Data Structures and Algorithms in Python

Michael T. Goodrich
Department of Computer Science
University of California, Irvine

Roberto Tamassia
Department of Computer Science
Brown University

Michael H. Goldwasser
Department of Mathematics and Computer Science
Saint Louis University

Instructor's Solutions Manual

WILEY

Chapter 11

Search Trees

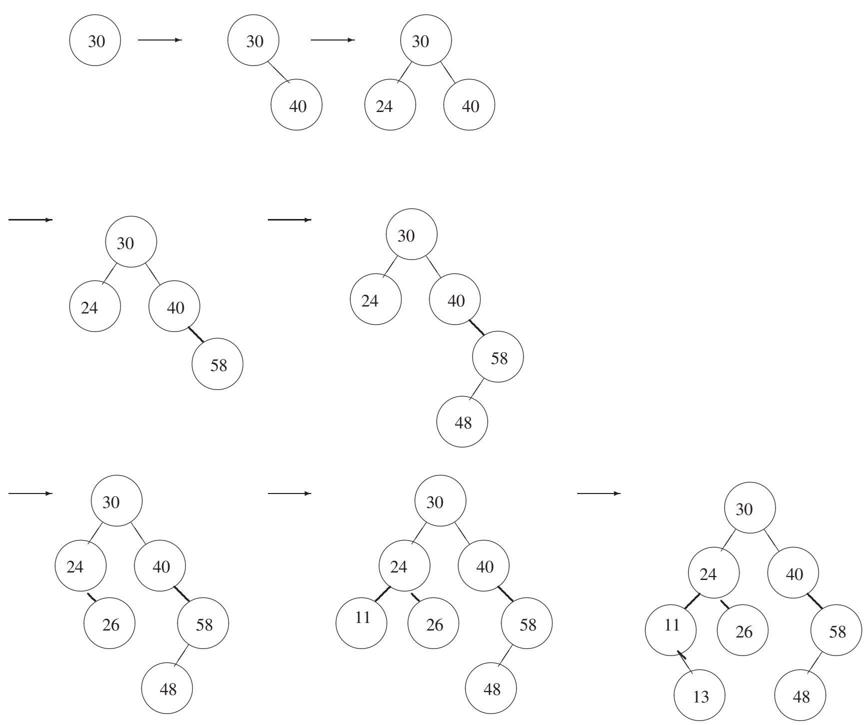
Hints and Solutions

Reinforcement

R-11.1) Hint Recall the definition of where we perform an insertion in a binary search tree.

R-11.2) Hint You will need to draw 8 trees, but they are all small.

R-11.2) Solution



R-11.3) Hint You can enumerate them with pictures.

R-11.3) Solution 5 (2 w/ 1 as root, 1 w/ 2 as root, 2 w/ 3 as root).

R-11.4) Hint Try a few examples of five-entry binary search trees.

R-11.4) Solution There are several solutions. One is to draw the binary search tree created by the input sequence: $9,5,12,7,13$. Now draw the tree created when you switch the 5 and the 7 in the input sequence: $9,7,12,5,13$.

R-11.5) Hint Try a few examples of five-entry AVL trees.

R-11.5) Solution There are several solutions. One is to draw the AVL tree created by the input sequence: $9,5,12,7,13$. Now draw the tree created when you switch the 5 and the 7 in the input sequence: $9,7,12,5,13$.

R-11.6) Hint Use a loop to express the repetition

R-11.6) Solution

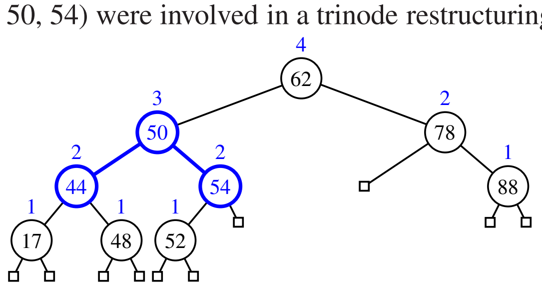
```
def _subtree_search(self, p, k):
  walk = p
  while True:
    if k < walk.key( ) and self.left(walk) is not None:
      walk = self.left(walk)
    elif k > walk.key( ) and self.right(walk) is not None:
      walk = self.right(walk)
    else:
      return walk
```

R-11.7) Hint There is one of each type. Which one is which?

R-11.7) Solution The rotation in Figure 11.12 is a double rotation. The rotation in Figure 11.14 is a single rotation.

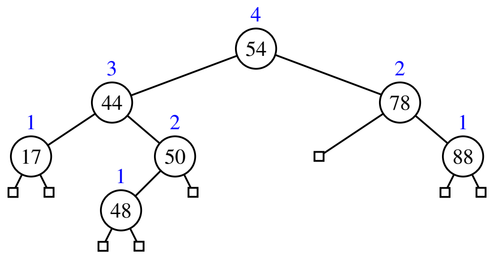
R-11.8) Hint Mimic the figure in the book.

R-11.8) Solution The updated tree follows. The highlighted nodes (44, 50, 54) were involved in a trinode restructuring.



R-11.9) Hint Mimic the figure in the book.

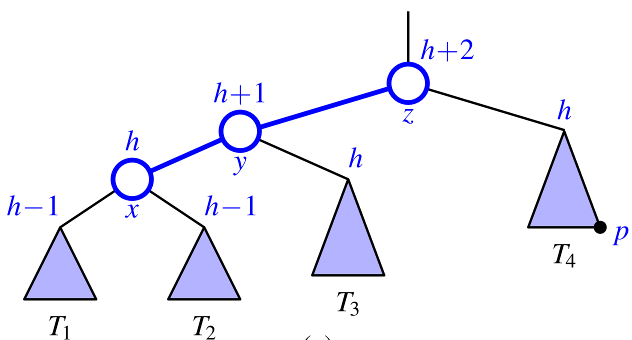
R-11.9) Solution The updated tree follows. Not much has changed, as by our implementation, the root node takes on value 54 (the max of the left subtree), and then the node that had held 54 is removed; however, no imbalance results from that deletion. (A more significant change would have occurred had the minimum value from the right subtree been used as a replacement for 62.)



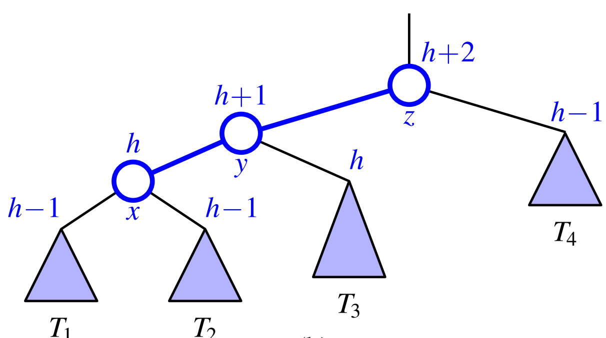
R-11.10) Hint Think about the data movements needed in an array list representation of a binary tree.

R-11.11) Hint Carefully note the heights of all subtrees before the deletion, and after the deletion but before the restructuring.

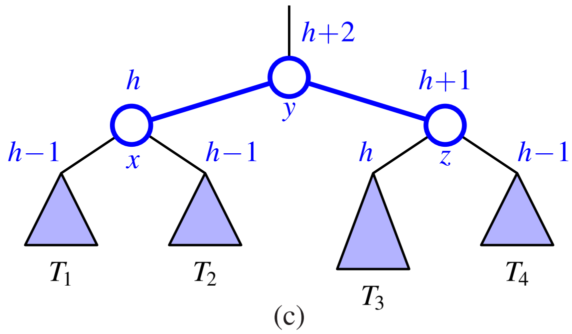
R-11.11) Solution



(a)



(b)

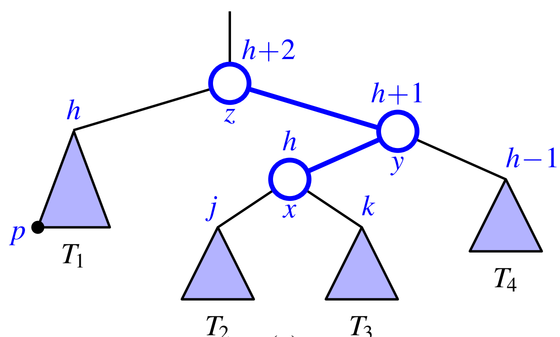


(c)

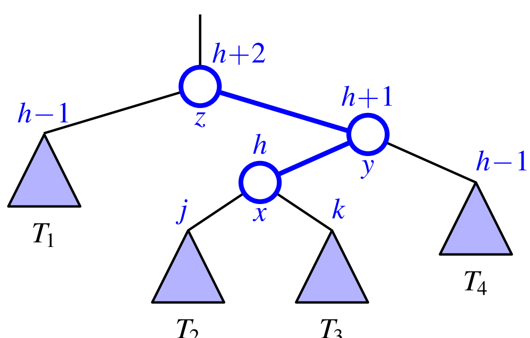
Rebalancing of a subtree during a deletion with both children of y having the same height: (a) before the deletion of p; (b) after deletion of p but before restructuring; (c) after trinode restructuring. Note that in the end, the height of the entire subtree is unchanged, and thus no further restructuring is needed.

R-11.12) Hint Carefully note the heights of all subtrees before the deletion, and after the deletion but before the restructuring.

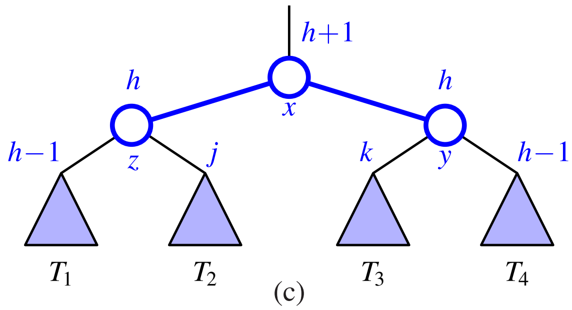
R-11.12) Solution



(a)



(b)



(c)

Rebalancing of a subtree during a deletion with one child of y having greater height: (a) before the deletion of p; (b) after deletion of p but before restructuring; (c) after trinode restructuring. In this configuration, one of j and k must be $h-1$, while the other is either $h-1$ or $h-2$. In either case, notice that the height of the overall subtree has been reduced by one, and therefore the imbalance might propagate upward.

R-11.13) Hint Carefully trace the potential heights of various subtrees.

R-11.13) Solution Look back at the solution we gave for the previous problem, but assume that T_4 has height h, such that the two children of y have the same height, yet the non-aligned one is chosen as x. The problem relates to the values of heights j and k. It may have been that $j = h-1$ and $k = h-2$. But in that case, after the restructuring, y is unbalanced as it would have children with heights $h-2$ and h.

R-11.14) Hint Use a pencil with a good eraser.

R-11.15) Hint Each entry is splayed to the root in increasing order.

R-11.16) Hint No. Why not?

R-11.16) Solution No. One property of a $(2,4)$ tree is that all external nodes are at the same depth. The multiway search tree of the example does not adhere to this property.

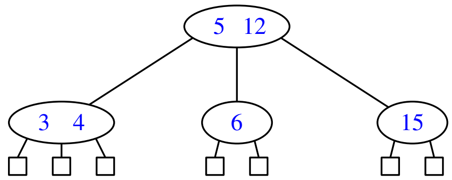
R-11.17) Hint It is not k_1. Why?

R-11.17) Solution The key k_2 would be stored at w's parent in this case. This is done in order to maintain the order of the keys within the tree. By storing k_2 at w's parent, all of the children would still be ordered such that the children to the left of the key are less than the key and those to the right of the key are greater than the key.

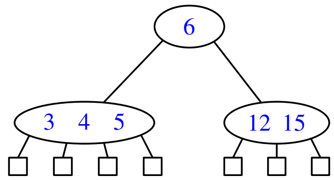
R-11.18) Hint You will need at list five entries to find a counterexample.

R-11.18) Solution

First, consider the insertion order 4, 6, 12, 15, 3, 5, which results in tree (as shown in Figure 11.27j):



Next, consider the insertion order 12, 3, 6, 4, 5, 15, which produces tree:



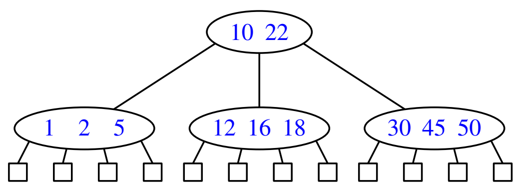
R-11.19) Hint Use the correspondence rules described in the chapter.

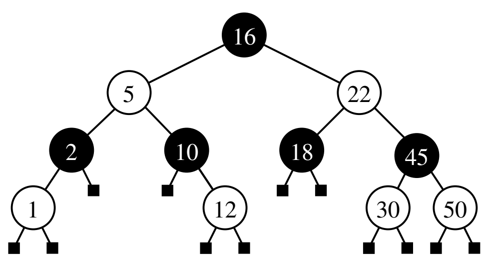
R-11.20) Hint Use a pencil with a good eraser.

R-11.20) Solution There is a $(2,4)$ tree with 4 internal nodes having height 2, with keys $\{4,8,12\}$ stored at the root, and four children, each holding 3 more keys. There is also a $(2,4)$ tree with 15 internal nodes, effectively a perfectly balanced binary tree (8 is at the root).

R-11.21) Hint Use a pencil with a good eraser.

R-11.21) Solution Based on our algorithm descriptions, the results are:





R-11.22) Hint Consider looking at the (2,4) tree and red-black tree definitions again.

R-11.23) Hint Recall the definition of a binary search tree, in general.

R-11.24) Hint Some have $O(\log n)$ worst-case height and some have $O(n)$ worst-case height. Make sure you know which. Also, try to get the constant factors right in this case.

R-11.24) Solution

a. The worst-case height for a binary search tree is n.

b. Based on the book's analysis, the AVL tree certainly has height no bigger than $2\log_2 n$. A tighter analysis shows that it is at most $1.44\log_2 n$.
c. A splay tree could have height n
d. A $(2,4)$ tree could have worst-case height of $\log_2 n$.
e. A red-black tree could have worst-case height $2\log_2 n$.

R-11.25) Hint You need to create a node that does not satisfy the AVL balance condition, but would be acceptable in a red-black tree. A good example would be a tree with at least 6 nodes, but no more than 16.

R-11.26) Hint Note that the black-path length must have been identical for each path downward from p.

R-11.26) Solution Since p has zero children now, it must have had exactly one child just before the deletion. At that time, the path to the non-child side has black-length zero, and therefore it must also be that the path through the to-be-deleted child also has black-length zero. Therefore, that deleted child must have been a red leaf.

R-11.27) Hint Note that the black-path length must have been identical for each path downward from p.

R-11.27) Solution If p has one child after the deletion, then it must have had 2 children just prior to the deletion, with the deleted child being a leaf. Just prior to the deletion, the two subtrees of p must have equal black-depth. If the deleted child had been a red leaf, it had black-depth 0 and thus the remaining subtree must be a single red leaf; in this case, there is no black deficit caused by the deletion. Otherwise, the deleted child must have been a black leaf, with the remaining subtree having black-depth 1. In this case, there is a black deficit of 1 due to the deleted black leaf.

R-11.28) Hint Note that the black-path length must have been identical for each path downward from p.

R-11.28) Solution If p has two children after the deletion, then its deleted child must have itself had one child which has since been promoted to be a child of p. Since the subtree rooted at the deleted node must have been balanced before the deletion, its one child must have been a red leaf (and it must have been black).

Creativity

C-11.29) Hint The method is similar to priority-queue sorting.

C-11.30) Hint Review what it means for splay trees to have $O(\log n)$ amortized time performance.

C-11.30) Solution Yes, splay trees can sort in $O(n\log n)$ time in the worst case: just insert all the elements and then do an inorder traversal of the splay tree. This runs in worst-case $O(n \log n)$ time because each individual insert operation runs in $O(\log n)$ amortized time and there are n insertions.

C-11.31) Hint You should make a single call to utility `_subtree_search`.

C-11.32) Hint Show that $O(n)$ rotations suffice to convert any binary tree into a ***left chain***, where each internal node has an external right child.

C-11.33) Hint Where might the search path for k diverge from a path to one of the other keys?

C-11.33) Solution For the sake of contradiction, assume that an unsuccessful search for key k does not pass through the node with key k' which is the greatest key less than k. Note that if k' were modified to make it equal to k, the tree must still be a valid binary search tree, as this modification does not change the outcome of any comparison between two keys. Furthermore, a search for k in this modified tree must follow the same path, since we assume that path did not pass through the node with k' and thus all comparisons in the search are unchanged. Yet in this case, the search does not find k; this violates the original correctness of the binary tree search algorithm. The rest of the argument is symmetric.

C-11.34) Hint Consider the maximum number of times the recursive method is called on a position that is not within the subrange.

C-11.35) Hint Consider a top-down recursive approach.

C-11.36) Hint Make sure that the result is a valid AVL tree.

C-11.37) Hint Note that this method returns a single integer, so it is not necessary to visit all s items that lie in the range. You will need to extend the tree data structure, adding a new field to each node.

C-11.37) Solution For each node of the tree, maintain the size of the corresponding subtree, defined as the number of internal nodes in that subtree. While performing the search operation in both the insertion and deletion, the subtree sizes can be either incremented or decremented. During the rebalancing, care must be taken to update the subtree sizes of the three nodes involved (labeled a, b, and c by the restructure algorithm).

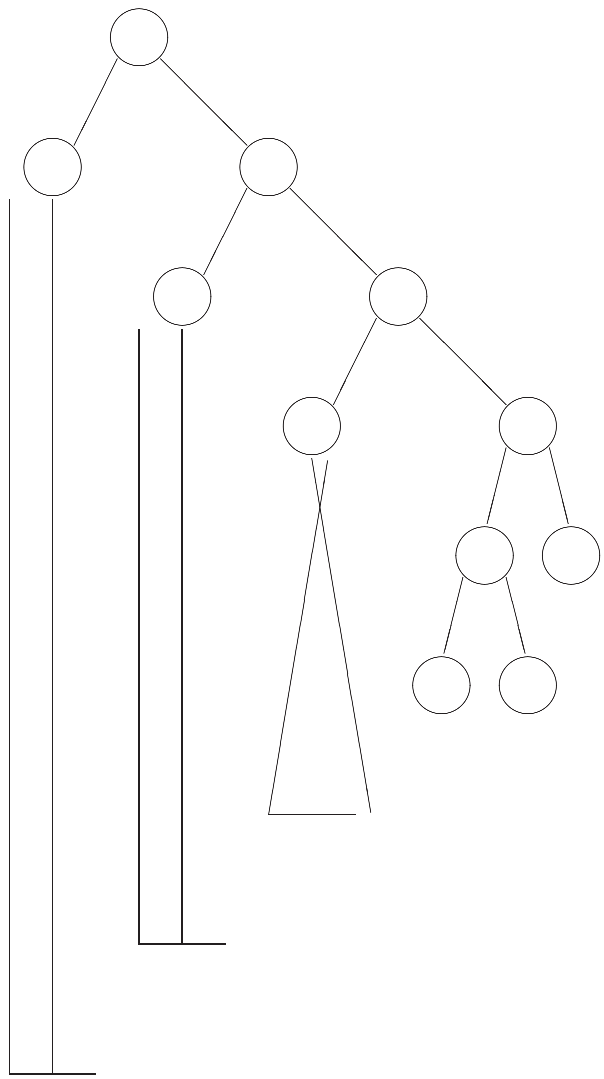
To calculate the number of nodes in a range (k_1, k_2), search for both k_1 and k_2, and let P_1 and P_2 be the associated search paths. Call v the last node common to the two paths. Traverse path P_1 from v to k_1. For each internal node $w \neq v$ encountered, if the right child of w is in not in P_1, add one plus the size of the subtree of the child to the current sum. Similarly, traverse path P_2 from v to k_2. For each internal node $w \neq v$ encountered, if the left child of w is in not in P_2, add one plus the size of the subtree of the left to the current sum. Finally, add one to the current sum (for the key stored at node v).

C-11.38) Hint How do the rebalancing actions affect the information stored at each node?

C-11.38) Solution Whenever a rotation is performed, the sizes of the two affected nodes must be recalculated. This can be implemented within TreeMap._rotate, so that the support is inherited by any of our balanced tree subclasses.

C-11.39) Hint Use triangles to represent subtrees that are not affected by this operation, and think of how to cascade the imbalances up the tree.

C-11.39) Solution



C-11.40) Hint Think carefully about how the balance of a node and its ancestors changes immediately after an insertion or deletion.

C-11.41) Hint Just consider the operations that could change the leftmost position or who points to it.

C-11.42) Hint How do the rebalancing actions affect the minimum?

C-11.42) Solution No changes are necessary. Rebalancing does not affect which node contains the smallest key.

C-11.43) Hint Have each node of the tree maintain references to its inorder neighbors.

C-11.44) Hint How do the rebalancing actions affect the inorder relationships?

C-11.44) Solution No changes are necessary. Rebalancing does not affect the inorder relationships among nodes.

C-11.45) Hint Carefully review the steps of deletion when given a direct reference to the position to be deleted.

C-11.45) Solution When given a direct reference to the position of a node to delete, the only nontrivial cost for the deletion arises when the node has two children and we must find its predecessor as a replacement. If the tree is threaded, that predecessor can be located in constant time, thereby reducing the entire deletion operation to $O(1)$ time. (Of course, if using a balanced search tree, the rebalancing may require additional time.)

C-11.46) Hint These operations will be easier if you know the size of each subtree.

C-11.47) Hint Is it possible for an splay tree to also be a red-black tree?

C-11.48) Hint Study closer the balance property of an AVL tree and the rebalance operation. Also, make a node high up in tree have its AVL balance depend on the node that just got inserted.

C-11.49) Hint Study closer the balance property of an AVL tree and the rebalance operation.

C-11.50) Hint Find the right place to "splice" one tree into the other to maintain the (2,4) tree property. Also, it is okay to destroy the old versions of T and U.

C-11.50) Solution Assume that T and U have heights h_t and h_u and $h_t > h_u$. Remove the smallest entry from U. Insert this entry into the rightmost node of tree T at height $h_t - h_u - 1$. Link this node to the root of tree U.

The remove operation on U takes $O(\log m)$ time and the insert operation on T takes $O(\log n)$ time.

C-11.51) Hint Find the right place to "splice" one tree into the other to maintain the red-black tree property.

C-11.52) Hint You don't need to use induction here.

C-11.53) Hint Think about a way of using the structure of the binary search tree itself to indicate color.

C-11.53) Solution Since the nodes store keys in order, when keys are distinct, we can detect when the children's subtrees are swapped. This indication can be used to mark nodes as red. Detecting this indication increases the search and update times by a constant factor, but the asymptotic running times remain the same.

C-11.54) Hint Search down for k and cut along this path. Now consider how to "glue" the pieces back together in the right order.

C-11.55) Hint Consider the red and black meaning of the three possible balance factors in an AVL tree.

C-11.55) Solution All nodes should initially be black. If a node is black and it's left and right sub trees have different heights, the node should be colored red (excluding the root).

C-11.56) Hint Since you know the node x will eventually become the root, maintain a tree of nodes to the left of x and a tree of nodes to the right of x, which will eventually become the two children of x.

C-11.57) Hint The analysis in the book works also for half-splay trees, with minor modifications.

C-11.58) Hint If you are having trouble with this problem, you may wish to gain some intuition about splay trees by "playing" with an interactive splay tree program on the Internet.

C-11.59) Hint Force such a replacement to take place and then attempt to use an existing position instance to the item that served as the replacement.

C-11.60) Hint Make sure that each item remains at its original node instance.

C-11.60) Solution Rather than replacing the item at the node to be deleted, restructure the tree so that the predecessor node is spliced in place of the node to be deleted.

Projects

P-11.61) Hint We've provided the implementations; you need to develop the experiment.

P-11.62) Hint In this case, you will need a skip list implementation to test.

P-11.63) Hint Remember that a single node of the tree might store multiple (key,value) pairs.

P-11.64) Hint The order is implicit in the tree, so adding these methods should not be hard.

P-11.65) Hint Think carefully about how to uniquely represent the position of a single (key,value) pair.

P-11.66) Hint Use a recursive method to do the conversion.

P-11.67) Hint The most significant challenge is how to handle the insertion of duplicate, given that the original tree search will stop when it finds the existing key.

P-11.68) Hint Review the cases for zig-zag, zig-zig, and zig. Make sure you do splaying right before doing anything else.

P-11.69) Hint First figure out a way that works assuming that all keys in existing mergeable heaps are distinct, and then work out how this is not strictly necessary.