Lecture 17: Face Recognition

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Stanford Vision Lab

What we will learn today

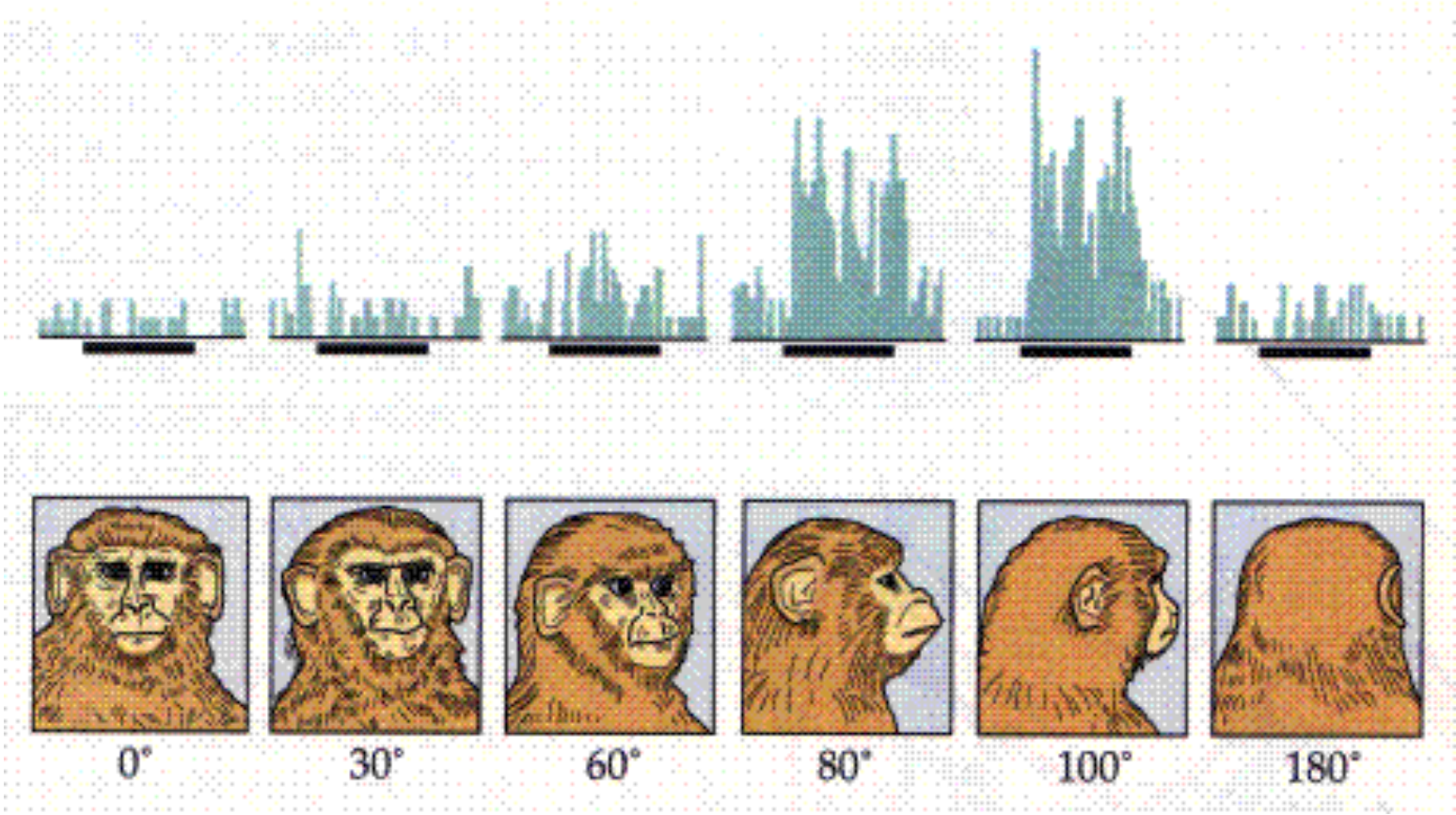
- Introduction to face recognition
- Principal Component Analysis (PCA)
- The Eigenfaces Algorithm
- Linear Discriminant Analysis (LDA)

Shree Nayar, First Principles of Computer Vision: Perception -> Appearance Matching -> (Principal Component Analysis, Finding Principal Compon. , PCA & SVD)

Turk and Pentland, Eigenfaces for Recognition, *Journal of Cognitive Neuroscience* **3** (1): 71–86.

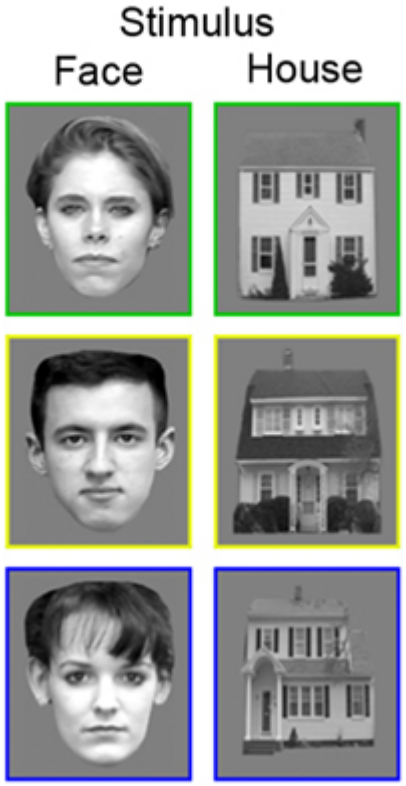
P. Belhumeur, J. Hespanha, and D. Kriegman. "Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection". *IEEE Transactions on pattern analysis and machine intelligence* **19** (7): 711. 1997.

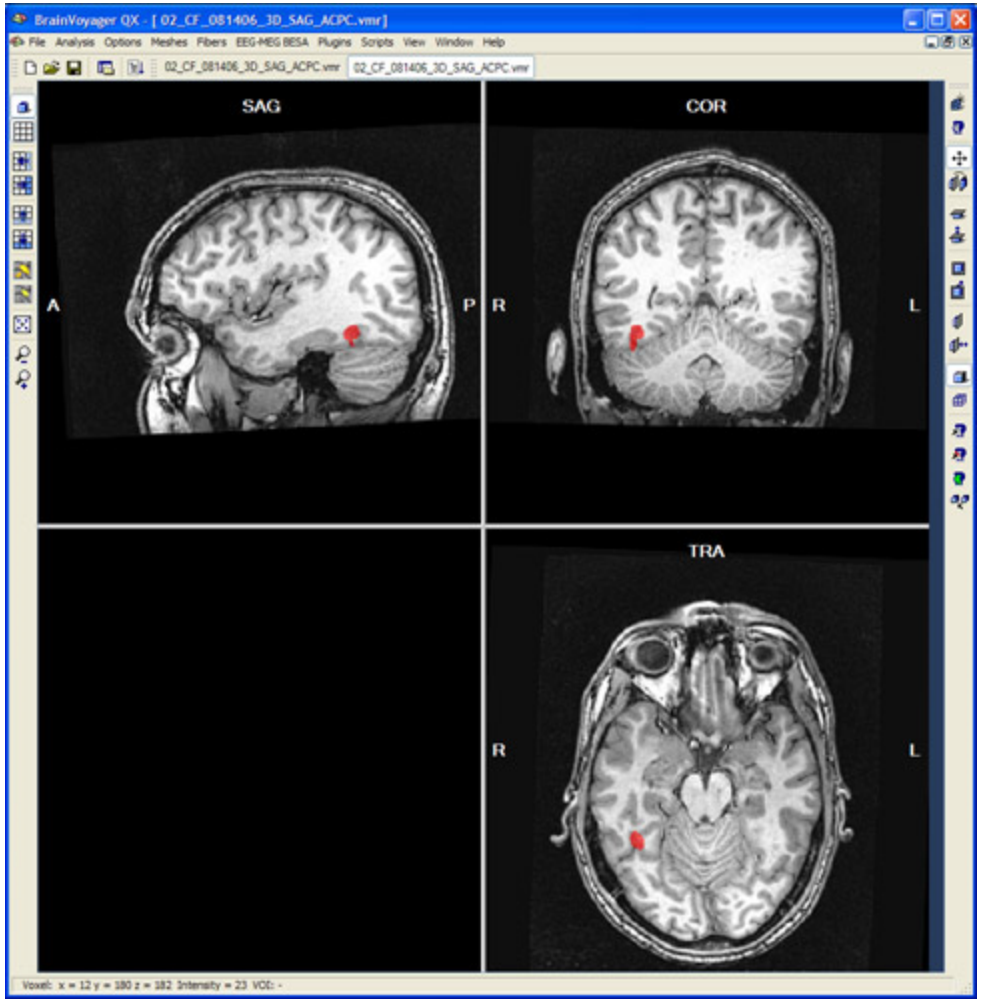
“Faces” in the brain



Courtesy of Johannes M. Zanker

“Faces” in the brain

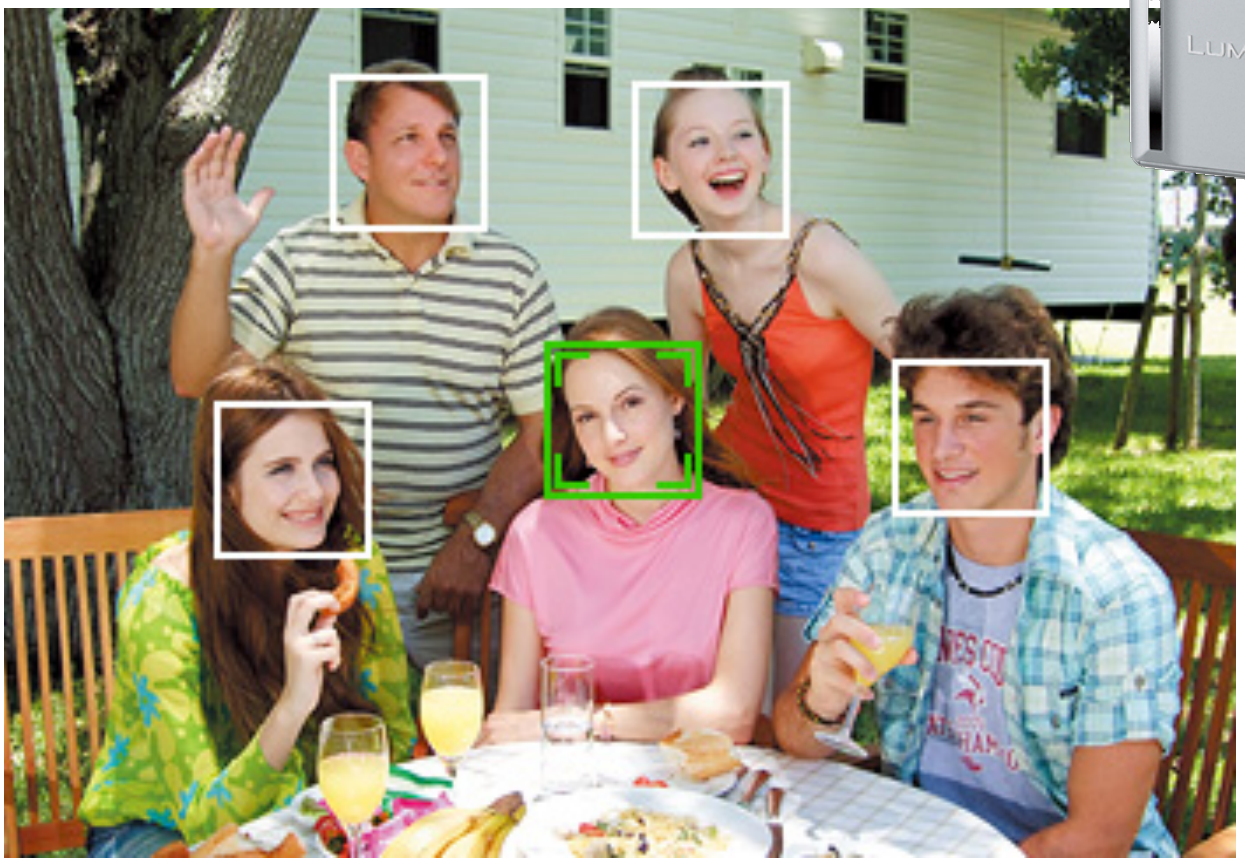




Kanwisher, et al. 1997

Face Recognition

- Digital photography



Face Recognition

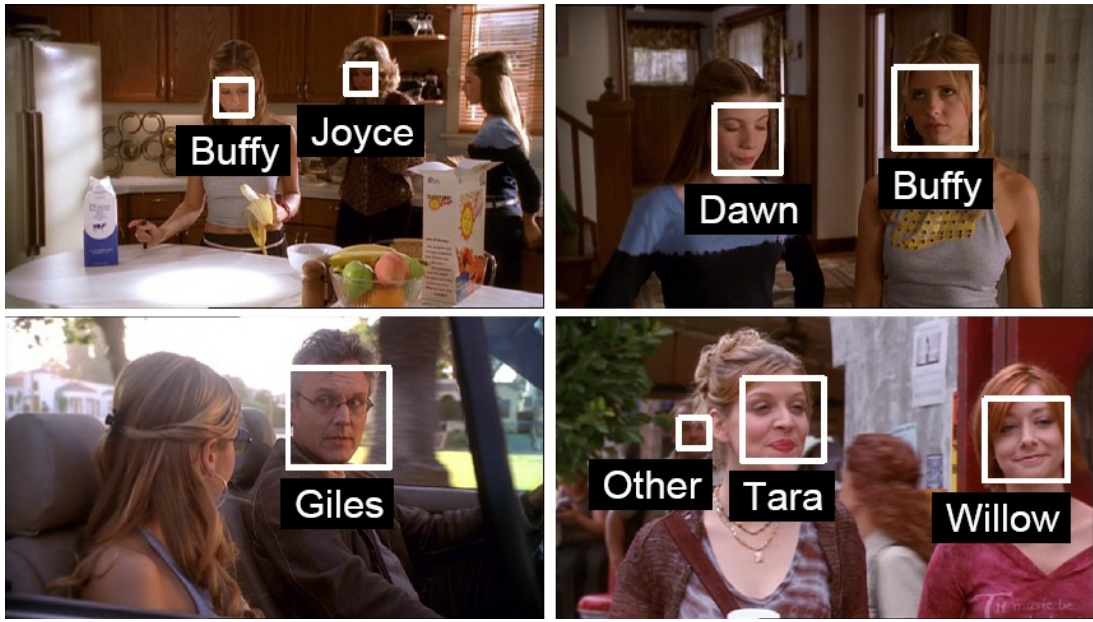
- Digital photography
- Surveillance

Face Recognition

- Digital photography
- Surveillance
- Album organization

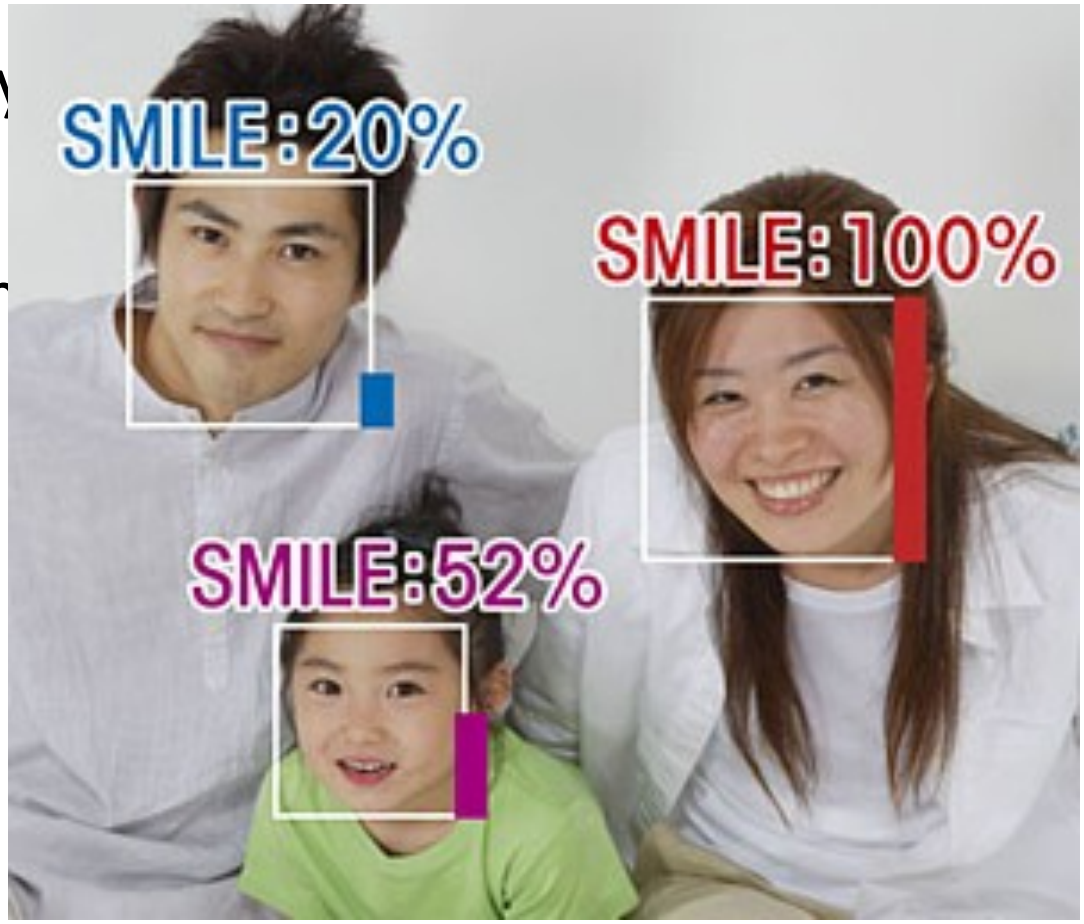
Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.



Face Recognition

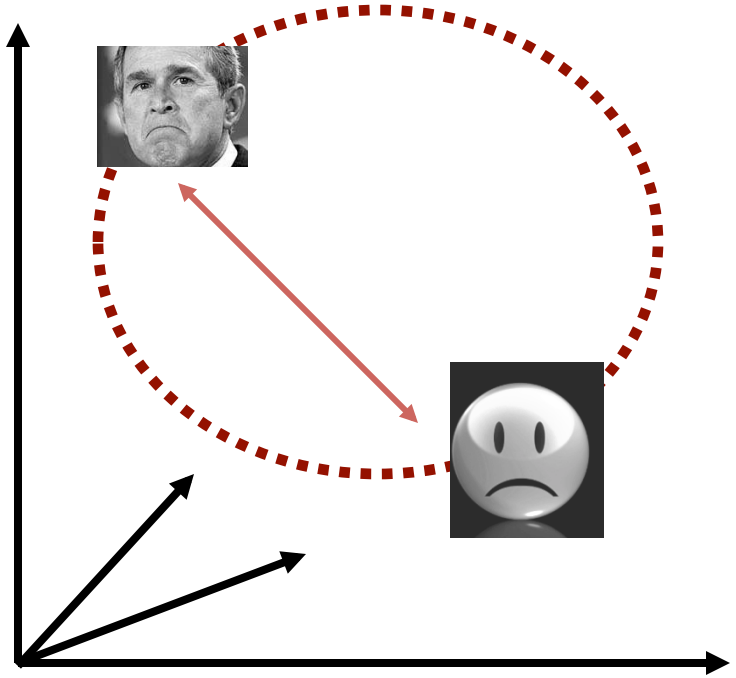
- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions



Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions
- Security/warfare
- Tele-conferencing
- Etc.

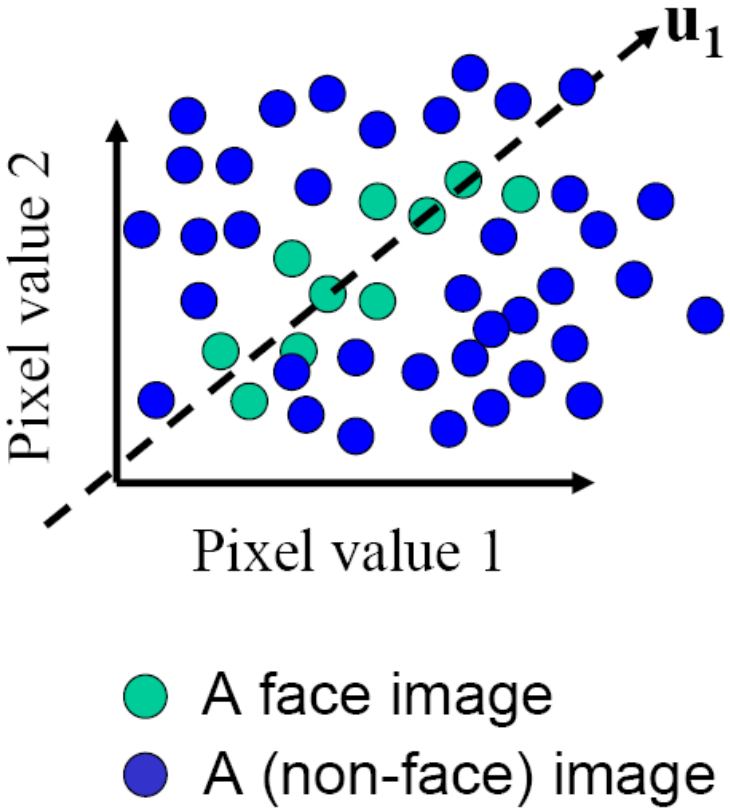
The Space of Faces



- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim

Slide credit: Chuck Dyer, Steve Seitz, Nishino

The Space of Faces



- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim
- However, relatively few high dimensional vectors correspond to valid face images
- We want to effectively model the subspace of face images

Slide credit: Chuck Dyer, Steve Seitz, Nishino

Image space

Face space

- Computes n-dim subspace such that the projection of the data points onto the subspace has the largest variance among all n-dim subspaces.
- Maximize the scatter of the training images in face space

Key Idea

- So, compress them to a low-dimensional subspace that captures key appearance characteristics of the visual DOFs.

- USE PCA for estimating the sub-space (dimensionality reduction)

- Compare two faces by projecting the images into the subspace and measuring the EUCLIDEAN distance between them.

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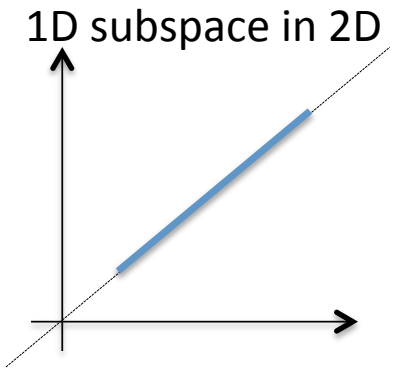
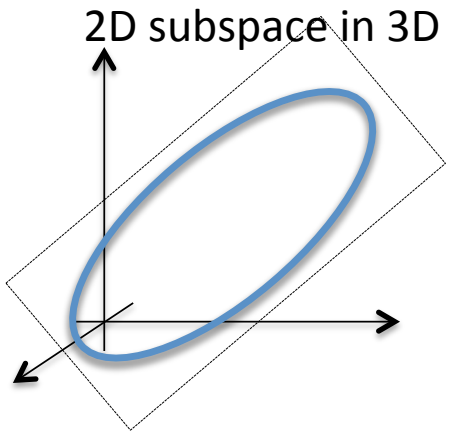
PCA Formulation

- Basic idea:
  - If the data lives in a subspace, it is going to look very flat when viewed from the full space, e.g.



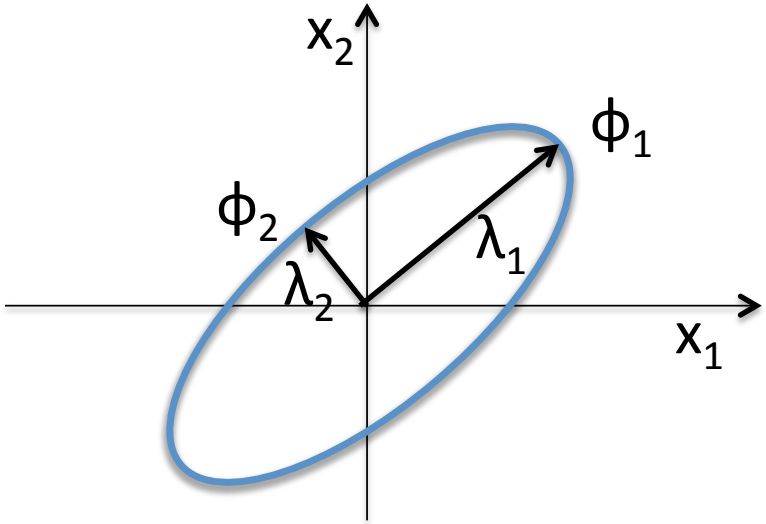


  - This means that if we fit a Gaussian to the data the equiprobability contours are going to be highly skewed ellipsoids

Slide inspired by N. Vasconcelos

PCA Formulation

- If x is Gaussian with covariance Σ, the equiprobability contours are the ellipses whose
  - Principal components ϕ_i are the eigenvectors of Σ
  - Principal lengths λ_i are the eigenvalues of Σ



- by computing the eigenvalues we know the data is
  - Not flat if $\lambda_1 \approx \lambda_2$
  - Flat if $\lambda_1 >> \lambda_2$

Slide inspired by N. Vasconcelos

PCA Algorithm (training)

- Given sample $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}, \;\; x_i \in \mathcal{R}^d$
  - compute sample mean: $\hat{\mu} = \frac{1}{n}\sum_i(\mathbf{x}_i)$
  - compute sample covariance: $\hat{\Sigma} = \frac{1}{n}\sum_i(\mathbf{x}_i - \hat{\mu})(\mathbf{x}_i - \hat{\mu})^T$
  - compute eigenvalues and eigenvectors of $\hat{\Sigma}$

$$\hat{\Sigma} = \Phi\Lambda\Phi^T, \;\; \Lambda = diag(\sigma_1^2, \ldots, \sigma_n^2) \;\; \Phi^T\Phi = I$$

  - order eigenvalues $\sigma_1^2 > \ldots > \sigma_n^2$
  - if, for a certain k, $\sigma_k << \sigma_1$ eliminate the eigenvalues and eigenvectors above k.

Slide inspired by N. Vasconcelos

PCA Algorithm (testing)

- Given principal compoenents $\phi_i, i \in 1, \ldots, k$ and a test sample $\mathcal{T} = \{\mathbf{t}_1, \ldots, \mathbf{t}_n\}, \ t_i \in \mathcal{R}^d$

  - subtract mean to each point $\mathbf{t}'_i = \mathbf{t}_i - \widehat{\mu}$

  - project onto eigenvector space $\mathbf{y}_i = \mathbf{A}\mathbf{t}'_i$ where

$$\mathbf{A} = \begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_k^T \end{bmatrix}$$

  - use $\mathcal{T}' = \{\mathbf{y}_1, \ldots \mathbf{y}_n\}$ to estimate class conditional densities and do all further processing on $\mathbf{y}$.

Slide inspired by N. Vasconcelos

PCA by SVD

- An alternative manner to compute the principal components, based on singular value decomposition
- Quick reminder: SVD
  - Any real n x m matrix (n>m) can be decomposed as

$$A = M\Pi N^T$$

  - Where M is an (n x m) column orthonormal matrix of left singular vectors (columns of M)
  - Π is an (m x m) diagonal matrix of singular values
  - N^T is an (m x m) row orthonormal matrix of right singular vectors (columns of N)

$$M^T M = I \qquad N^T N = I$$

Slide inspired by N. Vasconcelos

PCA by SVD

- To relate this to PCA, we consider the data matrix

$$X = \begin{bmatrix} | & & | \\ x_1 & \dots & x_n \\ | & & | \end{bmatrix}$$

- The sample mean is

$$\mu = \frac{1}{n}\sum_i x_i = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1 & \dots & x_n \\ | & & | \end{bmatrix}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{1}{n}X1$$

Slide inspired by N. Vasconcelos

PCA by SVD

- Center the data by subtracting the mean to each column of X
- The centered data matrix is

$$X_c = \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix} - \begin{bmatrix} | & & | \\ \mu & \cdots & \mu \\ | & & | \end{bmatrix}$$

$$= X - \mu 1^T = X - \frac{1}{n} X 1 1^T = X\left(I - \frac{1}{n} 1 1^T\right)$$

Slide inspired by N. Vasconcelos

PCA by SVD

- The sample covariance matrix is

$$\Sigma = \frac{1}{n}\sum_i (x_i - \mu)(x_i - \mu)^T = \frac{1}{n}\sum_i x_i^c (x_i^c)^T$$

where x_i^c is the i^{th} column of X_c

- This can be written as

$$\Sigma = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1^c & \dots & x_n^c \\ | & & | \end{bmatrix}\begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix} = \frac{1}{n} X_c X_c^T$$

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technical note

PCA by SVD

- The matrix

$$X_c^T = \begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix}$$

is real (n x d). Assuming n>d it has SVD decomposition

$$X_c^T = \mathrm{M}\Pi\mathrm{N}^T$$

$$\mathrm{M}^T\mathrm{M} = I \qquad \mathrm{N}^T\mathrm{N} = I$$

and

$$\Sigma = \frac{1}{n} X_c X_c^T = \frac{1}{n} \mathrm{N}\Pi\mathrm{M}^T\mathrm{M}\Pi\mathrm{N}^T = \frac{1}{n}\mathrm{N}\Pi^2\mathrm{N}^T$$

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technical note

PCA by SVD

$$\Sigma = \mathrm{N}\left(\frac{1}{n}\Pi^2\right)\mathrm{N}^T$$

- Note that N is (d x d) and orthonormal, and Π^2 is diagonal. This is just the eigenvalue decomposition of Σ
- It follows that
  - The eigenvectors of Σ are the columns of N
  - The eigenvalues of Σ are

$$\lambda_i = \frac{1}{n}\pi_i^2$$

- This gives an alternative algorithm for PCA

Slide inspired by N. Vasconcelos

PCA by SVD

- In summary, computation of PCA by SVD
- Given X with one example per column
  - Create the centered data matrix

$$X_c^T = \left(I - \frac{1}{n} 1 1^T \right) X^T$$

  - Compute its SVD

$$X_c^T = M \Pi N^T$$

  - Principal components are columns of N, eigenvalues are

$$\lambda_i = \frac{1}{n} \pi_i^2$$

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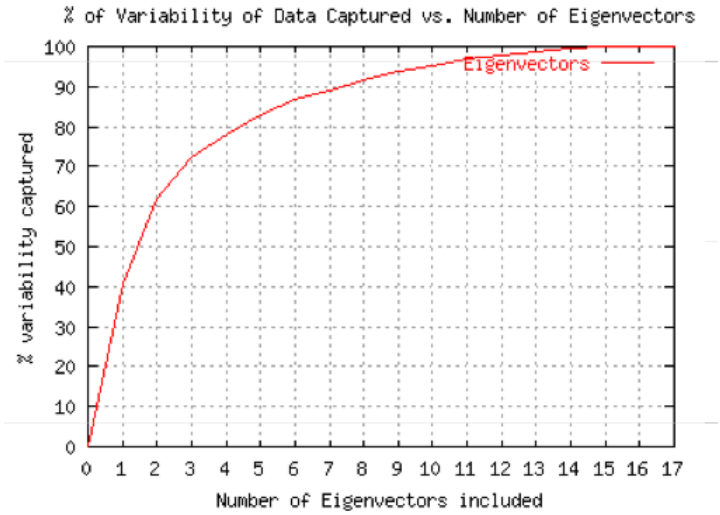
Rule of thumb for finding the number of PCA components

- A natural measure is to pick the eigenvectors that explain p% of the data variability
  - Can be done by plotting the ratio r_k as a function of k



$$r_k = \frac{\sum_{i=1}^{k} \lambda_i^2}{\sum_{i=1}^{n} \lambda_i^2}$$

  - E.g. we need 3 eigenvectors to cover 70% of the variability of this dataset

Slide inspired by N. Vasconcelos

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Eigenfaces: key idea

- Assume that most face images lie on a low-dimensional subspace determined by the first k (k<<d) directions of maximum variance
- Use PCA to determine the vectors or "eigenfaces" that span that subspace
- Represent all face images in the dataset as linear combinations of eigenfaces

M. Turk and A. Pentland, Face Recognition using Eigenfaces, CVPR 1991

Eigenface algroithm

- Training
  1. Align training images $x_1, x_2, \ldots, x_N$

Note that each image is formulated into a long vector!

2. Compute average face $\mu = \frac{1}{N}\sum x_i$

3. Compute the difference image (the centered data matrix)

$$X_c = \begin{bmatrix} | & & | \\ x_1 & \ldots & x_n \\ | & & | \end{bmatrix} - \begin{bmatrix} | & & | \\ \mu & \ldots & \mu \\ | & & | \end{bmatrix}$$

$$= X - \mu 1^T = X - \frac{1}{n} X 1 1^T = X\left(I - \frac{1}{n} 1 1^T\right)$$

Eigenface algroithm

4. Compute the covariance matrix

$$\Sigma = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1^c & \dots & x_n^c \\ | & & | \end{bmatrix}\begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix} = \frac{1}{n} X_c X_c^T$$

5. Compute the eigenvectors of the covariance matrix Σ
6. Compute each training image x_i 's projections as

$$x_i \rightarrow \left(x_i^c \cdot \phi_1, x_i^c \cdot \phi_2, \ldots, x_i^c \cdot \phi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

7. Visualize the estimated training face x_i

$$x_i \approx \mu + a_1\phi_1 + a_2\phi_2 + \ldots + a_K\phi_K$$

Eigenface algroithm

6. Compute each training image x_i 's projections as

$$x_i \rightarrow \left(x_i^c \cdot \phi_1, x_i^c \cdot \phi_2, \ldots, x_i^c \cdot \phi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

7. Visualize the estimated training face x_i

$$x_i \approx \mu + a_1\phi_1 + a_2\phi_2 + \ldots + a_K\phi_K$$

Eigenface algroithm

- Testing
  1. Take query image t
  2. Project y into eigenface space and compute projection

$$t \rightarrow \left((t-\mu)\cdot\phi_1, (t-\mu)\cdot\phi_2, \ldots, (t-\mu)\cdot\phi_K\right) \equiv \left(w_1, w_2, \ldots, w_K\right)$$

  3. Compare projection w with all N training projections
     - Simple comparison metric: Euclidean
     - Simple decision: K-Nearest Neighbor

       (note: this "K" refers to the k-NN algorithm, is different from the previous K's referring to the # of principal components)

Visualization of eigenfaces

Eigenfaces look somewhat like generic faces.

Reconstruction and Errors

K = 4

K = 200

K = 400

- Only selecting the top K eigenfaces → reduces the dimensionality.
- Fewer eigenfaces result in more information loss, and hence less discrimination between faces.

Summary for Eigenface

Pros

- Non-iterative, globally optimal solution

Limitations

- PCA projection is **optimal for reconstruction** from a low dimensional basis, but **may NOT be optimal for discrimination…**
  - See supplementary materials for "Linear Discriminative Analysis", aka "Fisherfaces"

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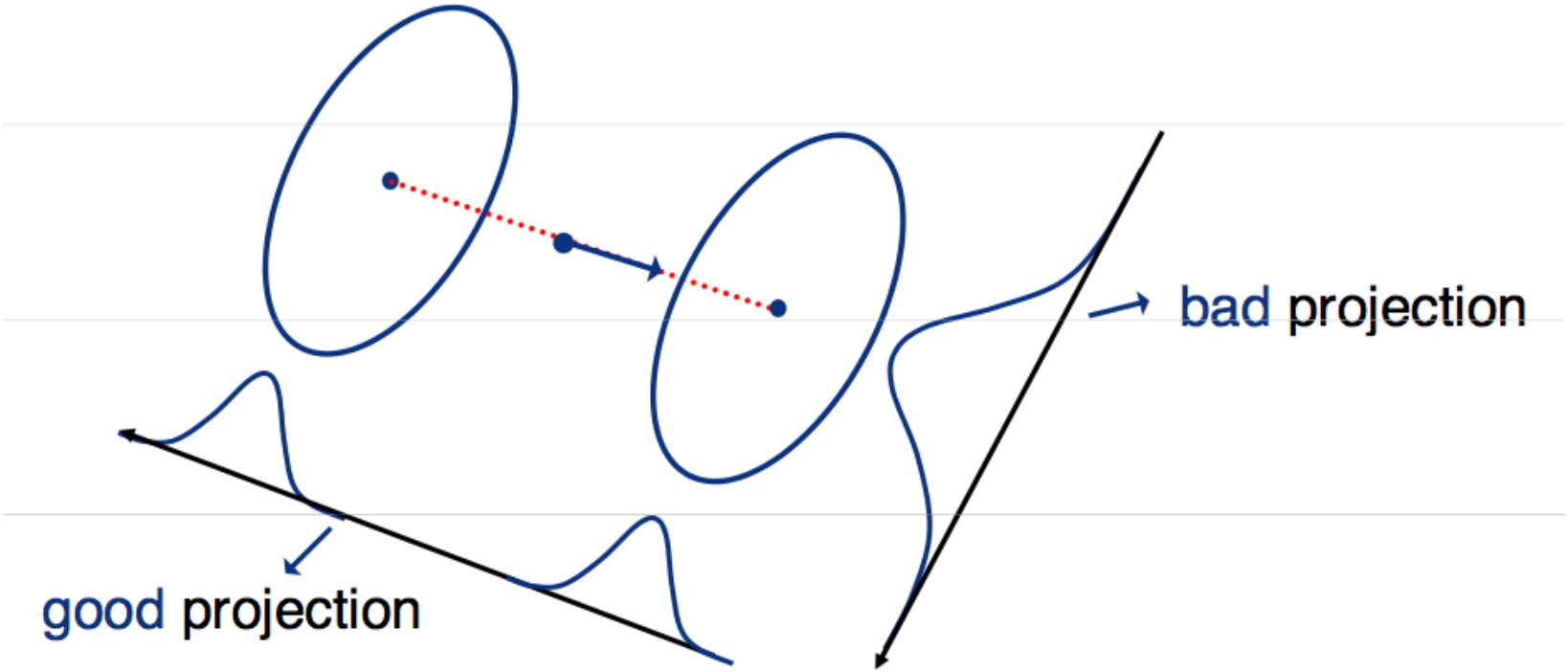
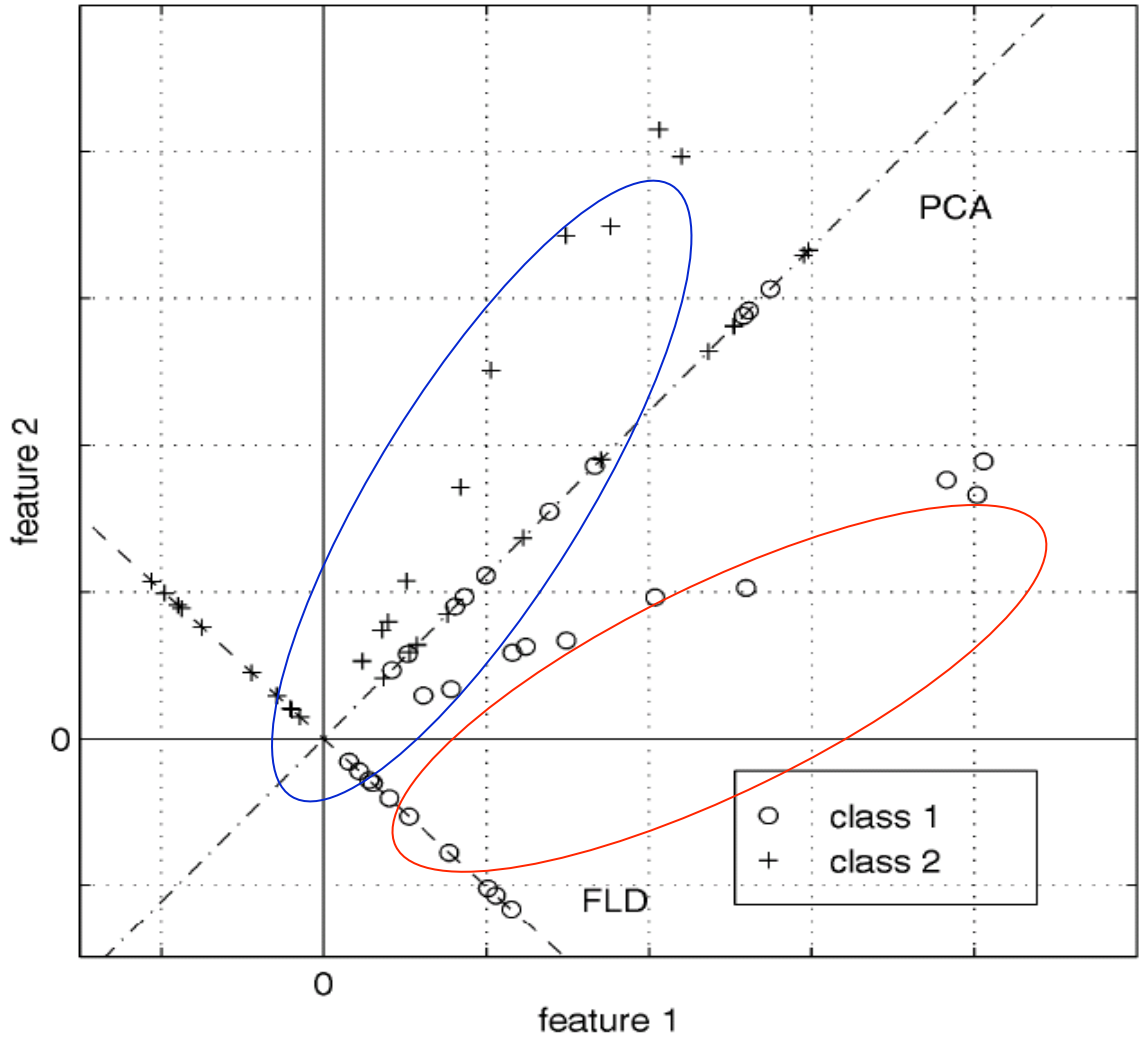
Fischer's Linear Discriminant Analysis

- Goal: find the best separation between two classes



Slide inspired by N. Vasconcelos

Basic intuition: PCA vs. LDA



Linear Discriminant Analysis (LDA)

- We have two classes such that

$$E_{X|Y}[X \mid Y = i] = \mu_i$$

$$E_{X|Y}[(X - \mu_i)(X - \mu_i)^T \mid Y = i] = \Sigma_i$$

- We want to find the line z that best separates them

$$z = w^T x$$

- One possibility would be to maximize

$$\left(E_{Z|Y}[Z \mid Y = 1] - E_{Z|Y}[Z \mid Y = 0]\right)^2 =$$

$$\left(E_{X|Y}[w^T x \mid Y = 1] - E_{X|Y}[w^T x \mid Y = 0]\right)^2 = \left(w^T[\mu_1 - \mu_0]\right)^2$$

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Linear Discriminant Analysis (LDA)

- However, this difference

$$\left(w^T\left[\mu_1-\mu_0\right]\right)^2$$

can be arbitrarily large by simply scaling w

- We are only interested in the direction, not the magnitude
- Need some type of normalization
- Fischer suggested

$$\max\frac{between\ \ class\ \ scatter}{within\ \ class\ \ scatter}=$$

$$\max_w\frac{\left(E_{Z|Y}\left[Z\mid Y=1\right]-E_{Z|Y}\left[Z\mid Y=0\right]\right)^2}{\mathrm{var}\left[Z\mid Y=1\right]+\mathrm{var}\left[Z\mid Y=0\right]}$$

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Linear Discriminant Analysis (LDA)

- We have already seen that

$$\left(E_{Z|Y}[Z \mid Y=1]-E_{Z|Y}[Z \mid Y=0]\right)^2=\left(w^T[\mu_1-\mu_0]\right)^2$$
$$=w^T[\mu_1-\mu_0][\mu_1-\mu_0]^T w$$

- also

$$\text{var}[Z \mid Y=i]=E_{Z|Y}\left\{\left(z-E_{Z|Y}[Z \mid Y=i]\right)^2 \mid Y=i\right\}$$
$$=E_{Z|Y}\left\{\left(w^T[x-\mu_i]\right)^2 \mid Y=i\right\}$$
$$=E_{Z|Y}\left\{w^T[x-\mu_i][x-\mu_i]^T w \mid Y=i\right\}$$
$$=w^T\Sigma_i w$$

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Linear Discriminant Analysis (LDA)

- And

$$J(w) = \frac{\left(E_{Z|Y}[Z \mid Y=1] - E_{Z|Y}[Z \mid Y=0]\right)^2}{\text{var}[Z \mid Y=1] + \text{var}[Z \mid Y=0]}$$

$$= \frac{w^T(\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w}{w^T(\Sigma_1 + \Sigma_0)w}$$

- which can be written as

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

between class scatter

$$S_B = (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T$$

$$S_W = (\Sigma_1 + \Sigma_0)$$

within class scatter

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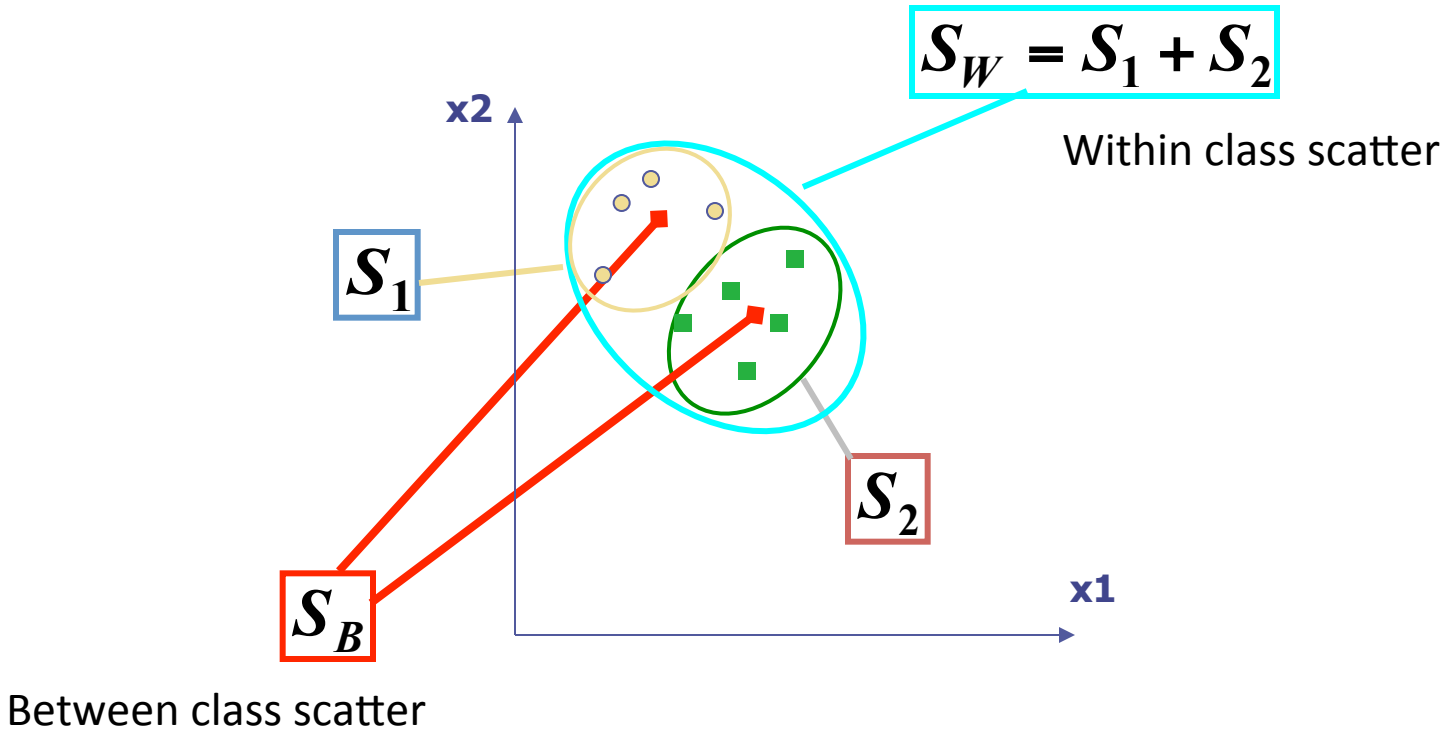
Visualization



Linear Discriminant Analysis (LDA)

- Maximizing the ratio

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

  - Is equivalent to maximizing the numerator while keeping the denominator constant, i.e.

$$\max_w w^T S_B w \quad \text{subject to} \quad w^T S_W w = K$$

  - And can be accomplished using Lagrange multipliers, where we define the Lagrangian as

$$L = w^T S_B w - \lambda\left(w^T S_W w - K\right)$$

  - And maximize with respect to both w and λ

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Linear Discriminant Analysis (LDA)

- Setting the gradient of

$$L = w^T (S_B - \lambda S_W) w + \lambda K$$

With respect to w to zeros we get

$$\nabla_w L = 2(S_B - \lambda S_W) w = 0$$

or

$$\boxed{S_B w = \lambda S_W w}$$

- This is a generalized eigenvalue problem
- The solution is easy when $\boxed{S_w^{-1} = (\Sigma_1 + \Sigma_0)^{-1}}$ exists

Slide inspired by N. Vasconcelos

Linear Discriminant Analysis (LDA)

- In this case

$$S_W^{-1} S_B w = \lambda w$$

- And using the definition of S

$$S_W^{-1} (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w = \lambda w$$

- Noting that $(\mu_1-\mu_0)^T w=\alpha$ is a scalar this can be written as

$$S_W^{-1} (\mu_1 - \mu_0) = \frac{\lambda}{\alpha} w$$

- and since we don't care about the magnitude of w

$$w^* = S_W^{-1} (\mu_1 - \mu_0) = (\Sigma_1 + \Sigma_0)^{-1} (\mu_1 - \mu_0)$$

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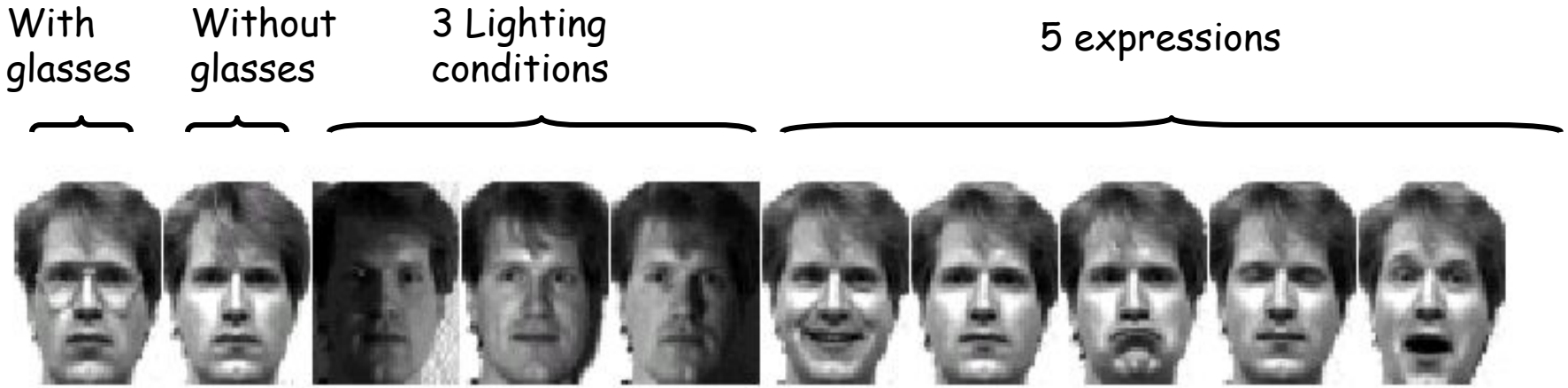
PCA vs. LDA

- Eigenfaces exploit the max scatter of the training images in face space
- Fisherfaces attempt to maximise the **between class scatter**, while minimising the **within class scatter**.

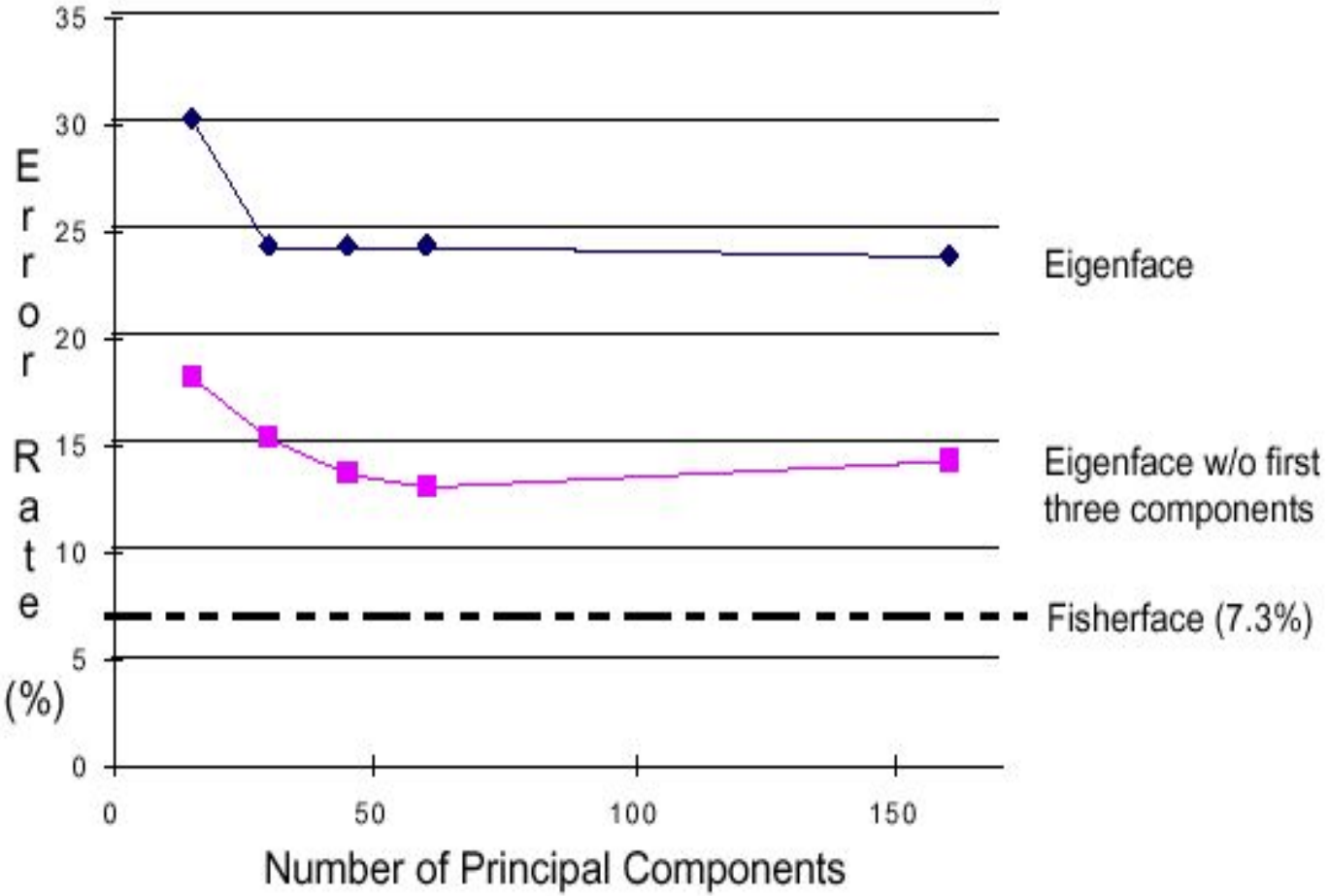
Results: Eigenface vs. Fisherface (1)

- Input: 160 images of 16 people
- Train: 159 images
- Test: 1 image

- Variation in Facial Expression, Eyewear, and Lighting



Eigenface vs. Fisherface (2)



What we have learned today

- Introduction to face recognition
- Principal Component Analysis (PCA)
- The Eigenfaces Algorithm
- Linear Discriminant Analysis (LDA)

Turk and Pentland, Eigenfaces for Recognition, *Journal of Cognitive Neuroscience* **3** (1): 71–86.

P. Belhumeur, J. Hespanha, and D. Kriegman. "Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection". *IEEE Transactions on pattern analysis and machine intelligence* **19** (7): 711. 1997.