Data Structures and Algorithms in Python

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Instructor's Solutions Manual

WILEY

Chapter

4 Recursion

Hints and Solutions

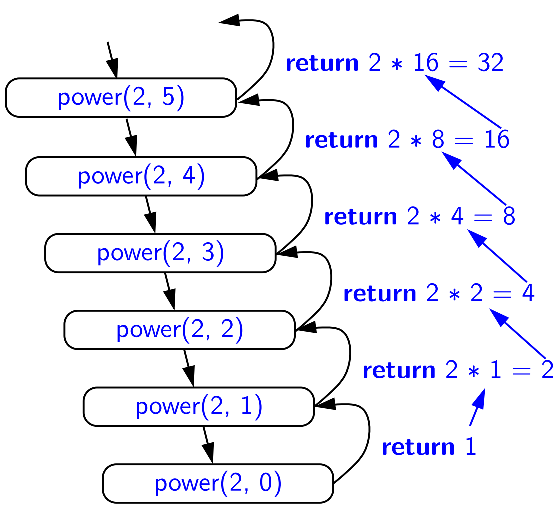
Reinforcement

R-4.1) Hint Don't forget about the space used by the function stack.

R-4.1) Solution If the sequence has 1 element, that is the maximum. Otherwise, consider the bigger of the first element or the maximum of the other $n-1$ elements. The running time and space usages is $O(n)$.

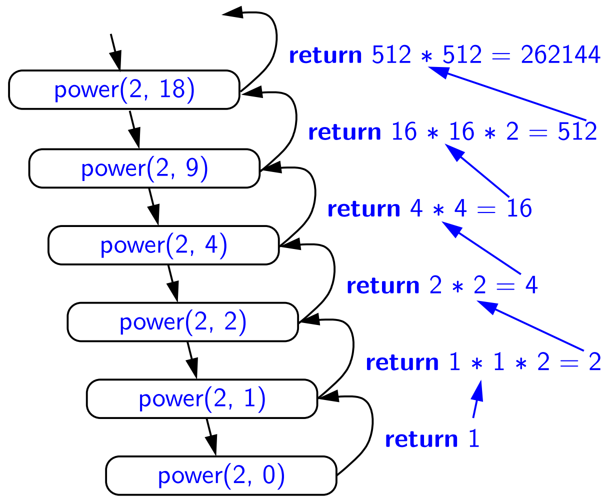
R-4.2) Hint This is probably the first power algorithm you were taught.

R-4.2) Solution



R-4.3) Hint Be sure to get the integer division right.

R-4.3) Solution



R-4.4) Hint You can model your figure after Figure 4.11.

R-4.4) Solution

0	1	2	3	4
4	3	6	2	6
6	3	6	2	4
6	2	6	3	4

R-4.5) Hint You should draw small boxes or use a big paper, as there are a lot of recursive calls.

R-4.6) Hint Start with the last term.

R-4.6) Solution The general case is $H_n = H_{n-1} + \frac{1}{n}$.

R-4.7) Hint Process the string left to right.

R-4.7) Solution Use a single-digit as the base case. For a multiple-digit string, let $s' = sd$ for digit d. We have that $value(s') = d + 10 * value(s)$.

R-4.8) Hint Look for a geometric series.

R-4.8) Solution The running time is $O(n)$, as it is $O(n + n/2 + n/4 + n/8 + \cdots)$.

Creativity

C-4.9) Hint Consider returning a tuple, which contains both the minimum and maximum value.

C-4.10) Hint The integer part of the base-two logarithm of n is the number of times you can divide by two before you get a number less than 2.

C-4.11) Hint Consider reducing the task of telling if the elements of a sequence are unique to the problem of determining if the last $n-1$ elements are all unique and different than the first element.

C-4.12) Hint You need subtraction to count down from m or n and addition to do the arithmetic needed to get the right answer.

C-4.12) Solution The recursive algorithm, ***product(n, m)***, for computing product using only addition and subtraction, is as follows: If $m = 1$ return n. Otherwise, return n plus the result of a recursive call to the function ***product*** with parameters n and $m-1$.

C-4.13) Hint Define a recurrence equation.

C-4.13) Solution let $R(c)$ denote the number of dashes drawn by draw_interval(c). We prove by induction that $R(c) = 2^{c+1} - c - 2$. As a base case, we note that draw_interval(0) does not produce any output, and that $R(0) = 2^{0+1} - 0 - 2 = 0$. For $c > 0$, we note that draw_interval(c) invokes two recursive calls of draw_interval($c-1$), and a call to drawLine that produces c dashes. Therefore, $R(c) = c + 2R(c-1)$, and by the inductive hypothesis, $R(c) = c + 2\left(2^{(c-1)+1} - (c-1) - 2\right) = c + 2\left(2^{c} - c - 1\right) = c + 2^{c+1} - 2c - 2 = 2^{c+1} - c - 2$.

C-4.14) Hint 1

C-4.15) Hint Start by removing the first element x and computing all the subsets that don't contain x.

C-4.16) Hint You can use syntax print(ch, end='') to print one character ch at a time, without extraneous spaces.

C-4.16) Solution

```
def _print_recurse(s, n=None):
  if n == None:
    n = len(s) − 1
  if n >= 0:
    print(s[n], end='')
    _print_recurse(s, n−1)

def print_reverse(s):
  _print_recurse(s, len(s)−1)
  print( )                          # final newline
```

C-4.17) Hint Check the equality of the first and last characters and recur (but be careful to return the correct value for both odd- and even-length strings).

C-4.18) Hint Write your recursive function to first count vowels and consonants.

C-4.19) Hint Consider whether the last element is odd or even and then put it at the appropriate location based on this and recur.

C-4.19) Solution

```
def _organize_recurse(data, low, high):
  if low < high:
    if data[high] & 1 == 0:                       # data[high] is even
      data[low],data[high] = data[high],data[low]
      _organize_recurse(data, low+1, high)        # data[low] is known to be even
    else:
      _organize_recurse(data, low, high−1)        # data[high] is known to be odd
def organize(data):
  _organize_recurse(data, 0, len(data) − 1)
```

C-4.20) Hint Begin by comparing the first and last elements in a range of indices in A.

C-4.20) Solution This problem can effectively be solved using the same technique as Exercise C-4.19.

C-4.21) Hint The beginning and the end of a range of indices in S can be used as arguments to your recursive function.

C-4.21) Solution The solution makes use of the function FindPair(A,i,j,k) below, which given the sorted subarray $A[i..j]$ determines whether there is any pair of elements that sums to k. First it tests whether $A[i]+A[j]<k$. Because A is sorted, for any $j' \leq j$, we have $A[i]+A[j']<k$. Thus, there is no pair involving $A[i]$ that sums to k, and we can eliminate $A[i]$ and recursively check the remaining subarray $A[i+1..j]$. Similarly, if $A[i]+A[j]>k$, we can eliminate $A[j]$ and recursively check the subarray $A[i..j-1]$. Otherwise, $A[i]+A[j]=k$ and we return true. If $i==j$, no such pair was found and we return false.

Algorithm FindPair(A,i,j,k):
 Input: An integer subarray $A[i..j]$ and integer k
 Output: Returns true if there are two elements of $A[i..j]$ that sum to k

```
if i == j then
  return false
else
  if A[i] + A[j] < k then
    return FindPair(A, i+1, j, k)
  else
    if A[i] + A[j] > k then
      return FindPair(A, i, j−1, k)
    else
      return true
```

C-4.22) Hint You can rely on bitwise operations to interpret n in binary.

C-4.22) Solution

```
def power(x, n):
  k = 0
  while (1 << k) <= n:
    k += 1

  answer = 1.0
  for j in range(k−1, −1, −1):
    answer *= answer
    if (1 << j) & n > 0:
      answer *= x
  return answer
```

Projects

P-4.23) Hint Review use of the os module.

P-4.24) Hint Use recursion in your main solution engine.

P-4.25) Hint Consider a small example to see why the binary representation of the counter is relevant.

P-4.26) Hint Note the recursive nature of the problem.

P-4.27) Hint Review use of the other methods of the os module.