

BOOK OF PROOF EXERCISES

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Purpose

I am studying the BOOK OF PROOF as preparation for the mathematical portion of Machine Learning studies that I have based on the Carnegie Mellon Machine Learning masters program. Specifically, this is part of my effort to prepare for studying materials covered in the '36-700 Probability & Mathematical Statistics or 36-705 Intermediate Statistics' course requirement.

After completing this survey in forming mathematical proofs, I will begin studying probability, mathematical statistics and calculus as they comprise the bulk of the mathematical underpinnings of machine learning.

Chapter 1

Sets

1.1 Exercises

A. Write each of the following sets by listing their elements between braces.

1. $\{5x - 1 : x \in \mathbb{Z}\} = \{\dots, -16, -11, -6, -1, 4, 9, 14, \dots\}$
2. $\{3x + 2 : x \in \mathbb{Z}\} = \{\dots, -7, -4, -1, 2, 5, 8, 11, \dots\}$
3. $\{x \in \mathbb{Z} : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
4. $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6\}$
5. $\{x \in \mathbb{R} : x^2 = 3\} = \{-\sqrt{3}, \sqrt{3}\}$
6. $\{x \in \mathbb{R} : x^2 = 9\} = \{-\sqrt{9}, \sqrt{9}\}$
7. $\{x \in \mathbb{R} : x^2 + 5x = -6\} = \{-2\}$
8. $\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\} = \{-3, -2, 0\}$
9. $\{x \in \mathbb{R} : \sin \pi x = 0\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
10. $\{x \in \mathbb{R} : \cos x = 1\} = \{\dots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \dots\}$
11. $\{x \in \mathbb{Z} : |x| < 5\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
12. $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$
13. $\{x \in \mathbb{Z} : |6x| < 5\} = \{0\}$
14. $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$
15. $\{5a + 2b : a, b \in \mathbb{Z}\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
16. $\{6a + 2b : a, b \in \mathbb{Z}\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

B. Write each of the following sets in set builder notation.

17. $\{2, 4, 8, 16, 32, 64, \dots\} = \{2x : x \in \mathbb{N}\}$
18. $\{0, 4, 16, 36, 64, 100, \dots\} = \{x^2 : x \in 2\mathbb{Z}, x \geq 0\}$
19. $\{\dots, -6, -3, 0, 3, 6, 9, 12, 15, \dots\} = \{3x : x \in \mathbb{Z}\}$
20. $\{\dots, -8, -3, 2, 7, 12, 17, \dots\} = \{5x + 2 : x \in 2\mathbb{Z}\}$
21. $\{0, 1, 4, 9, 16, 25, 36, \dots\} = \{x^2 : x \in 2\mathbb{Z}, x \geq 0\}$
22. $\{3, 6, 11, 18, 27, 38, \dots\} = \{x^2 + 2 : x \in 2\mathbb{Z}, x > 0\}$
23. $\{3, 4, 5, 6, 7, 8\} = \{x \in 2\mathbb{Z} : 3 \leq x \leq 8\}$
24. $\{-4, -3, -2, -1, 0, 1, 2\} = \{x \in 2\mathbb{Z} : -4 \leq x \leq 2\}$
25. $\{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \dots\} = \{2^x : x \in \mathbb{Z}\}$
26. $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\} = \{3^x : x \in \mathbb{Z}\}$
27. $\{\dots, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{2}{\pi}, \frac{5\pi}{2}, \dots\} = \{\frac{x\pi}{2} : x \in \mathbb{Z}\}$
28. $\{\dots, -\frac{3}{2}, -\frac{3}{4}, 0, \frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3, \frac{15}{4}, \frac{9}{2}, \dots\} = \{\frac{3x}{4} : x \in \mathbb{Z}\}$

C. Find the following cardinalities.

29. $|\{\{1\}, \{2, \{3, 4\}\}, \emptyset\}| = 3$
30. $|\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}| = 5$
31. $|\{\{\{1\}, \{2, \{3, 4\}\}, \emptyset\}\}| = 1$
32. $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}\}| = 1$
33. $|\{x \in \mathbb{Z} : |x| < 10\}| = 19$
34. $|\{x \in \mathbb{N} : |x| < 10\}| = 9$
35. $|\{x \in \mathbb{Z} : x^2 < 10\}| = 7$
36. $|\{x \in \mathbb{N} : x^2 < 10\}| = 3$
37. $|\{x \in \mathbb{N} : x^2 < 0\}| = 0, \emptyset$
38. $|\{x \in \mathbb{N} : 5x \leq 20\}| = 4$