Computational Vision

Radiometry and Reflectance

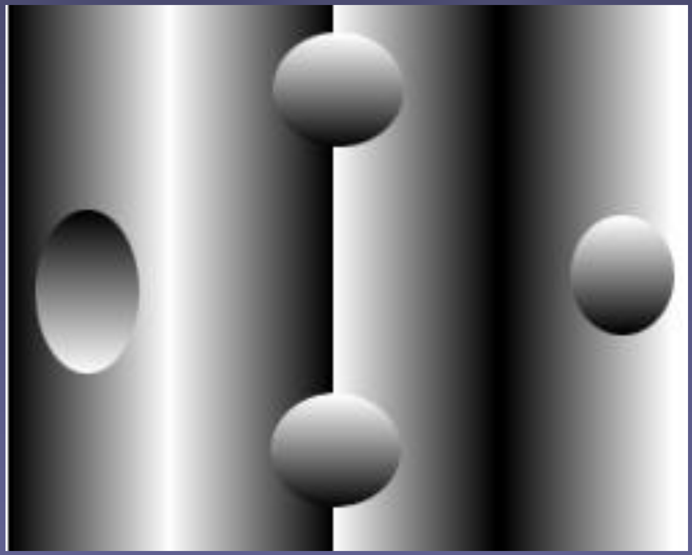
Szeliski: 2.2

Horn: Chapter 2: 22-24, Chapter 10: 202-215

From 2D to 3D

- DEPTH from TWO or MORE IMAGES
  - Stereo
  - Optical Flow -> Factorization Method.
- SHAPE from SINGLE IMAGE CUES
  - SHAPE from SHADING
  - SHAPE from TEXTURE
  - …

Shading Encodes Shape



From Robot Vision (Horn)

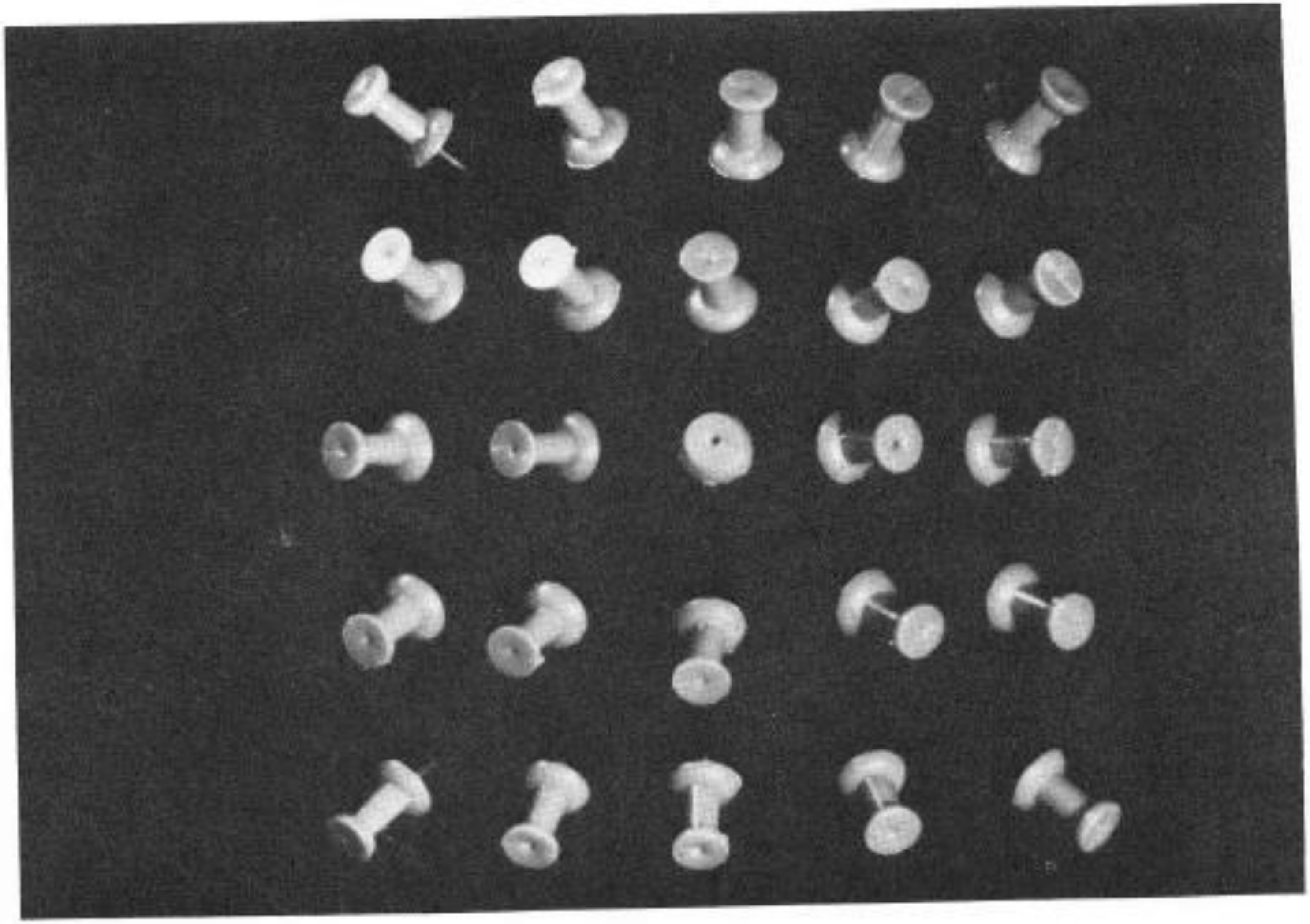


Figure 10-2. The appearance of an object depends greatly on its attitude in space relative to the viewer. Not only does the outline vary, but the brightness pattern within the silhouette changes.

Radiometry and Reflectance

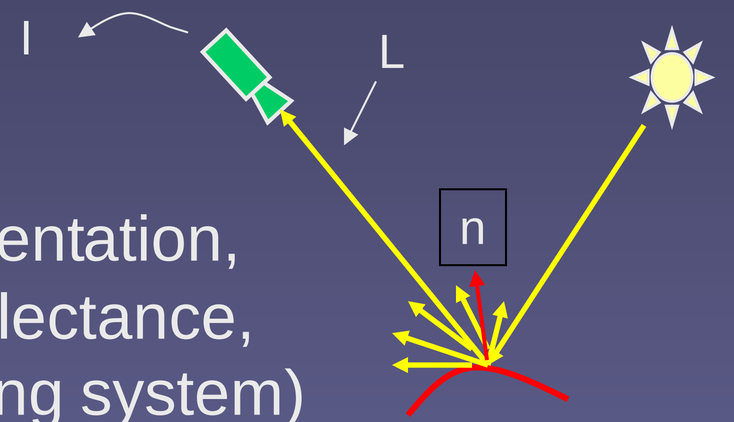


Image Intensity I = f (surface orientation,
surface reflectance,
illumination, imaging system)

Surface Element

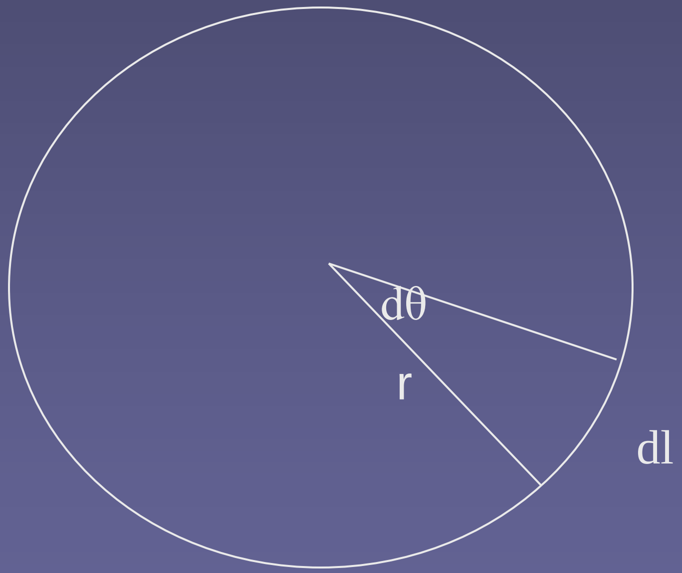
Note: Image Intensity Understanding is an **under-constrained** problem!

Angles in 2D & 3D

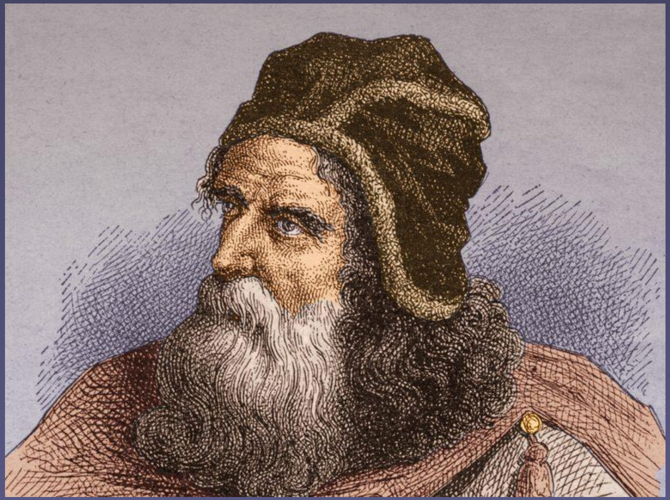
Angle definition on the plane

$d\theta = dl / r$ (radian)

$d\theta = dl$ in unit circle ($r = 1$)



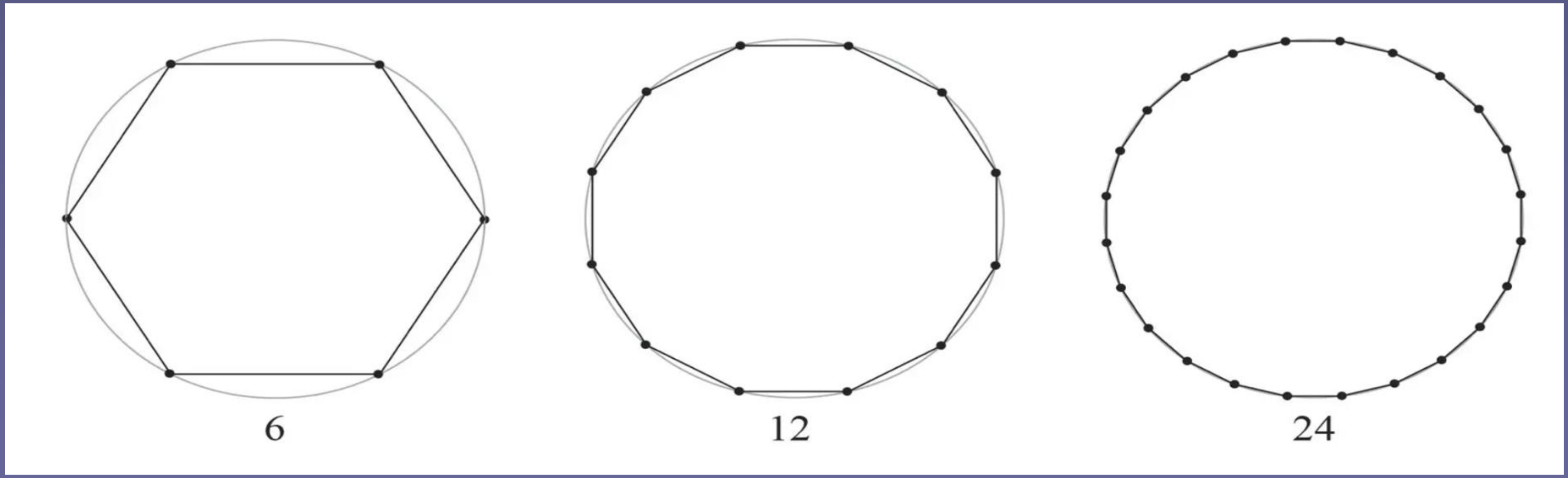
π day : 3/14



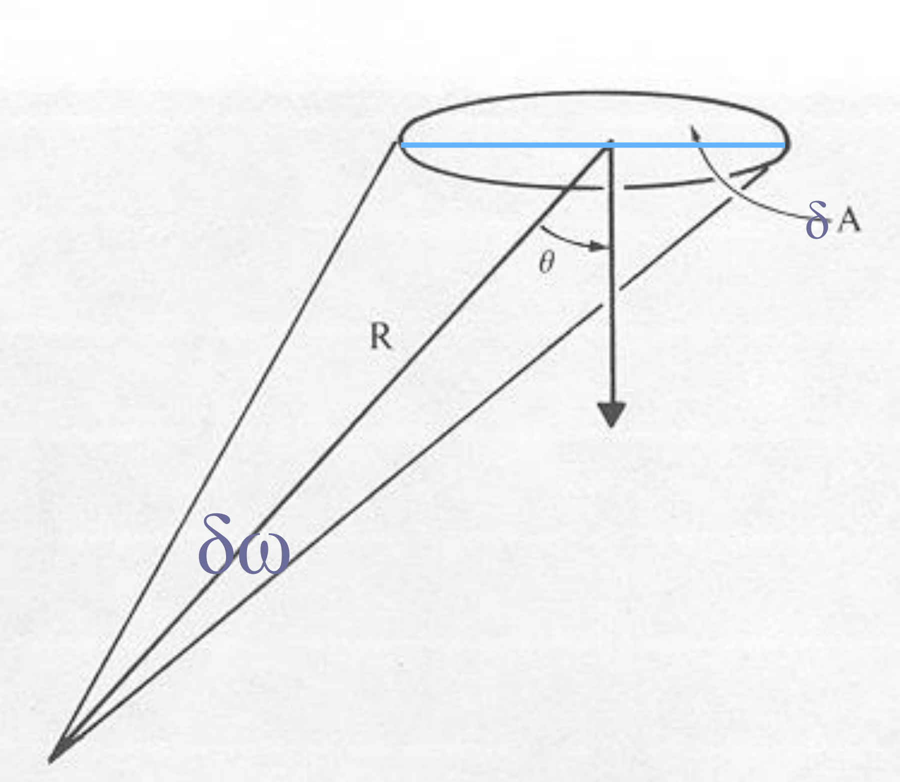
Archimedes (287-211BCE) computation of π

https://www.nytimes.com/article/pi-day-math-geometry-infinity.html?unlocked_article_code=1.ck0.WDnQ.5rBU-rR2oQHR&smid=url-shar

$$3 + \frac{10}{71} < \pi < 3 + \frac{10}{70}$$

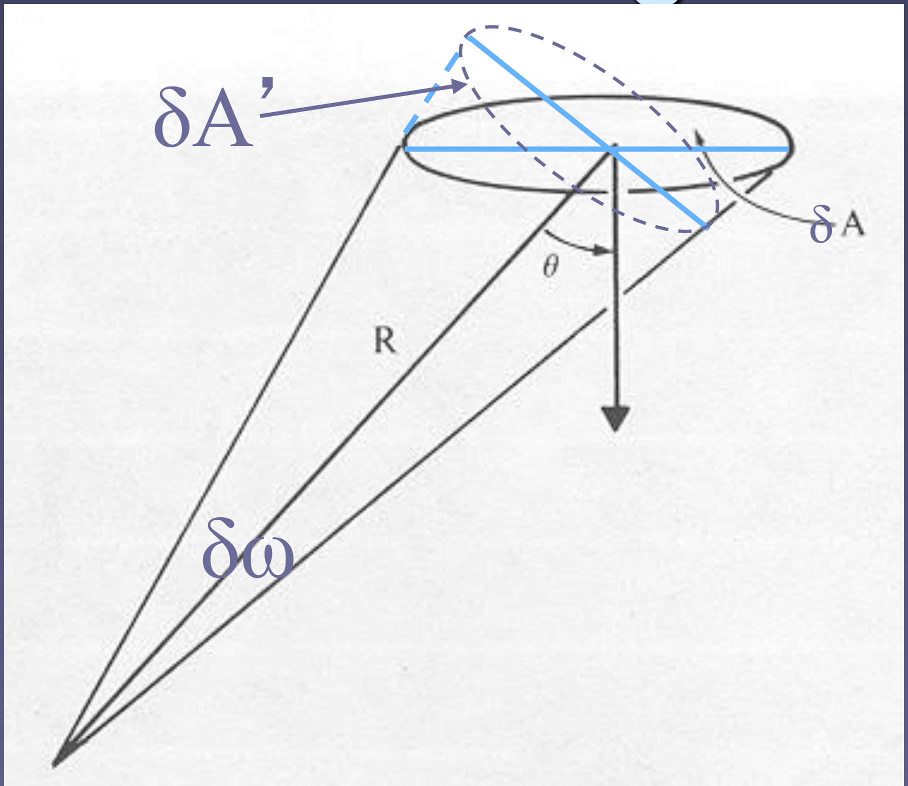


Solid Angle



$$\delta\omega = (\delta A \cos\theta)/R^2 \quad \text{(steradian)}$$

Solid Angle

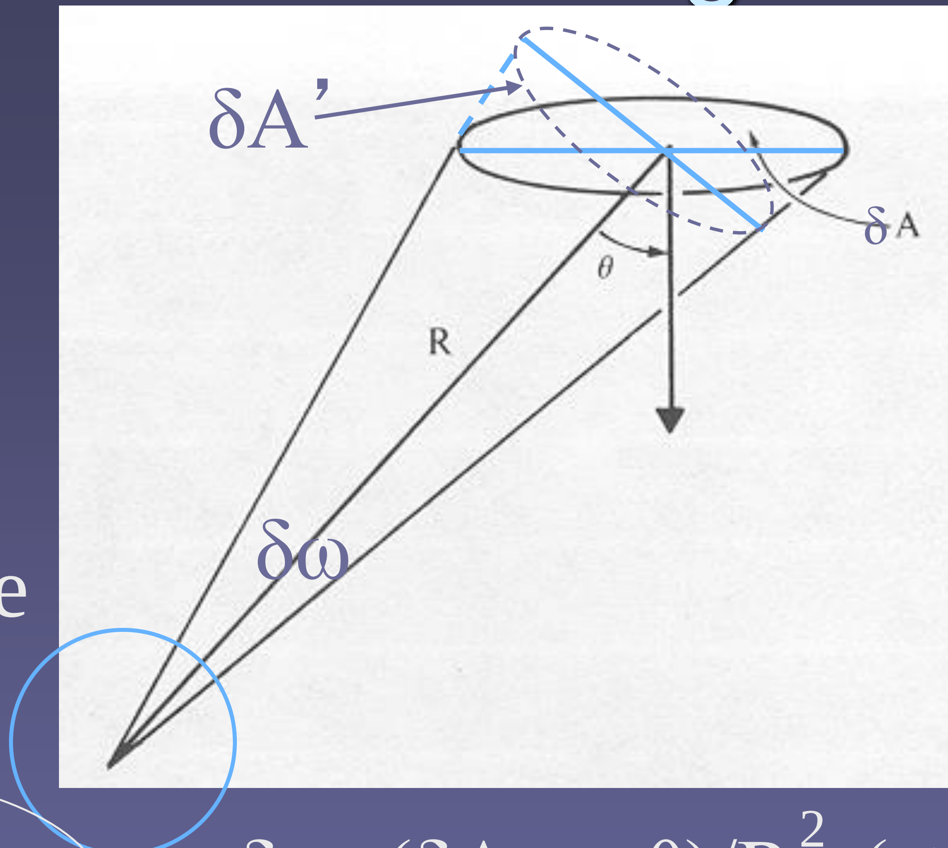


$$\delta\omega = (\delta A \cos\theta)/R^2 \quad \text{(steradian)}$$

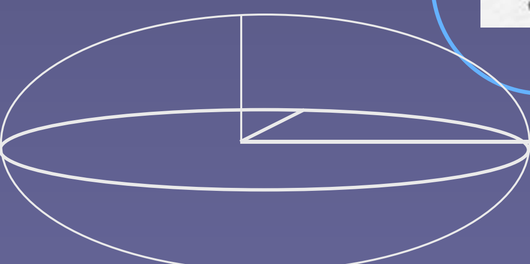
Foreshortened Area

$$\delta\omega = \delta A' / R^2 \quad \text{(steradian)}$$

Solid Angle



Unit Sphere

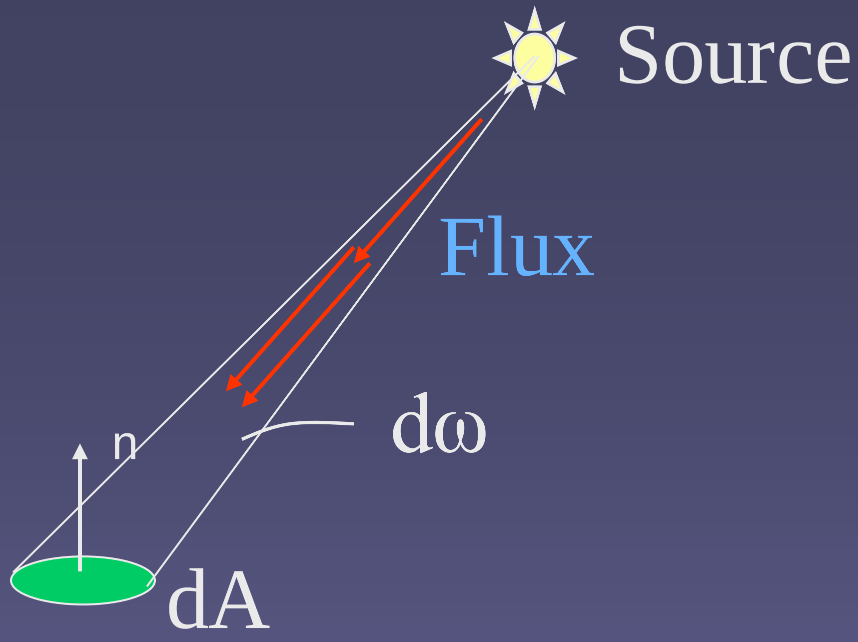


$$\delta\omega = (\delta A \cos\theta)/R^2 \quad \text{(steradian)}$$

Foreshortened Area

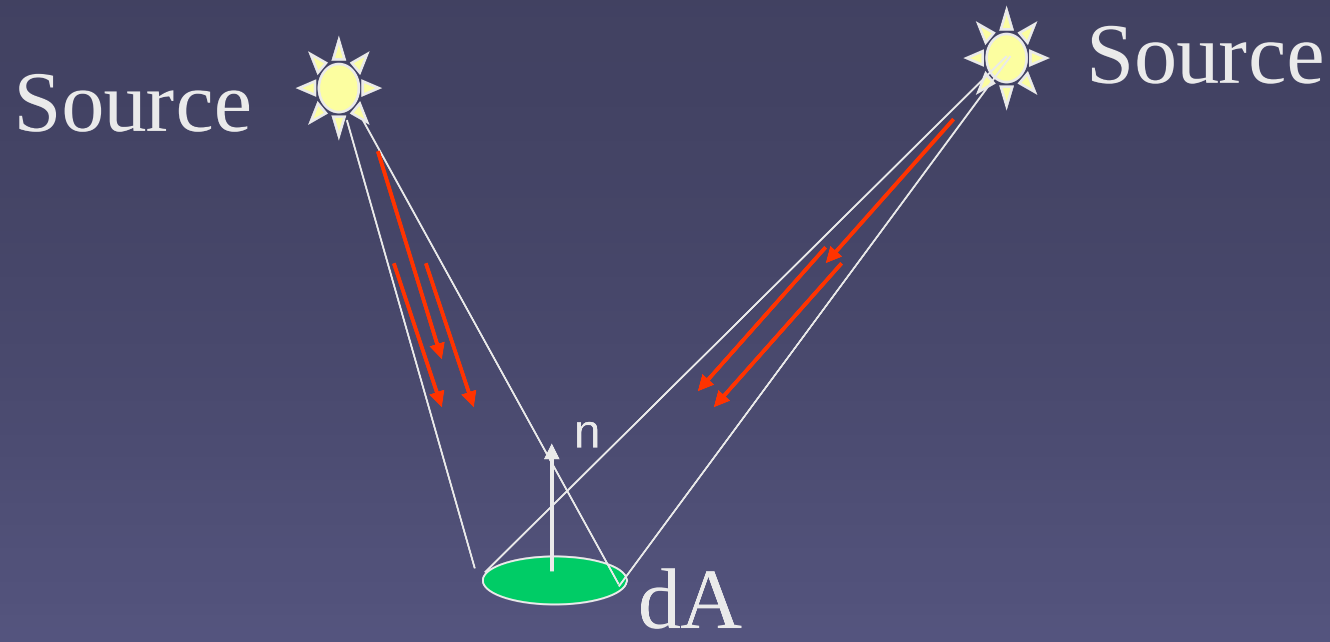
Solid Angle Sustained by a hemisphere = 2π

$$\delta\omega \quad = \delta A' / R^2 \quad \text{(steradian)}$$



<u>Radiant Intensity of Source</u>: Light flux (power) emitted per unit solid angle:

$$J = d\Phi / d\omega \text{ (watts/steradian)}$$

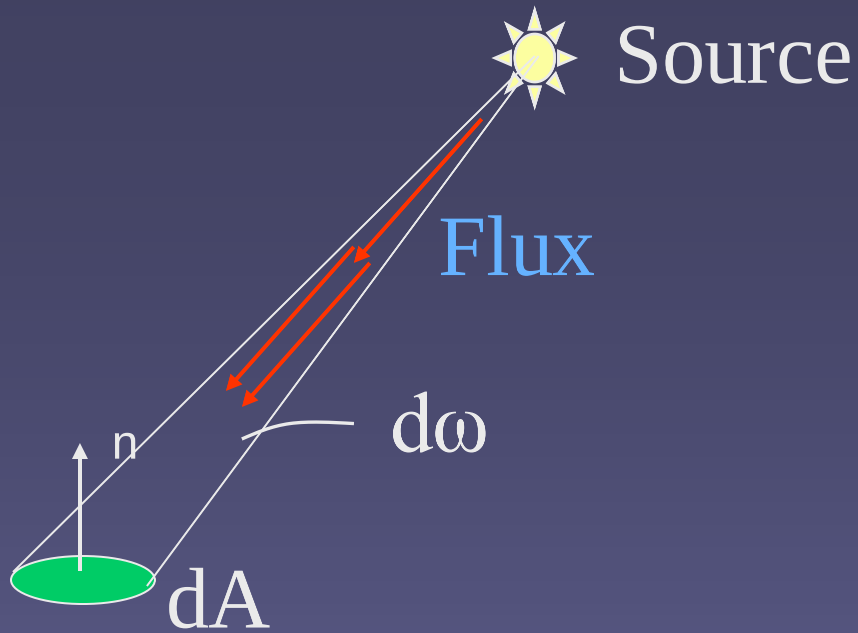


<u>Surface Irradiance</u>: Flux incident per unit surface area:

$$E = d\Phi/dA \text{ (watts/m}^2\text{)}$$

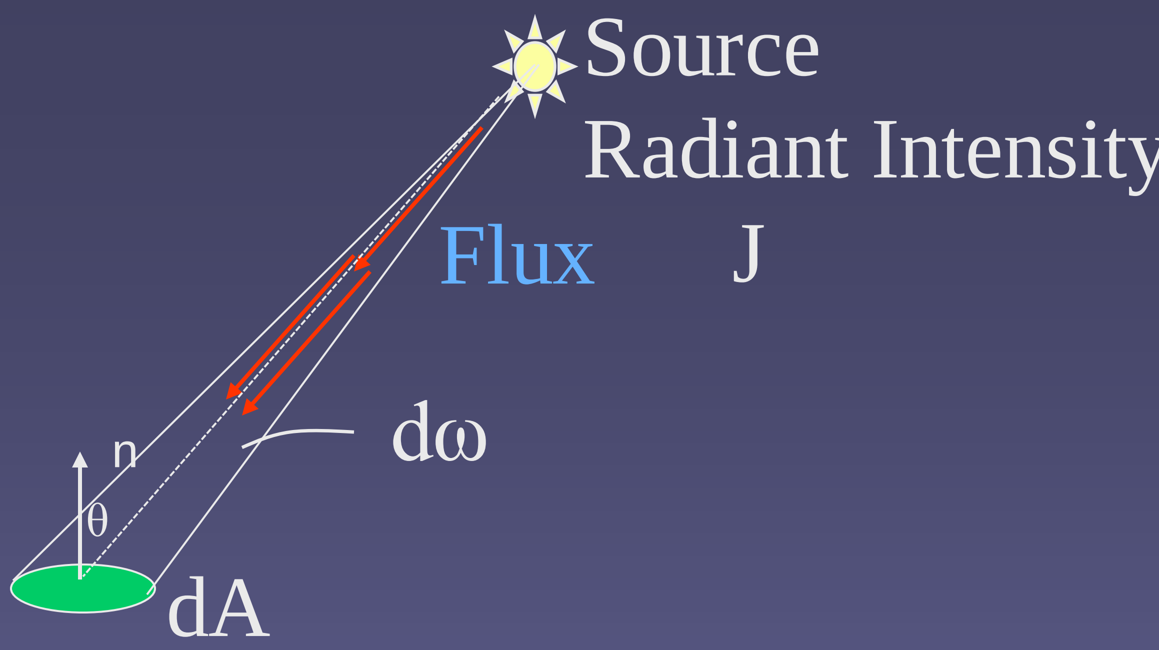
Does not depend on where the light is coming from!

One or more light sources.



Surface Irradiance: Flux incident per unit surface area:

$$E = d\Phi / dA \ (\text{watts}/m^2)$$

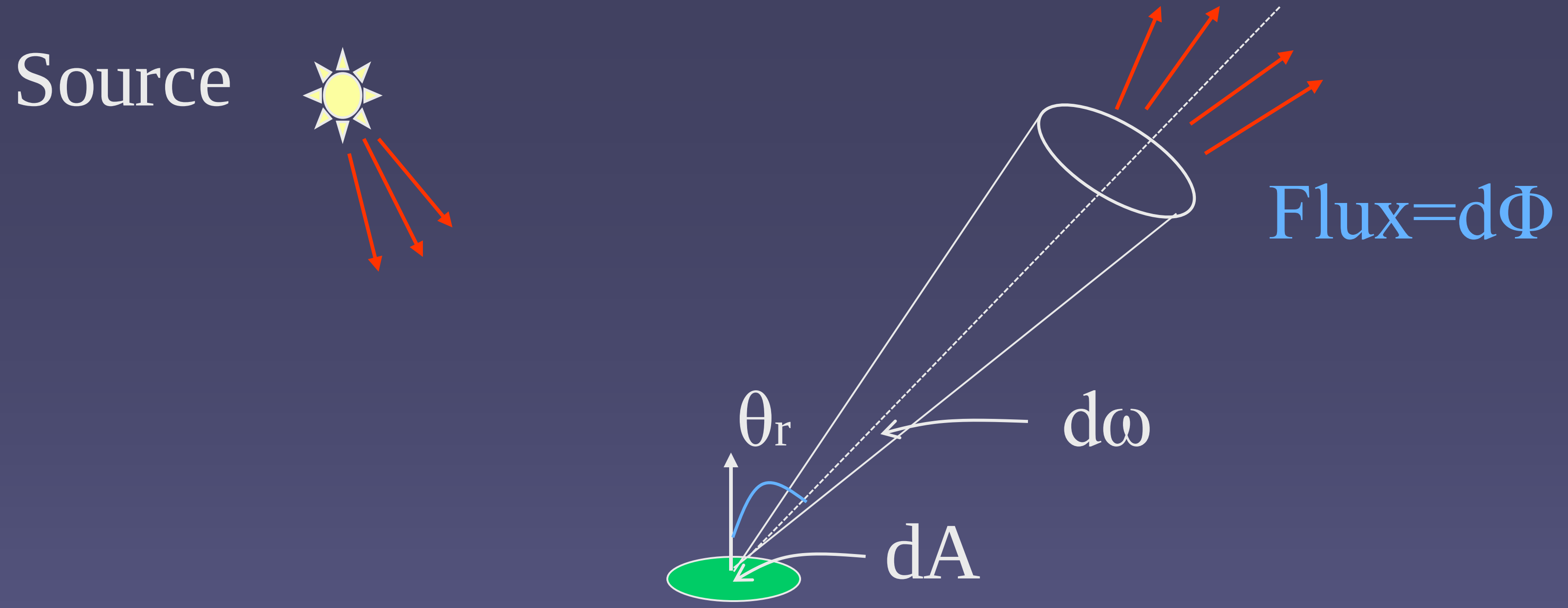


Surface Irradiance as a function of J (one source):

$E = d\Phi / dA \Rightarrow E = (J\, d\omega) / dA \Rightarrow E = (J \cos\theta) / R^2$

E inversely proportional to R

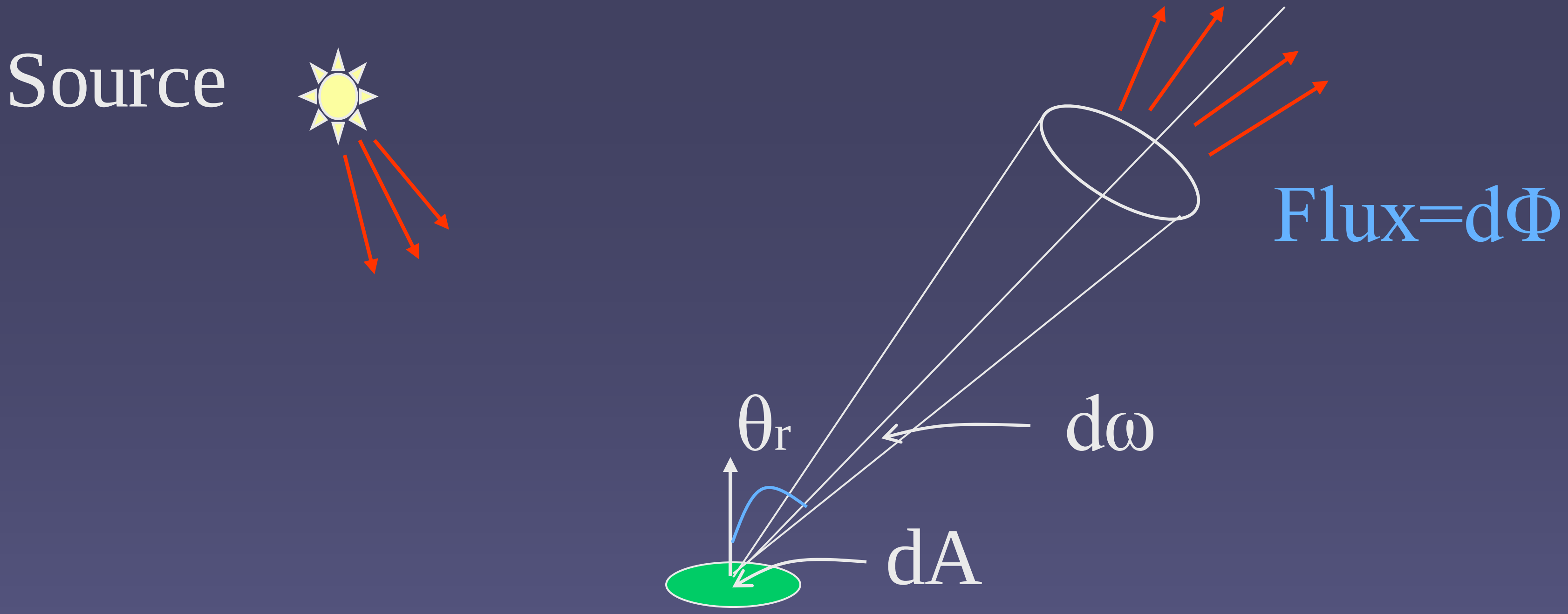
E proportional to cosθ (when is E maximized?)



Surface Radiance(Brightness):

Flux emitted per unit foreshortened area, per unit solid angle:

$$L = d\Phi / (dA \cos\theta_r) d\omega \text{ (watts/m}^2\text{ steradian)}$$



Surface Radiance(Brightness):

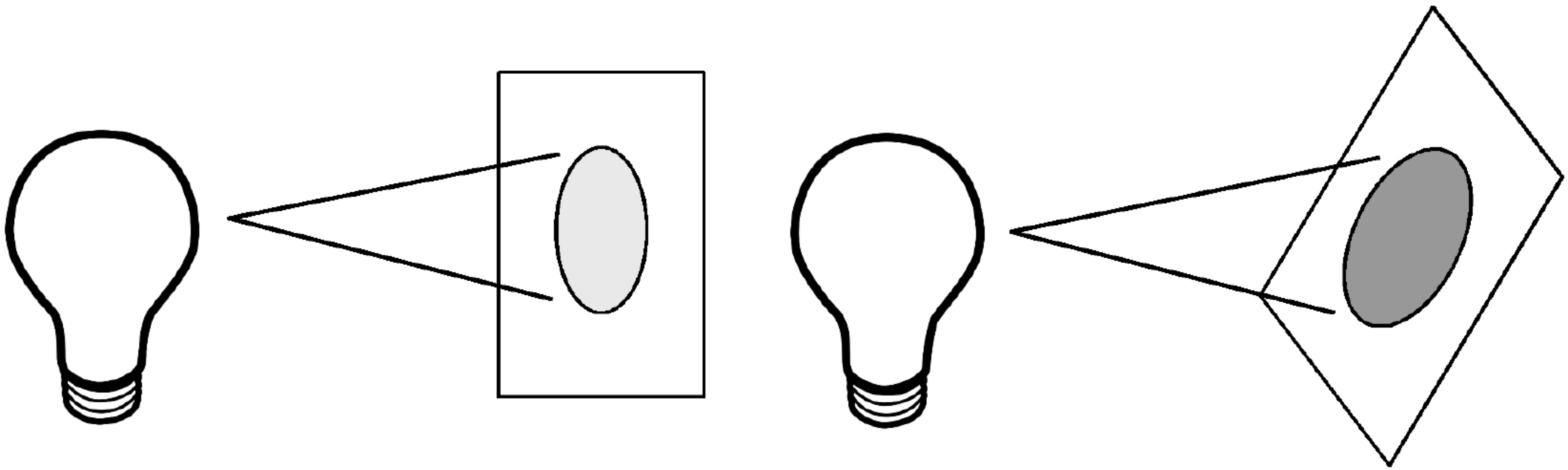
Flux emitted per unit foreshortened area,

per unit solid angle:

$$L = d\Phi / (dA \cos\theta_r) d\omega \ (\text{watts}/m^2 \cdot \text{steradian})$$

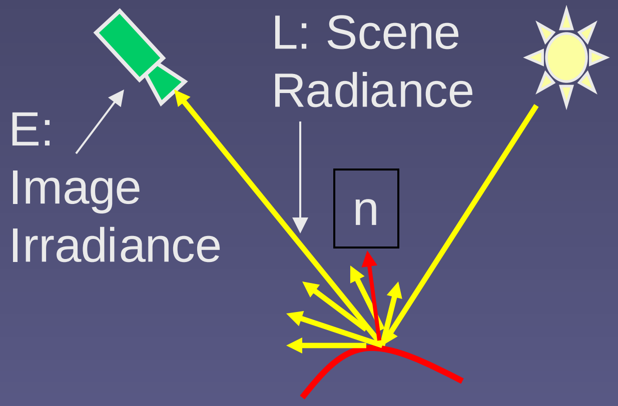
- L depends on direction θr
- Surface can radiate into whole hemisphere
- Depends on reflectance properties of surface

Foreshortening



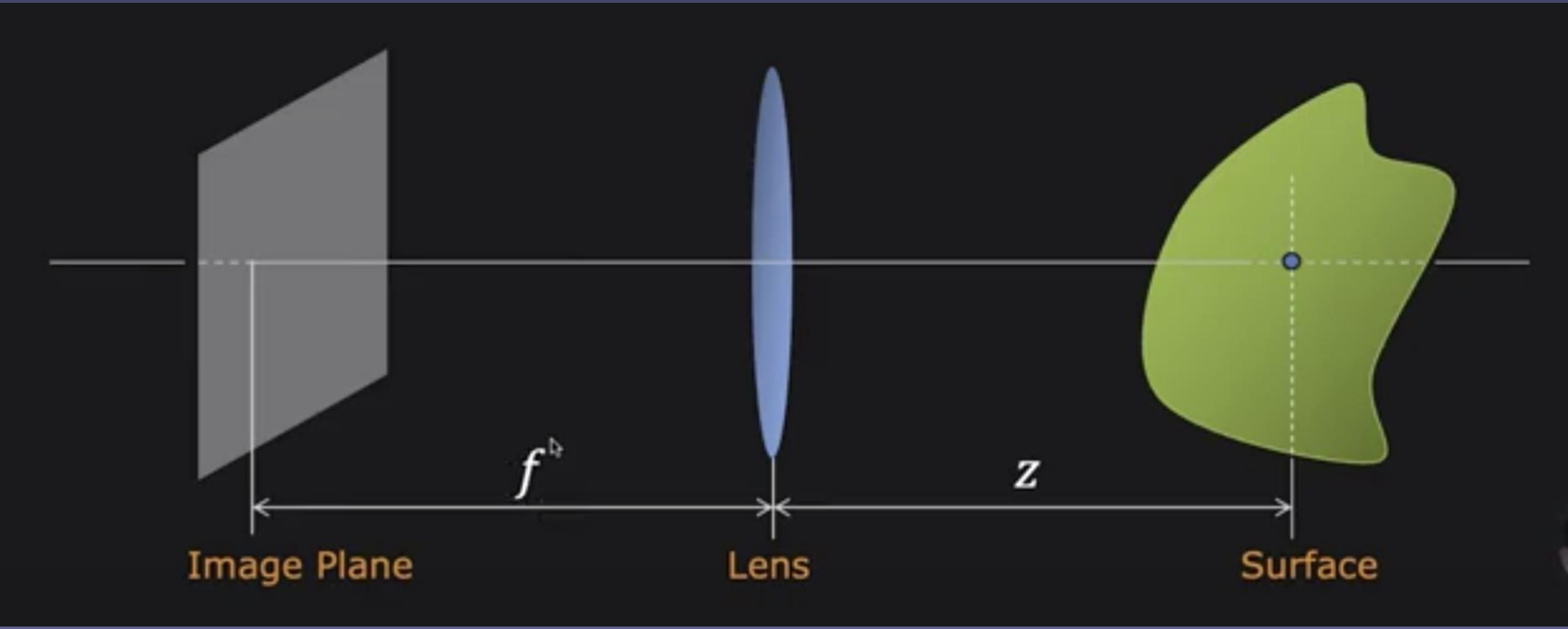
As the surface tilts away from the light source the same light energy is spread over a larger area, making the surface darker

Surface Radiance & Image Irradiance



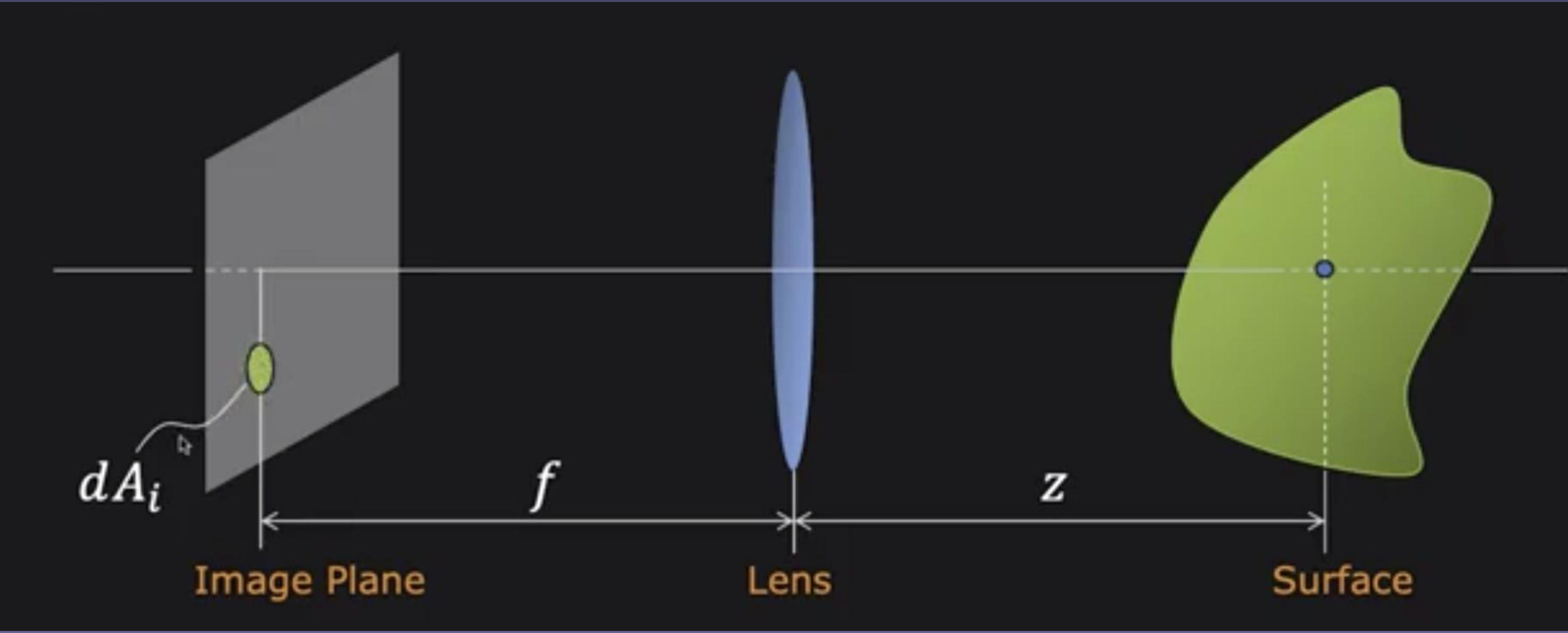
What is the relationship between L & E?

Surface Radiance & Image Irradiance



f: effective focal length

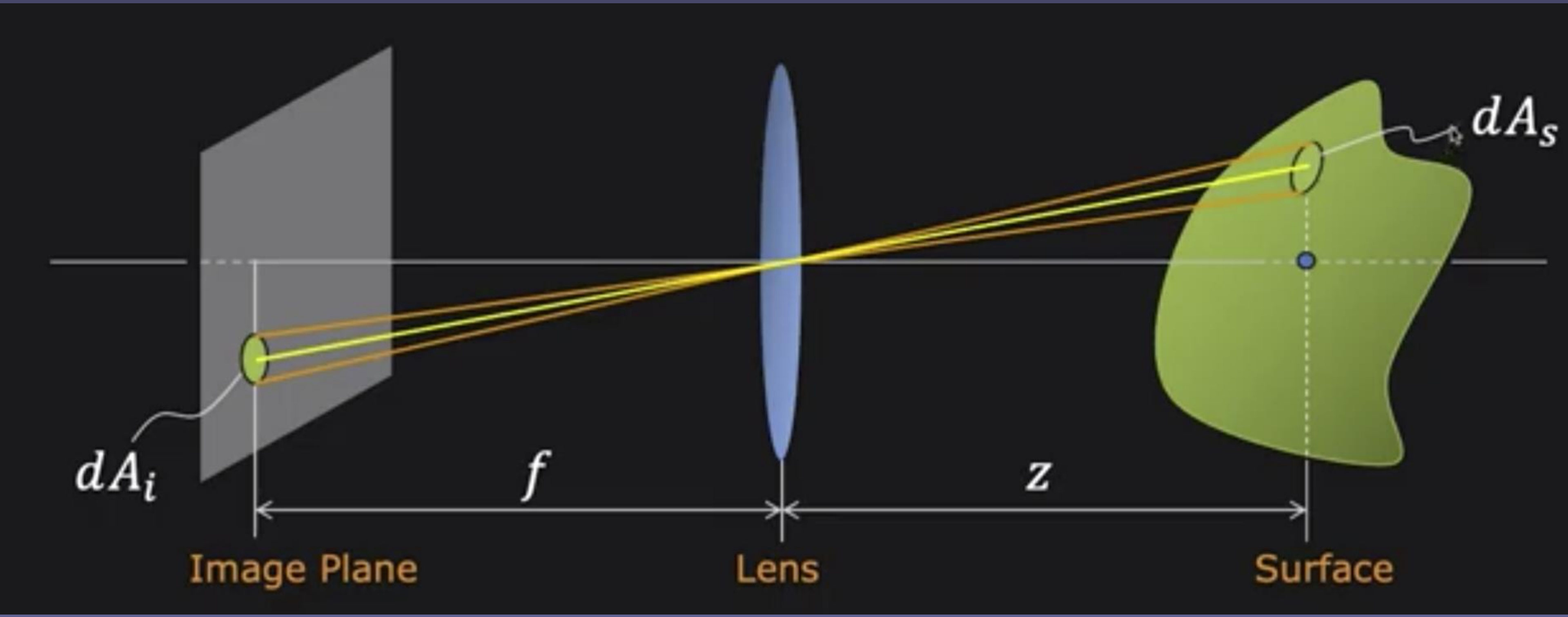
Surface Radiance & Image Irradiance



f: effective focal length

Shree Nayar

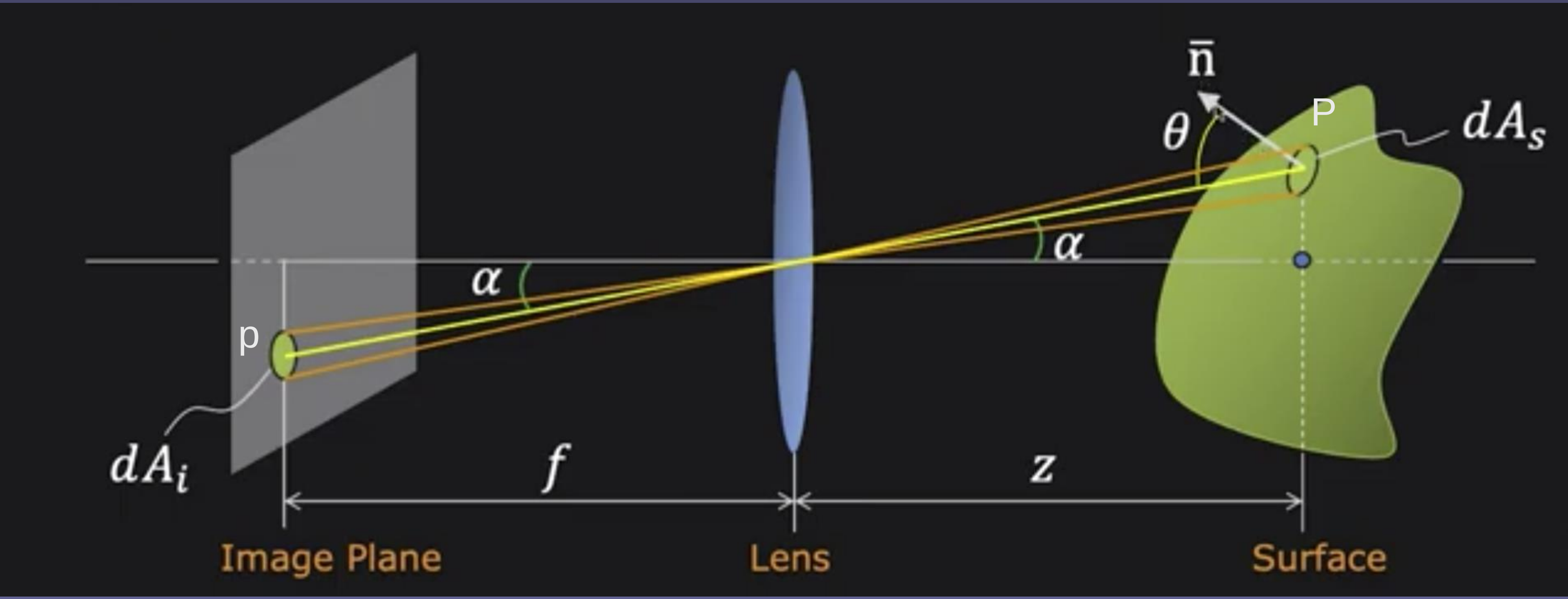
Surface Radiance & Image Irradiance



f: effective focal length

Shree Nayar

Surface Radiance & Image Irradiance



f: effective focal length
p is perspective projection of scene point P
dAi and dAs are very small surfaces

Shree Nayar

Surface Radiance & Image Irradiance

Trucco & Verri

L (Watts/m^2 * steradian)

Here

δI is dAi

δO is dAs

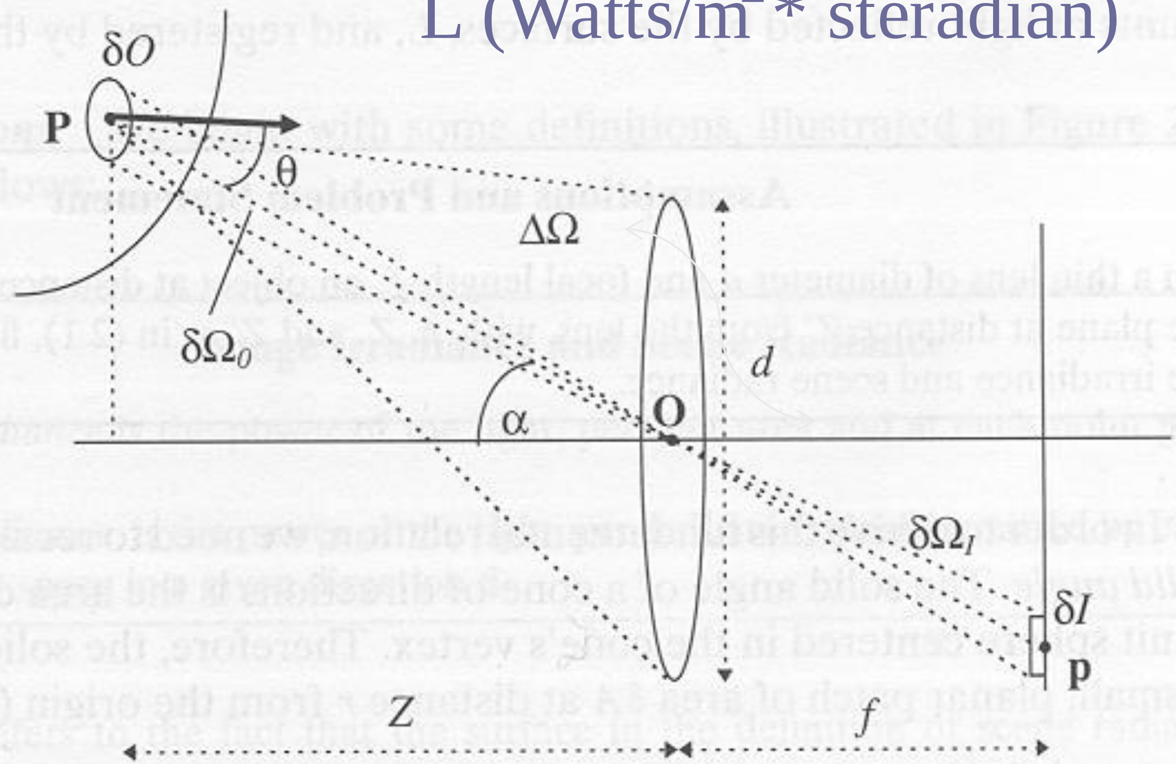


Figure 2.7 Radiometry of the image formation process.

Surface Radiance & Image Irradiance

Trucco & Verri

L (Watts/m^2 * steradian)

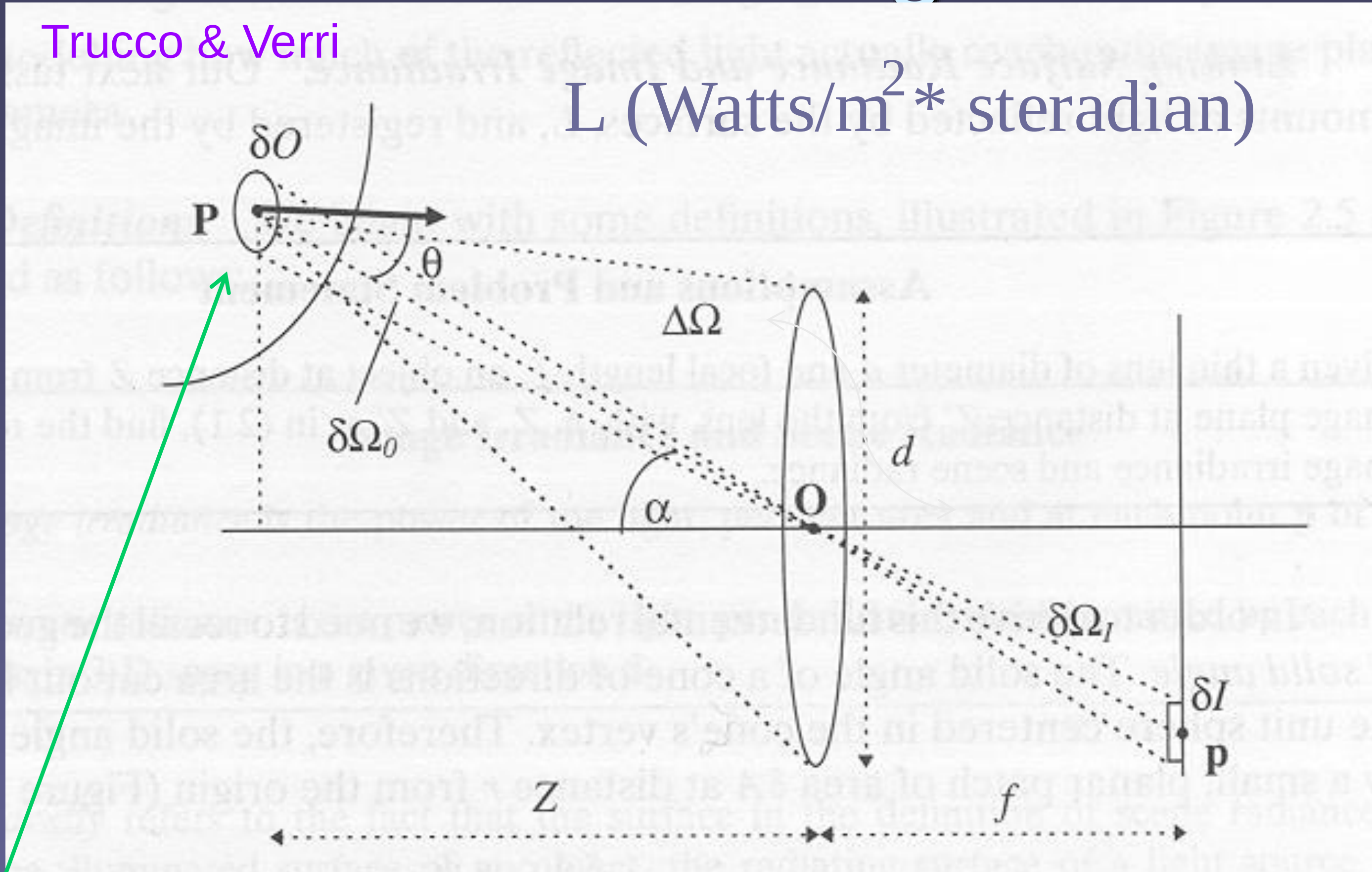


Figure 2.7 Radiometry of the image formation process.

$L = \delta\Phi / (\delta O * \cos\theta * \Delta\Omega)$, L scene radiance at **P**

Surface Radiance & Image Irradiance

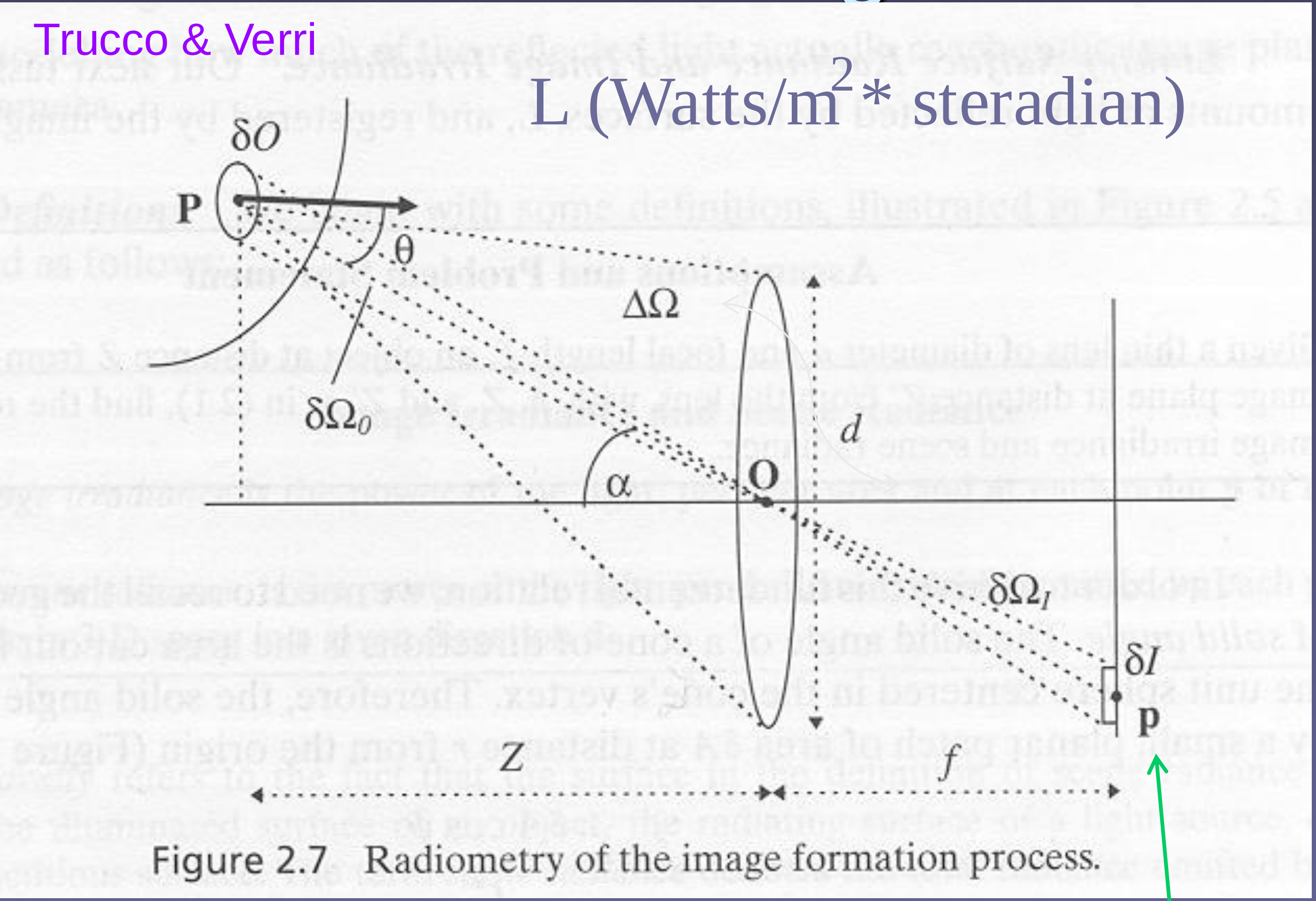


Figure 2.7 Radiometry of the image formation process.

$E = \delta\Phi / \delta I$, image irradiance at p

Surface Radiance & Image Irradiance

Trucco & Verri

L (Watts/m^2 * steradian)

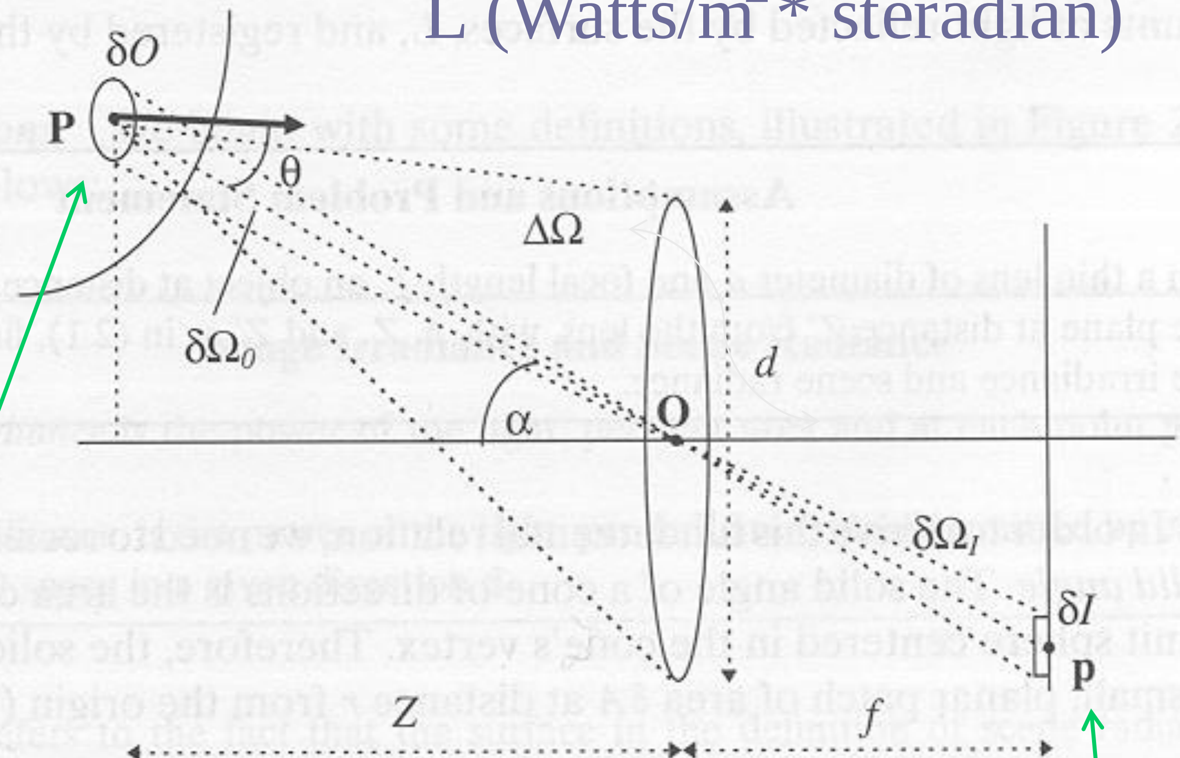


Figure 2.7 Radiometry of the image formation process.

$E = \delta\Phi / \delta I$, image irradiance at p.

$L = \delta\Phi / (\delta O * \cos\theta * \Delta\Omega)$, L scene radiance at **P**.

Fundamental Equation of Radiometric Image Formation

$$E = L * \Delta\Omega * \cos\theta * \delta O / \delta I$$

Solid angles:

$$\Delta\Omega = \delta A * \cos\alpha / R^2 = (\pi d^2/4) * \cos\alpha \,/\, (Z/\cos\alpha)^2$$
$$= (\pi/4)\, d^2 \cos^3\alpha \,/\, Z^2$$

$$\delta\Omega_o = \delta O * \cos\theta \,/\, (Z/\cos\alpha)^2$$

$$\delta\Omega_I = \delta I * \cos\alpha \,/\, (f/\cos\alpha)^2$$

$$\delta\Omega_o = \delta\Omega_I \Rightarrow \delta O / \delta I = (\cos\alpha / \cos\theta) * (Z/f)^2$$

Fundamental Equation of Radiometric Image Formation

$E = L * (\pi/4)\, d^2\, (\cos^3\alpha / Z^2) * \cos\theta * (\cos\alpha / \cos\theta) * (Z / f)^2$

Finally:

$$\boxed{E = L * (\pi / 4) * (d / f)^2 * \cos^4\alpha}$$

Surface Radiance & Image Irradiance

Image irradiance

Brightness falloff

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha)$$

1 / Effective F-number

Scene radiance

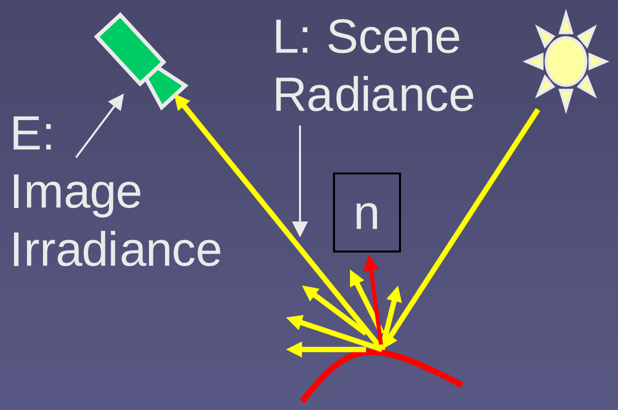


Image irradiance is proportional to scene radiance

$$E \propto L$$

Surface Radiance & Image Irradiance

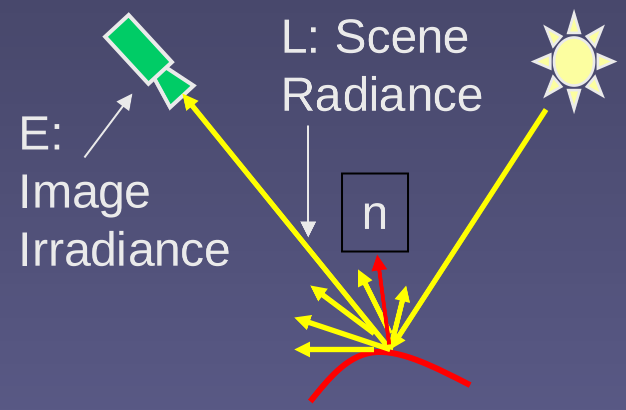
Image irradiance

Brightness falloff

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha)$$

1 / Effective F-number

Scene radiance



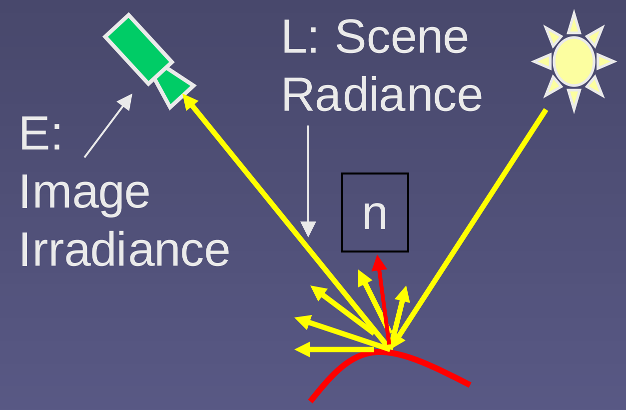
- Image irradiance is proportional to scene radiance $E \propto L$
- Image irradiance falls off as a function of $\cos^4(a)$
- Small field of view => small $\cos^4(a)$

Surface Radiance & Image Irradiance

Image irradiance

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha)$$

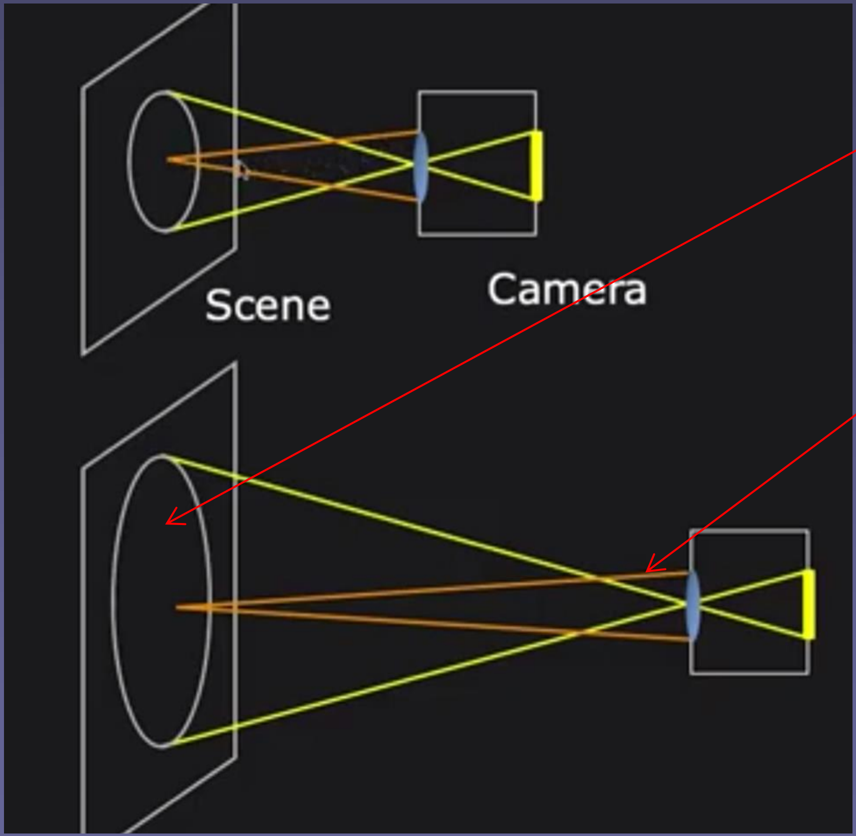
Scene radiance



DOES Image Irradiance VARY WITH SCENE DEPTH?

Image Irradiance Does Not Vary with Scene Depth

Varying Scene Depth



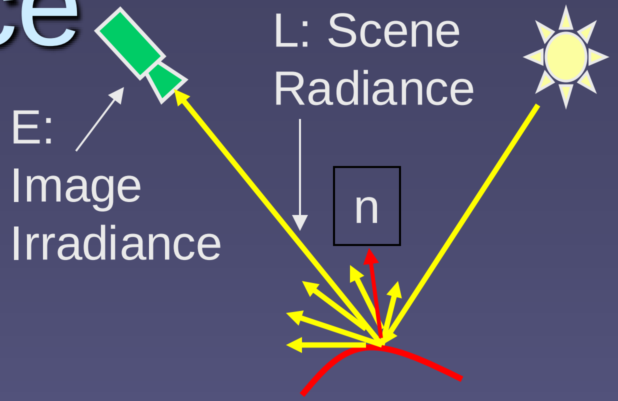
- Larger Scene Depth, Larger light accumulation
- Larger Scene Depth, Smaller Solid Angle subtended by each point

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha)$$

Surface Radiance & Image Irradiance

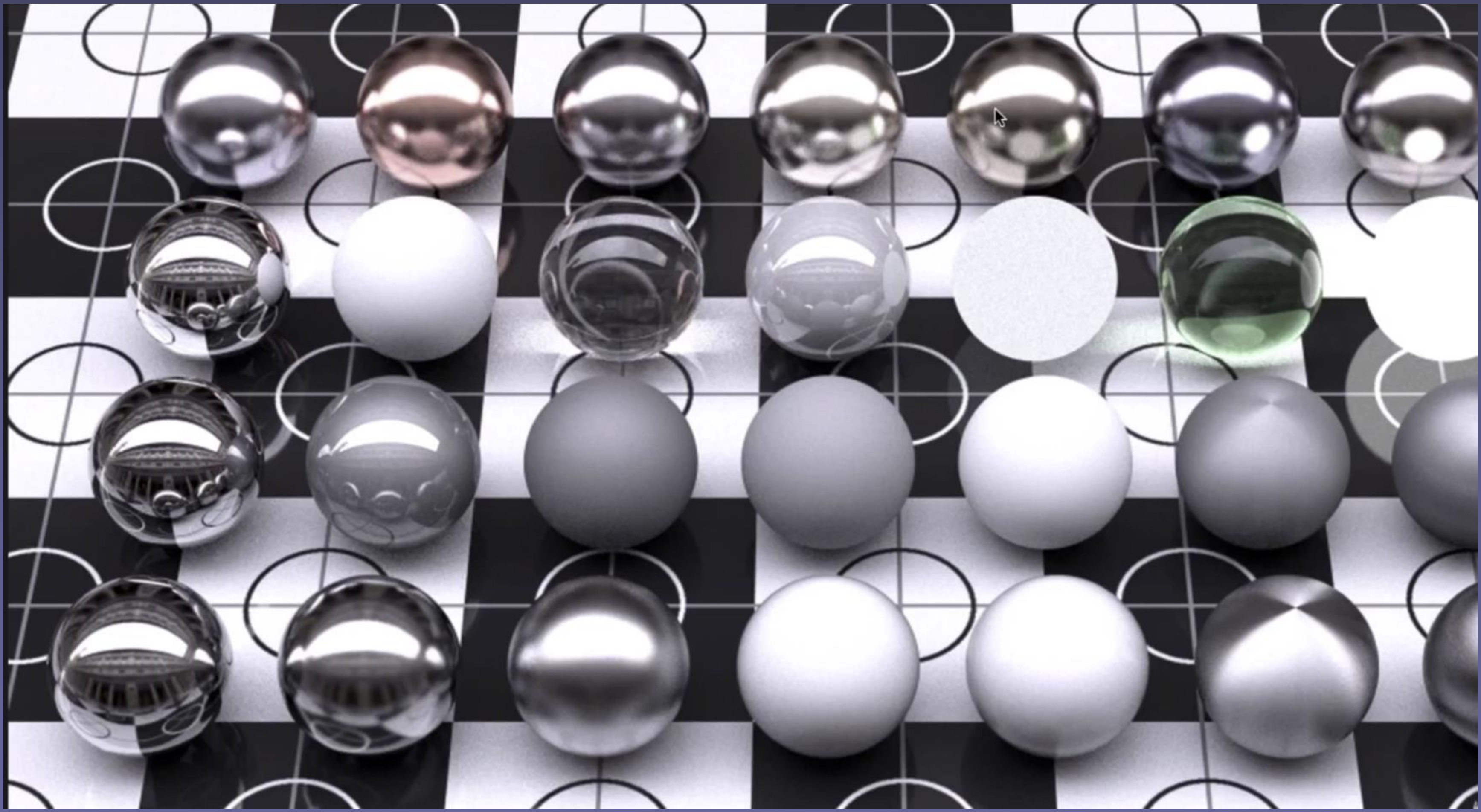
Image irradiance

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha)$$



DOES Image Irradiance VARY WITH SCENE DEPTH? NO

Surface Appearance



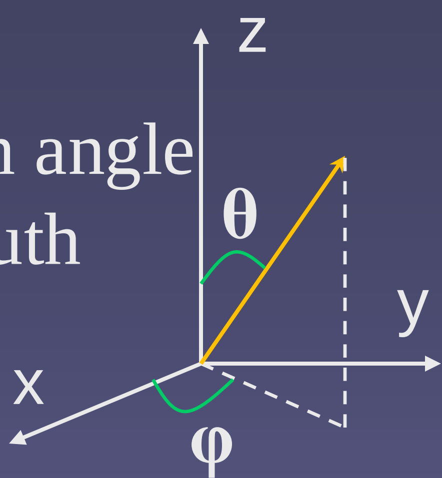
Measuring appearance

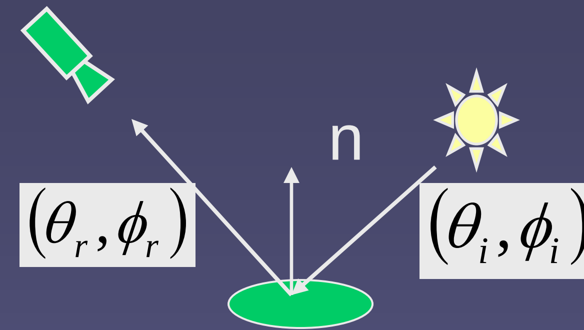
From “Fraunhober IOSB”

Bi-Directional Reflectance Distribution Function (BRDF)

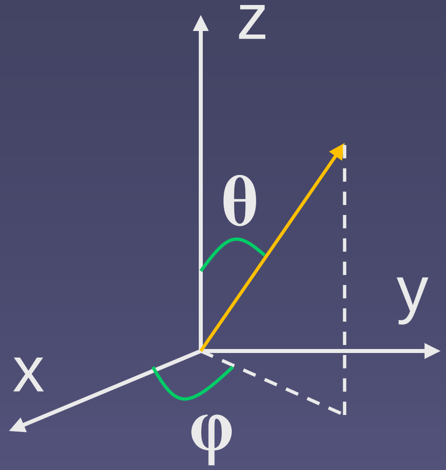
θ: zenith angle

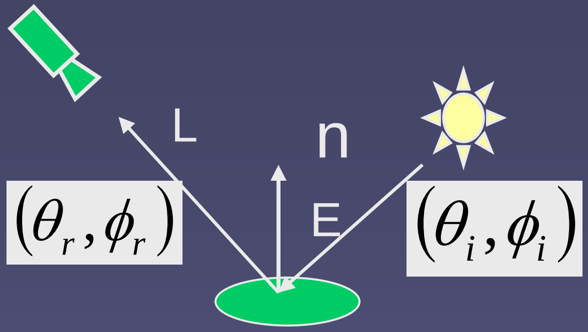
φ: azimuth angle





Bi-Directional Reflectance Distribution Function (BRDF)

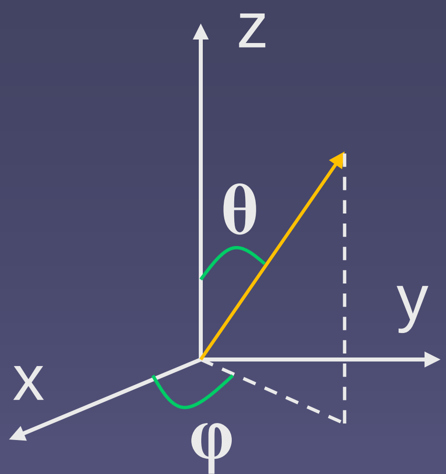


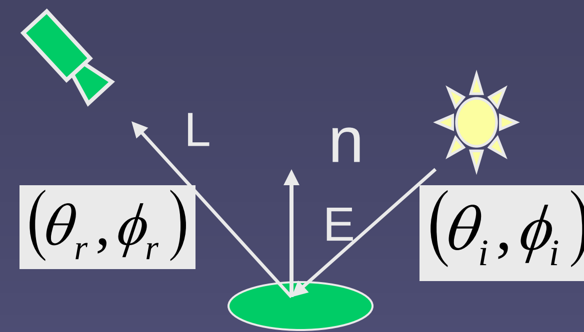


$E(\theta_i, \phi_i)$: Irradiance due to source in direction (θ_i, ϕ_i)

$L(\theta_r, \phi_r)$: Radiance of surface in direction (θ_r, ϕ_r)

Bi-Directional Reflectance Distribution Function (BRDF)





$E(\theta_i, \phi_i)$: Irradiance due to source in direction (θ_i, ϕ_i)

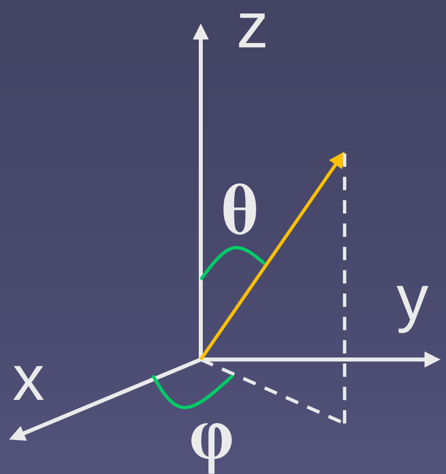
$L(\theta_r, \phi_r)$: Radiance of surface in direction (θ_r, ϕ_r)

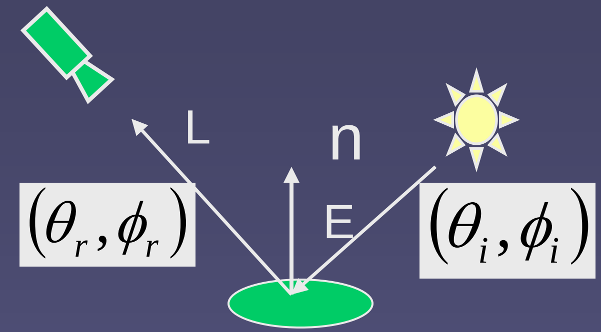
BRDF: $$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L(\theta_r, \phi_r)}{E(\theta_i, \phi_i)}$$

4-D function

Unit: 1 / steradian

Bi-Directional Reflectance Distribution Function (BRDF)





BRDF: $$f(\theta_i,\phi_i,\theta_r,\phi_r)=\frac{L(\theta_r,\phi_r)}{E(\theta_i,\phi_i)}$$

4-D function

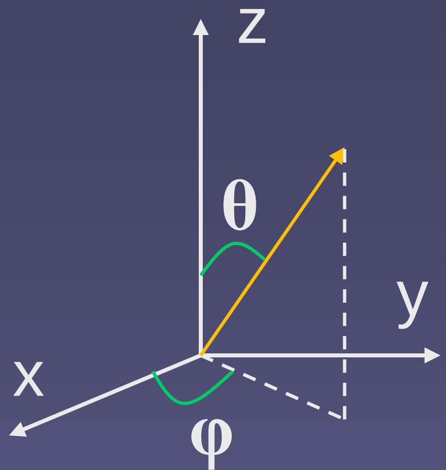
Unit: 1 / steradian

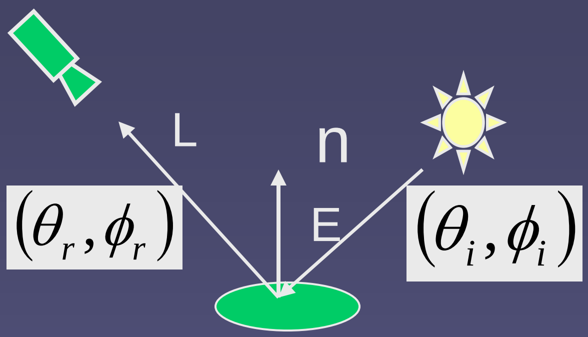
PROPERTIES:

1. BRDF f is always > 0

2. Helmholtz Reciprocity: $$f(q_i,j_i,q_r,j_r)=f(q_r,j_r,q_i,j_i)$$

Bi-Directional Reflectance Distribution Function (BRDF)





BRDF: $$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L(\theta_r, \phi_r)}{E(\theta_i, \phi_i)}$$

4-D function

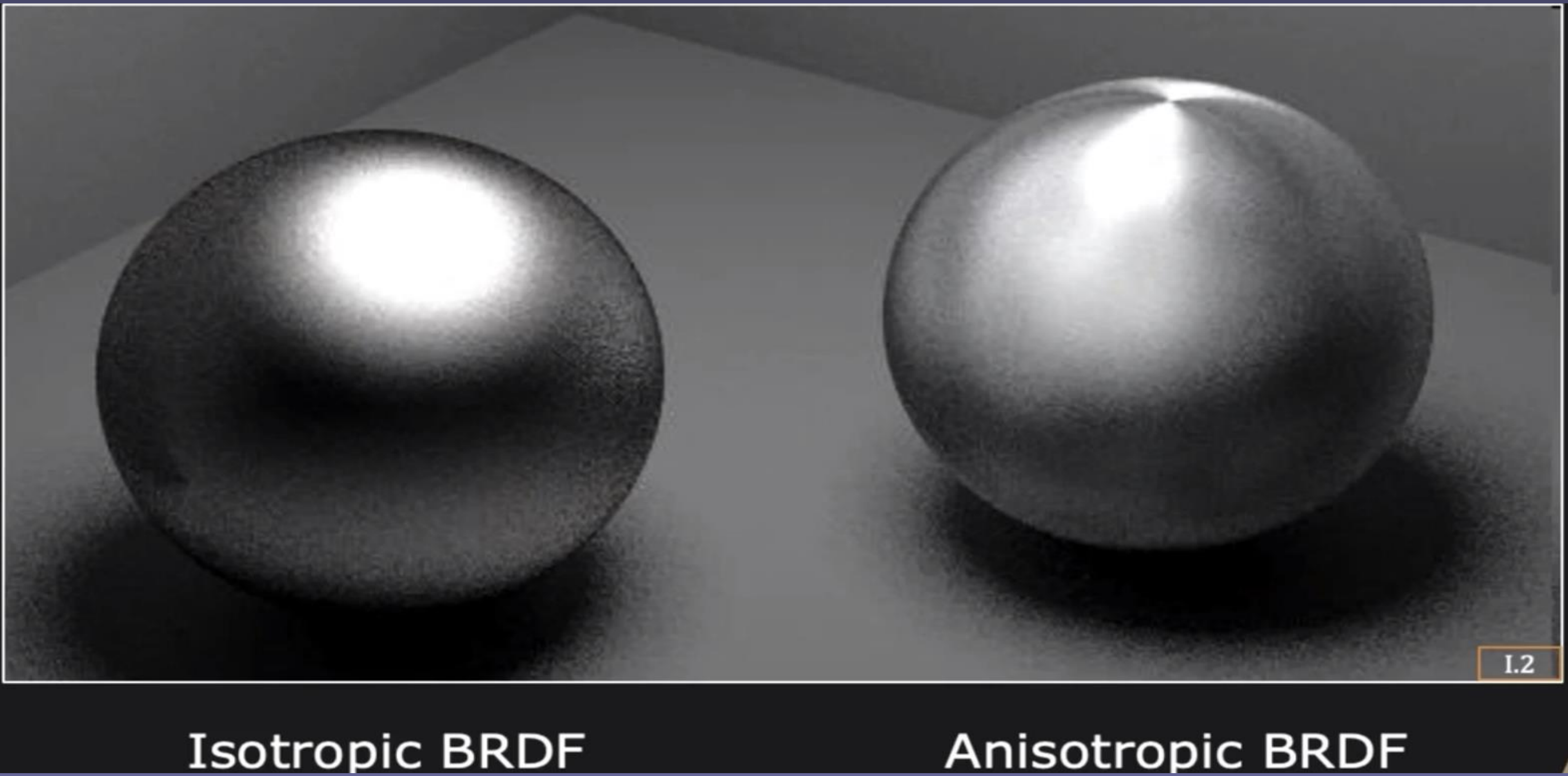
Unit: 1 / steradian

For Rotationally Symmetric Reflectance Properties :

BDRF: $$f(\theta_i, \theta_r, (\phi_r - \phi_i))$$

(ISOTROPIC SURFACES: 3-D)

Isotropic vs. Anisotropic BRDF

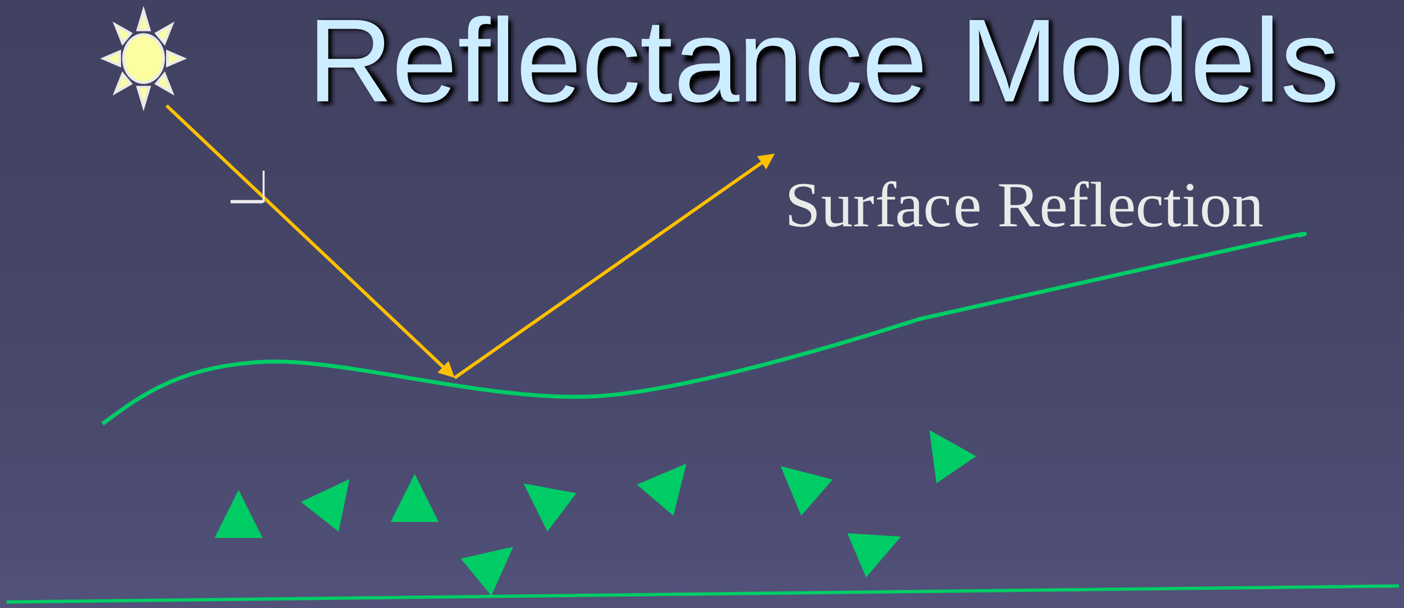


Anisotropic BRDF in nature

Butterfly wings

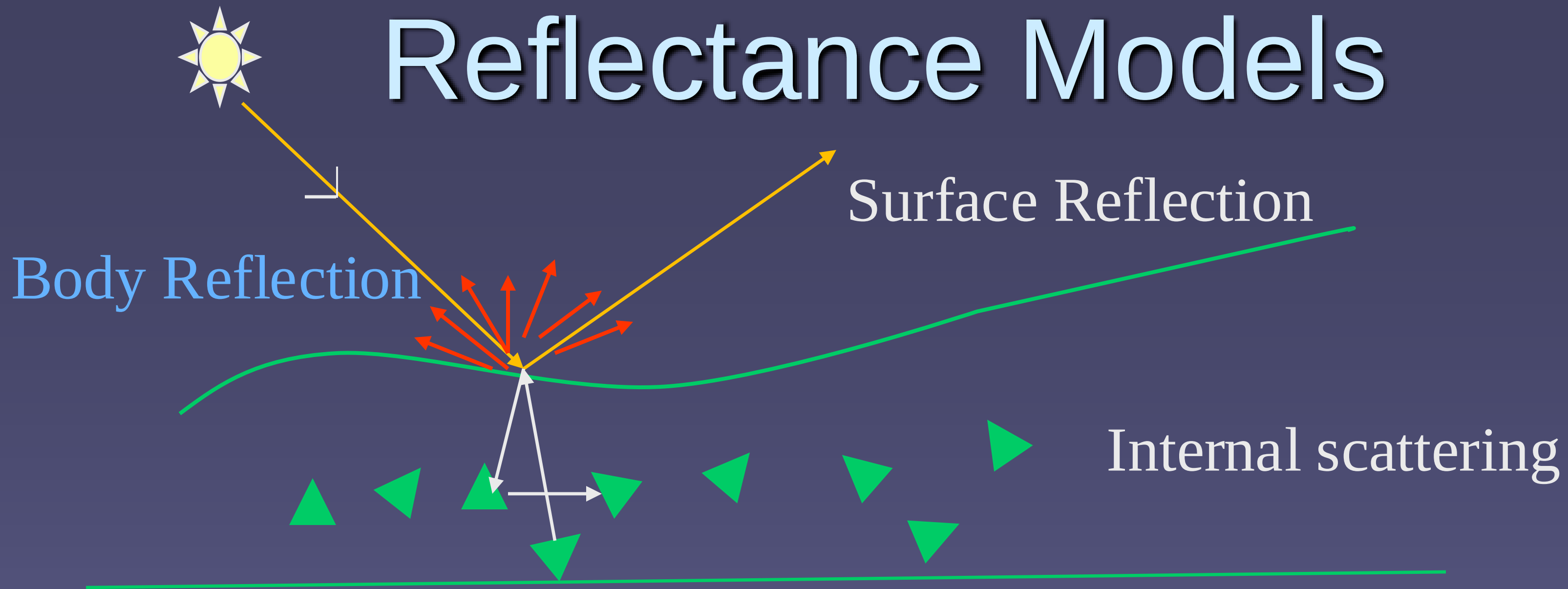
Peacock feathers

Reflectance Models



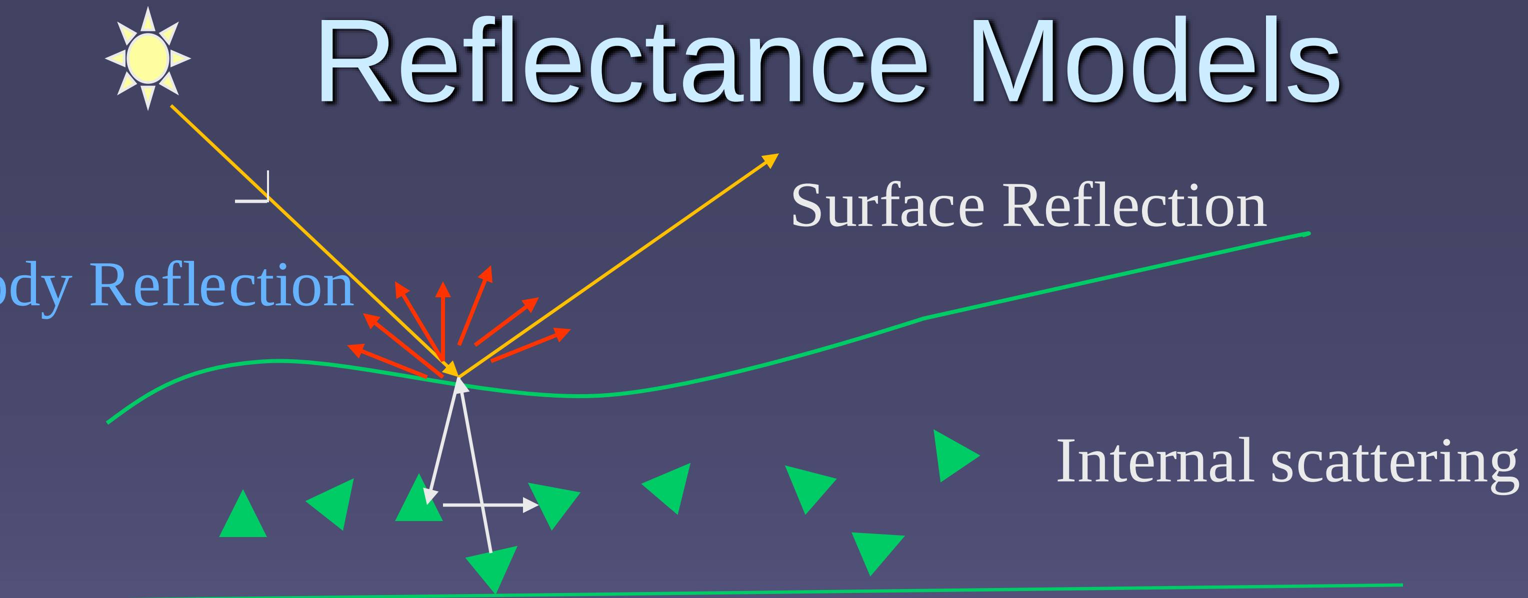
Surface Reflection: *Specular Reflection
*Glossy Appearance
*Highlights.
*Smooth surfaces (ex. metal mirror , glass)

Reflectance Models



Body Reflection: *Diffuse Reflection
*Matte Appearance
*Non-Homogeneous Medium (ex. clay, paper)

Reflectance Models

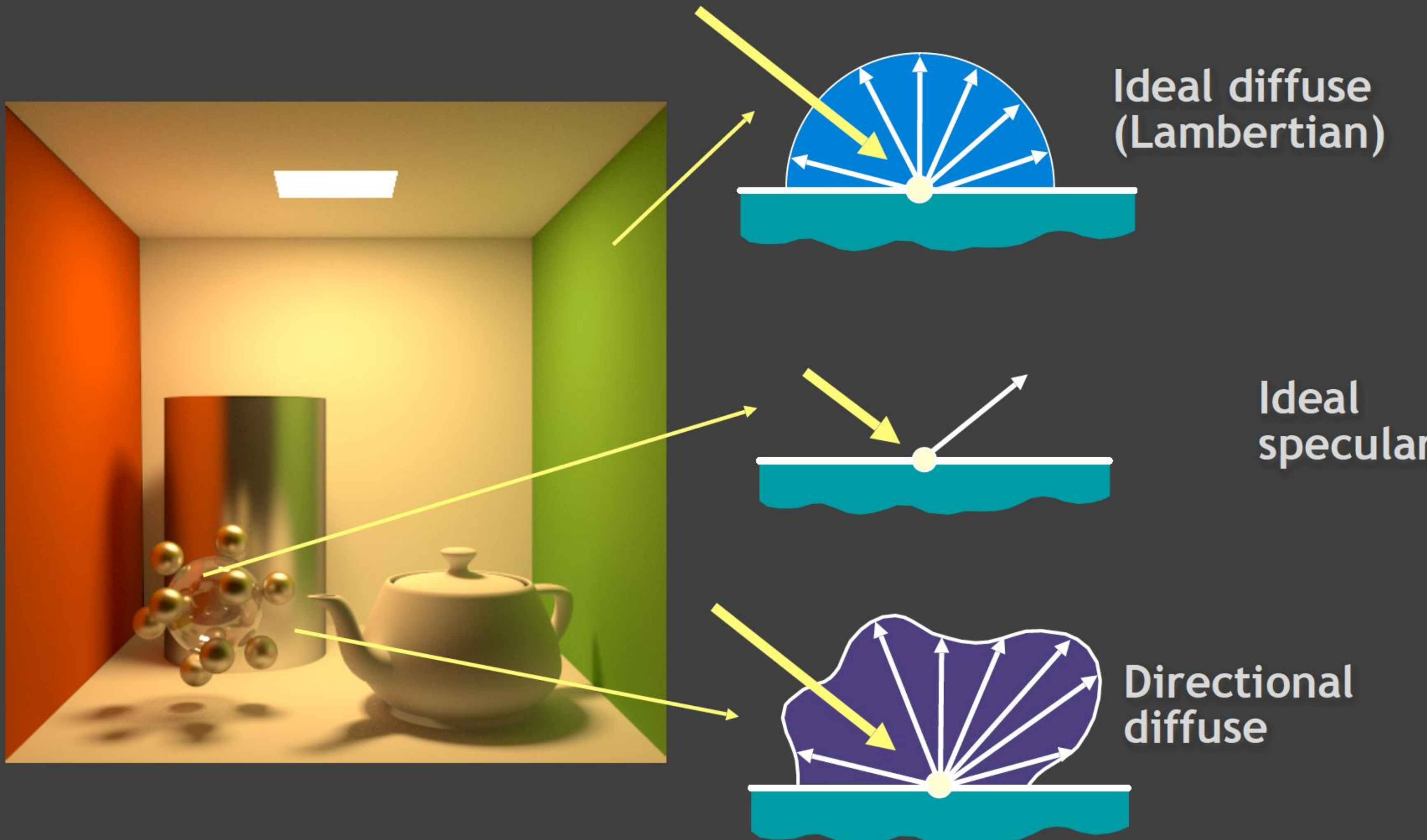


Body Reflection: *Diffuse Reflection
*Matte Appearance
*Non-Homogeneous Medium (ex. clay, paper)

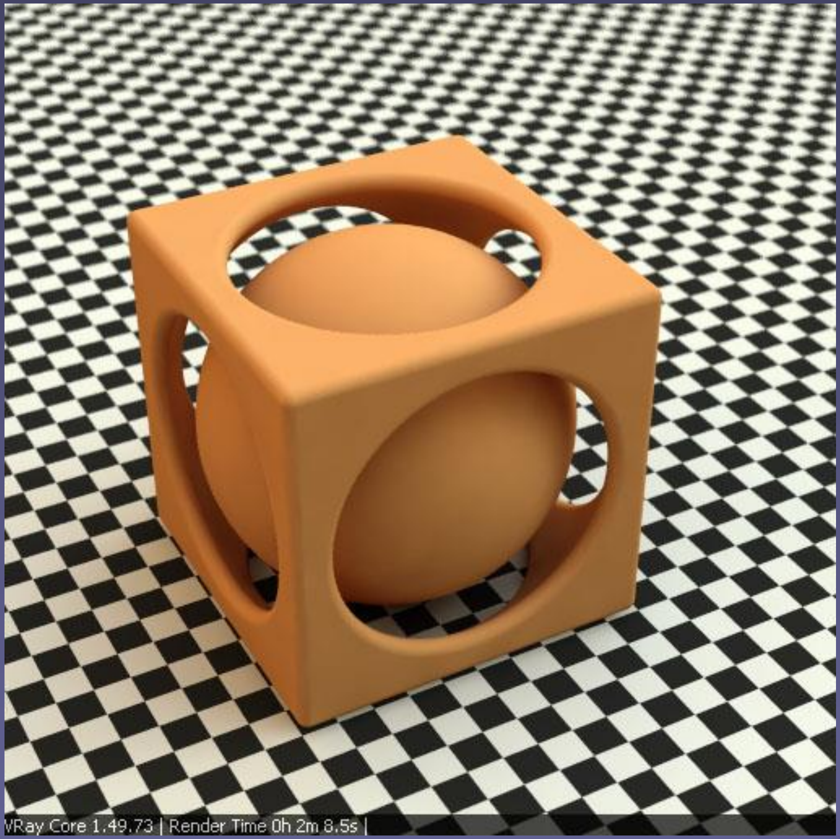
Surface Reflection: *Specular Reflection
*Glossy Appearance
*Highlights.
*Smooth surfaces (ex. metal mirror , glass)

Object appearance: Diffuse Reflection + Specular Reflection

Materials - Three Forms



Diffuse vs Specular Reflection



Mostly diffuse (no highlights)

Mostly specular (highlights)

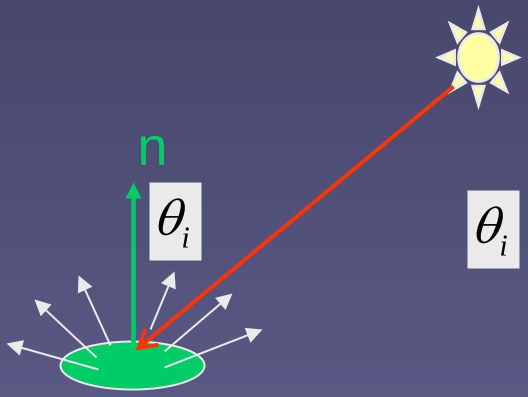
Body Reflection:

Surface Reflection:

Hybrid Reflection
(Body + Surface):

[Nayar 1991]

Lambertian Reflectance Model (Body Reflection)



θ_i is the angle between light source direction and surface normal

A Lambertian (i.e. diffuse) surface scatters light equally in all directions!

Lambert (1760)

Very widely used in Vision & Graphics

Lambertian Reflectance Model

A Lambertian (diffuse) surface scatters light equally in all directions

Constant **BRDF f**:

$$f = \frac{r}{\rho}$$

Albedo ρ:
intrinsic brightness of surface

$0 \leq \rho \leq 1$

0: black surface
1: fully reflective

Surface appears equally bright from all viewing directions

Lambertian Reflectance Model

A Lambertian (diffuse) surface scatters light equally in all directions

$$f(q_i, j_i, q_r, j_r) = \frac{r}{p} \Rightarrow$$

$$L(q_r, j_r) = \frac{r}{p} E(q_i, j_i)$$

$$L = \frac{r}{p} E$$

Albedo ρ:
intrinsic brightness of surface

$0 \leq \rho \leq 1$

Surface appears equally bright from all viewing directions

Lambertian Reflectance Model

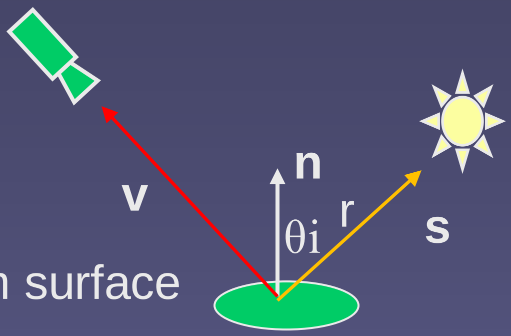
Unit Vectors

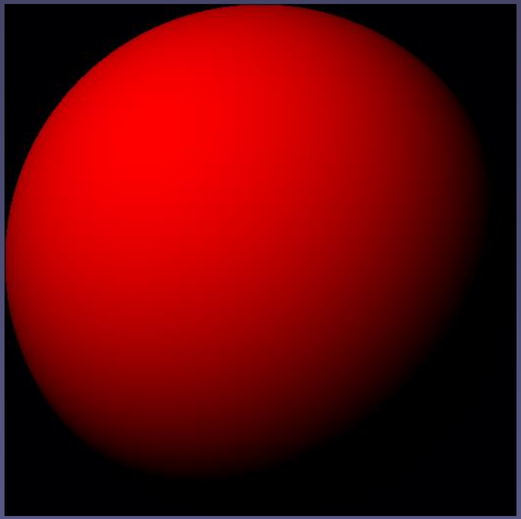
v: viewing direction

n: surface normal

s: light source direction

r: distance of light source from surface





A Lambertian sphere

Lambertian Reflectance Model

Unit Vectors

v: viewing direction

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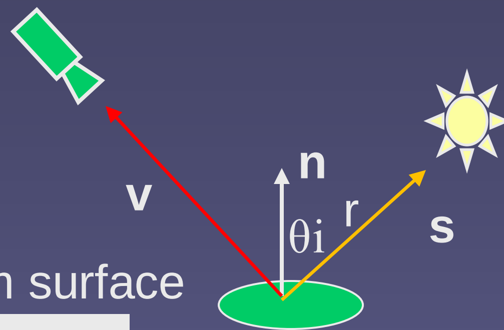


$$L = \frac{r}{p} E$$

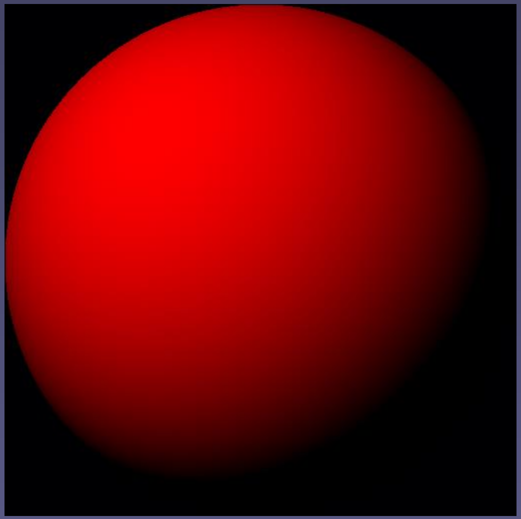
$$E = (J \cos q_i / r^2) = \frac{J}{r^2}(\mathbf{n} \times \mathbf{s}) =>$$

$$L = \frac{r}{p} \frac{J}{r^2}(\mathbf{n} \times \mathbf{s})$$

or

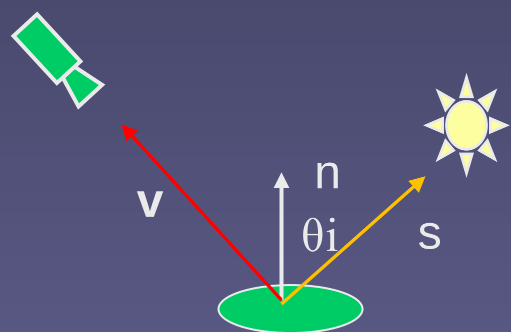
$$L = r k \left(\mathbf{n} \times \mathbf{s}\right), k = \frac{J}{p r^2}$$

Dot product between unit vectors = cos



A Lambertian sphere

Lambertian Reflectance Model



Surface normal n
Direction of illumination s

A Lambertian sphere

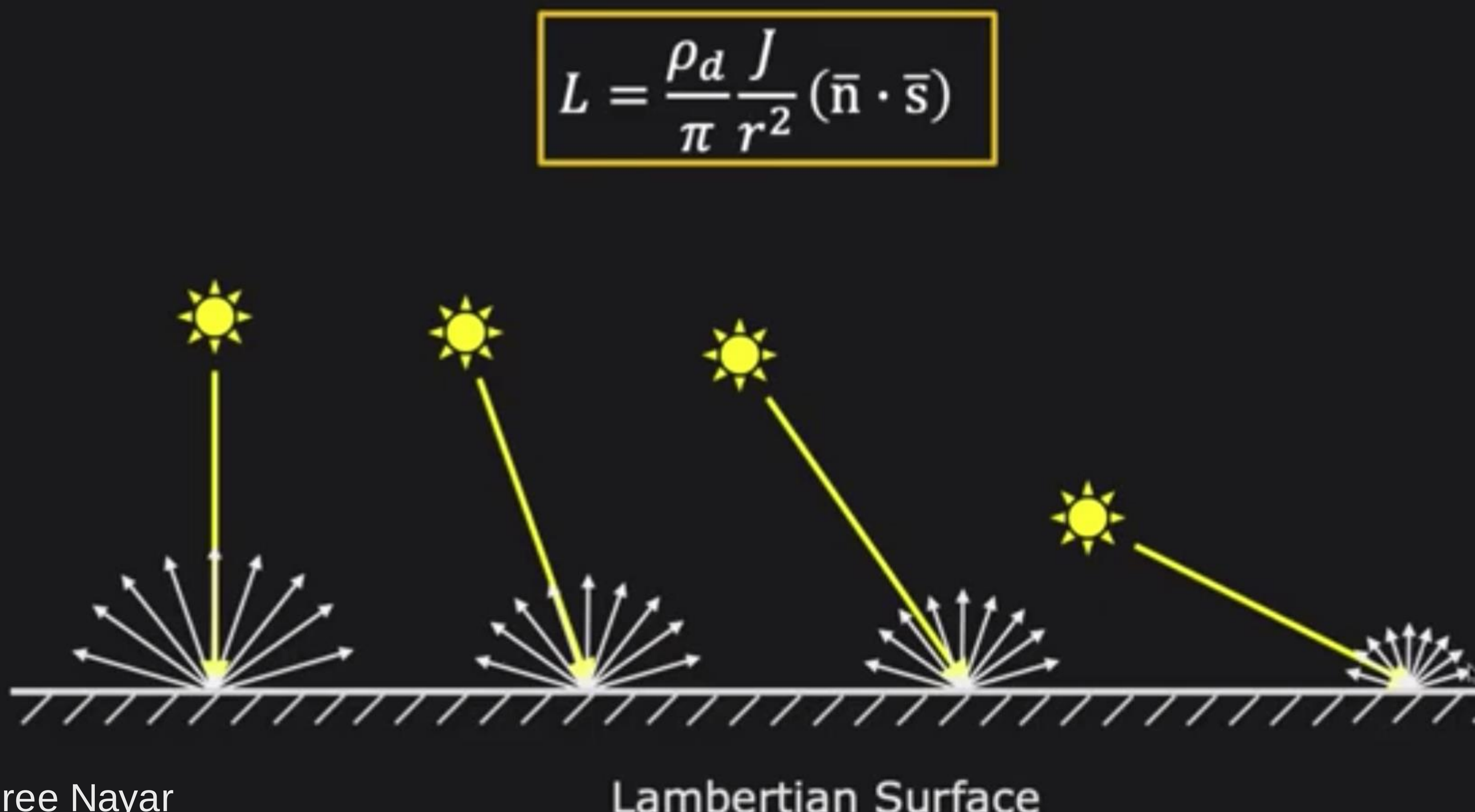
$$L = \rho k \cos\theta_i$$

$$L = \rho'(\mathbf{n} \cdot \mathbf{s})$$

Effective albedo

Commonly used in Computer Vision and Graphics

Lambertian Reflectance Model

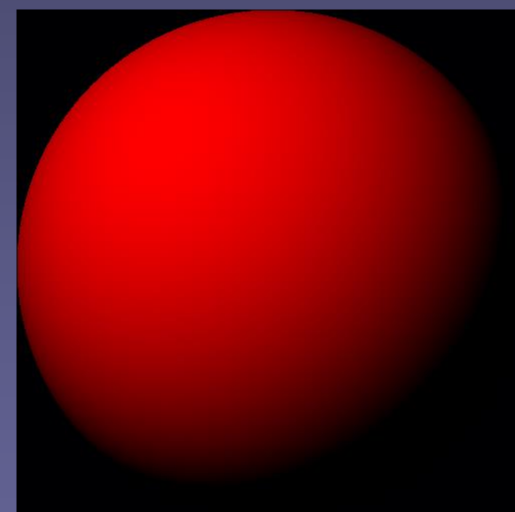


Shree Nayar

What Information Does Shading Encode

In regions of constant albedo, changes of intensity correspond to changes in the surface normal of the scene

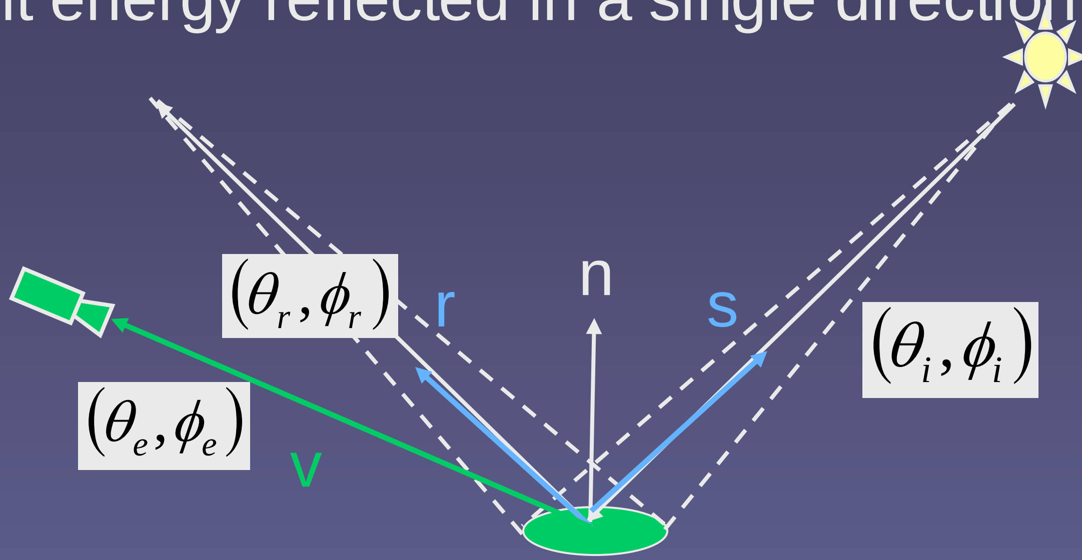
$$I \mu L = r'(\mathbf{n} \times \mathbf{s})$$



Ideal Specular Model (Mirrors)

*Very SMOOTH surface

*All incident energy reflected in a single direction



Perfect reflector

$$f(q_i, j_i, q_e, j_e) = \frac{d(q_i - q_e)\,d((j_i + p) - j_e)}{\cos q_i \sin q_i}$$

Viewer received light only when v = r

Glossy Surfaces

- Delta function δ() too harsh a BRDF model
  (valid only for highly polished mirrors and metals).

- Many glossy surfaces show broader highlights in addition to mirror reflection.

- Surfaces are not perfectly smooth – they show micro-surface geometry (roughness).

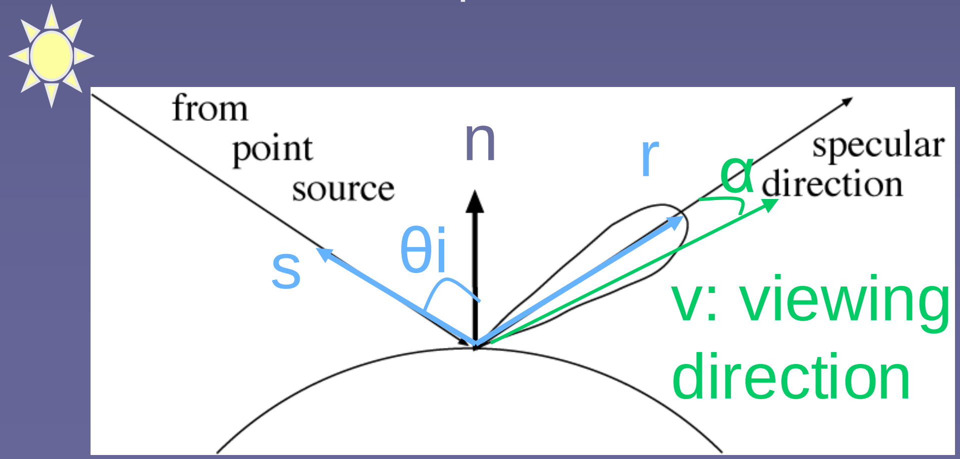
- Example Models : Phong model

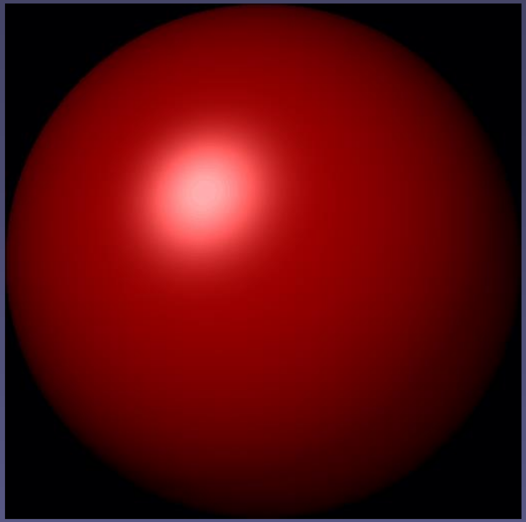
  Torrance Sparrow model

Phong Reflectance Model

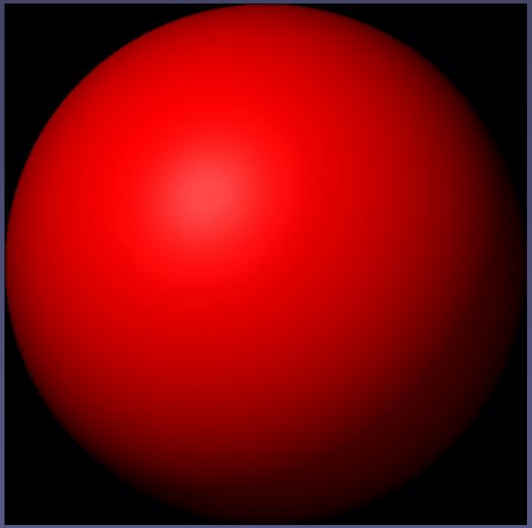
$$L = b\cos\theta_i + c\cos^n(\alpha)$$

diffuse specular

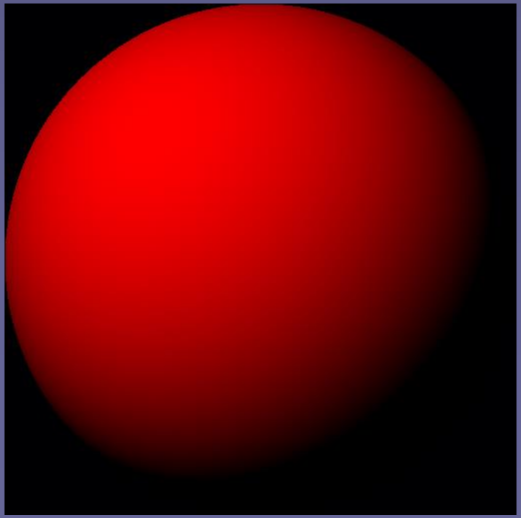




b=0.3, c=0.7, n=2



b=0.7, c=0.3, n=0.5

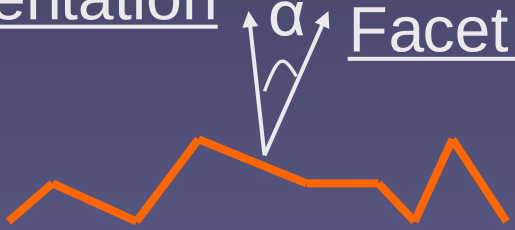


Torrance-Sparrow Model

Specular Reflection from Rough Surfaces.

Surface Micro-Structure Model – Each facet perfect mirror

Mean orientation α Facet orientation

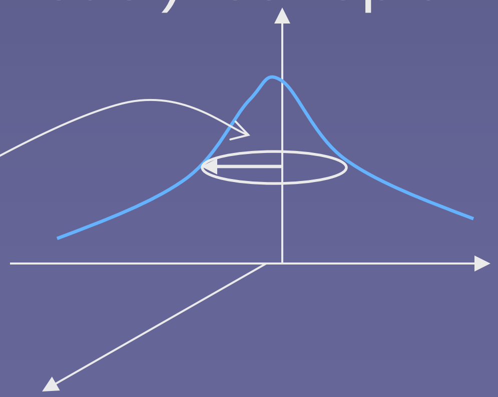


Micro-facet Orientation Model: (example)

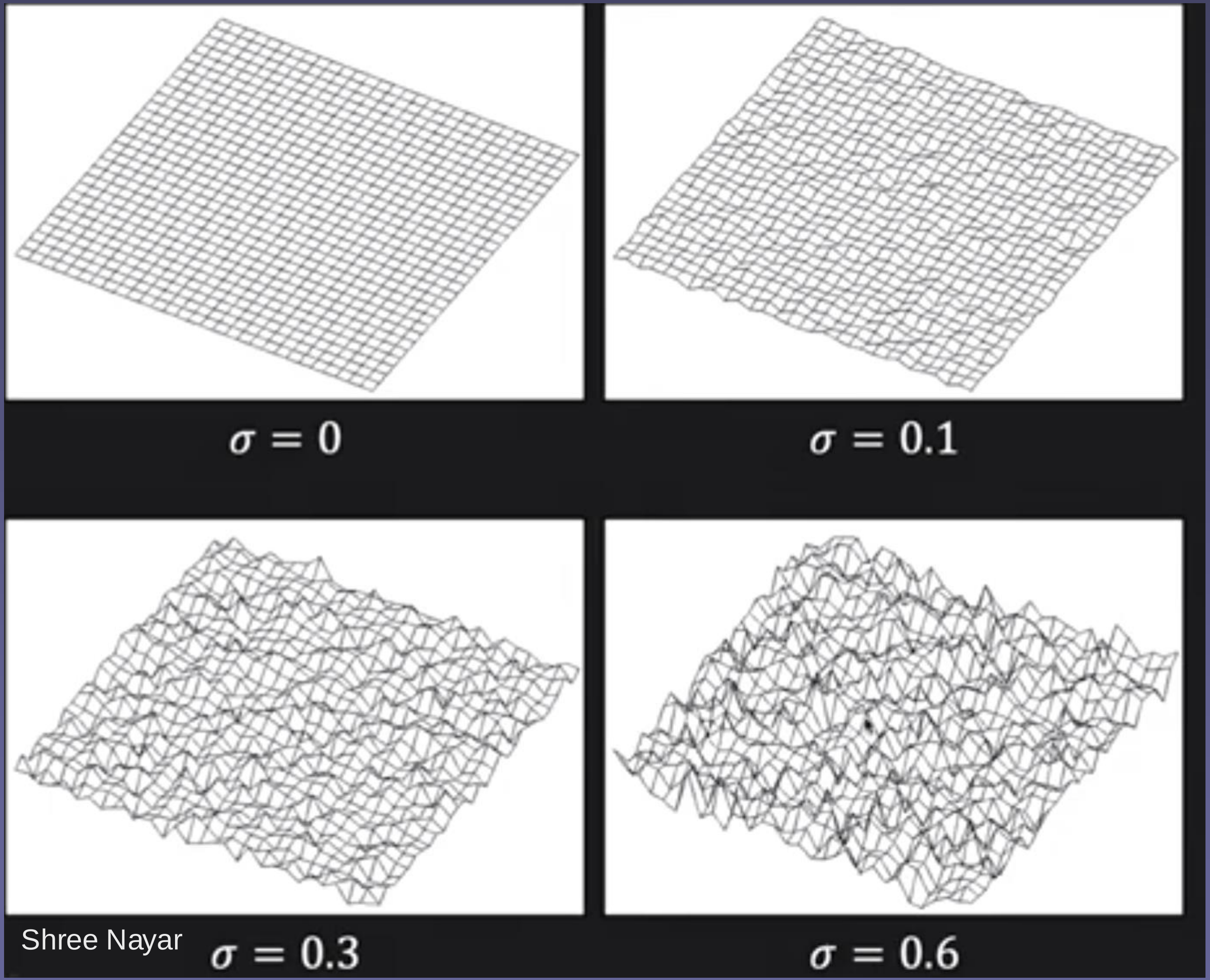
$$p(a) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{a^2}{\sigma^2}}$$

(Gaussian Model) Isotropic

σ: roughness parameter

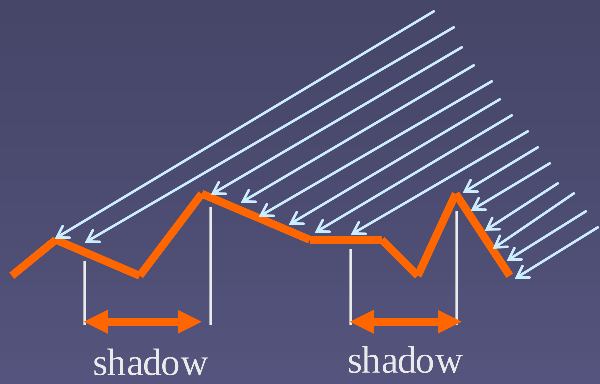


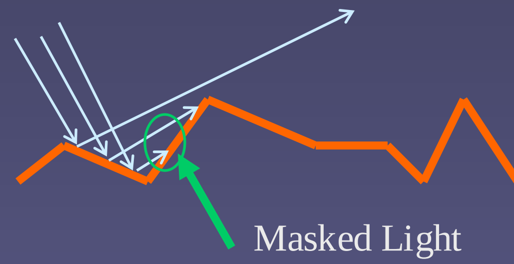
Micro-facet Orientation

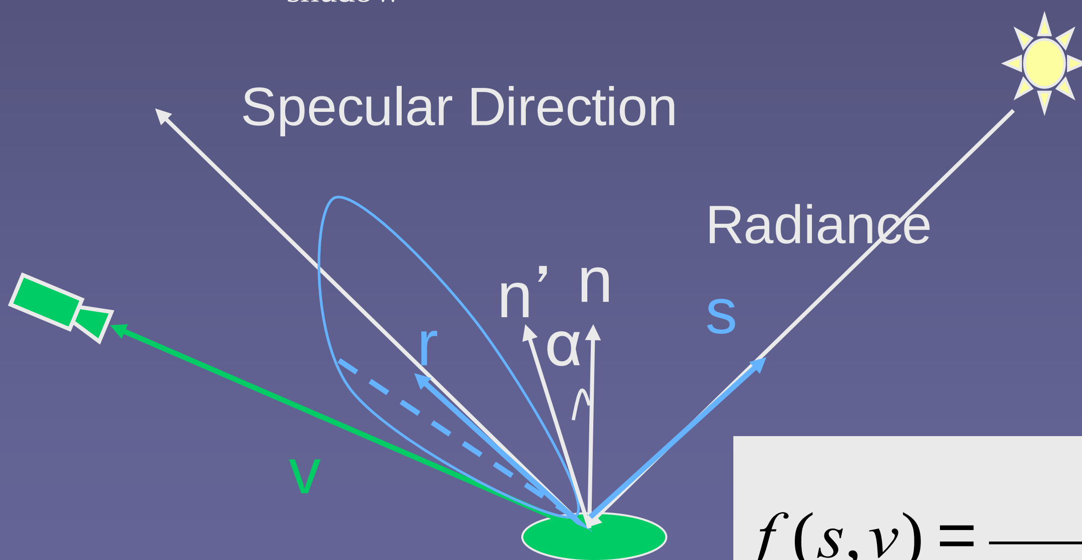


Torrance-Sparrow Model

Masking and Shadowing Effects





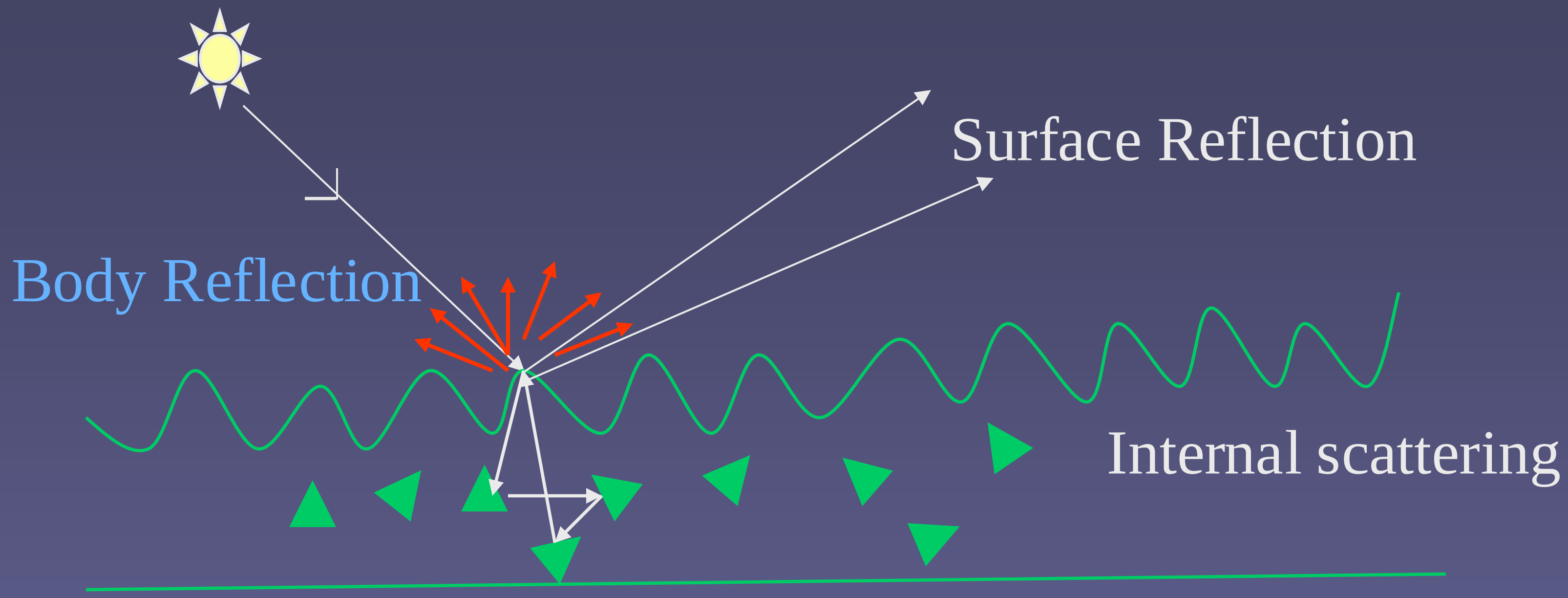


Specular albedo

Geometric Factor G
(Masking-Shadowing)

$$f(s,v)=\frac{r_{spec}}{(n\cdot v)(n\cdot s)}p(a,s)G(s,n,v)$$

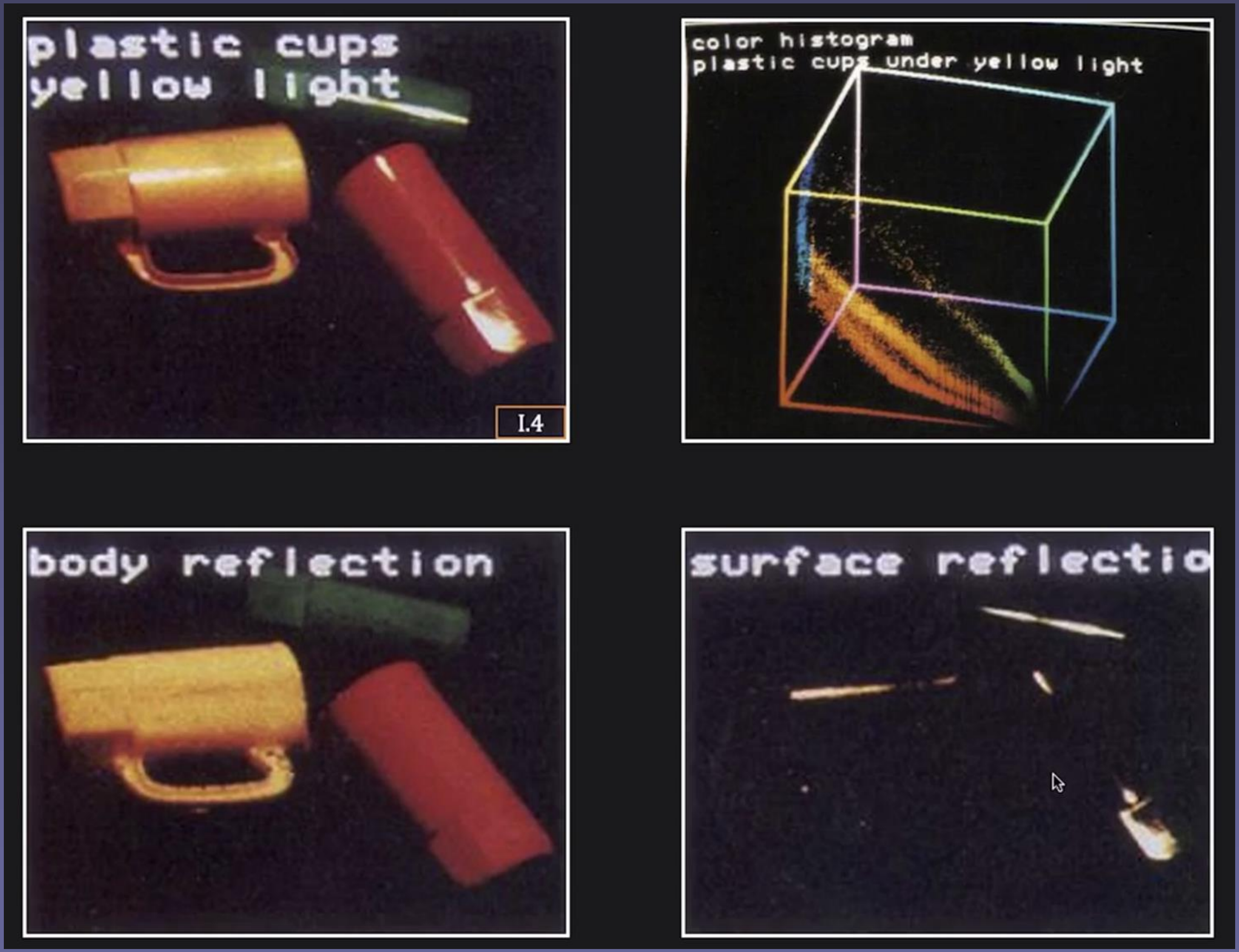
Dichromatic Model



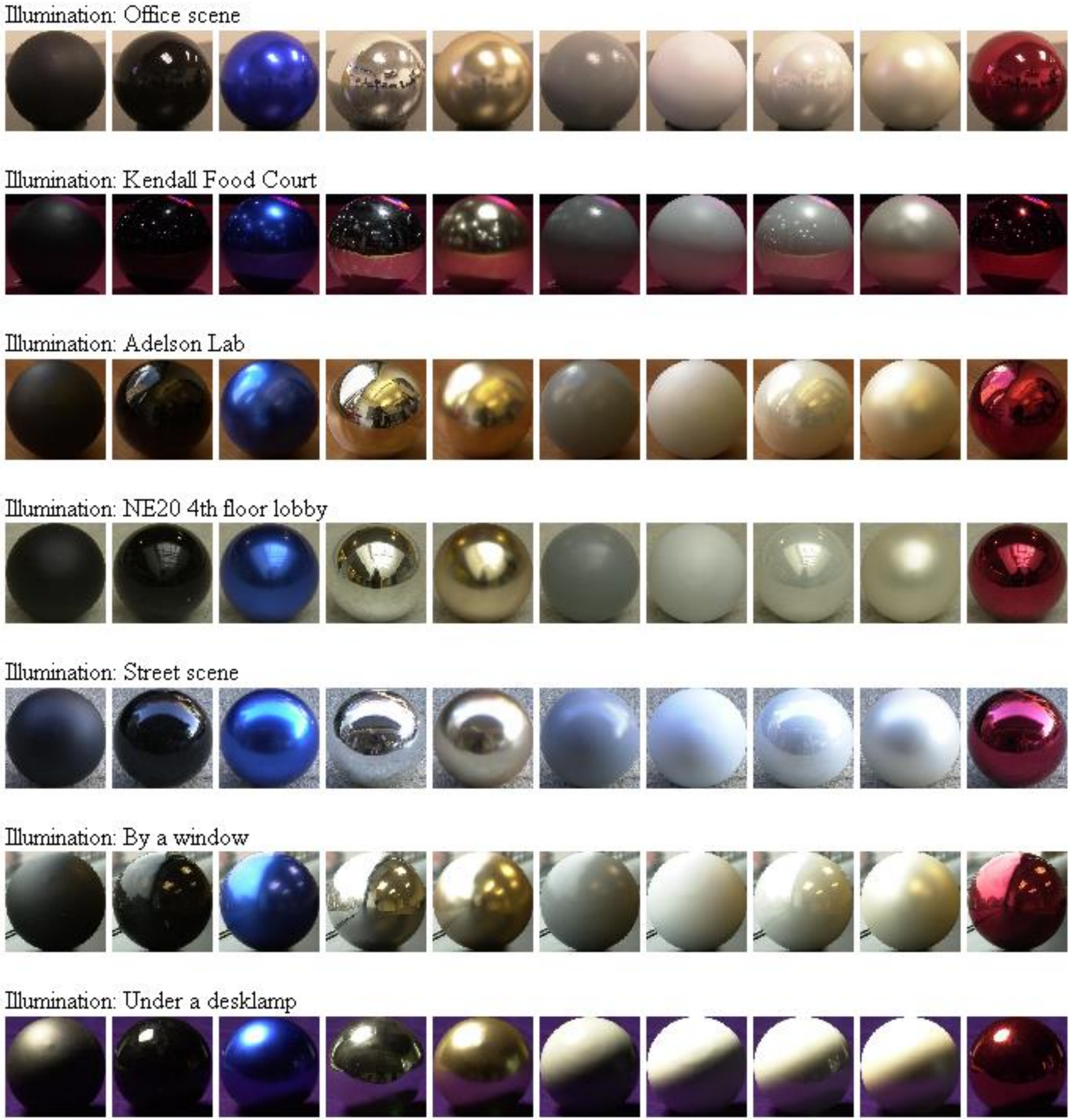
Color of body (diffuse) component = color of object x color of illumination
Color of surface (specular) component = color of illumination

We measure the sum of the diffuse and specular components.

Removing Specularities



Klinker, Shafer, Kanade, 1990, International Journal of Computer Vision



Dror, Adelson, Wilsky