Signals and Systems
Homework One

Due : Monday, March 29

1. Sums and Integrals of complex exponential

   (a) Find the sum
   $$\sum_{n=0}^{N-1} e^{j2\pi n/N}$$
   and explain your answer geometrically. $(N \in \mathbf{N}, N > 1)$

   (b) Derive the formula
   $$\sum_{k=-N}^{N} e^{j2\pi kt} = \frac{\sin(2\pi t(N+1/2))}{\sin(\pi t)}.$$

   (c) If n and m are integers, what is the result of the following integral?
   $$\int_0^T e^{j2\pi(n-m)\frac{1}{T}t} dt$$
   (Hint: Handle the cases for $n = m$ and $n \neq m$ separately.)

   (d) Derive the formula
   $$\int_{-T_1}^{T_1} e^{-j2\pi ft} dt = \frac{\sin(2\pi f T_1)}{\pi f}.$$

2. Convolution is my friend I: Calculating convolution sum directly

   (a) Let
   $$\Pi_a(t) = \begin{cases} 1, & \text{if } |t| < \frac{a}{2} \\ 0, & \text{otherwise.} \end{cases}$$
   What is $\Pi_a * \Pi_a$?

   (b) Let $f(x) = e^{-|x|}, -\infty < x < \infty$. Find $(f * f)(x)$.

   (c) Let $g(x) = e^{-\pi x^2}, -\infty < x < \infty$. Show that $(g * g)(x) = \frac{1}{\sqrt{2}} e^{-\pi x^2/2}$.

   (d) From the result in part (c), deduce the result of the n-fold convolution of g, i.e., $g * g * \cdots * g$ (with n factors of g).

$f * g(-t)$ 랑 $f(-t) * g$ 관계 보이는거임

3. Convolution is my friend II: Time reversals, shifts, and stretches
   Let $f(t)$ and $g(t)$ be signals.

   (a) If both $f(t)$ and $g(t)$ are reversed, what happens to their convolution? If one of $f(t)$ and $g(t)$ is reversed, what happens to their convolution?

   (b) Let's define the shift operator τ_b by
   $$\tau_b f(t) = f(t-b).$$
   Show that
   $$(\tau_b f) * g = \tau_b(f * g) = f * (\tau_b g).$$
   Use this result to deduce that if either f or g is periodic of period T then $f * g$ is periodic of period T.

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   (c) Let's define the stretch operator σ_a by
   $$\sigma_a f(t) = f(at).$$
   Show that
   $$(\sigma_a f) * g = \frac{1}{|a|}\sigma_a(f * (\sigma_{1/a} g)), \quad (\sigma_a f) * (\sigma_a g) = \frac{1}{|a|}\sigma_a(f * g).$$

4. MATLAB plotting, Convolution sum with MATLAB

   (a) Consider the signal
   $$x_k[n] = \sin(2\pi f_k n),$$
   where $f_k = k/5$. For $x_k[n]$ given gy $k = 1, 2, 4$ and 6, use `stem` to plot each signal on the interval $0 \leq n \leq 9$. All of the signals should be plotted with separate axes in the same figure using `subplot`. How many unique signals have you plotted? If two signals are identical, explain how different values of f_k can yield the same signal.

   (b) For the sequences $h[n] = 2\delta[n+1] - 2\delta[n-1]$, and $x[n] = \delta[n] + \delta[n-2]$ construct vectors `h` and `x`. Construct time indices in the vectors `nh` and `nx`. Plot them using `stem`.

   (c) Since the MATLAB function `conv` does not keep track of the time indices of the sequences that are convolved, you will have to do som extra bookkeeping in order to determine the proper indices for the result of the `conv` function. Define $y[n] = x[n] * h[n]$ and compute `y=conv(h,x)`. Determine the proper time indexing for `y` and store this set of time indices in the vector `ny`. Plot $y[n]$ as a function of n using `stem(ny, y)`.