11.1

Using the following equation:

$$\frac{4\gamma f}{D_h}x = \frac{M^2(x) - M_0^2}{M^2(x)M_0^2} + \frac{\gamma+1}{2}\ln\frac{M_0^2\left(1+\frac{\gamma-1}{2}M^2(x)\right)}{M^2(x)\left(1+\frac{\gamma-1}{2}M_0^2\right)} \tag{1}$$

the length of the pipe, x, can be found by substituting the following values that are given in the question.

$$\text{Mach number at the end of pipe, } M(x) = M_{pe} = 0.95$$
$$\text{Mach number at the start of pipe (exit of nozzle), } M_0 = 0.3$$
$$\text{Friction factor, } f = 0.005$$
$$\text{Pipe diameter, } D_h = 0.05\,\text{m}$$

Additionally, the value of γ for air is 1.4. Solving the equation, the value of x is found.

$$\text{Length of pipe, } L = x = 13.23994 \approx 13.24\,\text{m}$$

To find the pressure at the end of the pipe, we first calculate the pressure at the end of nozzle exit using the isentropic pressure equation which is as follows:

$$\frac{p}{p_t} = \left(1+\frac{\gamma-1}{2}\mathrm{M}^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{2}$$

in which p_t is the total pressure. Substituting Mach number at the exit of nozzle,M_{ne} of 0.3 as M and pressure at the large reservoir of 250kPa as p_t, the value of pressure at exit of nozzle, p_{ne} can be calculated and it is:

$$p_{ne} = 234.867\,\text{kPa}$$

Then, using the following equation

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}} \tag{3}$$

is used to find the ratio p_{ne}/p^* and p_{pe}/p^* where p_{ne} is the pressure at nozzle exit and p_{pe} is the pressure at pipe exit. They are then calculated with $M_{ne} = 0.3$ and $M_{pe} = 0.95$ respectively.

$$\frac{p_{ne}}{p^*} = 3.61906$$
$$\frac{p_{pe}}{p^*} = 1.06129$$

These lead to

$$\frac{p_{pe}}{p_{ne}} = \frac{p_{pe}}{p^*} \div \frac{p_{ne}}{p^*} = 0.29325$$

Then the pressure at the end of the pipe can be found.

$$p_{pe} = \frac{p_{pe}}{p_{ne}} \cdot p_{ne} = 68.875\,\text{kPa}$$

When the pipe is reduced to 0.75 of its original length, the new length is then

$$\text{New pipe length, } L_{\text{new}} = 0.75 \cdot L = 0.929955\,\text{m}$$

Using equation 1 whereby now x is the new pipe length, L_{new} the new Mach number at the end of the pipe $M(x)$ is found via root finding.

$$\text{New Mach number at end of pipe, new } M(x) = 0.47165$$

The subsonic value is chosen as M_{ne} is a subsonic value and the maximum Mach number the Fanno flow can reach is 1.
Then again, using equation 3, the new ratio between pressure at end of pipe and choked condition can be found.

$$\text{new } \frac{p_{pe}}{p^*} = 2.272576$$

With the other ratio staying constant,

$$\text{new } \frac{p_{pe}}{p_{ne}} = \frac{p_{pe}}{p^*} \div \frac{p_{ne}}{p^*} = 0.6279466$$

Similarly,

$$\text{new } p_{pe} = \frac{p_{pe}}{p_{ne}} \cdot p_{ne} = 147.484 \text{ kPa}$$

11.2

We again use the following equation:

$$\frac{4\gamma f}{D_h} L_p = \frac{M_e^2 - M_{\text{in}}^2}{M_e^2 M_{\text{in}}^2} + \frac{\gamma+1}{2} \ln \frac{M_{\text{in}}^2 \left(1 + \frac{\gamma-1}{2} M_e^2\right)}{M_e^2 \left(1 + \frac{\gamma-1}{2} M_{\text{in}}^2\right)} \tag{4}$$

where

$$\begin{aligned} \text{for air } \gamma &= 1.4 \\ \text{Friction factor, } f &= 0.005 \\ \text{Diameter of pipe, } D_h &= 0.05 \text{ m} \\ \text{Length of pipe, } L_p &= 0.6 \text{ m} \\ \text{Mach number at pipe inlet, } M_{\text{in}} &= 2 \end{aligned}$$

Via root finding, the value of Mach number at pipe exit, M_e can be found and it is

$$M_e = 1.30049$$

The supersonic value is chosen as M_{in} is a supersonic value and the minimum Mach number the Fanno flow can reach is 1.
Again, the following equation is used for the pressure ratio.

$$\frac{p}{p^*} = \frac{1}{M} \sqrt{\frac{\gamma+1}{2 + (\gamma-1)M^2}} \tag{5}$$

We then have

$$\begin{aligned} \frac{p_{\text{in}}}{p^*} &= 0.408248 \\ \frac{p_e}{p^*} &= 0.7281388 \end{aligned}$$

and together with given value of inlet pressure, $p_{\text{in}} = 80\text{kPa}$ gives

$$\begin{aligned} \frac{p_e}{p_{\text{in}}} &= 1.78357 \\ p_e &= \frac{p_e}{p_{\text{in}}} \cdot p_{\text{in}} = 142.6856 \text{ kPa} \end{aligned}$$

Similarly, the following equation is used for the temperature ratio.

$$\frac{T}{T^*} = \frac{\gamma+1}{2 + (\gamma-1)M^2} \tag{6}$$

We have

$$\begin{aligned} \frac{T_{\text{in}}}{T^*} &= 0.66667 \\ \frac{T_e}{T^*} &= 0.89669 \end{aligned}$$

and together with the given value of inlet temperature, $T_{\text{in}} = 20°C = 293.15$K gives

$$\frac{T_e}{T_{\text{in}}} = 1.345035$$
$$T_e = 394.297\,\text{K} = 121.146°C$$

Now as for the nozzle, an isentropic relation between area of a part of nozzle, A, to area of throat, A^* can be used and it is as follows.

$$\frac{A}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(y-1)}}}{M} \tag{7}$$

where M is the Mach number at that specific part of nozzle corresponding to A. Substituting

$$\frac{A_{\text{ne}}}{A^*} = 3$$

where A_{ne} is the area of nozzle exit, Mach number at nozzle exit is found to be

$$M_{\text{ne}} = 0.1974488$$

Since the flow undergoes isentropic process in the nozzle, the following isentropic relation for pressure can be used again.

$$\frac{p}{p_t} = \left(1+\frac{\gamma-1}{2}\text{M}^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{8}$$

Using Mach number at pipe exit, $M_e = 1.30049$ and that at nozzle exit, $M_{\text{ne}} = 0.1974488$, we have

$$\frac{p_e}{p_t} = 0.36067$$
$$\frac{p_{\text{ne}}}{p_t} = 0.97318$$

resulting in

$$\frac{p_{\text{ne}}}{p_e} = 2.6964$$
$$p_{\text{ne}} = 385.0023\,\text{kPa}$$

11.3

a)

I.

Since the maximum Mach number at the nozzle throat is 1, to place a shock in the nozzle throat, we use the fact that this shock is an infinitely weak normal shock.

Given that the area ratio of nozzle is 3.0, the following equation can be used to calculate the Mach number at the exit of the nozzle, M_{ne}.

$$\frac{A}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{1}{M} \cdot \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{9}$$

where $\gamma = 1.4$ for air. It is then calculated that

$$M_{\text{ne}} = 0.1974488$$

and since the flow undergoes isentropic process in the nozzle, the isentropic relation for pressure can once again be used which is as follows

$$\frac{p}{p_t} = \left(1+\frac{\gamma-1}{2}\text{M}^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{10}$$

giving,

$$\frac{p_{\text{ne}}}{p_t} = 0.9731818$$

where p_{ne} is the pressure at nozzle exit. Using the given value of pressure of stagnation condition of $p_t = 10$bar, pressure at nozzle exit is found to be

$$p_{\text{ne}} = 9.731818 \text{ bar}$$

Then, we make use of the following equation of Fanno flow to calculate the Mach number at the duct exit (receiver), M_{rec}.

$$\frac{4\gamma f}{D}L = \frac{M_{\text{rec}}^2 - M_{\text{in}}^2}{M_{\text{rec}}^2 M_{\text{in}}^2} + \frac{\gamma+1}{2}\ln\frac{M_{\text{in}}^2\left(1+\frac{\gamma-1}{2}M_{\text{rec}}^2\right)}{M_{\text{rec}}^2\left(1+\frac{\gamma-1}{2}M_{\text{in}}^2\right)} \tag{11}$$

where $M_{\text{in}} = M_{\text{ne}}$, $L/D = 12$ and $f = 0.0025$ (given). Solving for root, the Mach number at duct exit (receiver) is found to be

$$M_{rec} = 0.1981305$$

The subsonic value is chosen as M_{ne} is a subsonic value and the maximum Mach number the Fanno flow can reach is 1. Using the following equation which relates the pressure at a point to choked pressure,

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}} \tag{12}$$

We can find the following ratios

$$\frac{p_{\text{rec}}}{p^*} = 5.50733$$
$$\frac{p_{\text{ne}}}{p^*} = 5.526492$$

Then we can calculate for

$$p_{\text{rec}} = \frac{p_{\text{rec}}}{p^*} \div \frac{p_{\text{ne}}}{p^*} \cdot p_{\text{ne}} = 9.698075 \text{ bar}$$

II.

To place a shock at the nozzle exit, the flow in the downstream of the nozzle throat needs to be supersonic. Thus, the nozzle acts as a supersonic nozzle, speeding up the flow. We use the same equation 9 to find the roots. The difference here is that we choose the supersonic Mach number instead of the subsonic as we have done in part I. The equation with the same area ratio gives the following Mach number at the nozzle exit **before the shock**, M_{ne1}:

$$M_{\text{ne1}} = 2.637416$$

and this relates to the following pressure using equation 10:

$$p_{\text{ne1}} = 0.472987 \text{ bar}$$

At this point, the nozzle exit, the shock occurs and thus we use the following shock relation over a shock:

$$\frac{p_{\text{ne2}}}{p_{\text{ne1}}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{\text{ne1}}^2 - 1\right) \tag{13}$$

where p_{ne2} is the pressure of the flow **after the shock** at the nozzle exit. Substituting the values we have, we get

$$p_{\text{ne2}} = 3.28661867 \text{ bar}$$

We can also get the Mach number after the shock at the nozzle exit using the following equation

$$M_{\text{ne2}}^2 = \frac{1+[(\gamma-1)/2]M_{\text{ne1}}^2}{\gamma M_{\text{ne1}}^2 - (\gamma-1)/2} \tag{14}$$

and we end up with

$$M_{\text{ne2}} = 0.50069$$

We then make use of Fanno line equation to find the Mach number at the end of duct (receiver), M_{rec}, which is

$$\frac{4\gamma f}{D}L = \frac{M_{\text{rec}}^2 - M_{\text{ne2}}^2}{M_{\text{rec}}^2 M_{\text{ne2}}^2} + \frac{\gamma+1}{2}\ln\frac{M_{\text{ne2}}^2\left(1+\frac{\gamma-1}{2}M_{\text{rec}}^2\right)}{M_{\text{rec}}^2\left(1+\frac{\gamma-1}{2}M_{\text{ne2}}^2\right)} \tag{15}$$

with the same f and L/D as it was in part I. Solving for roots, we get

$$M_{\text{rec}} = 0.51635$$

and again, the subsonic Mach number of the two roots had to be chosen as M_{ne} is of a subsonic value and the maximum Mach number the flow can reach is 1. Using this Mach number and equation 12, we can get the following ratios:

$$\frac{p_{\text{ne2}}}{p^*} = 2.135$$
$$\frac{p_{\text{rec}}}{p^*} = 2.0671194$$

Thus we can achieve

$$p_{\text{rec}} = \frac{p_{\text{rec}}}{p^*} \div \frac{p_{\text{ne2}}}{p^*} \cdot p_{\text{ne2}} = 3.1821187 \text{ bar}$$

III.

To place the shock at the exit of duct, the flow has to be supersonic when it reaches the duct exit (thus upstream of the duct exit). Similar to part II., we can use the following values

$$M_{\text{ne}} = 2.637416$$
$$p_{\text{ne}} = 0.472987 \text{ bar}$$

where we used the values at nozzle exit before shock of part II. Again, we use the Fanno line equation to find the Mach number at the exit of the duct (receiver) **before shock**, $M_{\textbf{rec1}}$:

$$\frac{4\gamma f}{D} L = \frac{M_{\text{rec1}}^2 - M_{\text{ne}}^2}{M_{\text{rec1}}^2 M_{\text{ne}}^2} + \frac{\gamma+1}{2} \ln \frac{M_{\text{ne}}^2 \left(1 + \frac{\gamma-1}{2} M_{\text{rec1}}^2\right)}{M_{\text{rec1}}^2 \left(1 + \frac{\gamma-1}{2} M_{\text{ne}}^2\right)} \tag{16}$$

then we get

$$M_{\text{rec1}} = 2.1219411$$

Additionally, using equation 12, we get the following ratios:

$$\frac{p_{\text{ne}}}{p^*} = 0.2685992$$
$$\frac{p_{\text{rec1}}}{p^*} = 0.3744728$$

we are then able to calculate the pressure of the flow at the duct exit before the shock:

$$p_{\text{rec1}} = 0.65942406 \text{ bar}$$

Now, we use the shock relations:

$$M_{\text{rec}}^2 = \frac{1 + [(\gamma-1)/2] M_{\text{rec1}}^2}{\gamma M_{\text{rec1}}^2 - (\gamma-1)/2} \tag{17}$$

and

$$\frac{p_{\text{rec}}}{p_{\text{rec1}}} = 1 + \frac{2\gamma}{\gamma+1} \left(M_{\text{rec1}}^2 - 1\right) \tag{18}$$

where M_{rec} and p_{rec} are the Mach number and pressure of the flow **after the shock** respectively. Calculating, we get

$$M_{\text{rec}} = 0.55801$$
$$\frac{p_{\text{rec}}}{p_{\text{rec1}}} = 5.0864064$$
$$p_{\text{rec}} = 3.3541 \text{ bar}$$

b)

In order to allow no shocks within the system while having supersonic flow throughout the duct, we can make use of the values in part a) III. before the shock which has pressure value of

$$p = 0.65942406 \text{ bar}$$

If the receiver pressure is equivalent or lower than this value, there would not be a shock at the duct exit. Thus, the receiver pressure needs to be:

$$p_{\text{rec}} \leq 0.65942406 \text{ bar}$$

c)

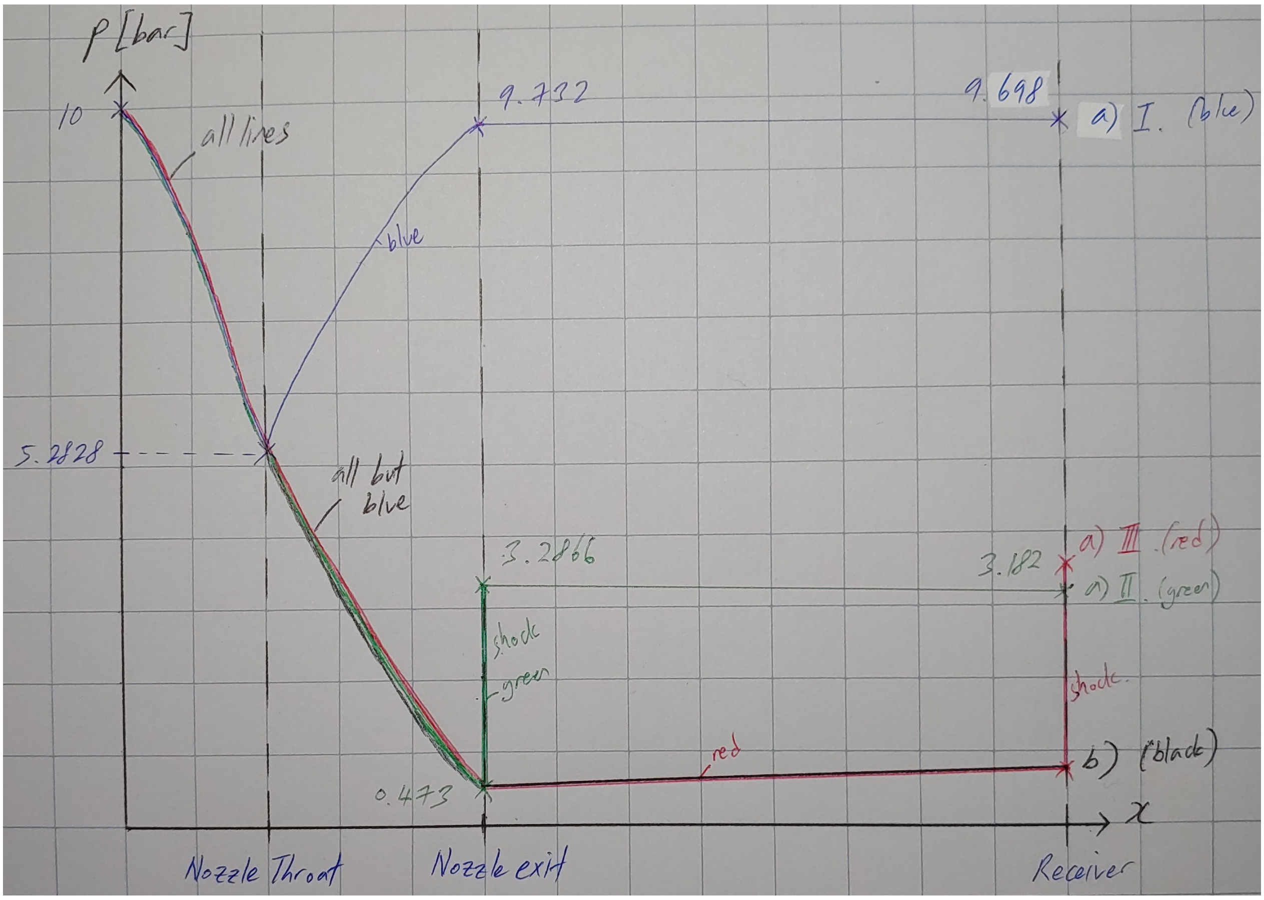
In order to plot the pressure distribution as a function of the streamwise coordinate, the pressure value at the throat has to be found and that can be done using equation 10 which is:

$$\frac{p}{p_t} = \left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\frac{-\gamma}{\gamma-1}}$$

Substituting $M = 1$ (nozzle throat) and $p_t = 10$ bar (given), we get

$$\text{Pressure at nozzle throat, } p_n t = 5.2828\,\text{bar}$$

The following plot is then achieved.



11.4

a)

Given that the flow is choked at the exit of the duct, the following equation is used to calculate for Mach number at the start of the duct (the end of the converging nozzle), M_2 as indicated in the given figure.

$$\frac{4\gamma f}{D_h}L_{\max} = \frac{1 - M_2^2}{M_2^2} + \frac{\gamma + 1}{2}\ln\frac{(\gamma + 1)M_2^2}{2\left(1 + \frac{\gamma - 1}{2}M_2^2\right)} \tag{19}$$

where by $fD_h/L = 5.3$ (given) and for oxygen $\gamma = 1.4$. M_2 is then found to be

$$M_2 = 0.1696987$$

Then using the isentropic pressure relation:

$$\frac{p}{p_t} = \left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{20}$$

and using the given value of $p_t = 7$bar, we have pressure at start of duct, p_2

$$p_2 = 6.8608\,\text{bar}$$

Next, we use the Fanno flow pressure ratio equation:

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}} \tag{21}$$

and since the exit of the duct is choked, $p_{\text{rec}} = p^*$. We then have

$$\frac{p_2}{p_{\text{rec}}} = 6.436726$$
$$p_{\text{rec}} = 1.06588 \text{ bar}$$

b)

Now, with four-fifth of the duct removed, the new value of $fD_h/L = 5.3/5$ and using exactly the same procedure,

$$M_2 = 0.325548$$
$$\frac{p_2}{p_t} = 0.9292195$$
$$p_2 = 6.504536 \text{ bar}$$
$$\frac{p_2}{p_{\text{rec}}} = 3.329855$$
$$p_{\text{rec}} = 1.9534 \text{ bar}$$

c)

In order to plot the T-s diagram, temperature values of parts a) and b) need to be calculated. To calculate these, the isentropic temperature ratio and Fanno flow temperature ratio equations need to be utilised. The isentropic relation is:

$$\frac{T}{T_t} = \left(1 + \frac{\gamma-1}{2}\mathrm{M}^2\right)^{-1} \tag{22}$$

where T_t is the total temperature and its given value is $T_t = 555\text{K}$, and the Fanno flow ratio is:

$$\frac{T}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M^2} \tag{23}$$

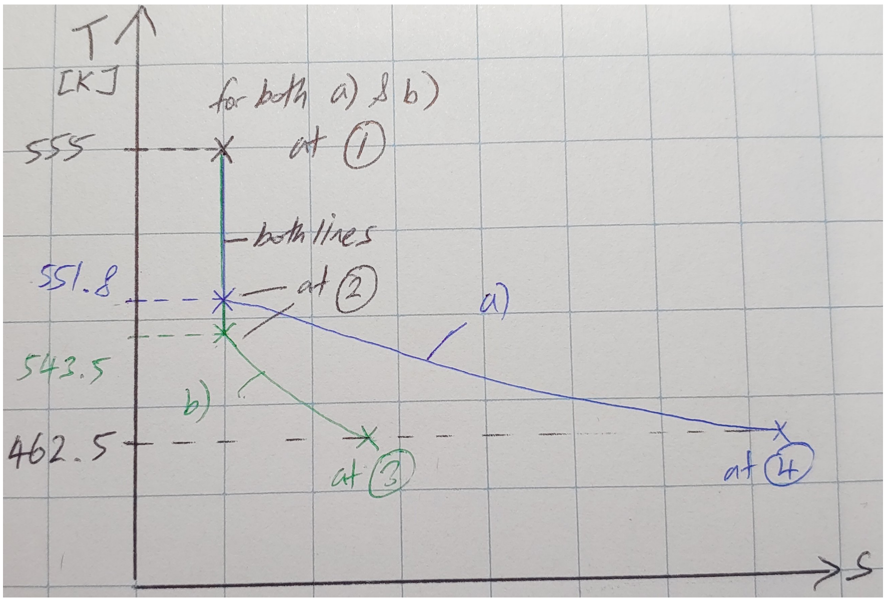
where T^* is the temperature of the choked region which in this case is at the receiver.
for part a) we have

$$\frac{T_2}{T_t} = 0.99427$$
$$T_2 = 551.822 \text{ K}$$
$$\frac{T_2}{T_{\text{rec}}} = 1.193128$$
$$T_{\text{rec}} = 462.5 \text{ K}$$

for part b) we have

$$\frac{T_2}{T_t} = 0.979244$$
$$T_2 = 543.48 \text{ K}$$
$$\frac{T_2}{T_{\text{rec}}} = 1.175093$$
$$T_{\text{rec}} = 462.5 \text{ K}$$

Using these values, the T-s diagram can be plotted.



As seen in the plot, the flow for part a) has a much greater increase in entropy compared to the flow in part b) as it goes over a larger viscous displacement.

12.1

$$\text{Mach number at inlet, } M_{\text{in}} = \frac{V_{\text{in}}}{\gamma R T_{\text{in}}} = 0.229222$$

$$\frac{T_{\text{t,in}}}{T_{\text{in}}} = 1 + \frac{\gamma - 1}{2} = 1.0105085$$

$$T_{\text{t,in}} = 306.33566 \text{ K}$$

where $\gamma = 1.4$ for air. Using the isentropic relations:

$$p_{\text{t,in}} = p_{\text{in}}(1 + \frac{\gamma - 1}{2} M_{\text{in}}^2)^{\frac{\gamma}{\gamma - 1}}$$
$$= 51.86328 \text{ kPa}$$

Since the ratio of mass of fuel and air is 1 to 40, and taking $c_p = \gamma R/(\gamma - 1) = 1.0045$ kJ/kgK,

$$q = c_p(T_{\text{t,e}} - T_{\text{t,in}})$$

$$\frac{1}{40} \cdot 40 \cdot 10^3 = 1.005(T_{\text{t,e}} - 306.33566)$$

$$\text{Stagnation temperature at exit, } T_{\text{t,e}} = 1301.85582 \text{ K} = 1028.706°C$$

Using the following equation:

$$\frac{T_{t,2}}{T_{t,1}} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left[\frac{M_2}{M_1}\right]^2 \left(\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_1^2}\right)$$

and taking condition 1 as inlet and condition 2 as exit,

$$\text{Mach number at exit, } M_e = 0.74935363$$

Using the following equation:

$$\frac{p_{t,2}}{p_{t,1}} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \left(\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_1^2}\right)^{\frac{\gamma}{\gamma - 1}}$$

and again taking condition 1 as inlet and condition 2 as exit,

$$\frac{p_{\text{t,e}}}{p_{\text{t,in}}} = 0.84101786$$

$$\text{Stagnation pressure at exit, } p_{\text{t,e}} = 43.617945 \text{ kPa}$$

Using the isentropic relation:

$$\frac{p_e}{p_{\text{t,e}}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}$$
$$= 0.689412$$
$$\text{Stagnation at exit, } p_e = 30.0707 \text{ kPa}$$

12.2

Let

- Region 1: Duct inlet
- Region 2: Duct exit before shock
- Region 3: Duct exit after shock (nozzle inlet)
- Region *: Nozzle exit

Using isentropic relation:

$$\frac{A}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M}$$

and the given ratio:

$$\frac{\text{Nozzle exit area}}{\text{Duct area}} = \frac{A^*}{A} = 0.98$$
$$M_2 = 0.8509$$

Using shock relation:

$$M_3^2 = \frac{1 + [(\gamma-1)/2] M_2^2}{\gamma M_2^2 - (\gamma-1)/2}$$
$$M_e = 1.18618$$

As for the temperature,

$$T_{t,1} = T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$
$$= 435 \text{ K}$$
$$\frac{T_{t,2}}{T_{t,1}} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left[\frac{M_2}{M_1}\right]^2 \left(\frac{2 + (\gamma-1) M_2^2}{2 + (\gamma-1) M_1^2}\right) = 1.079119$$
$$T_{t,2} = 469.4168 \text{K}$$

Then, taking $c_p = \gamma R/(\gamma - 1) = 1.0045$ kJ/kgK, we are able to achieve

$$q = c_p(T_{t,2} - T_{t-1})$$
$$= 34.572 \text{ kJ/kg}$$

Thus, heat is **added** to the flow.

12.3

a)

From the plot of stagnation curve, we know that

$$T_{\text{t,supersonic}} > T_{\text{t,subsonic}}$$
$$T_{t,2} > T_{t,1}$$

and since total temperature remains constant through an isentropic process,

$$T_{t,4} > T_{t,3}$$

and since $T_4 = T_3$, using the isentropic relation of

$$\frac{T_t}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

we have that

$$M_4 > M_3$$

and also from

$$a = \sqrt{\gamma R T}$$

we know that

$$a_4 = a_3$$

which means

$$M_4 > M_3$$
$$\frac{V_4}{a_4} > \frac{V_3}{a_3}$$
$$V_4 > V_3$$

Furthermore, since 1, 2, 3 and 4 are on the same line of entropy on the $p - \nu$ diagram, and on this line 3 and 4 coincide, we have that

$$\nu_4 = \nu_3$$
$$\rho_4 = \rho_3 = \rho$$

Ultimately, since the mass flow is constant,

$$\dot{m}_4 = \dot{m}_3$$
$$\rho_4 V_4 A_4 = \rho_3 V_3 A_3$$
$$\rho V_4 A_4 = \rho V_3 A_3$$
$$V_4 A_4 = V_3 A_3$$

since it was proved that $V_4 > V_3$, it has to be that

$$A_4 < A_3$$

b)

I.

Now,

$$T_3 \neq T_4 M_3 = M_4 = 1$$

and it remains that

$$T_{t,3} < T_{t,4}$$

From the isentropic relation:

$$\frac{T_t}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

we know that

$$T_3 < T_4$$

Then we know that since

$$a = \sqrt{\gamma R T}$$

we have

$$a_3 < a_4$$

Using this inequality, we ultimately get

$$M_3 = M_4$$
$$\frac{V_3}{a_3} = \frac{V_4}{a_4}$$
$$V_3 < V_4$$

II.

We have the fact that

$$s_3 = s_4$$
$$T_3 < T_4$$

From $p - \nu$ diagram, we then have that

$$p_3 < p_4$$
$$\nu_3 > \nu_4$$
$$\rho_3 < \rho_4$$

Since mass flow has to be equal, we have

$$\dot{m}_3 = \dot{m}_4$$
$$\rho_3 V_3 A_3 = \rho_4 V_4 A_4$$

and using the ρ and V inequalities, it can be seen that we need

$$A_3 > A_4$$

12.4

To plot the T-s diagram, the following characteristics of the various flows need to be mentioned.

Fanno :

$$\frac{T}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M^2}$$
$$\frac{T_2}{T_1} = \frac{\gamma+1}{2+(\gamma-1)M_2^2} \div \frac{\gamma+1}{2+(\gamma-1)M_1^2} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}$$

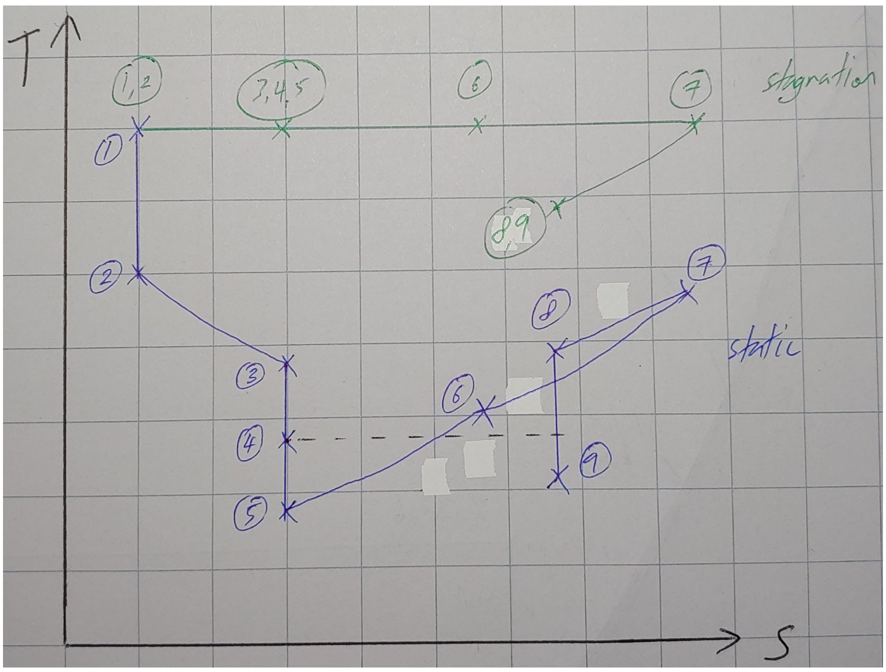
- The higher the M, the lower the T

Shock :

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right] \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}$$

- $M_1 > 1$: $T_2/T_1 > 1 \rightarrow$ Supersonic upstream: temperature increases over shock

We can then achieve the follow plot.



It is to be noted that due to the insufficient information regarding the values across the shock and magnitude of heat removed, the entropy difference between point 6 and 8,9 as to which is larger is uncertain. Furthermore, the temperature at 9 is lower than temperature 4 even though they both have Mach number of 1 due to the removal of heat from 7 to 8.