Tensor computation Assignment Set1 Solution

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0.1 Multiply two Tensor:

Suppose two tensor A and B such that :

$$A \in \mathbf{R}^{(4,5,6)} \quad B \in \mathbf{R}^{(5,6,6)}$$

Einstein Product:

As per **Einstein Product**, $A *_2 B \in \mathbf{R}^{(4,6)}$ How to calculate : Tensor A last two index 5,6 should be same as Tensor B first two index to compute $A *_2 B$ of 3rd order tensors.Einstein Product can be computed by below formula where these two indexs will merge to create size of output tensor as 4*6 .

$$c_{k,l} = \sum_{i=0}^{5} \sum_{j=0}^{6} a_{kij}\, b_{ijl}$$

Algorithm 1 Einstein Product Tensor Computation

Initialize $C[k,l]$ with all elements as 0
for $k = 1$ to 4 **do**
  for $l = 1$ to 6 **do**
    for $i = 1$ to 5 **do**
      for $j = 1$ to 6 **do**
        $C[k,l] = A[k,i,j] \times B[i,j,l]$
      end for
    end for
  end for
end for
return C

Checkout the code: Click here

T-Product:

As per **T-Product**, for same $A \in \mathbf{R}^{(4,5,6)}$ *and* $B \in \mathbf{R}^{(5,6,6)}$

$$A * B \in \mathbf{R}^{(4,6,6)}$$

Use FFT to transform A from tensor domain to Fourier domain :

$$A_{FFT} = (F_6 \bigotimes I_4)\ bcirc(A)(F_6 \bigotimes I_5)$$

Here F_6 is a $6*6$ normalize DFT matrix defined as : $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$

Algorithm 2 T-Product Tensor Computation

Set $A = FFT(A, axis = 2)$
Set $B = FFT(B,\ axis = 2)$
Initialize C with all element as 0
for $i = 1$ to 6 **do**
  C[:,:,i]=A[:,:,i]*B[:,:,i]
end for
Set $C = iFFT(C, axis = 2\ )$
return C

Checkout the code: Click here

0.2 2-mode Product of a Tensor

Suppose a tensor A and matrix B such that :

$$A \in \mathbf{R}^{(J_1,J_2,J_3)} \quad B \in \mathbf{R}^{(I,J_2)}$$

2-mode Product:

As per **2-mode Product**,

$$A *_2 B \in \mathbf{R}^{(J_1,I,J_3)}$$

How to calculate : 2-mode product of tensor A can be computed with a matrix as below formula, when tensor second index will match with matrix last index . In this case tensor second index J_2 matchs matrix last index J_2.

$$c_{k,j,p} = \sum_{i=0}^{J_2} a_{kip}\, b_{j,i}$$

Algorithm 3 2-mode Product Tensor Computation

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Initialize C of size (J1, I, J3) with all Element 0
for k = 0 to J1 do
  for j = 0 to I do
    for p = 0 to J3 do
      for i = 0 to J2 do
        C[k,j,p]=A[k,i,p]*B[j,i]
      end for
    end for
  end for
end for
return C
```

Checkout the code: Click here

0.3 Einstein Product Identity

Suppose two tensor A and B such that :

$$A \in \mathbf{R}^{(J_1,J_2,J_3,J_4)} \quad B \in \mathbf{R}^{(J_3,J_4,J_5,J_6)}$$

Einstein Product:

As per **Einstein Product**,

$$A *_2 B \in \mathbf{R}^{(J_1,J_2,J_5,J_6)}$$

How to calculate : Tensor A last two index 5,6 should be same as Tensor B first two index to compute $A *_2 B$ of 3rd order tensors.Einstein Product can be computed by below formula where these two indexs will merge to create size of output tensor as 4*6 .

$$c_{k,l,m,n} = \sum_{i=0}^{J_3} \sum_{j=0}^{J_4} a_{klij}\, b_{ijmn}$$

Algorithm 4 Einstein Product Tensor Computation

```
Initialize C[k, l, m, n] with all elements as 0
for k = 1 to 4 do
  for l = 1 to 6 do
    for m = 1 to 6 do
      for n = 1 to 6 do
        for i = 1 to 5 do
          for j = 1 to 6 do
            C[k, l, m, n] = A[k, l, i, j] × B[i, j, m, n]
          end for
        end for
      end for
    end for
  end for
end for
return C
```

Given

$$A \in \mathbf{R}^{(J_1,J_2,J_3,J_4)} \quad X \in \mathbf{R}^{(J_3,J_4,J_5,J_6)} \quad Y \in \mathbf{R}^{(J_3,J_4,J_5,J_6)}$$

Evaluate :

$$A * (X + Y) = A * X + A * Y$$

LHS : $A * (X + Y)$, First add $X + Y$ which will be $\in \mathbf{R}^{J_3,J_4,J_5,J_6}$ element to element addition uing for loop . Then after using above Algorithm-4 multiply by tensor A* (X+Y) $\in \mathbf{R}^{J_1,J_2,J_5,J_6}$.

RHS : using above Algorithm-4 find A * X $\in \mathbf{R}^(J_1, J_2, J_5, J_6)$ same for A* Y $\in \mathbf{R}^(J_1, J_2, J_5, J_6)$. Now you have two tensor of same size (J_1, J_2, J_5, J_6) and you can find element wise addition uing for loop and you will get (A*X)+(A*Y) $\in \mathbf{R}^{J_1,J_2,J_5,J_6}$.

Checkout the code: Click here

0.4 T-Product to check Symmetric ,Diagonal and identity

Symmetric:

Any Tensor A $\in \mathbf{R}^{(n,m,p)}$ is Symmetric if $A^T \in \mathbf{R}^{m,n,p}$

$$A = A^T$$

Algorithm 5 Find a tensor is Symmetric or not using T-Product

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A = FFT(A,axis=2)
S=Initialise a tesor of similar order with all element as 0 with complex dtype
for $i = 1$ to 3 **do**
  S[:,:,i]=Transpose of A[:,:,i]
end for
S=iFFT(S,axis=2)
if A equal to S **then**
  A is Symmetric
else
  A is not Symmetric
end if

Diagonal:

Any Tensor D $\in \mathbf{R}^{(n,n,p)}$ is Diagonal when forall $A \in \mathbf{R}^{n,n,p}$

$$D * A = A * D$$

Algorithm 6 Find a tensor D is Diagonal or not using T-Product

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A= Initialise any tesor of similar order
Multiply $A * D$ using Algorithm-2
Multiply $D * A$ using Algorithm-2
if $A * D$ equal to $D * A$ **then**
  D is Diagonal
else
  D is not Diagonal
end if

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identity:

Any Tensor I $\in \mathbf{R}^{(n,n,p)}$ is identity when forall $A \in \mathbf{R}^{n,n,p}$

$$A * I = I * A = A$$

Checkout the code: Click here

Algorithm 7 Find a tensor I is Indentity or not using T-Product

A= Initialise any tesor of similar order
Multiply $A * I$ using Algorithm-2
Multiply $I * A$ using Algorithm-2
if $A * I$ equal to A **then**
  I is Identity
else
  I is not Identity
end if