

ML Homework1

Description :

Part 1

- Please write down the mathematical derivation of the following methods:
 1. Closed-form LSE approach
 2. Steepest descent method
 3. Newton's method
- NOTE :

During the demo, we will ask you about which parts of the code correspond to the sections in the mathematical derivations.

Part2

Please write a program that can do **regularized linear model regression** (polynomial basis) and **visualization**.

You should do it by **closed-form LSE approach**, **Steepest descent**, and **Newton's method**.

- Input parameters:

1. The path and name of a file which consists of data points (comma separated: x,y):

```
1,12
122,34
-12,323
...
```

2. The number of polynomial bases n.

$$\phi_0(x) = x^0, \phi_1(x) = x^1, \phi_2(x) = x^2, \dots, \phi_{n-1}(x) = x^{n-1},$$

3. **lambda** λ (only for LSE case)

- Program Behavior: For example, if the number of bases is set to 3, it means that the program is going to find a curve that best fits the data points by $ax^2 + bx^1 + cx^0 = y$

- Required functions:

- a. For closed-form LSE approach:

1. Use **LU decomposition** to find the inverse of $(A^T A + \lambda I)$, **Gauss-Jordan elimination** will also be accepted. (A is the design matrix).
2. **Print out the equation of the best fitting line and the error.**

- b. For steepest descent method:

1. Use **steepest descent with LSE and L1 norm** to find the best fitting line.
2. Print out the equation of the best fitting line and the error.
3. **reference** (hint : Consider using a **smaller learning rate**.)

- c. For Newton's method:

1. Please use the method mentioned in the lesson.
2. Print out the equation of the best fitting line and the error, and **compare to LSE**.

- d. For visualization:

1. Please **visualize the data points** which are the input of program, **and the best fitting curve**.
2. It's free to use any existing package.

- NOTE:

- Use whatever programming language you prefer.
- You should use **as few functions from any library as possible**. That would be great if you **implement all detail operations (like matrix operations)** by yourself.
- Time complexity is not what we care for now, but if you like to improve it in that regard,

it is always good for you.

- Grading policy
- You **must implement matrix inverse operation by yourself**. Please do not use the built in package or you'll not get 100.
- Sample input & output (for reference only, please **note that it doesn't include results from the steepest descent method**)
 - Input: A file (here shows the content of the file)

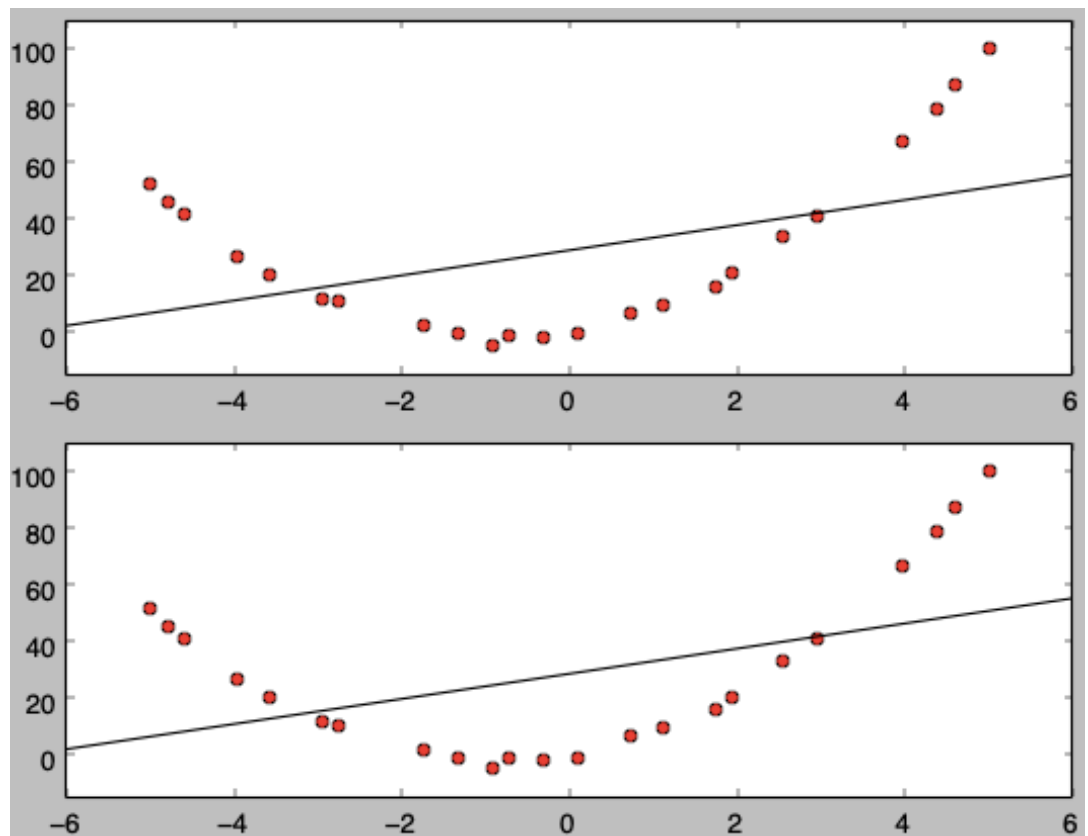
```
$ cat testfile.txt
-5.0,51.76405234596766
-4.795918367346939,45.42306433039972
-4.591836734693878,41.274448104888755
-3.979591836734694,26.636216497466364
-3.571428571428571,20.256806057008426
-2.9591836734693877,11.618429243797276
-2.7551020408163263,10.450525068812203
-1.7346938775510203,1.8480982318414874
-1.3265306122448979,-1.0405349639051173
-0.9183673469387754,-4.614630798757861
-0.7142857142857144,-1.3871977310902517
-0.3061224489795915,-1.9916444039966117
0.1020408163265305,-0.912924608376358
0.7142857142857144,6.63482003068499
1.1224489795918373,9.546867459016372
1.7346938775510203,15.72016146597016
1.9387755102040813,20.62251683859554
2.5510204081632653,33.48059725819715
2.959183673469388,40.76391965675495
3.979591836734695,66.8997605629381
4.387755102040817,78.44316465660981
4.591836734693878,86.99156782355371
5.0,99.78725971978604
```

- Output

- Case 1: $n = 2$, $\lambda = 0$

```
LSE:
Fitting line: 4.43295031008X^1 + 29.3064047061
Total error: 16335.123165

Newton's Method:
Fitting line: 4.43295031008X^1 + 29.3064047061
Total error: 16335.123165
```



- Case 2: $n = 3$, $\lambda = 0$

LSE:

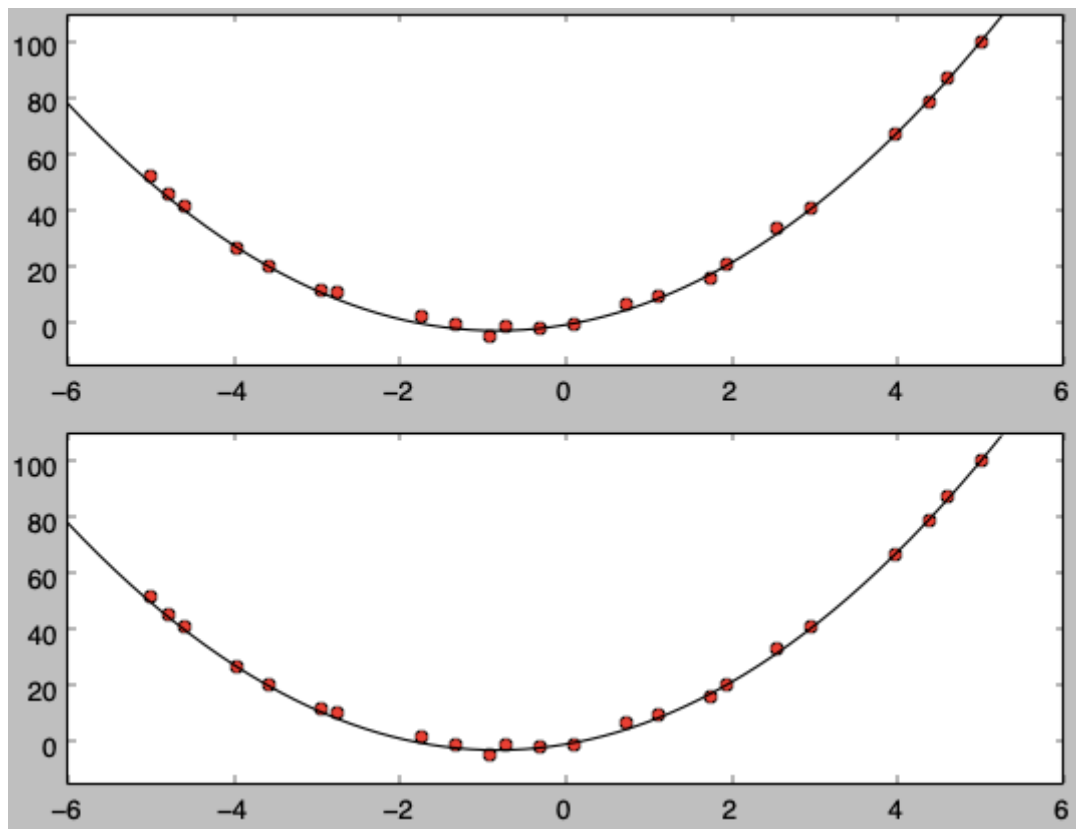
Fitting line: $3.02385339349X^2 + 4.90619026386X^1 - 0.231401756088$

Total error: 26.5599594993

Newton's Method:

Fitting line: $3.02385339349X^2 + 4.90619026386X^1 - 0.231401756088$

Total error: 26.5599594993



- Case 3: $n = 3$, $\lambda = 10000$

LSE:

Fitting line: $0.8345332827X^2 + 0.0931481983192X^1$
 $+ 0.0469506992735$

Total error: 22649.738493

Newton's Method:

Fitting line: $3.02385339349X^2 + 4.90619026386X^1$
 -0.231401756088

Total error: 26.5599594993

