

Tensor computation Assignment Set1 Solution

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0.1 Multiply two Tensor:

Suppose two tensor A and B such that :

$$A \in \mathbf{R}^{(4,5,6)} \quad B \in \mathbf{R}^{(5,6,6)}$$

Einstein Product:

As per **Einstein Product**,

$$A *_2 B \in \mathbf{R}^{(4,6)}$$

How to calculate : Tensor A last two index 5,6 should be same as Tensor B first two index to compute $A *_2 B$ of 3rd order tensors. Einstein Product can be computed by below formula where these two indexes will merge to create size of output tensor as 4*6 .

$$c_{k,l} = \sum_{i=0}^5 \sum_{j=0}^6 a_{kij} b_{ijl}$$

Algorithm 1 Einstein Product Tensor Computation

```
1: Initialize  $C[k, l]$  with all elements as 0
2: for  $k = 1$  to 4 do
3:   for  $l = 1$  to 6 do
4:     for  $i = 1$  to 5 do
5:       for  $j = 1$  to 6 do
6:          $C[k, l] = A[k, i, j] \times B[i, j, l]$ 
7:       end for
8:     end for
9:   end for
10: end for
11: return C
```

Checkout the code: [Click here](#)

T-Product:

As per **T-Product**,

$$A * B \in \mathbf{R}^{(4,6,6)}$$

Algorithm 2 T-Product Tensor Computation

```
1: Set  $A = FFT(A, axis = 2)$ 
2: Set  $B = FFT(B, axis = 2)$ 
3: Initialize C with all element as 0
4: for  $i = 1$  to 6 do
5:    $C[:, :, i] = A[:, :, i] * B[:, :, i]$ 
6: end for
7: Set  $C = iFFT(C, axis = 2)$ 
8: return C
```

Checkout the code: [Click here](#)

0.2 2-mode Product of a Tensor

Suppose a tensor A and matrix B such that :

$$A \in \mathbf{R}^{(J_1, J_2, J_3)} \quad B \in \mathbf{R}^{(I, J_2)}$$

2-mode Product:

As per **2-mode Product**,

$$A *_2 B \in \mathbf{R}^{(J_1, I, J_3)}$$

How to calculate : 2-mode product of tensor A can be computed with a matrix as below formula, when tensor second index will match with matrix last index . In this case tensor second index J_2 matches matrix last index J_2 .

$$c_{k,j,p} = \sum_{i=0}^{J_2} a_{kip} b_{j,i}$$

Algorithm 3 2-mode Product Tensor Computation

Initialize C of size (J_1, I, J_3) with all Element 0

```
for k = 0 to  $J_1$  do
  for j = 0 to  $I$  do
    for p = 0 to  $J_3$  do
      for i = 0 to  $J_2$  do
        C[k,j,p] = A[k,i,p] * B[j,i]
      end for
    end for
  end for
end for
return C
```

Checkout the code: [Click here](#)

0.3 Einstein Product Identity

Suppose two tensor A and B such that :

$$A \in \mathbf{R}^{(J_1, J_2, J_3, J_4)} \quad B \in \mathbf{R}^{(J_3, J_4, J_5, J_6)}$$

Einstein Product:

As per **Einstein Product**,

$$A *_2 B \in \mathbf{R}^{(J_1, J_2, J_5, J_6)}$$

How to calculate : Tensor A last two index 5,6 should be same as Tensor B first two index to compute $A *_2 B$ of 3rd order tensors. Einstein Product can be computed by below formula where these two indexes will merge to create size of output tensor as $4*6$.

$$c_{k,l,m,n} = \sum_{i=0}^{J_3} \sum_{j=0}^{J_4} a_{klij} b_{ijmn}$$

Algorithm 4 Einstein Product Tensor Computation

```
1: Initialize  $C[k, l, m, n]$  with all elements as 0
2: for  $k = 1$  to 4 do
3:   for  $l = 1$  to 6 do
4:     for  $m = 1$  to 6 do
5:       for  $n = 1$  to 6 do
6:         for  $i = 1$  to 5 do
7:           for  $j = 1$  to 6 do
8:              $C[k, l, m, n] = A[k, l, i, j] \times B[i, j, m, n]$ 
9:           end for
10:        end for
11:       end for
12:      end for
13:    end for
14:  end for
15: return C
```

Given

$$A \in \mathbf{R}^{(J_1, J_2, J_3, J_4)} \quad X \in \mathbf{R}^{(J_3, J_4, J_5, J_6)} \quad Y \in \mathbf{R}^{(J_3, J_4, J_5, J_6)}$$

Evaluate :

$$A * (X + Y) = A * X + A * Y$$

LHS : $A * (X + Y)$, First add $X + Y$ which will be $\in \mathbf{R}^{J_3, J_4, J_5, J_6}$ element to element addition using for loop . Then after using above Algorithm-4 multiply by tensor $A * (X+Y) \in \mathbf{R}^{J_1, J_2, J_5, J_6}$.

RHS : using above Algorithm-4 find $A * X \in \mathbf{R}^{(J_1, J_2, J_5, J_6)}$ same for $A * Y \in \mathbf{R}^{(J_1, J_2, J_5, J_6)}$. Now you have two tensor of same size (J_1, J_2, J_5, J_6) and you can find element wise addition using for loop and you will get $(A*X)+(A*Y) \in \mathbf{R}^{J_1, J_2, J_5, J_6}$.

Checkout the code: [Click here](#)

0.4 T-Product to check Symmetric ,Diagonal and identity

Symmetric:

Algorithm 5 Find a tensor is Symmetric or not using T-Product

```
A = FFT(A,axis=2)
S=Initialise a tesor of similar order with all element as 0 with complex dtype
for  $i = 1$  to 3 do
    S[:, :, i]=Transpose of A[:, :, i]
end for
S=iFFT(S,axis=2)
if A equal to S then
    A is Symmetric
else
    A is not Symmetric
end if
```

Diagonal:

Algorithm 6 Find a tensor D is Diagonal or not using T-Product

```
A= Initialise any tesor of similar order
Multiply  $A * D$  using Algorithm-2
Multiply  $D * A$  using Algorithm-2
if  $A * D$  equal to  $D * A$  then
    D is Diagonal
else
    D is not Diagonal
end if
```

identity:

Algorithm 7 Find a tensor I is Indentity or not using T-Product

```
A= Initialise any tesor of similar order
Multiply  $A * I$  using Algorithm-2
Multiply  $I * A$  using Algorithm-2
if  $A * I$  equal to  $A$  then
    I is Identity
else
    I is not Identity
end if
```

Checkout the code: [Click here](#)