

# ML Homework4

## Description :

### 1. Logistic regression

- Input:

1.  $N$  (number of data points)
2.  $mx_1, vx_1, my_1, vy_1, mx_2, vx_2, my_2, vy_2$  ( $m$ : mean,  $v$  variance)

- Function:

1. Generate  $n$  data point:  $D1 = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , where  $x$  and  $y$  are independently sampled from  $N(mx_1, vx_1)$  and  $N(my_1, vy_1)$  respectively.
2. Generate  $n$  data point:  $D2 = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , where  $x$  and  $y$  are independently sampled from  $N(mx_2, vx_2)$  and  $N(my_2, vy_2)$  respectively.
3. Use **Logistic regression** to separate  $D1$  and  $D2$ . You should implement both **Newton's** and **steepest gradient descent method during optimization**.
  - In other words, **when the Hessian is singular, use steepest descent for instead**. You should come up with a reasonable rule to determine convergence.(a simple run out of the loop should be used as the ultimatum)

- Output:

1. The **confusion matrix and the sensitivity and specificity** of the logistic regression applied to the training data  $D$ .
2. Visualization
  - Plot the **ground truth**
  - Plot the **predict result**
    - Gradient descent
    - Newton's method

**Use the Gaussian random number generator in homework 3.**

- Sample input & output (for reference only)

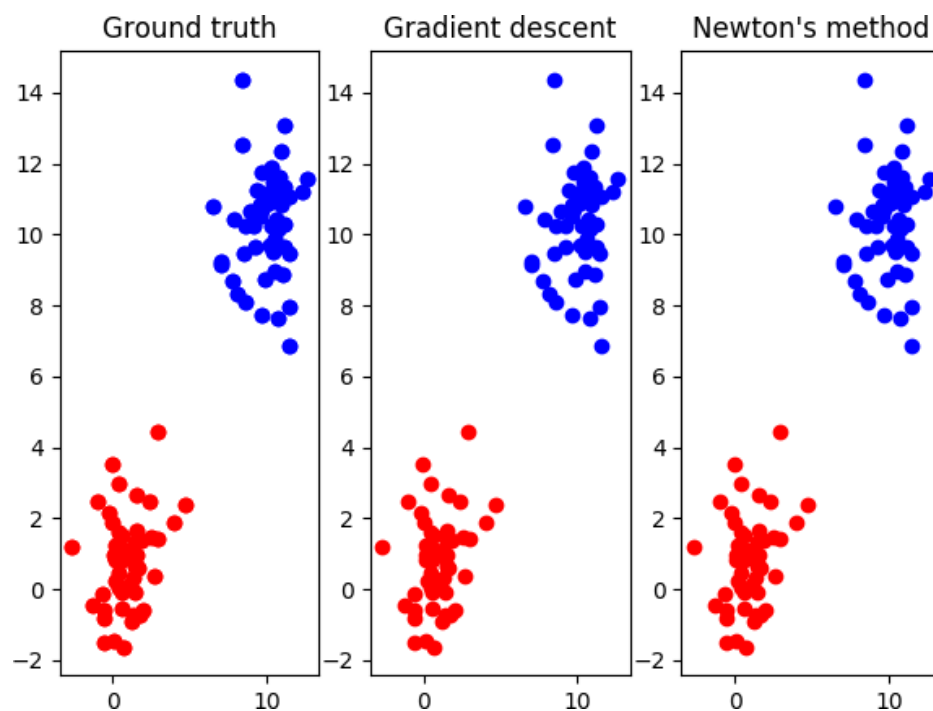
- Case 1:  $N = 50, mx_1 = my_1 = 1, mx_2 = my_2 = 10, vx_1 = vy_1 = vx_2 = vy_2 = 2$

```
1 Gradient descent:
2
3 w:
```

```

4  -78.1766393662
5    6.7233419236
6   11.2430677919
7
8  Confusion Matrix:
9          Predict cluster 1 Predict cluster 2
10 Is cluster 1          50          0
11 Is cluster 2          0          50
12
13 Sensitivity (Successfully predict cluster 1): 1.00000
14 Specificity (Successfully predict cluster 2): 1.00000
15
16 -----
17 Newton's method:
18
19 w:
20 -118.3601516394
21   8.7747332848
22  10.1954120077
23
24 Confusion Matrix:
25          Predict cluster 1 Predict cluster 2
26 Is cluster 1          50          0
27 Is cluster 2          0          50
28
29 Sensitivity (Successfully predict cluster 1): 1.00000
30 Specificity (Successfully predict cluster 2): 1.00000

```

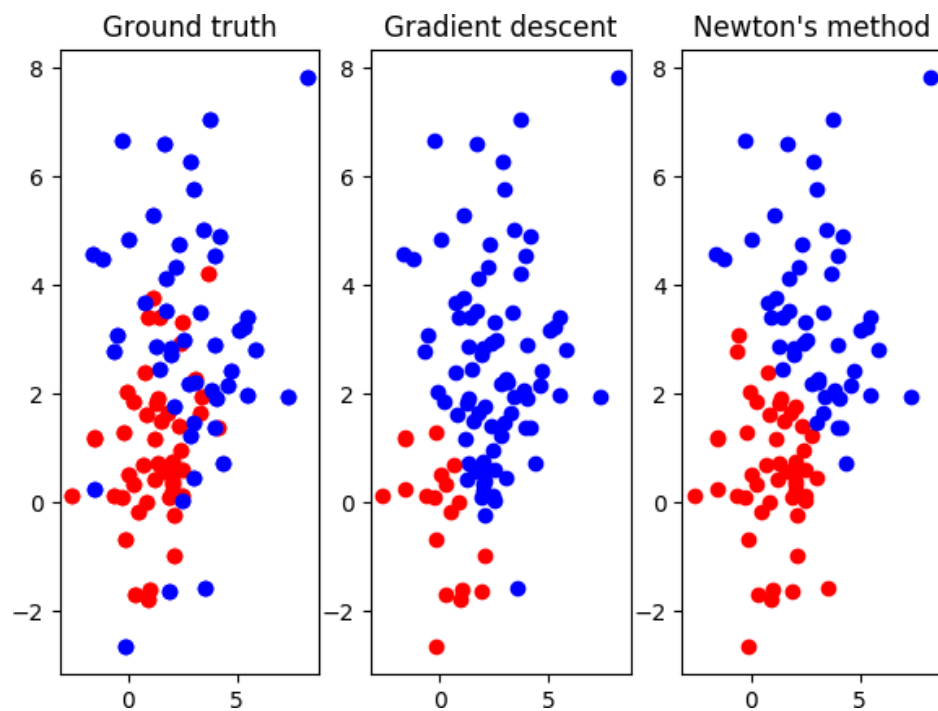


- Case 2:  $N = 50, m_{x_1} = m_{y_1} = 1, m_{x_2} = m_{y_2} = 3, v_{x_1} = v_{y_1} = 2, v_{x_2} = v_{y_2} = 4$

```

1 Gradient descent:
2
3 w:
4 -71.1902536008
5 46.0123814025
6 54.6803199701
7
8 Confusion Matrix:
9 Predict cluster 1 Predict cluster 2
10 Is cluster 1 16 34
11 Is cluster 2 3 47
12
13 Sensitivity (Successfully predict cluster 1): 0.32000
14 Specificity (Successfully predict cluster 2): 0.94000
15
16 -----
17 Newton's method:
18
19 w:
20 -1.9045831451
21 0.3940876974
22 0.5695243849
23
24 Confusion Matrix:
25 Predict cluster 1 Predict cluster 2
26 Is cluster 1 40 10
27 Is cluster 2 10 40
28
29 Sensitivity (Successfully predict cluster 1): 0.80000
30 Specificity (Successfully predict cluster 2): 0.80000

```



## 2. EM algorithm

- Input: [MNIST training](#) data and label sets. (Same as HW02)
- Function:
  1. Binning the gray level value into **two bins**. Treating all pixels as random variables following Bernoulli distributions. Note that **each pixel follows a different Binomial distribution independent to others**.
  2. Use EM algorithm to cluster each image into **ten groups**. You should come up with a reasonable rule to determine convergence. (a simple run out of the loop should be used as the ultimatum)
- Output:
  1. For each digit, output a **confusion matrix and the sensitivity and specificity** of the clustering applied to the training data.
  2. Print out the **imagination of numbers** in your classifier
    - Just like before, about the details please refer to HW02
- Hint: The algorithm is a kind of unsupervised learning, so the labels are not used during training. But you can **use these labels to help you to figure out which class belongs to which number**.

In other words, you should **find a way to assign label to each class** which you classified **before you compute the confusion matrix**
- Sample input & output (**for reference only**)

```
1 class 0:  
2   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
3   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

[illegible]

```

53      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
54      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
55      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
56      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
57      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
58      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
59      0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
60
61 ... all other unlabeled imagination of numbers goes here ..
62
63 class 9:
64     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
65     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
66     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
67     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
68     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0
69     0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
70     0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
71     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
72     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
73     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
74     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0
75     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0
76     0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
77     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
78     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
79     0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
80     0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
81     0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0
82     0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
83     0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
84     0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0
85     0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0
86     0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0
87     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
88     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
89     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
90     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
91     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
92 .
93 No. of Iteration: 1, Difference: 3176.579389514846
94
95 -----
96
97 class 0:
98
99     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
100    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
101    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

[illegible]

```
151 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0
152 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
153 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
154 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
155 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
156 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
157 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

158

```
159 No. of Iteration: 10, Difference: 19.89546432548733
```

160

```
161 -----
```

```
162 -----
```

163

```
164 labeled class 0:
```

```
165 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
166 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
167 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
168 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
169 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
170 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0
171 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
172 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
173 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
174 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
175 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
176 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
177 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
178 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
179 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
180 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
181 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
182 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
183 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
184 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0
185 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0
186 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
187 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0
188 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
189 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
190 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
191 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
192 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

193

```
194 labeled class 1:
```

```
195 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
196 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
197 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
198 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
199 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```



```
200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
201 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
202 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0
203 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
204 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
205 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
206 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
207 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0
208 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
209 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
210 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
211 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
212 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
213 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
214 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
215 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
216 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0
217 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
218 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
219 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
220 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
221 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
222 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
223
224 ... all other labeled imagination of numbers goes here ...
225
226 labeled class 9:
227 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
228 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
229 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
230 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
231 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
232 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
233 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
234 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
235 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
236 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 0 0 0 0 0 0
237 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0
238 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
239 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
240 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
241 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0
242 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
243 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
244 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0
245 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
246 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
247 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
248 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
```

```

249 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
250 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
251 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
252 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
253 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
254 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
255
256
257
258 Confusion Matrix 0:
259         Predict number 0 Predict not number 0
260 Is number 0          3023          2900
261 Isn't number 0       113          53964
262
263 Sensitivity (Successfully predict number 0)      : 0.51038
264 Specificity (Successfully predict not number 0): 0.99791
265
266 -----
267
268 Confusion Matrix 1:
269         Predict number 1 Predict not number 1
270 Is number 1          5986          756
271 Isn't number 1       800          52458
272
273 Sensitivity (Successfully predict number 1)      : 0.88787
274 Specificity (Successfully predict not number 1): 0.98498
275
276 -----
277
278 ... all other confusion matrix goes here ...
279
280 -----
281
282 Confusion Matrix 9:
283         Predict number 9 Predict not number 9
284 Is number 9          2718          3231
285 Isn't number 9       5147          48904
286
287 Sensitivity (Successfully predict number 9)      : 0.45688
288 Specificity (Successfully predict not number 9): 0.90478
289
290 Total iteration to converge: 10
291 Total error rate: 0.5081666666666667

```

### 3. EM algorithm (Handwritten)

Given two coins ( $C_0, C_1$ ) and in each trial, a certain coin is chosen and tossed for three times. There are totally three trial outcomes: {HHH, HHT, TTT}. Note that the chance of choosing  $C_0$  is  $k$ , the chance of  $C_0$  showing H is  $P_0$ , and the chance of  $C_1$  showing H is  $P_1$ . Use EM algorithm to update the estimates of the parameters for just one round. You should show the process step by step with the initial values of parameters:  $k = 0.5, P_0 = 0.6, P_1 = 0.1$

- Note:
  1. During the demo, you will be required to explain how you arrived at your calculations.
  2. Upload the handwritten file to e3 (it can be in .pdf or any image format).