

# Symbolic computation in Spiking Neural Networks:

## Spike-frequency adaptation enables cognitive computations

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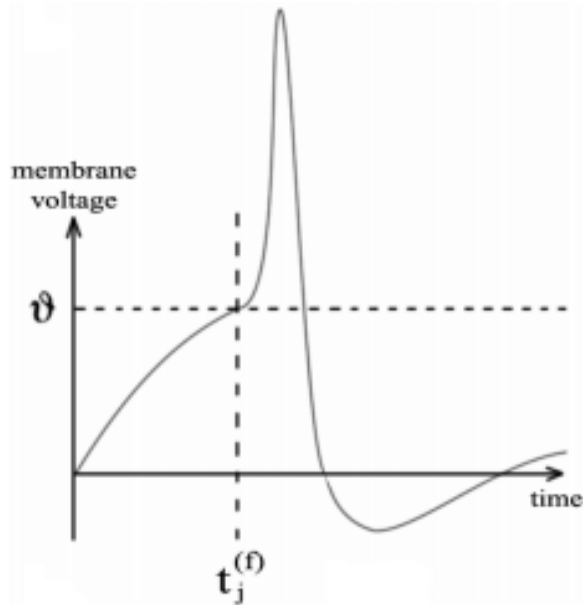
Wolfgang Maass

D. Salaj, A. Subramoney, C. Krašniković, G. Bellec, R. Legenstein, and W. Maass. [Spike-frequency adaptation provides a long short-term memory to networks of spiking neurons.](#) *bioRxiv*, 2020.

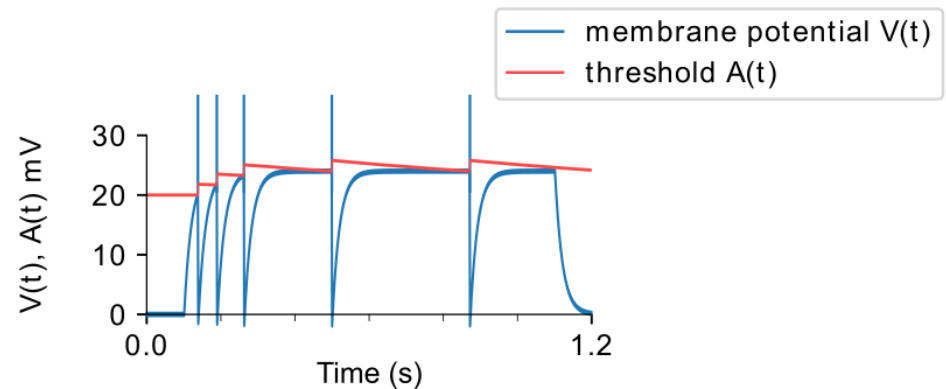
# On symbolic computation

- “Symbolic computation” uses symbolic expressions.
  - Examples:  $x + y$ , instruction/rule, algorithm
- Challenges for artificial neural networks:
  - Flexible cognitive control  
**RED, GREEN, BLUE, PURPLE**
  - Free generalization  
*e.g., wug - wugged*

# Leaky Integrate-and-Fire neuron



Firing response to a step input current of simulated ALIF neuron model



LIF neurons are augmented with an adaptive threshold (**ALIF**).

Wulfram Gerstner and Werner Kistler. *Spiking Neuron Models: An Introduction*. Cambridge University Press, New York, NY, USA, 2002.

Allen Institute. Allen Cell Types Database Technical white paper: GLIF models.  
<http://help.brainmap.org/download/attachments/8323525/glifmodels.pdf>. Technical report, October 2017. v4.

# 12AX task

- First introduced in (O'Reilly and Frank, 2006)
- Input symbols: {1, 2, A, X, B, Y, C, Z}
- Output symbols: {L, R}
- Input: 1 ... A [CZ] X ... B ... Y  
Target: L ... L ... R ... L ... L
- Input: 2 ... A ... X ... B [CZ] Y  
Target: L ... L ... L ... L ... R
- Performance: 97.79% fully correct episodes.

R. C. O'Reilly and M. J. Frank. Making working memory work: a computational model of learning in the prefrontal cortex and basal ganglia. *Neural computation*, 18(2):283-328, 2006.

# Brain-like operations on sequences

- Input and output symbols: English alphabet
- Commands: DUPLICATE and REVERSE string
- An example input/output pair for DUPLICATE operation:

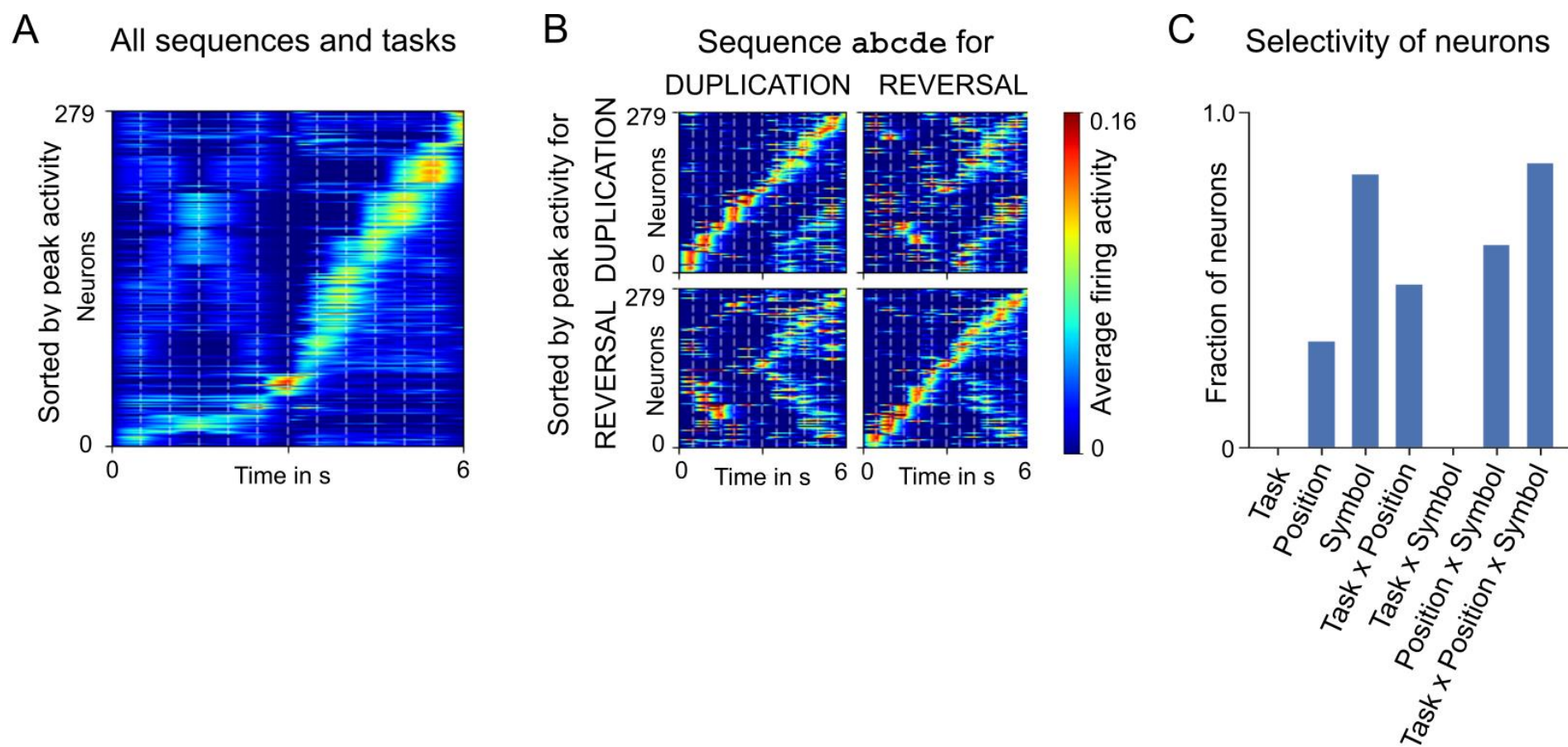
Input	a	b	c	x	y	*	?	?	?	?	?	?
Output							a	b	c	x	y	*

- An example input/output pair for REVERSE operation:

Input	a	b	c	x	y	*	?	?	?	?	?	?
Output							y	x	c	b	a	*

- Performance: 95.88% fully correct output strings, tested on previously unseen strings.

# Emergence of neural codes





# Thank you for your attention! Questions?



# Neuron Model

## LIF

$$\tau_m \frac{du}{dt} = -u(t) + RI(t)$$

$$I_i(t) = \sum_j w_{ij} \sum_f \alpha(t - t_j^{(f)})$$

$$\alpha(s) = A \exp\left(-\frac{s}{\tau_s}\right) \Theta(s)$$

$\tau_m$  - membrane time constant,  
 $u(t)$  - membrane voltage,  
 $I(t)$  - current,  $w_{ij}$  - efficacy,  
 $t_j^{(f)}$  - firing time of neuron j,  
 $\tau_s$  - time decay constant,  
 $\Theta(s)$  - Heaviside step function

## Adaptive LIF

$$\tau_m \frac{du_j}{dt} = -u_j(t) + R_m I_j(t)$$

$$\tau_{a,j} \frac{db_j}{dt} = b_j^0 - b_j(t)$$

$\tau_m$  - membrane time constant  
 $\tau_{a,j}$  - adaptation time constant

## In discrete time:

$$u_j(t + dt) = \alpha u_j(t) + (1 - \alpha) R_m I_j(t) - b_j(t) z_j(t)$$

$$b_j(t + dt) = \rho_j b_j(t) + (1 - \rho_j) z_j(t)$$

$$\alpha = \exp\left(-\frac{dt}{\tau_m}\right)$$

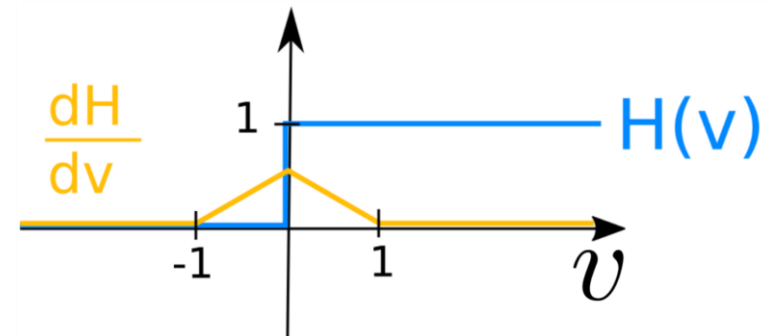
$$\rho_j = \exp\left(-\frac{dt}{\tau_{a,j}}\right)$$

# Backpropagation Through Time (BPTT)

- BPTT is used to train RNNs.
  - Gradients are propagated through many steps.
- Outputs of spiking neurons are non-differentiable.
  - Dampened pseudo-derivative can be used instead.

$$\frac{dH}{dv} = \gamma \max\{0, 1 - |v|\},$$

$v$  – normalized membrane potential,  $\gamma$  – dampening factor.

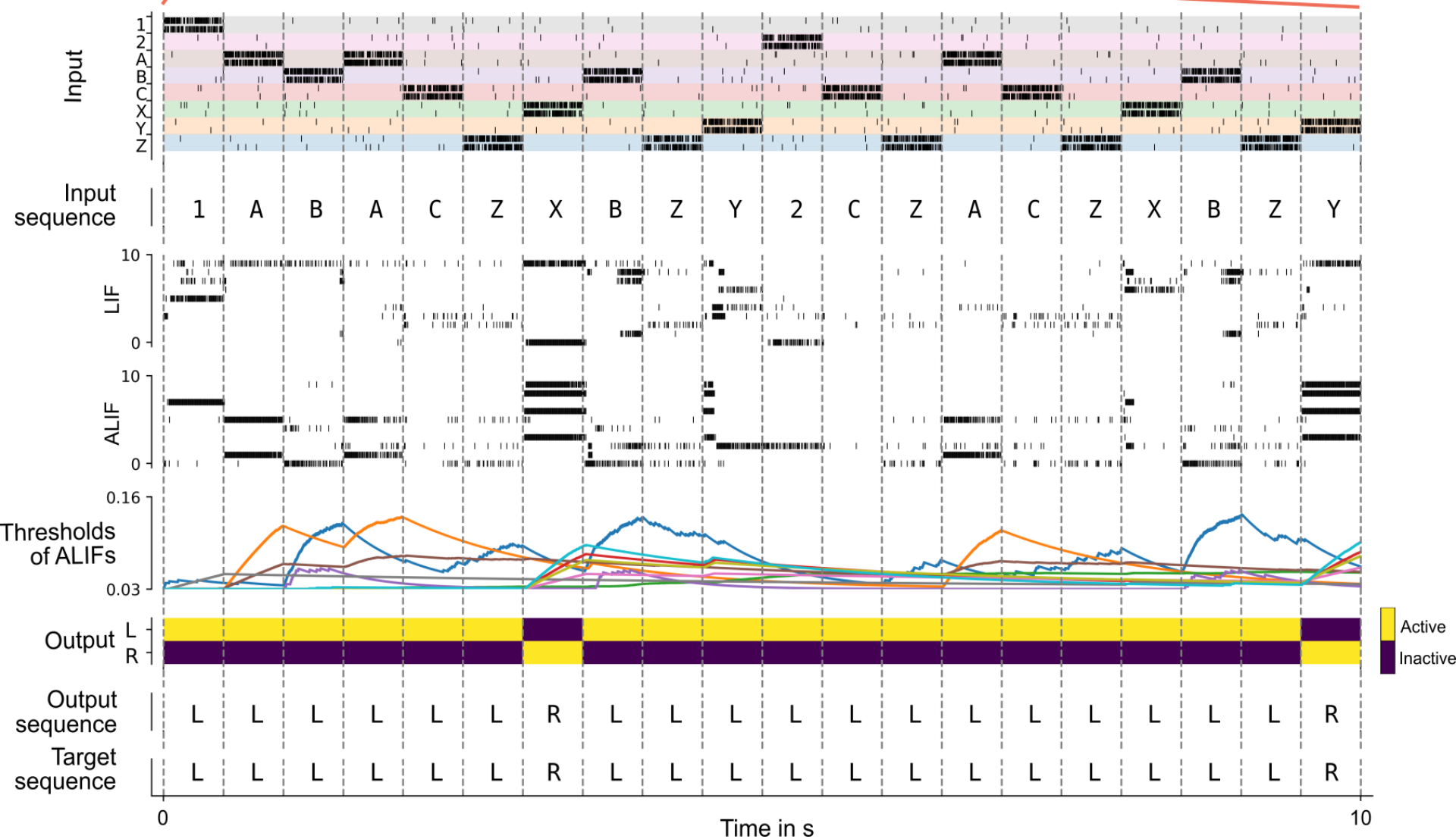


[G. Bellec, D. Salaj, A. Subramoney, R. Legenstein, and W. Maass. Long short-term memory and learning-to-learn in networks of spiking neurons. *32nd Conference on Neural Information Processing Systems (NIPS 2018), Montreal, Canada, 2018.*]

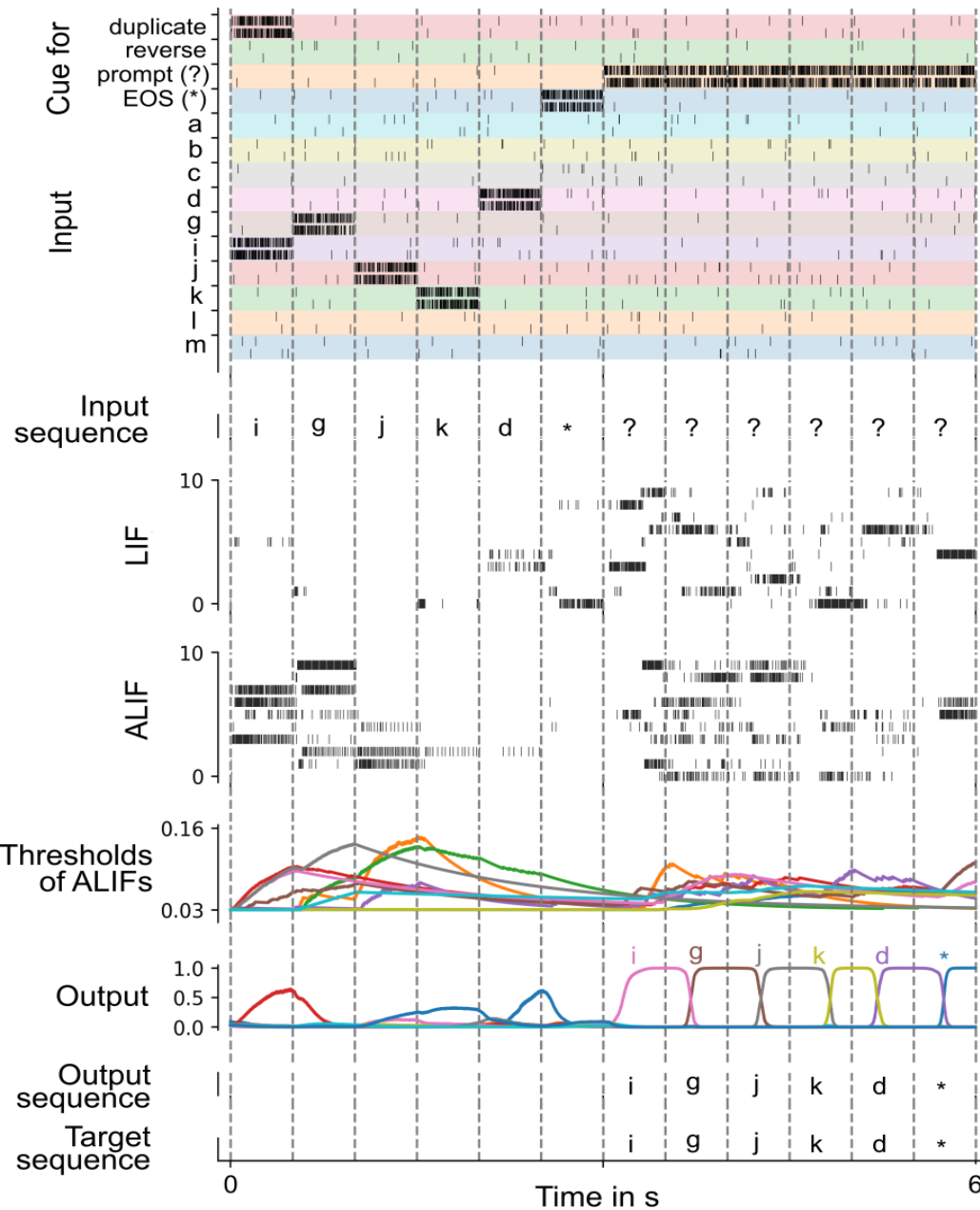
Full input  
sequence

1A B A C X Z B Z Y 2 C Z A C X Z B Z Y 2 A C B B Y 2 A Z B Y Y C X Y B A Y C Y 1 C B B C Z Z C C C Y A C C Z C X Z 2 C Y X B B Z X B C Z Y C Y 1 C C A C X Z 1 A X A C Z X B X B Y  
L L L L L R L L L L L L L L L L R L L L L R L L L L R L R L L L L L L L L L L R L L L L L L L L L L R L L R L L L R L L L

Target output  
sequence



## DUPLICATION



## REVERSAL

