Computational Vision

Camera Calibration

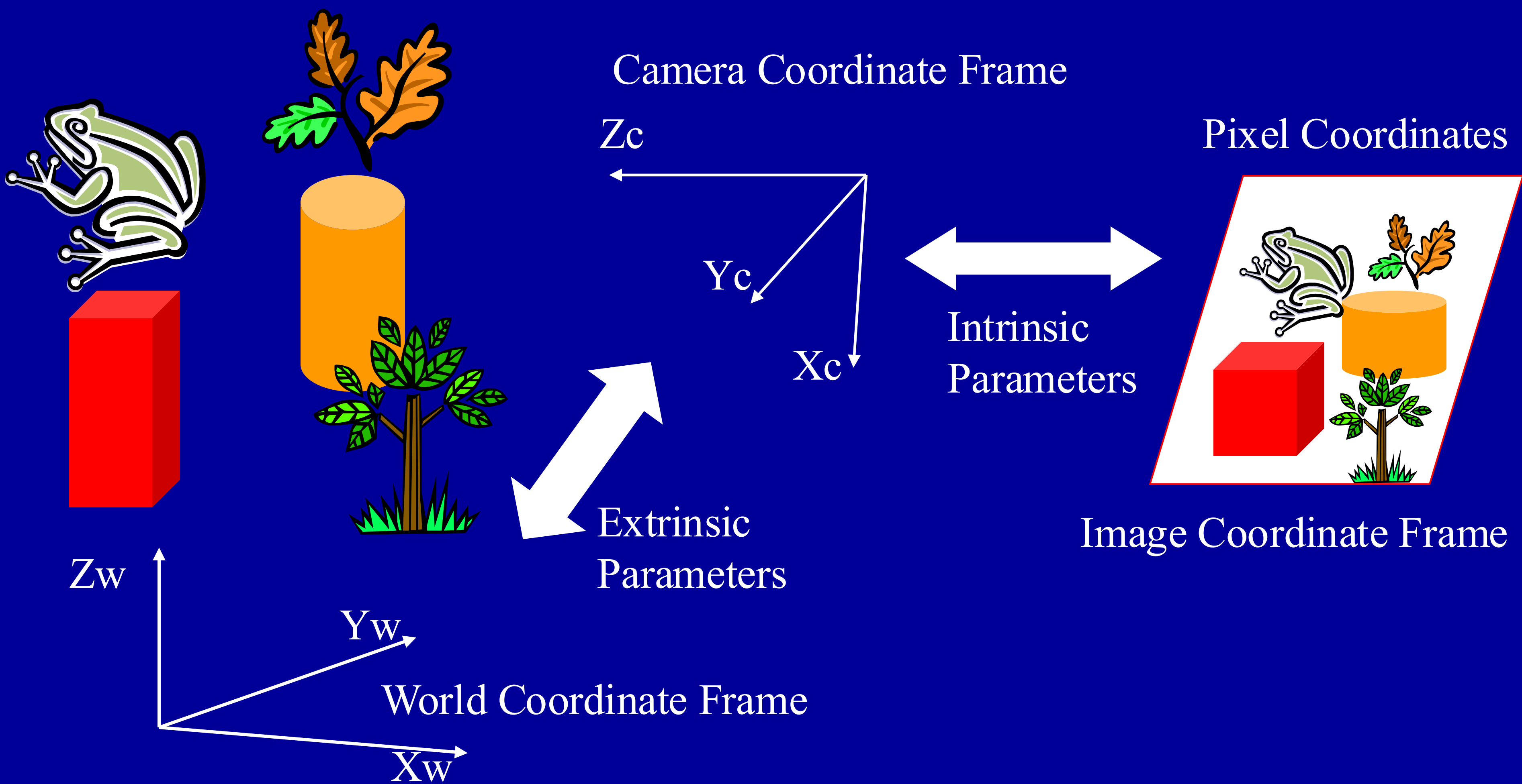
Szeliski 6.2/6.3

Trucco, chapter 6

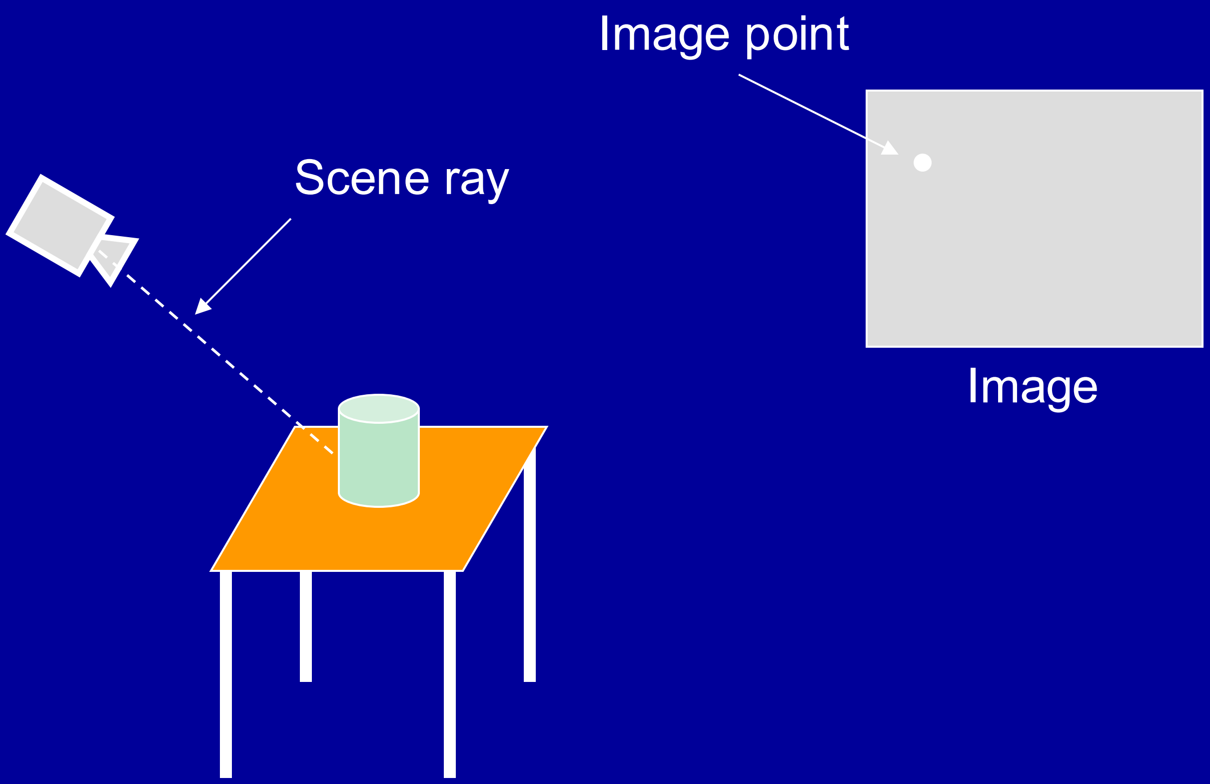
Camera Calibration

- Problem: Estimate camera's extrinsic & intrinsic parameters.
- Method: Use image(s) of known scene.
- Tools:
  - Geometric camera models.
  - SVD and constrained least-squares.
  - Line extraction methods.

Coordinate Frames

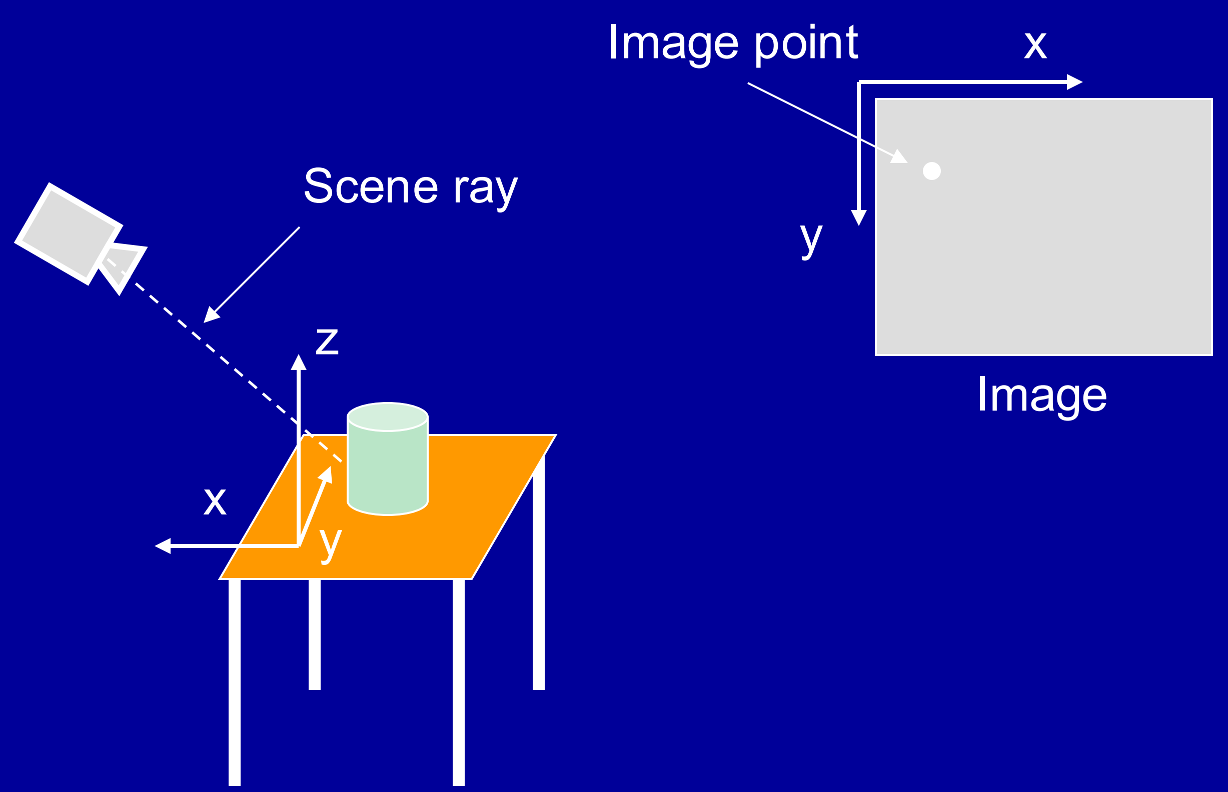


Why Calibrate?



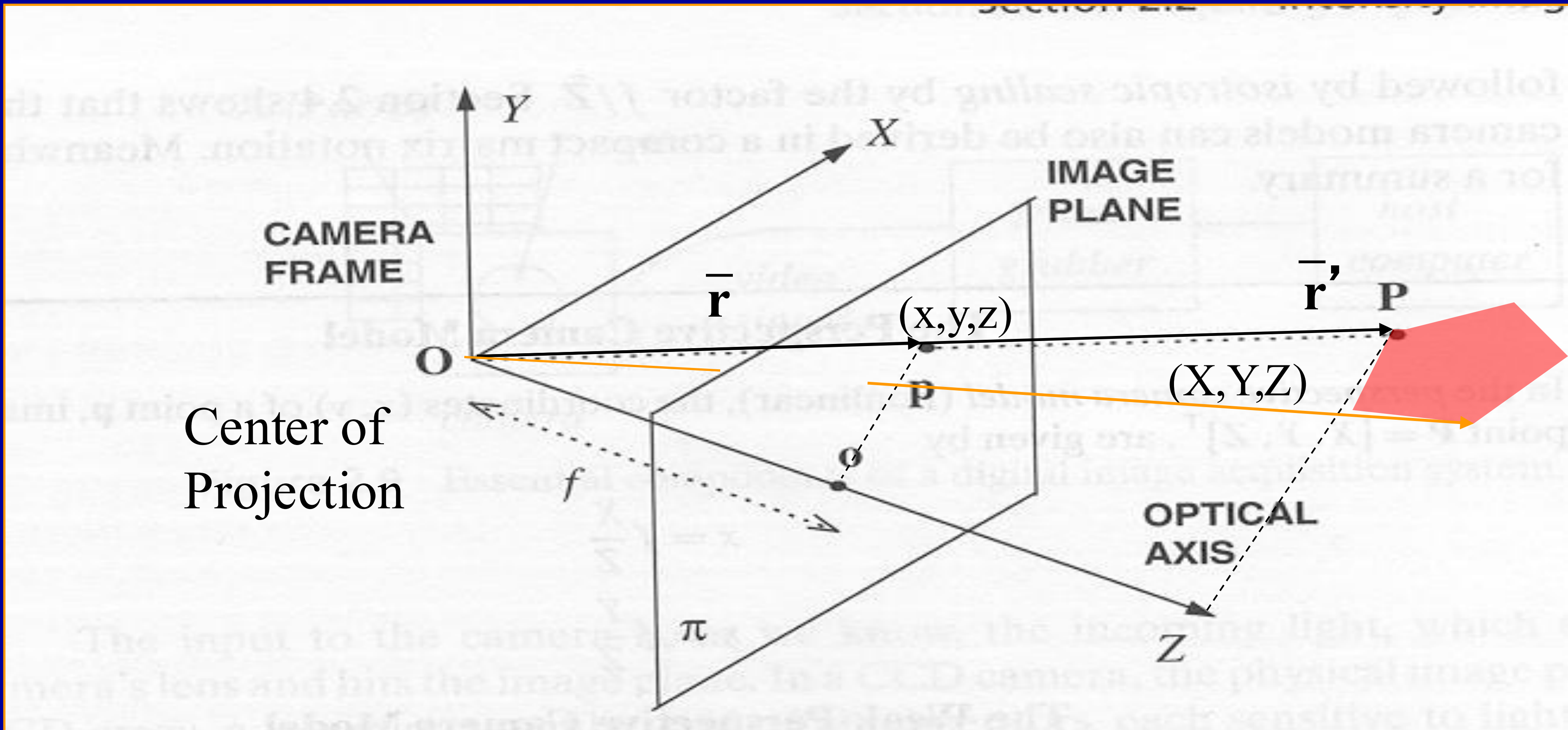
Calibration: relates points in the image to rays in the scene

Why Calibrate?



Calibration: relates points in the image to rays in the scene

Perspective Camera



$\bar{r}$ =(x,y,z)

$\bar{r}/f = \bar{r}'/Z$

x=f * X/Z
y=f * Y/Z
z=f

$\bar{r}'$ =(X,Y,Z)

f: effective focal length:
distance of image plane from O.

Extrinsic Parameters

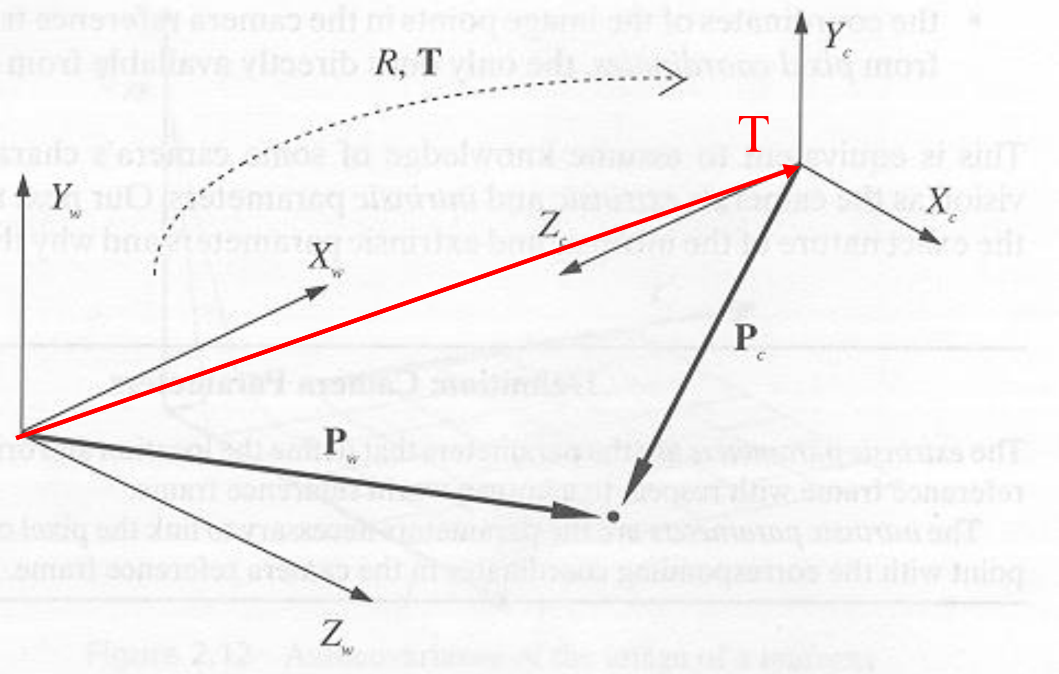


Figure 2.13 The relation between camera and world coordinate frames.

Pc=R(Pw-T)

Translation followed by rotation

Extrinsic Parameters (2^{nd} formulation)

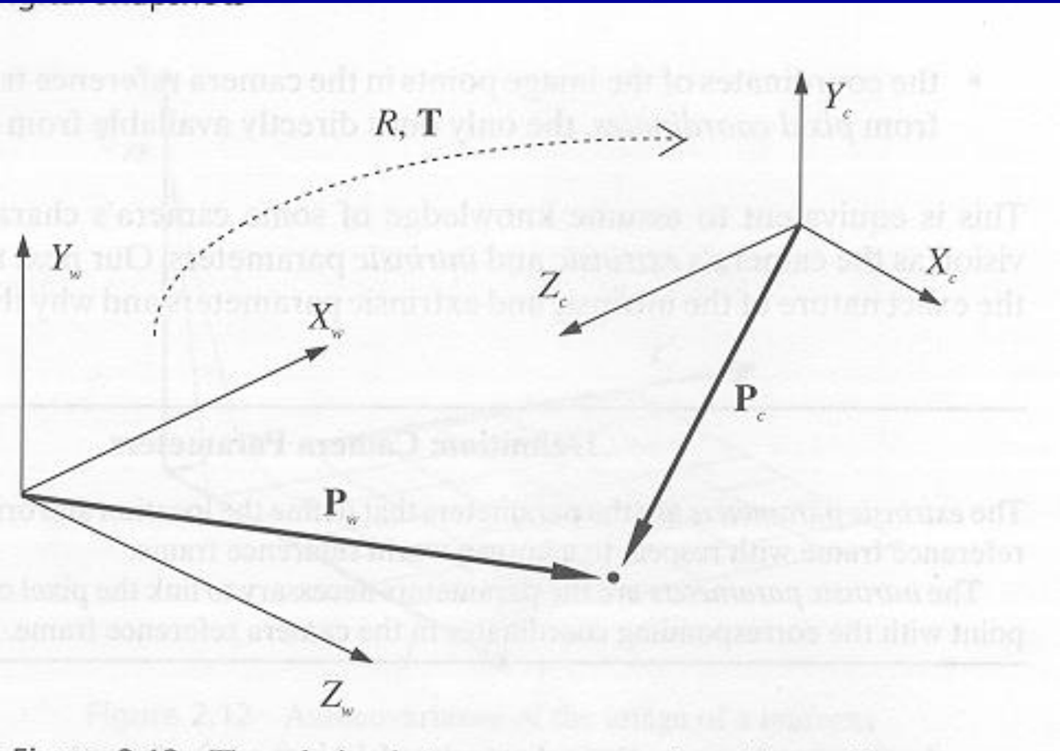


Figure 2.13 The relation between camera and world coordinate frames.

R same as before
T different

Pc=R Pw +T

Rotation followed by translation

The Rotation Matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}.$$

$R * R^T = R^T * R = I \Rightarrow$

$R^{-1} = R^T$

Orthonormal Matrix

Degrees of freedom?

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Basic 3D rotations (about axes)

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation about x, angle θ

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation about y, angle θ

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation about z, angle θ

Basic 3D rotation (example)

$$R_z(90^\circ)\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}\cos 90^\circ & -\sin 90^\circ & 0\\ \sin 90^\circ & \cos 90^\circ & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$$

Does it make sense geometrically?

Basic 3D rotations: can be combined

$$R = Rz(\alpha) * Ry(\beta) * Rx(\gamma)$$

3D Rotation

$$
\begin{aligned}
R = R_z(\alpha)\,R_y(\beta)\,R_x(\gamma) &= \overset{\text{yaw}}{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{\text{pitch}}{\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}} \overset{\text{roll}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}} \\
&= \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}
\end{aligned}
$$

From Wikipedia

Order of rotations is important: Applied from right to left:

First Rx, then Ry and finally Rz.

Rotation: angle, axis representation (optional)

$$R = \begin{bmatrix} \cos\theta + u_x^2\left(1-\cos\theta\right) & u_x u_y\left(1-\cos\theta\right) - u_z\sin\theta & u_x u_z\left(1-\cos\theta\right) + u_y\sin\theta \\ u_y u_x\left(1-\cos\theta\right) + u_z\sin\theta & \cos\theta + u_y^2\left(1-\cos\theta\right) & u_y u_z\left(1-\cos\theta\right) - u_x\sin\theta \\ u_z u_x\left(1-\cos\theta\right) - u_y\sin\theta & u_z u_y\left(1-\cos\theta\right) + u_x\sin\theta & \cos\theta + u_z^2\left(1-\cos\theta\right) \end{bmatrix}.$$

Angle: θ

Axis: unit vector (ux, uy, uz)

Intrinsic Parameters

$$x = \frac{f}{Z} X$$

$$y = \frac{f}{Z} Y$$

The Transformation between Camera and Image Frame Coordinates

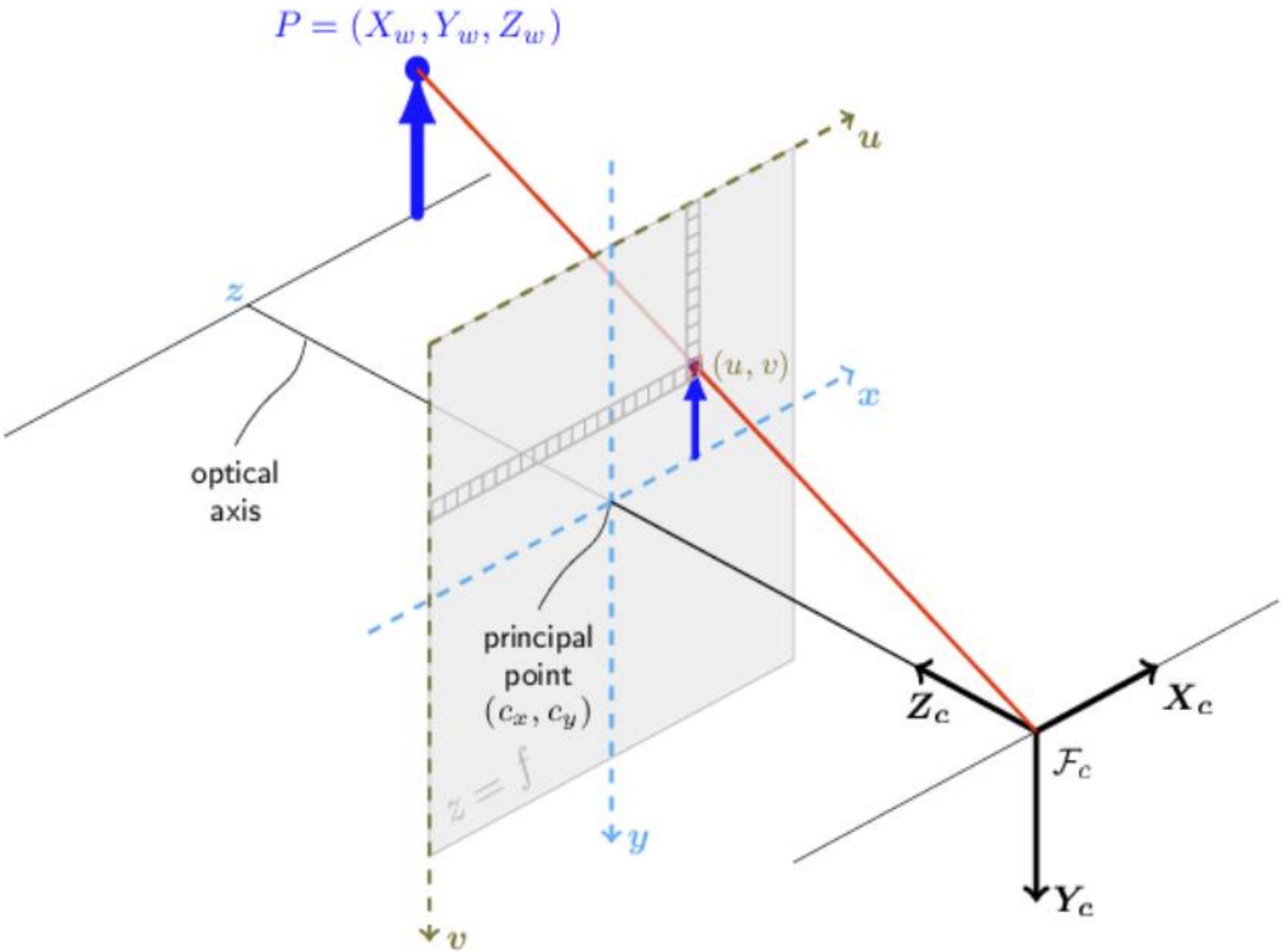
Neglecting any geometric distorsions possibly introduced by the optics and in the assumption that the CCD array is made of a rectangular grid of photosensitive elements, we have

$$\begin{aligned} x &= -(x_{im} - o_x)s_x \\ y &= -(y_{im} - o_y)s_y \end{aligned} \qquad (2.20)$$

with (o_x, o_y) the coordinates in pixel of the image center (the principal point), and (s_x, s_y) the effective size of the pixel (in millimeters) in the horizontal and vertical direction respectively.

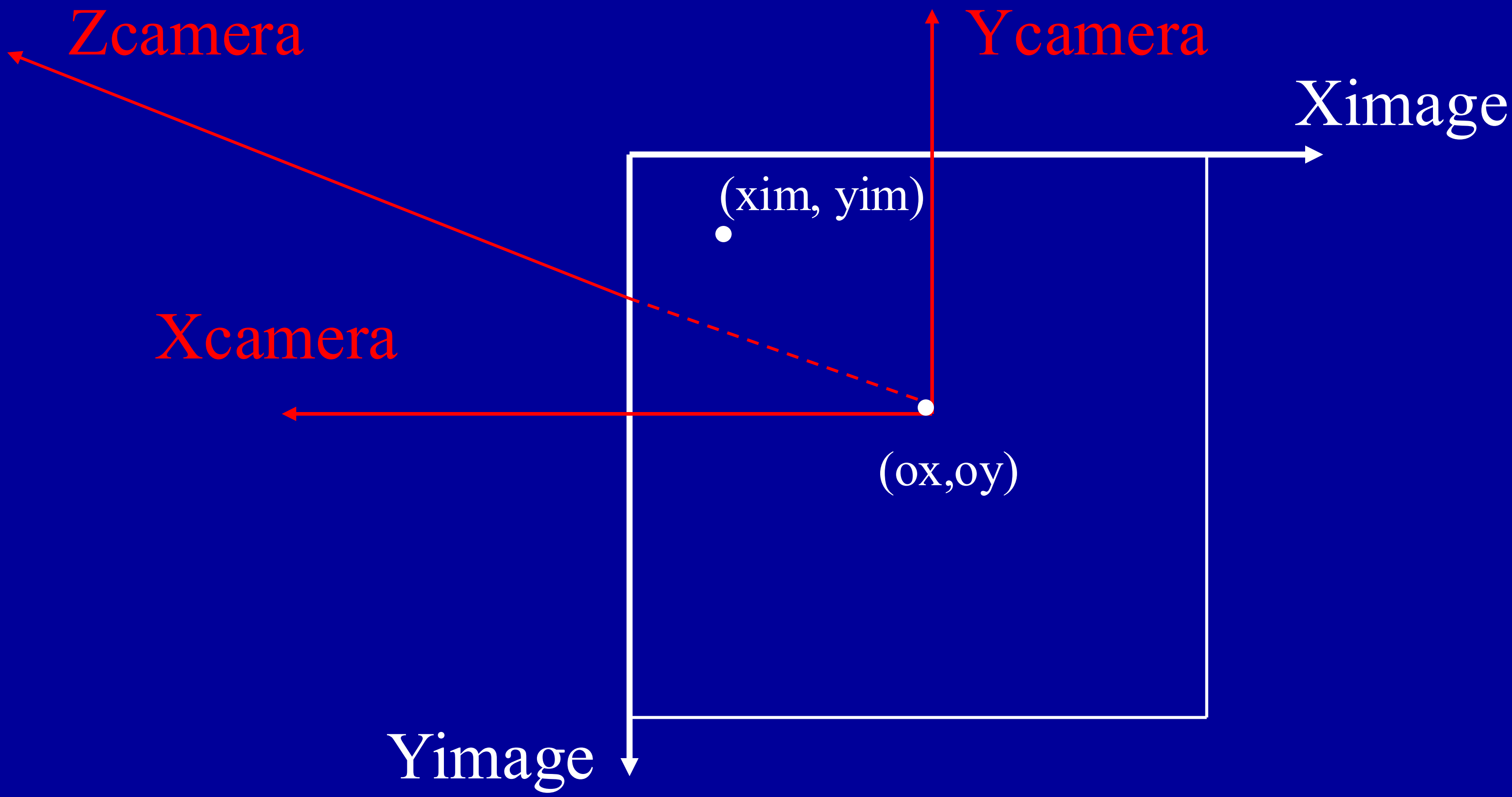
Therefore, the current set of intrinsic parameters is f, o_x, o_y, s_x, s_y.

Pinhole Camera Model



Pinhole camera model

Image and Camera Frames



Geometric Model

$$x = -(x_{im} - o_x)s_x = f\frac{Xc}{Zc}$$

$$y = -(y_{im} - o_y)s_y = f\frac{Yc}{Zc}$$

3D Point in Camera Coordinate Frame

- Transformation from Image to Camera Frame. (ox,oy,sx,sy)
- No distortion!

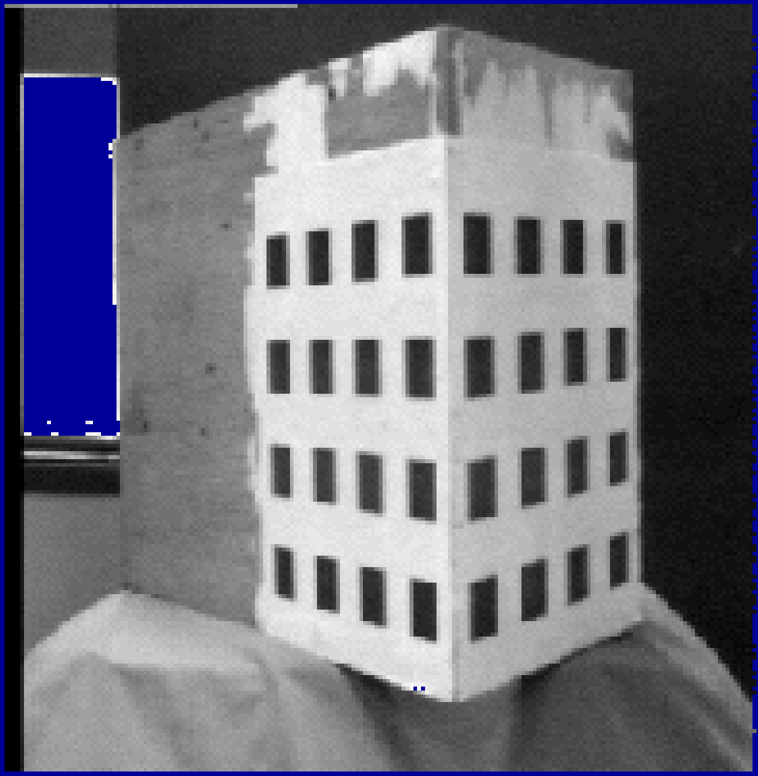
- Transformation from World to Camera Frame.
- Perspective projection (f, R, T)

Point in Camera Frame

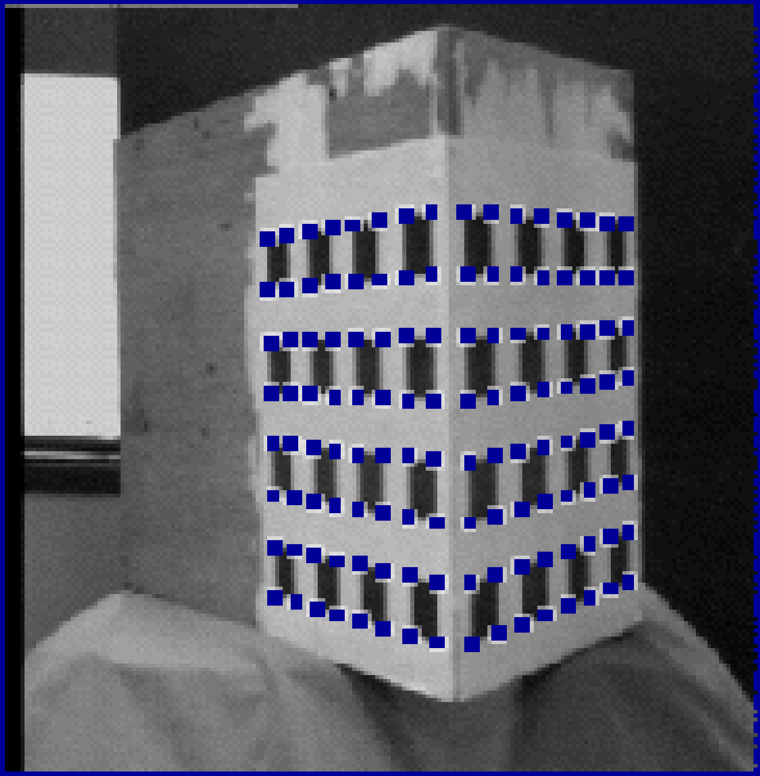
Camera Calibration: Issues

- Which parameters need to be estimated.
  - Focal length, image center, aspect ratio
  - Radial distortions
- What kind of accuracy is needed.
  - Application dependent
- What kind of calibration object is used.
  - One plane, many planes
  - Complicated three dimensional object

Camera Calibration

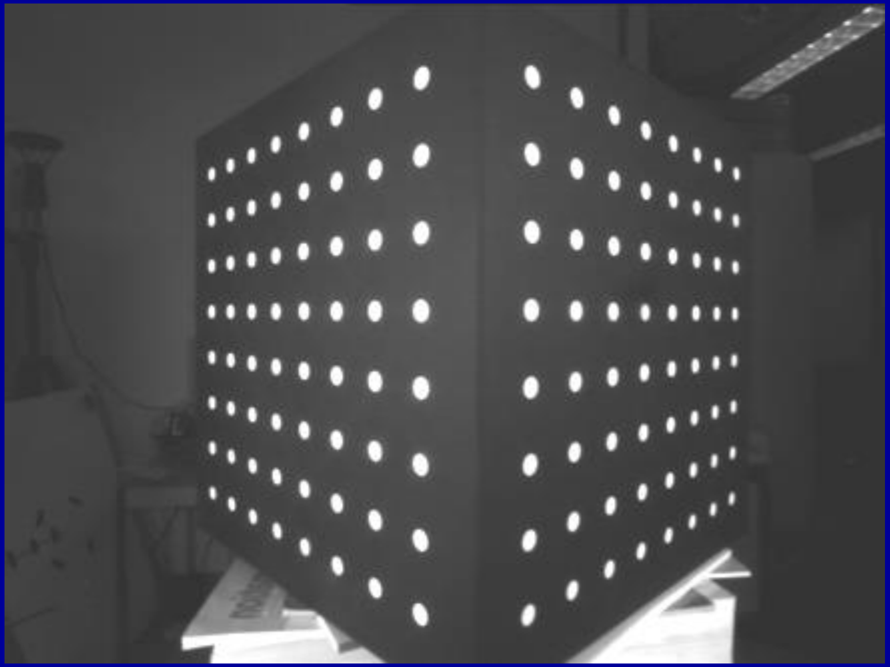


Calibration object



Extracted features

Camera Calibration



Extract centers of circles

Basic Equations

$$x_{im} = -\frac{f}{s_x}\frac{X^c}{Z^c} + o_x$$

$$y_{im} = -\frac{f}{s_y}\frac{Y^c}{Z^c} + o_y$$

$$X^c = r_{11}X^w + r_{12}Y^w + r_{13}Z^w + T_x$$

$$Y^c = r_{21}X^w + r_{22}Y^w + r_{23}Z^w + T_y$$

$$Z^c = r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z$$

Basic Equations

$$x = -f_x \frac{X^c}{Z^c} + o_x$$

$$y = -f_y \frac{Y^c}{Z^c} + o_y$$

Focal length expressed in x-pixel size units

$$f_x = \frac{f}{s_x}, f_y = \frac{f}{s_y}$$

Focal length expressed in y-pixel size units

$$X^c = r_{11}X^w + r_{12}Y^w + r_{13}Z^w + T_x$$
$$Y^c = r_{21}X^w + r_{22}Y^w + r_{23}Z^w + T_y$$
$$Z^c = r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z$$

Basic Equations

$$x = -f_x \frac{X^c}{Z^c} + o_x$$

$$y = -\frac{f_x}{\alpha} \frac{Y^c}{Z^c} + o_y$$

Aspect ratio

$$\alpha = \frac{s_x}{s_y}$$

$\alpha = 1$ for square pixels

$$X^c = r_{11} X^w + r_{12} Y^w + r_{13} Z^w + T_x$$

$$Y^c = r_{21} X^w + r_{22} Y^w + r_{23} Z^w + T_y$$

$$Z^c = r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z$$

Basic Equations

Extrinsic Parameters

1) Rotation matrix R (3x3)
2) Translation vector **T** (3x1)

Intrinsic Parameters

1) fx=f/sx, length in effective horizontal pixel size units.
2) α=sy/sx, aspect ratio.
3) (ox,oy), image center coordinates.
4) Radial distortion coefficients.

Total number of parameters (excluding distortion & image center coordinates): 8

Basic Equations

$$x - o_x = -f_x \frac{r_{11}X^w + r_{12}Y^w + r_{13}Z^w + T_x}{r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z}$$

$$y - o_y = -f_y \frac{r_{21}X^w + r_{22}Y^w + r_{23}Z^w + T_y}{r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z}$$

1) **Assume that image center is known.**
2) **Solve for the remaining parameters.**
3) **Use N image points** (x_i, y_i) **and their corresponding**
   N world points $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x = -f_x \frac{r_{11}X^w + r_{12}Y^w + r_{13}Z^w + T_x}{r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z}$$

$$y = -f_y \frac{r_{21}X^w + r_{22}Y^w + r_{23}Z^w + T_y}{r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z} \quad (1)$$

1) Assume that image center is known.
2) Solve for the remaining parameters.
3) Use N image points (x_i, y_i) and their corresponding N world points $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x_i f_y (r_{21} X_i^w + r_{22} Y_i^w + r_{23} Z_i^w + T_y) = y_i f_x (r_{11} X_i^w + r_{12} Y_i^w + r_{13} Z_i^w + T_x) \quad (2)$$

1) **Assume that image center is known.**
2) **Solve for the remaining parameters.**
3) **Use N image points** (x_i, y_i) **and their corresponding N world points** $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x_i X_i^w v_1 + x_i Y_i^w v_2 + x_i Z_i^w v_3 + x_i v_4$$
$$- y_i X_i^w v_5 - y_i Y_i^w v_6 - y_i Z_i^w v_7 - y_i v_8 = 0 \qquad \textbf{(3)}$$

$$v_1 = r_{21}, v_5 = \alpha r_{11}$$
$$v_2 = r_{22}, v_6 = \alpha r_{12}$$
$$v_3 = r_{23}, v_7 = \alpha r_{13}$$
$$v_4 = T_y, v_8 = \alpha T_x$$

1 linear equation, 8 unknowns

Basic Equations

$$x_i X_i^w v_1 + x_i Y_i^w v_2 + x_i Z_i^w v_3 + x_i v_4 - y_i X_i^w v_5 - y_i Y_i^w v_6 - y_i Z_i^w v_7 - y_i v_8 = 0 \quad \mathbf{(3)}$$

$$v_1 = r_{21}, v_5 = \alpha r_{11}$$
$$v_2 = r_{22}, v_6 = \alpha r_{12}$$
$$v_3 = r_{23}, v_7 = \alpha r_{13}$$
$$v_4 = T_y, v_8 = \alpha T_x$$

$$A\mathbf{v} = \mathbf{0}$$

A is a N x 8 matrix (N correspondences, i.e. N equations)

v is the 8 x 1 vector of unknowns [v1, v2, …, v8]

Basic Equations

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

N x 8 matrix: Known from correspondences

$$\boldsymbol{v} = \begin{matrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \end{matrix}$$

$$\boxed{A\mathbf{v} = \mathbf{0}}$$

How would we solve this system?

8 unknowns

Basic Equations

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

N x 8 matrix: Known from correspondences

$$\boxed{A\mathbf{v} = \mathbf{0}} \quad (3)$$

How would we solve this system?

Rank of matrix A?

Solution up to a scale factor.

Singular Value Decomposition

Any matrix A can be decomposed as follows:

$$A = UDV^T$$

Orthonormal matrices   Diagonal matrix

A: m x n matrix

U: m x m matrix, columns orthogonal unit vectors.

V: n x n matrix, -//-

D: m x n , diagonal matrix. The diagonal elements

σ_i are the singular values

$\sigma_1 >= \sigma_2 >= \ldots >= \sigma_n >= 0$

SVD: Some properties

From Appendix A.6, Trucco & Verri book

$$A = UDV^T$$

- Square A is non-singular iff σi != 0
- For square A, C=σ1/σn is the condition number
- For rectangular <u>**A # of non-zero σi is the rank**</u>
- For square non-singular A: $A^{-1} = VD^{-1}U^T$
- For square A, pseudoinverse: $A^{+} = VD_0^{-1}U^T$
- Singular values of A = square roots of
  eigenvalues of AA^T and $A^T A$
- 7. Columns of U, V
  eigenvectors of AA^T and $A^T A$
- 8. Frobenius norm of a matrix

$$||A||_F = \sum_{i,j} a_{i,j}^2 = \sum_{i=1}^{n} \sigma_i^2$$

Singular Value Decomposition

$$A = UDV^T$$

$$A\mathbf{v} = 0$$

A ix a N x 8 matrix. Assume that N >= 8

What is the maximum rank of A?
Think of special case when N = 8

Singular Value Decomposition

$$A = UDV^T$$

$$A\mathbf{v} = 0$$

A ix a N x 8 matrix. Assume that N >= 8

Can A have rank 8? What does it mean for **v**?

Singular Value Decomposition

$$A = UDV^T$$

$$A\mathbf{v} = 0$$

A is N x 8
U is N x N
D is N x 8
V is 8 x 8

Rank of A is 7 => $\boldsymbol{\sigma_1 \geq \sigma_2 \geq \cdots > \sigma_8 = 0}$

Last singular value is 0 (or effectively close to 0)

Solution **v** is the last column of matrix V

Also note: Solution is up to a scale factor!

Algorithm for finding v

$$A = UDV^T$$

$$A\mathbf{v} = 0$$

A is N x 8
U is N x N
D is 8 x 8
V is 8 x 8

Rank of A is 7 => $\sigma_1 \geq \sigma_2 \geq \cdots > \sigma_8 = 0$

1) Construct Nx8 matrix A
2) Compute the SVD of A
3) Verify that last singular value is 0 – if not error
   What would an error mean?
4) Solution is the last column of matrix V

Solving for v

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

$$\boxed{A\mathbf{v} = \mathbf{0}} \qquad \textbf{(3)}$$

How would we solve this system: **SVD.**

Solution: $\bar{\mathbf{v}} = \gamma(r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x)$

Uknown scale factor γ=?

Aspect ratio α=?

Solving for v

Solution:

$$\bar{\mathbf{v}} = \gamma(r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x)$$

So $v_1 = \gamma r_{21}, v_2 = \gamma r_{22}, v_3 = \gamma r_{23}$

$$v_1{}^2 + v_2{}^2 + v_3{}^2 = \gamma^2(r_{21}{}^2 + r_{22}{}^2 + r_{23}{}^2) = \gamma^2$$

Sum of squares of rotation column is 1 due to orthonormality

From the first three elements of v you can get the scale factor γ, but with an unknown sign (why?)

Solving for α

$$\overline{\mathbf{v}} = \gamma(r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x)$$

$v_5 = \alpha\gamma r_{11}$, $v_6 = \alpha\gamma r_{12}$, $v_7 = \alpha\gamma r_{13}$

Compute $v_5^2 + v_6^2 + v_7^2 = (\alpha\gamma)^2(r_{11}^2 + r_{12}^2 + r_{13}^2) = (\alpha\gamma)^2$

Similarly, from elements 5, 6 and 7 you can solve for aspect ratio α

Finally, all other elements can be derived.

For example: $r_{21} = {}^{v_1}/_{\gamma}$, …, $T_x = v_8/(\gamma * \alpha)$

What we have computed

- Aspect ratio α
- Tx, Ty
- Rotation matrix R
  - We have computed two rows
    - Third row is cross-product of the first two

Solving for Tz and fx?

Note, that Tz and fx have not been computed yet

Solving for Tz and fx?

$$x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w + T_z) = -f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)$$

Going back to original equations. **Only** two unknowns: (Tz, fx).

Linear system of N equations and 2 unknowns:

$$A\begin{pmatrix} T_z \\ f_x \end{pmatrix} = \mathbf{b}$$

A is known matrix of size N x 2

b is a known vector of size N x 1

How would we solve this system?

$$x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w) + x_i \cdot T_z + f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x) = 0$$

$$\Longleftrightarrow \underbrace{x_i}_{\text{KNOWN}} \cdot T_z + f_x \underbrace{(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)}_{\text{KNOWN}} = \underbrace{-x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w)}_{\text{KNOWN}}$$

$$\Longleftrightarrow \alpha_1^i T_z + \alpha_2^i T_x = b_i, \quad \alpha_1^i = x_i, \quad \alpha_2^i = r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x$$

$$b_i = -x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w)$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} \alpha_1^1 & \alpha_2^1 \\ \alpha_1^2 & \alpha_2^2 \\ \vdots & \\ \alpha_1^N & \alpha_2^N \end{bmatrix}}_{A_{N\times 2}} \cdot \underbrace{\begin{bmatrix} T_z \\ f_x \end{bmatrix}}_{\text{unknowns}} = \underbrace{\begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix}}_{b_{N\times 1}}$$

Solving for Tz and fx?

$$x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w + T_z) = -f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)$$

$$A\begin{pmatrix} T_z \\ f_x \end{pmatrix} = \mathbf{b}$$

How would we solve this system?

$$\begin{pmatrix} \hat{T}_z \\ \hat{f}_x \end{pmatrix} = (A^T A)^{-1} A^T \mathbf{b}$$

Solution in the **least squares sense.**

Camera Center

Camera Models (linear versions)

Internal parameters:

$$M_{int} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}$$

Elegant decomposition.
No distortion!

Rotation

Translation

$$M_{ext} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & Tx \\ r_{21} & r_{22} & r_{23} & Ty \\ r_{31} & r_{32} & r_{33} & Tz \end{pmatrix}.$$

The Linear Matrix Equation of Perspective Projections

Homogeneous
Coordinates

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = M_{int} M_{ext} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}.$$

Measured Pixel
$(x_{im}=x_1/x_3,\ y_{im}=x_2/x_3)$

World Point
(Xw, Yw,Zw)

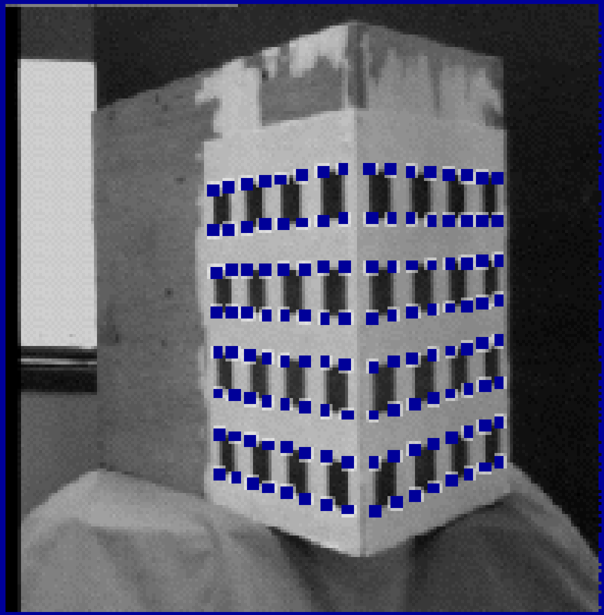
?

Camera Calibration – Other method

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{u}{w}$$

$$y = \frac{v}{w}$$



Extracted features

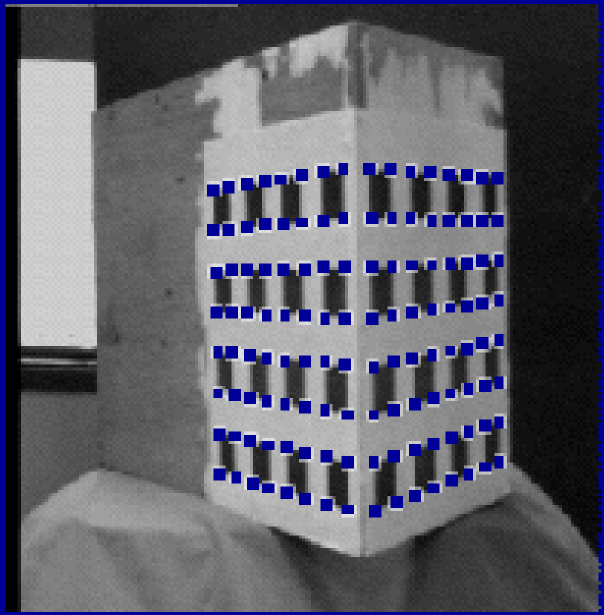
Step 1: Estimate P (3x4 matrix)
Step 2: Decompose P into internal and external parameters R,T,C

Camera Calibration: Step 1

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{u}{w}$$

$$y = \frac{v}{w}$$



Extracted features

$$wx = u$$

$$wy = v$$

w

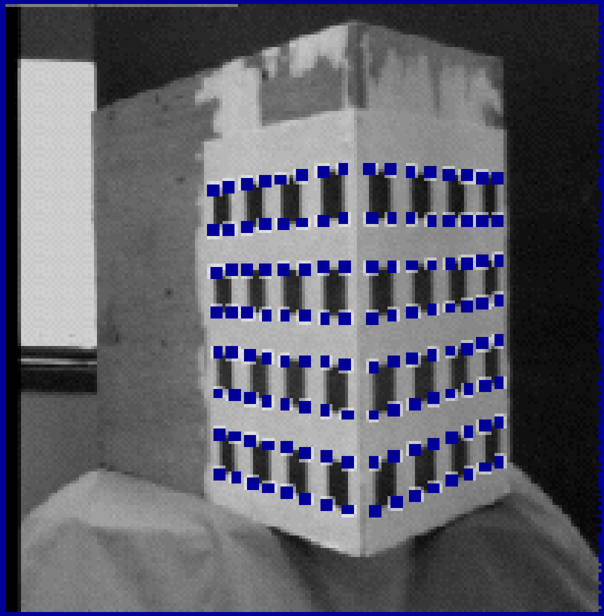
$$x(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{11}X + p_{12}Y + p_{13}Z + p_{14} \longleftarrow u$$

$$y(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{21}X + p_{22}Y + p_{23}Z + p_{24} \longleftarrow v$$

Each point (x,y) gives us two equations

Camera Calibration: Step 1

$$\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



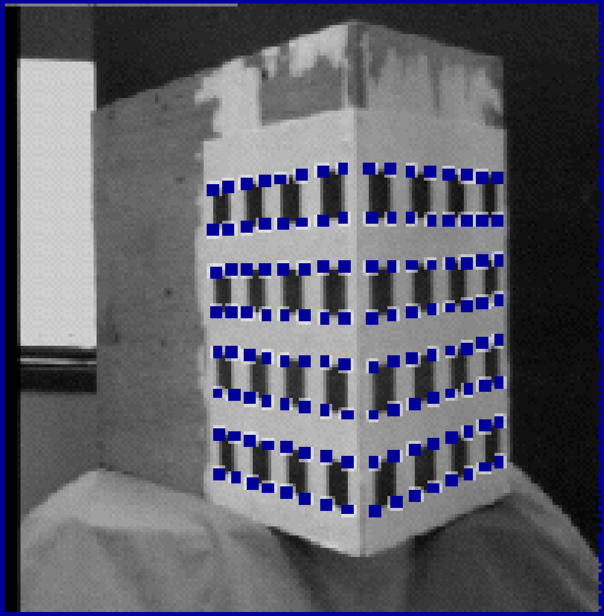
Extracted features

$$x(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{11}X + p_{12}Y + p_{13}Z + p_{14}$$
$$y(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{21}X + p_{22}Y + p_{23}Z + p_{24}$$

Each corner (x,y) gives us two equations

Camera Calibration: Step 1

$$2n\left\{\underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \\ \cdots & & & & & & & & & & & \\ \vdots & & & & & & & & & & & \\ \cdots & & & & & & & & & & & \\ X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix}}_{A}\right. \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

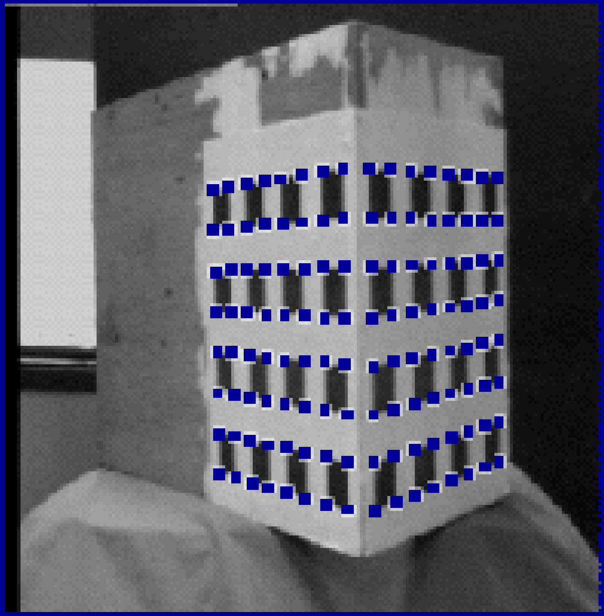


Extracted features

n points gives us 2n equations

Camera Calibration: Step 1

$$2n\left\{\underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \\ \cdots &&&&&&&&&&& \\ \vdots &&&&&&&&&&& \\ \cdots &&&&&&&&&&& \\ X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix}}_{A}\right. \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$



Extracted features

We need to solve $A\mathbf{p} = 0$

In the presence of noise we need to solve $\min_{\mathbf{p}} \|A\mathbf{p}\|$

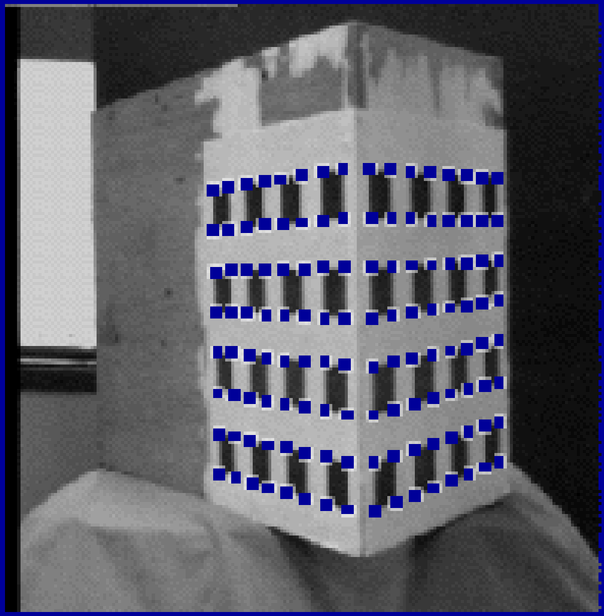
The solution is given by the eigenvector with the smallest eigenvalue of $A^T A$

Camera Calibration: Step 1

The result can be improved through non-linear minimization.

$$\min_{\mathbf{p}} \sum_i \left(\left(x_i - \frac{u_i}{w_i} \right)^2 + \left(y_i - \frac{v_i}{w_i} \right)^2 \right)$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$



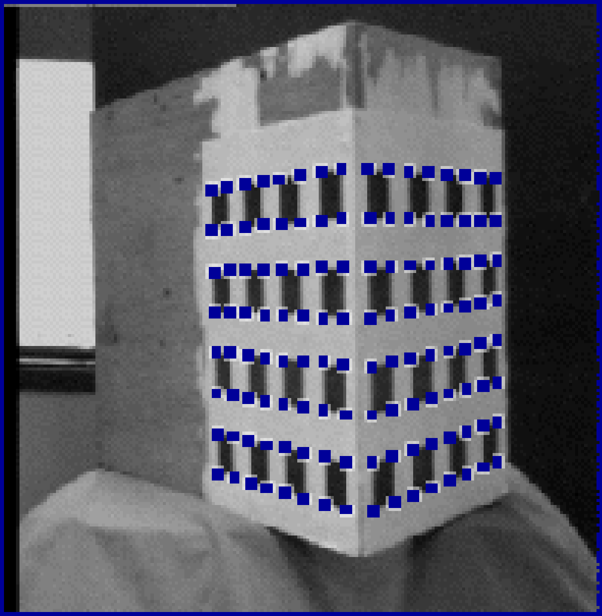
Extracted features

Camera Calibration: Step 1

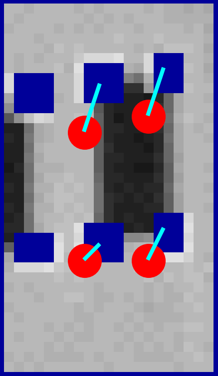
The result can be improved through non-linear minimization.

$$\min_{\mathbf{p}} \sum_i \left(\left(x_i - \frac{u_i}{w_i} \right)^2 + \left(y_i - \frac{v_i}{w_i} \right)^2 \right)$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$



Extracted features



Minimize the distance between the predicted and detected features.