STAT W4640

Assignment One

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Chapter 1: Exercise 7

There are three boxes: Box A, Box B, Box C. The probability of three box has the big prize will be:

$$Pr(A) = Pr(B) = Pr(C) = \frac{1}{3}$$

Without the loss of generality,If Box A was chosen. Let the Host Open Box C.The probability of the host opening the Box C will be: If A has the big prize, the host could open Box C or B: $Pr(OpenC|A) = \frac{1}{2}$.If B has the big prize,the host has to open Box C, $Pr(OpenC|B) = 1$. If C has the big prize, the host could never open Box C $Pr(OpenC|C) = 0$.

$$Pr(OpenC) = \frac{1}{3} * (\frac{1}{2} + 1 + 0) = \frac{1}{2}$$

Therefore, the probability A has the big prize and the host open Box C will be:

$$Pr(A|OpenC) = \frac{Pr(OpenC|A)Pr(A)}{Pr(OpenC)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

The probability B has the big prize and the host open Box C will be:

$$Pr(B|OpenC) = \frac{Pr(OpenC|B)Pr(B)}{Pr(OpenC)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

. After all, we suggest change the choice to the other unchosen box.

Computational Problem

The prior density will be:

$$p(\theta) = \frac{\alpha - \alpha^d}{1 - \alpha} \cdot \frac{\theta}{\alpha} + \frac{1 - \alpha}{1 - \alpha^d} \cdot \theta$$

The likelihood function will be

$$p(y|\theta) \propto \theta^y (1 - \theta)^{n-y}$$

Therefore the posterior density will be:

$$\begin{aligned} p(\theta|y) \propto p(y|\theta) \cdot p(\theta) &= \frac{1 - \alpha^{d-1}}{1 - \alpha} \cdot \theta^{1+y}(1 - \theta)^{n-y} + \frac{1 - \alpha}{1 - \alpha^d} \cdot \theta^{1+y}(1 - \theta)^{n-y} \\ &= \frac{(1 - \alpha)^2 + (1 - \alpha^d)(1 - \alpha^{d-1})}{(1 - \alpha)(1 - \alpha^d)} \theta^{1+y}(1 - \theta)^{n-y} \end{aligned}$$

Chapter 2: Exercise 19

(a)

$$
\begin{aligned}
&p(\theta) \propto \ \theta^{\alpha-1}\exp(-\beta\theta) \\
&p(y|\theta) \propto \theta\exp(\theta y) \\
&p(\theta|y) \propto p(\theta)\cdot p(y|\theta) = \theta^{\alpha}\exp[(y-\beta)\theta]
\end{aligned}
$$

Therefore, the posterior density will have gamma distribution with $\alpha+1$ and $\beta-y$ two parameters. The posterior density and the prior density are both gamma distributed.

(b)

(c)

(d)

Chapter 2: Exercise 21

(a)

(b)

(c)

Problem 8.3

Problem 9.3