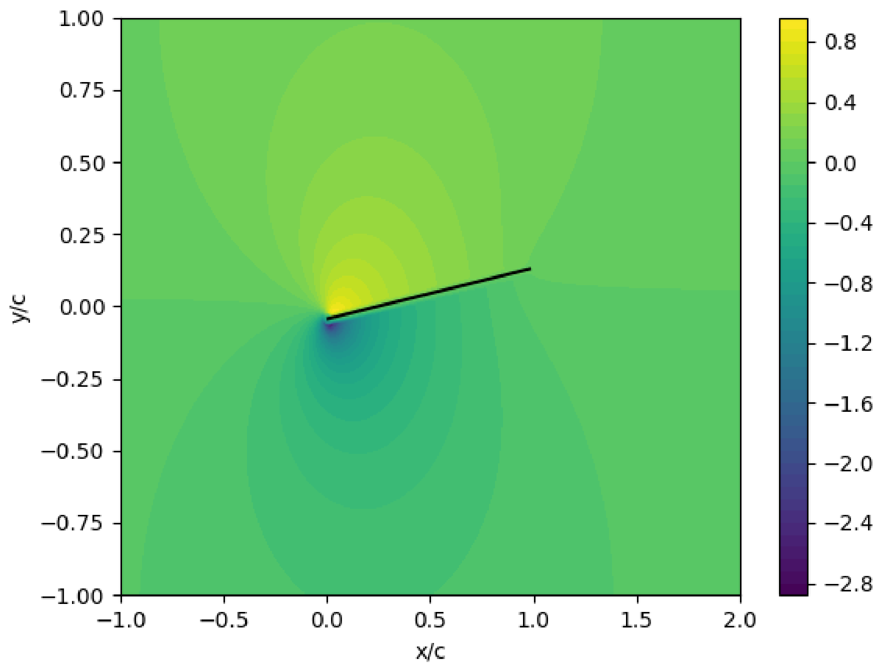

Fig. 7 Pressure field around the flat plate at $\alpha = -10°$

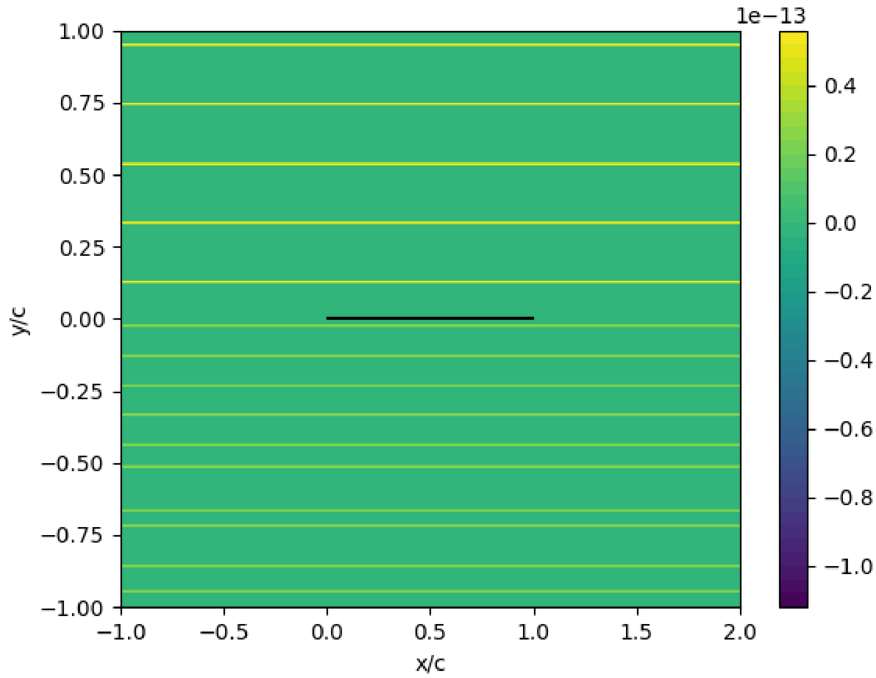

Fig. 8 Pressure field around the flat plate at $\alpha = 0°$

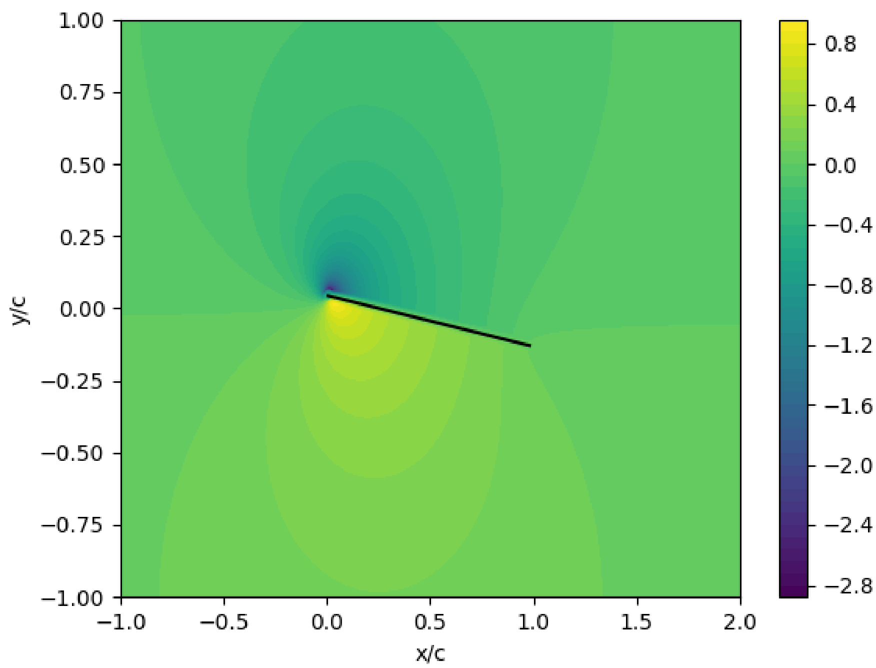

Fig. 9 Pressure field around the flat plate at $\alpha = 10°$

Apart from the exact magnitudes, the pressure field exhibits almost identical behaviour as it was describes for the velocity field.

3.2. Second flow case: Pitching airfoil at different reduced frequencies

The second flow case involves modelling the unsteady flow around the flat plate that is in pitching motion which is represented by a sine function. This is done for different values of reduced frequency k and the load cycle is shown in Figure 10.

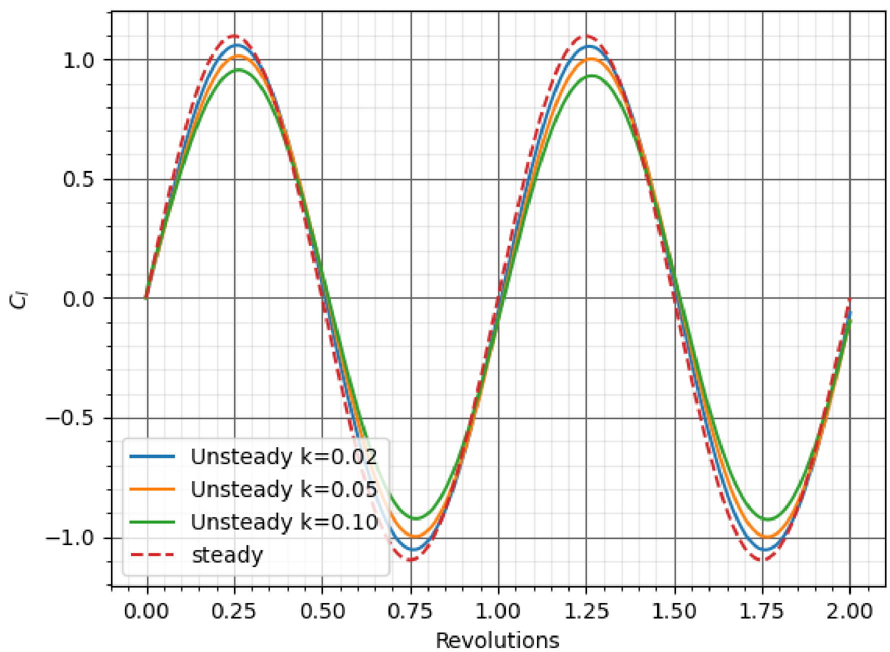

Fig. 10 Load cycle in unsteady flow

It can be observed that with an increasing reduced frequency in the unsteady case, the amplitudes of the sinusoidal behaviour of lift coefficient C_l gets smaller, thus further from the steady case. This behaviour is also shown in Figure 3 where the plot of lift coefficient against angle of attack is presented and for the unsteady cases, they oscillate around the line of steady case, going from angle of attack of -10 to 10, forming ellipses.

For this unsteady flow case, three angles of attacks are used to evaluate the velocity fields, just like it was done for the steady case in the previous flow case presented. For each angle of attack, results of reduced frequency of $k = 0.02$, 0.05 and 0.1 are presented and analysed.

$\alpha = -10$ **degrees** In Figure 11, Figure 12 and Figure 13, the velocity field around the flat plate at $k = 0.02$, 0.05 and 0.1 are presented respectively.

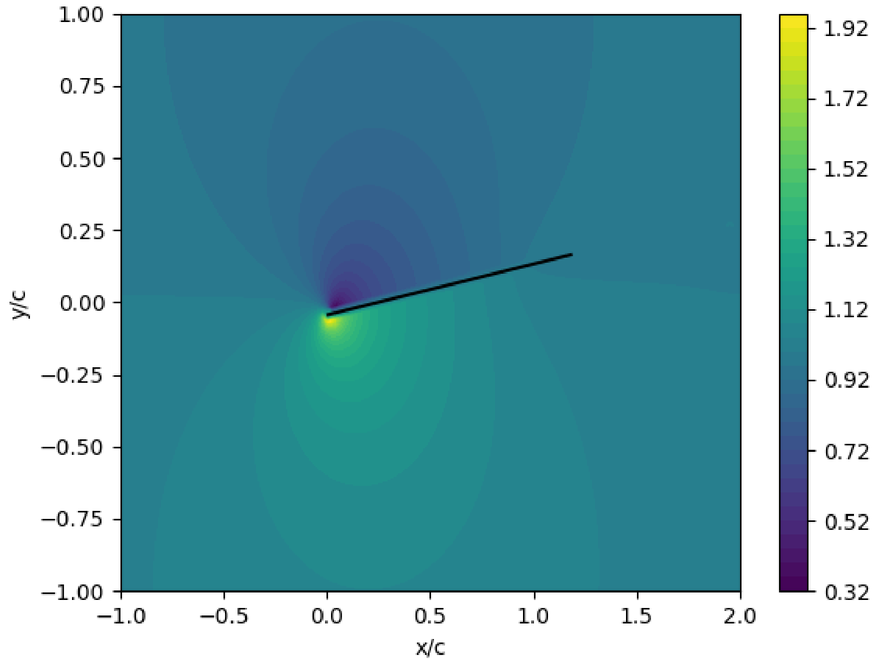

Fig. 11 Velocity field around the flat plate at $\alpha = -10°$, $k = 0.02$

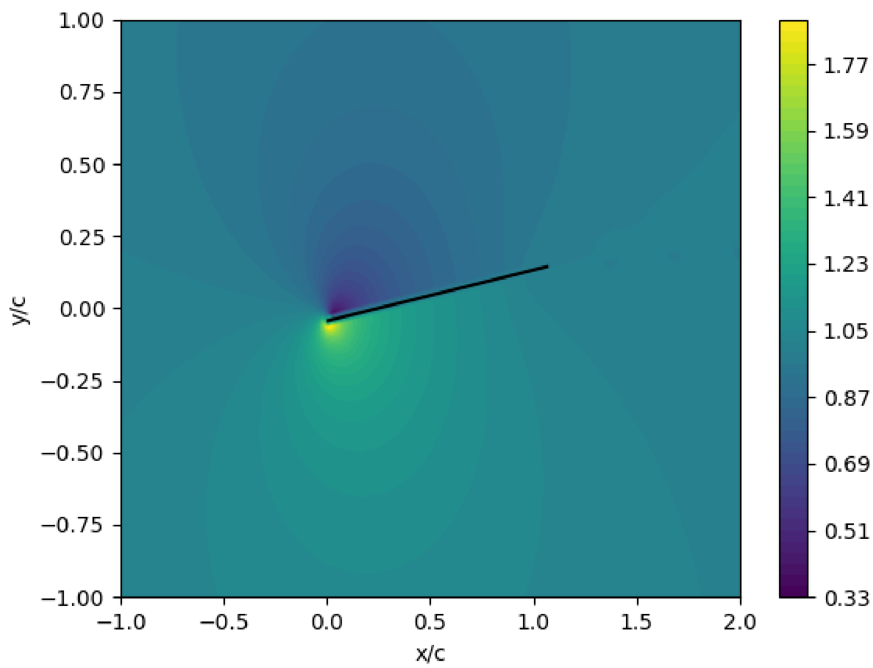

Fig. 12 Velocity field around the flat plate at $\alpha = -10°$, $k = 0.05$

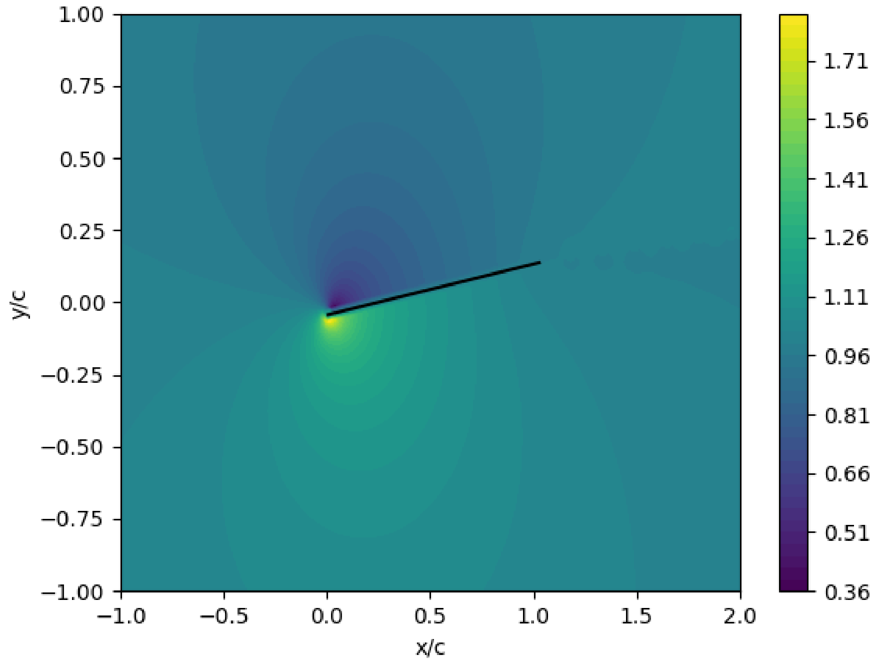

Fig. 13 Velocity field around the flat plate at $\alpha = -10°$**,** $k = 0.1$

For this case of $\alpha = -10$ degrees, the velocity field does not portray any significant difference from the steady flow counter part. One minor difference observed is that the range of velocities is smaller for the unsteady case and gets smaller with increasing k and this follows the trend in loading that was discussed early for Figure 10. Furthermore, since for a sinusoidal behaviour of C_l the maximums and minimums that correspond to $\alpha = 10$ and -10 respectively have time derivative of 0, there is no rate of change of circulation present. Thus, it is expected and also observed from the results that there is no symmetrical circulation that is present in the wake.

$\alpha = 0$ **degree** Similarly for angle of attack of 0 degree, the results are presented in Figure 14, Figure 15 and Figure 16.

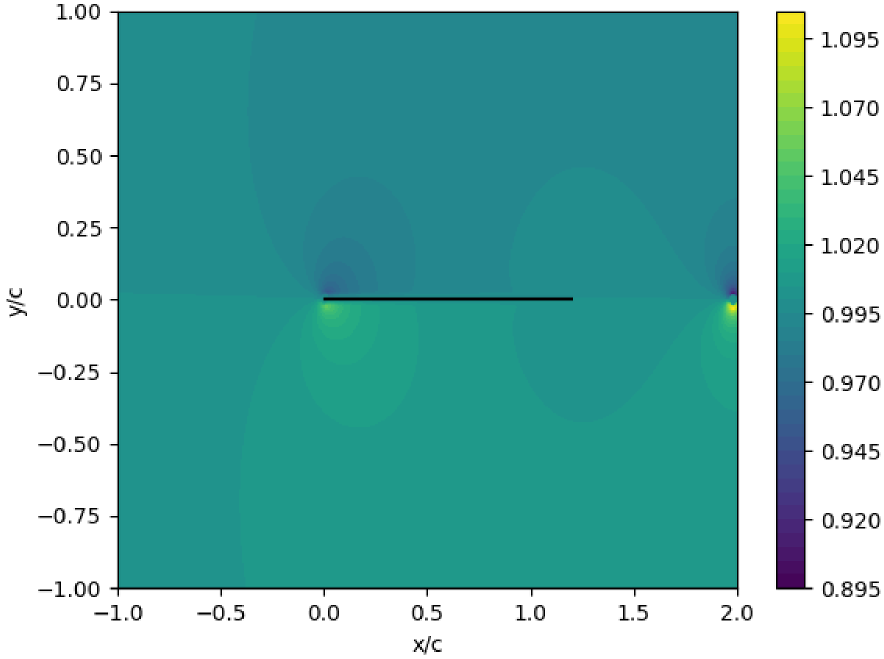

Fig. 14 Velocity field around the flat plate at $\alpha = 0°$**,** $k = 0.02$

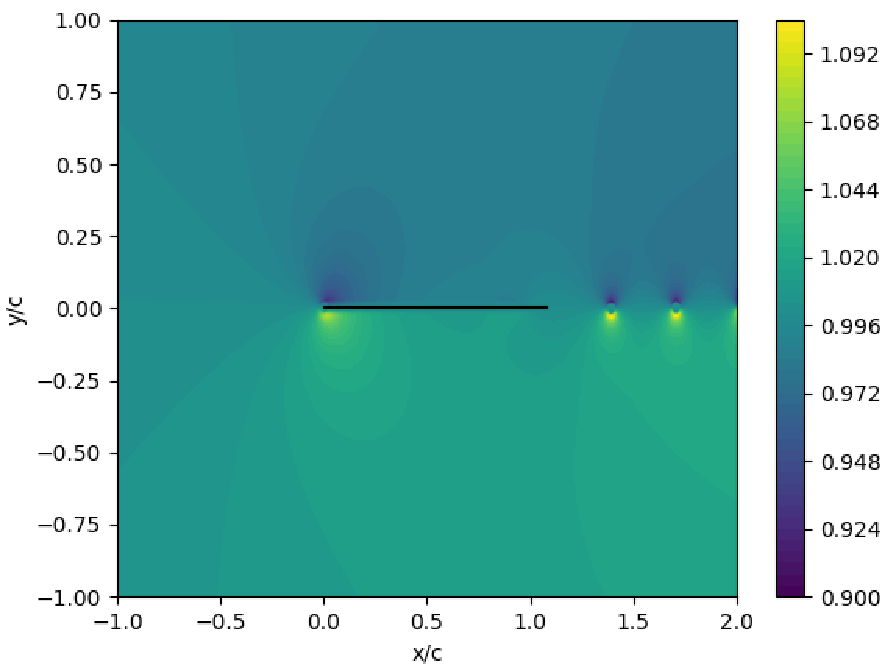

Fig. 15 Velocity field around the flat plate at $\alpha = 0°$**,** $k = 0.05$

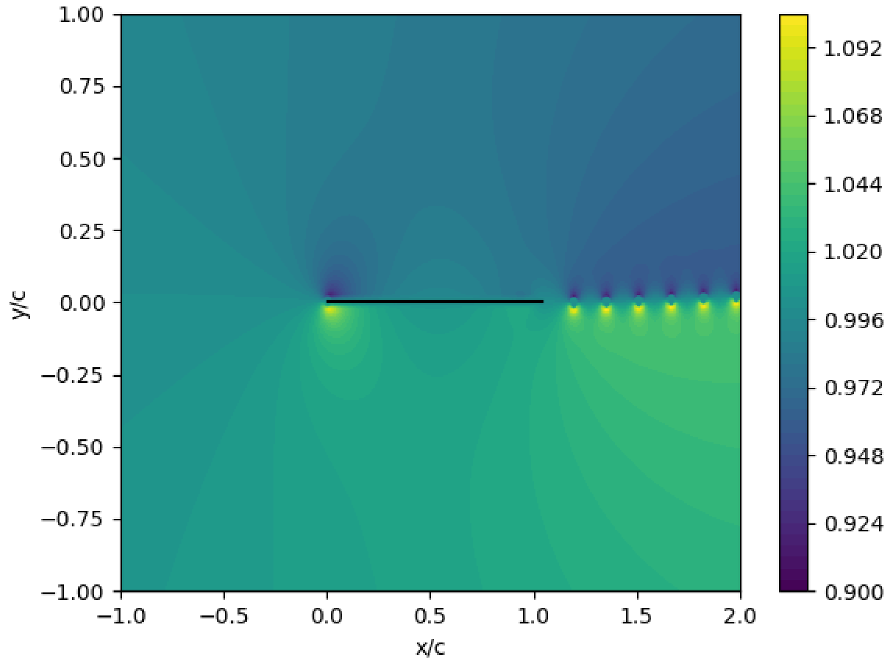

Fig. 16 Velocity field around the flat plate at $\alpha = 0°$, $k = 0.1$

A large difference in velocity fields of unsteady flow at $\alpha = 0$ from the steady case can be observed. There are circulations present in the leading edge of the flat plate which the magnitude increases for increasing reduced frequency k. Furthermore, increasing k results in larger number of circulations in the wake that are more closely packed. This is due to non-zero time derivative of angle of attack at $\alpha = 0$ thus inducing rate of change of circulation at the leading edge of airfoil. Thus, according to Kelvin's theorem, the symmetrical circulation is present in the wake. Moreover, with larger k value, the oscillations occur at a higher rate and hence the presence of higher number of circulations that are shed. Summing up all the circulations in the domain would theoretical result in 0 magnitude.

$\alpha = 10$ **degrees** Likewise for the last angle of attack under analysis of 10 degrees, the results are presented in Figure 17, Figure 18 and Figure 19.

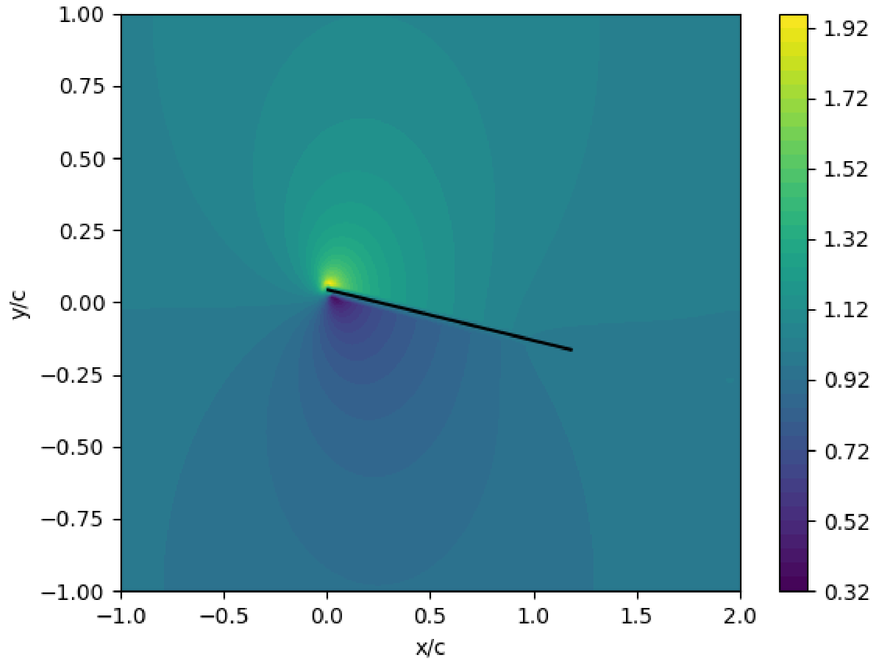

Fig. 17 Velocity field around the flat plate at $\alpha = 10°$**,** $k = 0.02$

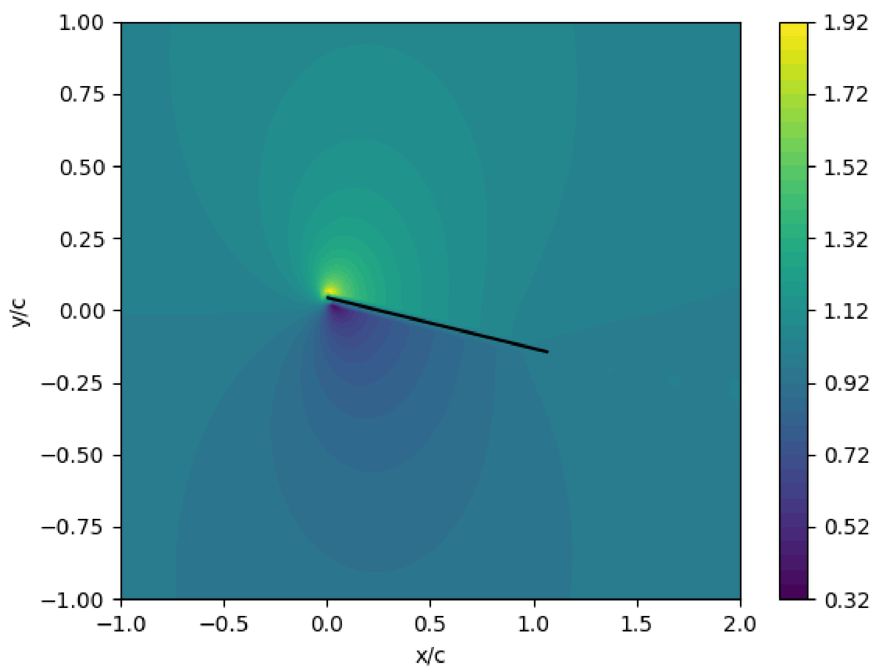

Fig. 18 Velocity field around the flat plate at $\alpha = 10°$**,** $k = 0.05$

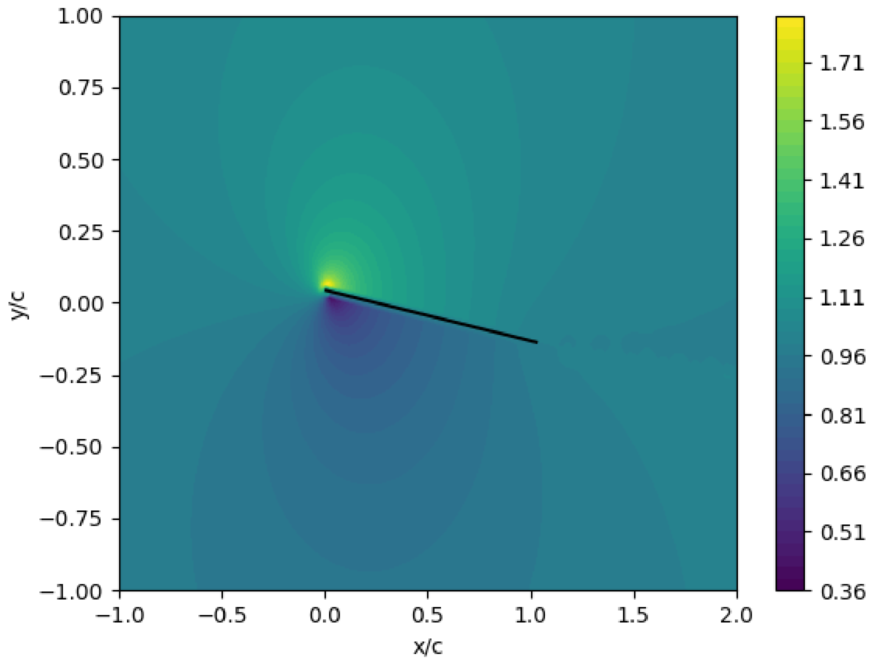

Fig. 19 Velocity field around the flat plate at $\alpha = 10°$, $k = 0.1$

The observations for $\alpha = 10$ is exactly the same as ones made for $\alpha = -10$ since now it is at the maximum of the sinusoidal behaviour of angle of attack, again resulting in zero time derivative which does not induce any rate of change of circulation at the leading edge of the airfoil.

4. Limitations and possible improvements

The model is based on inviscid, irrotational and incompressible flow. This potential flow does not represent reality exactly, however does provide a good approximation for low Mach number and high Reynolds number flows around streamlines bodies at low angles of attack. Furthermore the wake and airfoil are modelled as discrete vortices, this is not an exact solution to the unsteady case, where the vorticity is smoothly distributed along the wake.

5. Difference between shed and trailing vorticity

The shed velocity in the wake originates from the change of circulation with respect to time of the airfoil. This is different to the trailing vorticity of the lifting line model, which originates to satisfy Helmholtz theorem. In theory and 3 dimensions, each shed shed vortex will be part of a closed vortex ring, that is advected with the flow. Similarly to this, the trailing tip and root vortices of a lifting line model belong to a closed vortex ring, which at the end is connected by the same circulation the airfoil experiences. As the flow however is steady this vortex has been advected away to infinity and thus does not influence the airfoil or wing. In the lifting line model, the inclusion of unsteady wake shedding can be included easily, which however requires a change from the frozen to a free wake geometry.

6. Conclusion

In this report, a flat plate in unsteady flow has been modelled in which a sinusoidal variation of angle of attack has been imposed. The results of loading and velocity fields have been compared against those of steady flow field and it was observed that at maximum and minimum angles of attack in the oscillations, there was almost no difference but at zero angle of attack that the angle of attack was oscillating about, significant differences were observed which are in line with Kelvin's theorem that relates to the rate of change of circulation. Moreover, limitations of the model were determined and discussed. Furthermore, the difference between shed and trailing vorticity was discussed with respect to a lifting line model.