Data Structures and Algorithms in Python

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Instructor's Solutions Manual

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Chapter 8 Trees

Hints and Solutions

Reinforcement

R-8.1) Hint Be sure not to get confused between depth (which is for a single node) and height (which is for the entire tree).

R-8.2) Hint Recall that the worst-case running time for the depth method is $O(n)$.

R-8.3) Hint Use the definitions of height and depth, and note that the height of the tree has to be realized by some path from the root to an external node. In addition, using induction is the easiest justification technique to use with this proposition.

R-8.4) Hint Use a new parameter, n_p, which refers to the number of positions in the subtree rooted at p.

R-8.4) Solution The running time is $O(n_p)$, where n_p is the number of positions in the subtree rooted at position p.

R-8.5) Hint Review the binary tree ADT.

R-8.6) Hint Consider using dummy nodes.

R-8.7) Hint You need to give four values—the minimum and maximum number of internal nodes and the same for external nodes. In addition, one of these four values is 1.

R-8.7) Solution A tree that is one long path would have $n-1$ internal nodes and 1 external node. A tree that is proper (and thus odd n) would have $\frac{n-1}{2}$ internal nodes and $\frac{n+1}{2}$ external nodes.

R-8.8) Hint Use the formulas presented in the book that relate external nodes, internal nodes, and height.

R-8.9) Hint Consider how any proper binary tree with n nodes can be related to a proper binary tree with $n-2$ nodes.

R-8.10) Hint Although this is an abstract class, you may rely on its other (abstract) methods.

R-8.10) Solution

```
def num_children(self, p):
  count = 0
  if self.left(p) is not None:
    count += 1
  if self.right(p) is not None:
    count += 1
  return count
```

R-8.11) Hint Each subtree is rooted at a node of the tree; give the value associated with each such node.

R-8.12) Hint This one is a real puzzler, and it doesn't even use the operators $+$ and $\times$.

R-8.12) Solution The tree expresses the formula $(6/(1-(5/7)))$.

R-8.13) Hint Review how arithmetic expressions can be represented with binary trees.

R-8.14) Hint Assume that the container of children is implemented as a positional list.

R-8.14) Solution The size and isEmpty methods run in $O(1)$ assuming we maintain a top-level count of the number of nodes in the tree. The methods root, parent, isRoot, isInternal, isExternalare also constant since we need only to check local fields of the node or the tree's root to determine these. The num_children method is constant-time, because it is simply the size method of the secondary container of children. Reporting children(p) requires $O(c_p+1)$ time because it requires iterating through the entire collection of children (even if the container were empty). The analysis of depth and height were given in Section 8.1.3.

R-8.15) Hint We'll get you started...

```
class MutableLinkedBinaryTree(LinkedBinaryTree):
  def add_root(self, e):
    return self._add_root(e)
```

R-8.16) Hint Think about a tree that is really a path.

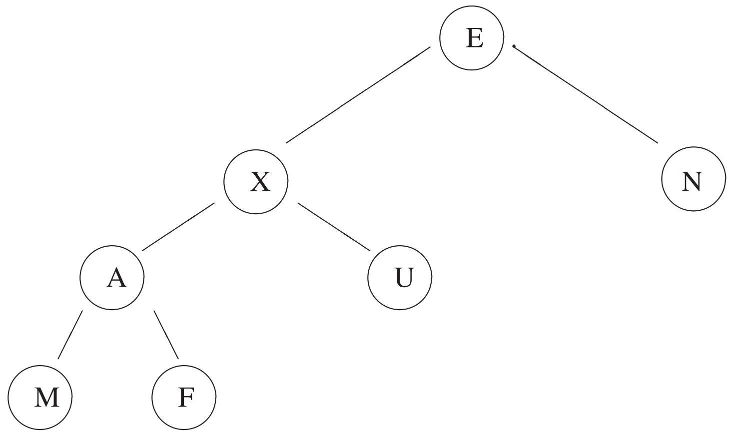
R-8.17) Hint Recall the definition of the level numbering and use this definition when passing information from a node to its children.

R-8.18) Hint All of these methods can be easily performed using formulas involving level numbers.

R-8.19) Hint Use a substitution of $g(p) = f(p) + 1$.

R-8.20) Hint Draw the tree starting with the beginning and end characters of the two character strings.

R-8.20) Solution The tree is drawn below



R-8.21) Hint Draw the tree on a separate sheet of paper and then mark nodes as they are visited, writing down the output produced for each visit.

R-8.22) Hint Draw the tree on a separate sheet of paper and then mark nodes as they are visited, writing down the output produced for each visit.

R-8.22) Solution 3 1 + 3 x 9 5 - 2 + - 3 7 4 - x 6 + -

R-8.23) Hint Draw a tree with one node, one with two nodes, and then one with three nodes.

R-8.23) Solution It is not possible for the postorder and preorder traversal of a tree with more than one node to visit the nodes in the same order. A preorder traversal will always visit the root node first, while a postorder traversal node will always visit an external node first.

It is possible for a preorder and a postorder traversal to visit the nodes in the reverse order. Consider the case of a tree with only two nodes.

R-8.24) Hint Draw a tree with one node and then one with three nodes (and note it is impossible to draw a proper binary tree with exactly two nodes).

R-8.24) Solution It is not possible for the post- and preorder traversals to visit the nodes of a proper binary tree in the same order for the same reason in the previous question.

It is not possible for the post- and preorder traversals to visit the nodes of a proper binary tree in the reverse order. Let a be the root of a proper binary tree and let T_1 and T_2 be the left and right subtrees. A postorder traversal would visit the postorder traversal of T_1, the postorder traversal of T_2 and then node a while the preorder traversal would visit node a, the preorder traversal of T_1 and then the preorder traversal of T_2. Clearly the postorder and preorder traversals cannot be the reverse of each other since in both cases, all the nodes of T_1 are visited before all the nodes of T_2.

R-8.25) Hint We already got you started...

R-8.25) Solution

$\{1\}$
$\{2,3,4\}$
$\{3,4,5,6\}$
$\{4,5,6,7,8,9,10\}$
and so on.

R-8.26) Hint You can add all of c's children to the fringe at once.

R-8.27) Hint Draw the tree again and use a pencil and paper to mark how the recursive calls move around the tree.

R-8.28) Hint Argue that this method basically performs the same computations as a familiar tree traversal method.

R-8.28) Solution The running is $O(n)$ since parenthesize is just a preorder traversal with output.

R-8.29) Hint Consider the information and computations that need to be made in each of the three visits during an Euler tour traversal.

R-8.30) Hint Regular expressions can be helpful for parsing such strings.

Creativity

C-8.31) Hint Use the fact that we can build T from a single root tree via a series of pairs of insertLeft and insertRight operations.

C-8.32) Hint The minimum value will occur when T has one external node.

C-8.33) Hint Consider how you could possibly combine a tree with maximum depth with a tree with minimum depth.

C-8.33) Solution Let T_1 be a tree of $n/2$ nodes in a single path from the root to a single external node v. And let T_2 be a tree of $n/2$ nodes having $\lfloor n/2 \rfloor + 1$ external nodes. Now, create T by attaching T_2 to T_1 at v.

C-8.34) Hint First show that there is a node with three external node children, and then consider what changes when the children of that node are all removed.

C-8.34) Solution We will show this using induction. For $n_I = 0$, then $n_E = 2n_I + 1 = 1$. This is obviously true. For $n_I = 1$, then $n_E = 2n_I + 1 = 2 + 1 = 3$. Again, this is obviously true from our problem definition. Now let us assume that the n_E equation holds true for $k' < k$, i.e., for any $n_I = k' < k$, $n_E = 2n_I + 1$.

Now consider $n_I = k$. Then, $n_E = 2(k-1)+1+(3-1)$. That is, the number of external nodes is equal to the number of external nodes for a tree with $k-1$ internal nodes plus 3 (we added an internal node which must have 3 children) minus 1 (in creating the new internal node, we made an external node into an internal node). Thus, $n_E = 2k-2+3 = 2k+1$. This is what we needed to show.

C-8.35) Hint Use the definition to derive a recursive algorithm.

C-8.36) Hint Explore the different ways you can have empty nodes on the bottom level.

C-8.37) Hint Explore with pen and paper.

C-8.37) Solution There are 5 such trees.

C-8.38) Hint Can you tell how many elements are removed?

C-8.38) Solution Based on our current representation, this would require $O(n_p)$ time where n_p is the number of nodes in the subtree rooted at position p.

C-8.39) Hint There are many special cases to consider when p and q neighbor each other.

C-8.40) Hint Try to avoid conditionals when possible.

C-8.41) Hint Recursion can be very helpful.

C-8.42) Hint Build the new tree in preorder fashion.

C-8.43) Hint The answer to the first part of (c) is ”yes”.

C-8.43) Solution

a) yes
b) no
c) yes, postorder.

C-8.44) Hint Use a tree traversal.

C-8.44) Solution We can accomplish the task of printing the element stored at p along with the height of the subtree rooted at p by using a postorder traversal. During this traversal, we will find the height of each subtree. The height for a subtree at p will be 0 if p is a leaf and otherwise one more than the height of the max child. We can print out the element at p and its computed height during the postorder visit.

C-8.45) Hint Derive a formula that relates the depth of a position p to the depths of positions adjacent to p.

C-8.45) Solution This can be done using a preorder traversal. When doing a “visit” in the traversal, simply store the depth of the position’s parent incremented by 1. Now, every node will contain its depth.

C-8.46) Hint Modify an algorithm for computing the depth of each position so that it computes path lengths at the same time.

C-8.47) Hint Try to compute node heights and balance factors at the same time.

C-8.47) Solution One way to do this is the following: in the _hook_previsit method, set height and balance to be zero. Then, alter the _hook_postvisit method as follows:

Algorithm _hook_postvisit(v):
  if v is internal **then**
    if $v.leftchild.height > v.rightchild.height$ **then**
      $v.height = v.leftchild.height + 1$;
    else
      $v.height = v.rightchild.height + 1$;
    $v.balance = abs(v.rightchild.height - v.leftchild.height)$;
  print $v.balance$

C-8.48) Hint Draw a picture of a proper binary tree and its preorder traversal and then hold this picture up to mirror.

C-8.49) Hint First derive a formula for $post(p) - pre(p)$.

C-8.49) Solution $post(p) = desc(p) - 1 - depth(p) + pre(p)$. To prove this, consider the difference, $post(p) - pre(p)$. This difference is equal to all the positions counted by postorder but not preorder (the strict descendants of p, whose number is $desc(p) - 1$) minus all the positions counted by preorder but not postorder (the ancestors of p, whose number is $depth(p)$).

C-8.50) Hint Think about what could be the worst-case number of nodes that would have to be traversed to answer each of these queries.

C-8.50) Solution The algorithms are as follows:

Algorithm preorder_next(v):
  if v is internal **then**
    return v's left child
  else
    $p =$ parent of v
    if v is left child of p **then**
      return right child of p
    else
      while v is **not** left child of p **do**
        $v = p$
        $p = p.parent$
      return right child of p

Algorithm inorder_next(v):
  if v is internal **then**
    p = left child of v
    while p has left child **do**
      p = left child of p
      return p
  else
    p = parent of v
    if v is left child of p **then**
      return p
    else
      while v is **not** left child of p **do**
        $v = p$
        $p = p.parent$
      return p

Algorithm postorder_next(v):
  if v is internal **then**
    p = parent of v
    if v is right child of p **then**
      return p
    else
      v = right child of p
      while v is **not** external **do**
        v = left child of v
      return v
  else
    p = parent of v
    if v is left child of p **then**
      return right child of p
    else
      return p

The worst-case running times for these algorithms are all $O(h)$ where h is the height of the tree T.

C-8.51) Hint See Section 2.3.4 for discussion of iterators.

C-8.52) Hint Use induction to show that no crossing edges are produced.

C-8.53) Hint Use induction to show that no crossing edges are produced.

C-8.54) Hint The y coordinates can be the same as in the binary tree case. The trick, then, is to find a good substitute for the inorder number used in the binary tree drawing algorithm.

C-8.55) Hint **import** os; help(os.walk).

C-8.56) Hint Consider a recursive algorithm.

C-8.56) Solution The algorithm is given below.

Algorithm indented_parenthetic_representation(T, v, $indent$):

```
print out indent number of tabs
if T is external then
   print v.element()
else
   indent += 1
   print v.element() + "("
   for each child w in T.children(v) do
      indented_parenthetic_representation(T, w, indent)
   indent -= 1
   print out indent number of tabs
```

C-8.57) Hint Consider a recursive algorithm.

C-8.58) Hint It helps to know the relative depths of p and q.

C-8.58) Solution The algorithm is given below.

Algorithm LCA(v, w):

```
int v_depth = v.depth
int w_depth = w.depth
while v_depth > w_depth do
   v = v.parent
while w_depth > v_depth do
   w = w.parent
while v ≠ w do
   v = v.parent
   w = w.parent
return v
```

C-8.59) Hint Be careful—the path establishing the diameter might not include the root of the tree.

C-8.60) Hint Consider what numbers $f(p)+1$, $f(q)+1$, and $f(a)+1$ look like in binary.

C-8.61) Hint Parameterize your recursion by an index into the list of tokens which begins a subtree.

C-8.62) Hint For the base case, you must determine whether the token is a numeric literal or a string variable.

C-8.63) Hint Use the inherited postorder method.

Projects

P-8.64) Hint What are natural update methods for this representation?

P-8.65) Hint Use a Python list for the children.

P-8.66) Hint You will need to reimplement the _make_position utility.

P-8.67) Hint The essential part of this structure is just a binary tree, so take each part one step at a time.

P-8.68) Hint This is a huge project, so plan accordingly.

P-8.69) Hint Review the definition of this representation to get the details right.

P-8.70) Hint Use recursion where appropriate.