# ML Homework 3

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# Description:

## 1. Random Data Generator

- a. Univariate gaussian data generator
  - Input
    - Expectation value or mean: $m$
    - Variance: $s$
  - Output: A data point from $N(m, s)$
  - HINT
    - **Generating values from normal distribution**
    - You have to handcraft your geneartor based on one of the approaches given in the hyperlink.
    - You can use uniform distribution function (Numpy)
- b. Polynomial basis linear model data generator
  - $y = W^T \phi(x) + e$
    - $W$ is a $n \times 1$ vector
    - $e \sim N(0, a)$
  - Input: $n$ (basis number), $a, w$
    - e.g. $n = 2 \rightarrow y = w_0 x^0 + w_1 x^1$,
  - Output: $x$ $y$ (a point)
  - Internal constraint
    - $-1.0 < x < 1.0$
    - $x$ is uniformly distributed.

## 2. Sequential Estimator

- Sequential estimate the mean and variance
  - Data is given from the univariate gaussian data generator (1.a).
- Input: $m, s$ as in (1.a)
- Function:
  - Call (1.a) to get a new data point from $N(m, s)$
  - Use sequential estimation to find the current estimates to $m$ and $s$
  - Repeat steps above until the estimates converge.
- Output: Print the new data point and the current estimiates of $m$ and $s$ in each iteration.
- Notes

    - You should derive the recursive function of mean and variance based on the sequential esitmation.
    - Hint: [Online algorithm]
- Sample input & output (⚠️ ***for reference only*** ⚠️)

```
Data point source function: N(3.0, 5.0)

Add data point: 1.220492527761238
Mean = 1.220492527761238    Variance = 0.0
Add data point: 3.6967805272943366
Mean = 2.458636527527787    Variance = 1.53300056415791
Add data point: 2.7258100985704146
Mean = 2.5476943845419964  Variance = 1.0378629798971994
Add data point: 2.2138523069477527
Mean = 2.4642338651434352  Variance = 0.7992942098177336
Add data point: 2.2113035958584453
Mean = 2.4136478112864372  Variance = 0.6496711632334788
Add data point: 0.05399706095719692
Mean = 2.020372686231564    Variance = 1.3147192559625305
Add data point: 4.3538771826058
Mean = 2.3537304714278835  Variance = 1.7936666971024264

...

Add data point: 4.233592159021013
Mean = 2.961576104513964    Variance = 5.045715437349161
Add data point: 3.529990930040463
Mean = 2.961883688294010    Variance = 5.043159812425648
Add data point: 1.125210345431449
Mean = 2.960890354955524    Variance = 5.042255747918937
```

## 3. Derive the Posterior

- Derive the posterior mean and variance for prior given by $w \sim N(0, b^{-1}I)$

NOTE

- During the demo, you will be required to explain the entire mathematical proof.
- Upload the handwritten file to e3 (it can be in .pdf or any image format).
- This part may help you to solve question 4.

## 4. Baysian Linear regression

- Input
  - The precision (i.e., b) for initial prior $w \sim N(0, b^{-1}I)$
  - All other required inputs for the polynomial basis linear model geneartor (1.b)
- Function
  - Call (1.b) to generate one data point
  - Update the prior, and calculate the parameters of predictive distribution
  - Repeat steps above until the posterior probability converges.
- Output
  - Print the new data point and the current paramters for posterior and predictive distribution.

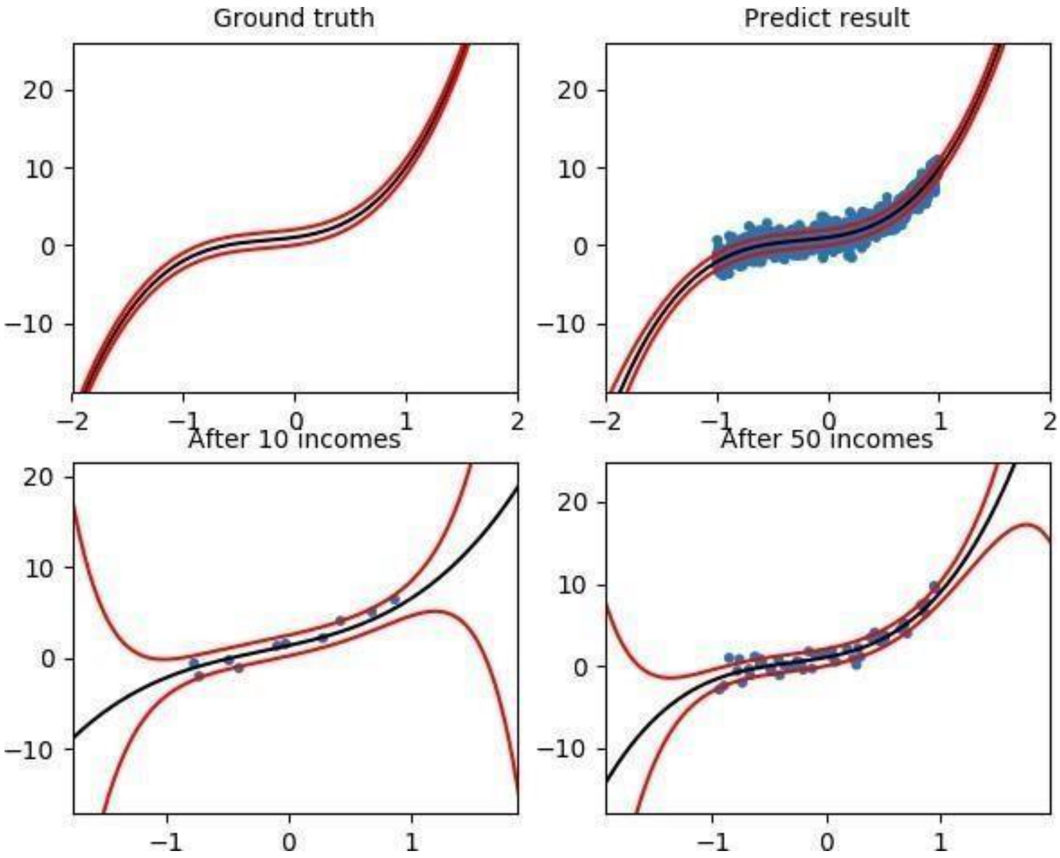
    - After probability converged, do the visualization
        - Ground truth function (from linear model generator)
        - Final predict result
        - At the time that have seen 10 data points
        - At the time that have seen 50 data points
        - Note
            - Except ground truth, you have to draw those data points which you have seen before
            - Draw a black line to represent the mean of function at each point
            - Draw two red lines to represent the variance of function at each point
                - In other words, distance between red line and mean is **ONE** variance
- Hint: Online learning
- Sample input & output (***for reference only***)

1. b = 1, n = 4, a = 1, w = [1, 2, 3, 4]

```
Add data point (-0.64152, 0.19039):

 Postirior mean:
    0.0718294547
   -0.0460797888
    0.0295609502
   -0.0189638408

 Posterior variance:
    0.6227289276,   0.2420256620,  -0.1552634839,   0.0996041049
    0.2420256620,   0.8447365161,   0.0996041049,  -0.0638976884
   -0.1552634839,   0.0996041049,   0.9361023116,   0.0409914289
    0.0996041049,  -0.0638976884,   0.0409914289,   0.9737033172

Predictive distribution ~ N(0.00000, 2.65061)

Add data point (0.07122, 1.63175):

 Postirior mean:
    0.6736864869
    0.2388980107
   -0.1054659080
    0.0710615952

 Posterior variance:
    0.3765992302,   0.1254838660,  -0.1000441911,   0.0627881634
    0.1254838660,   0.7895542671,   0.1257503020,  -0.0813299447
   -0.1000441911,   0.1257503020,   0.9237138418,   0.0492510997
    0.0627881634,  -0.0813299447,   0.0492510997,   0.9681964094
```

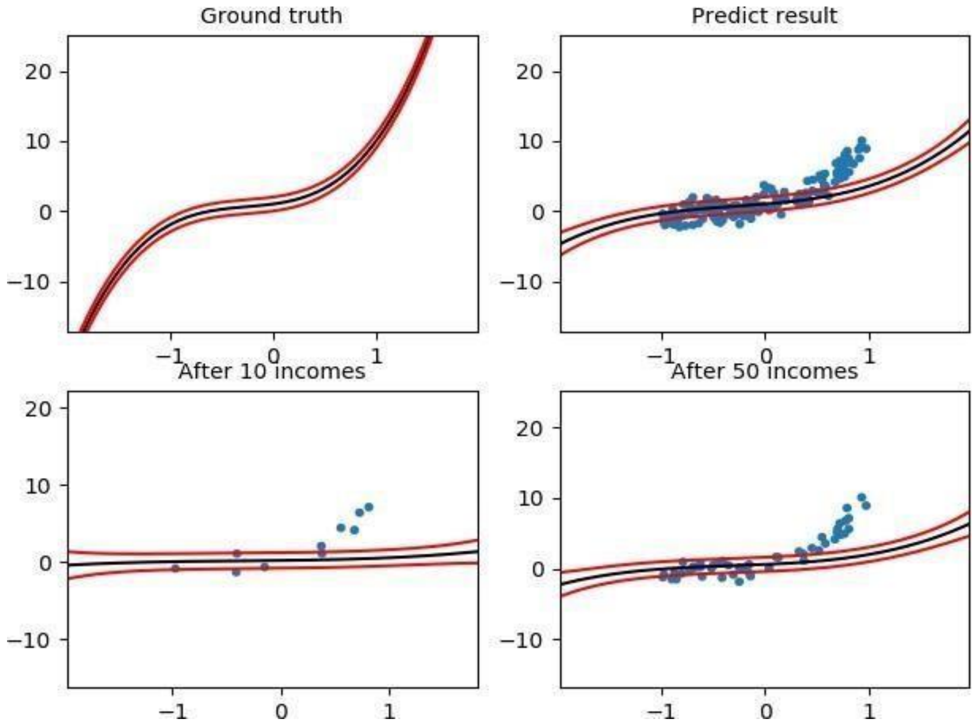
```

Predictive distribution ~ N(0.06869, 1.66008)
-----------------------------------------------------------------------------------------------
Add data point (-0.19330, 0.24507):

Postirior mean:
    0.5760972313
    0.2450231522
   -0.0801842453
    0.0504992402

Posterior variance:
    0.2867129751,    0.1311255325,   -0.0767580827,    0.0438488542
    0.1311255325,    0.7892001707,    0.1242887609,   -0.0801412282
   -0.0767580827,    0.1242887609,    0.9176812972,    0.0541575540
    0.0438488542,   -0.0801412282,    0.0541575540,    0.9642058389

Predictive distribution ~ N(0.62305, 1.34848)


...


Add data point (-0.76990, -0.34768):

Postirior mean:
    0.9107496675
    1.9265499885
    3.1119297129
    4.1312375189

Posterior variance:
    0.0051883836,   -0.0004416700,   -0.0086000319,    0.0008247001
   -0.0004416700,    0.0401966605,    0.0012708906,   -0.0554822477
   -0.0086000319,    0.0012708906,    0.0265353911,   -0.0031205875
    0.0008247001,   -0.0554822477,   -0.0031205875,    0.0937197255

Predictive distribution ~ N(-0.61566, 1.00921)

Add data point (0.36500, 2.22705):

Postirior mean:
    0.9107404583
    1.9265225090
    3.1119408740
    4.1312734131

Posterior variance:
    0.0051731092,   -0.0004872471,   -0.0085815201,    0.0008842340
```

```
  -0.0004872471,    0.0400606628,    0.0013261280,   -0.0553046044
  -0.0085815201,    0.0013261280,    0.0265129556,   -0.0031927398
   0.0008842340,   -0.0553046044,   -0.0031927398,    0.0934876838

Predictive distribution ~ N(2.22942, 1.00682)

```



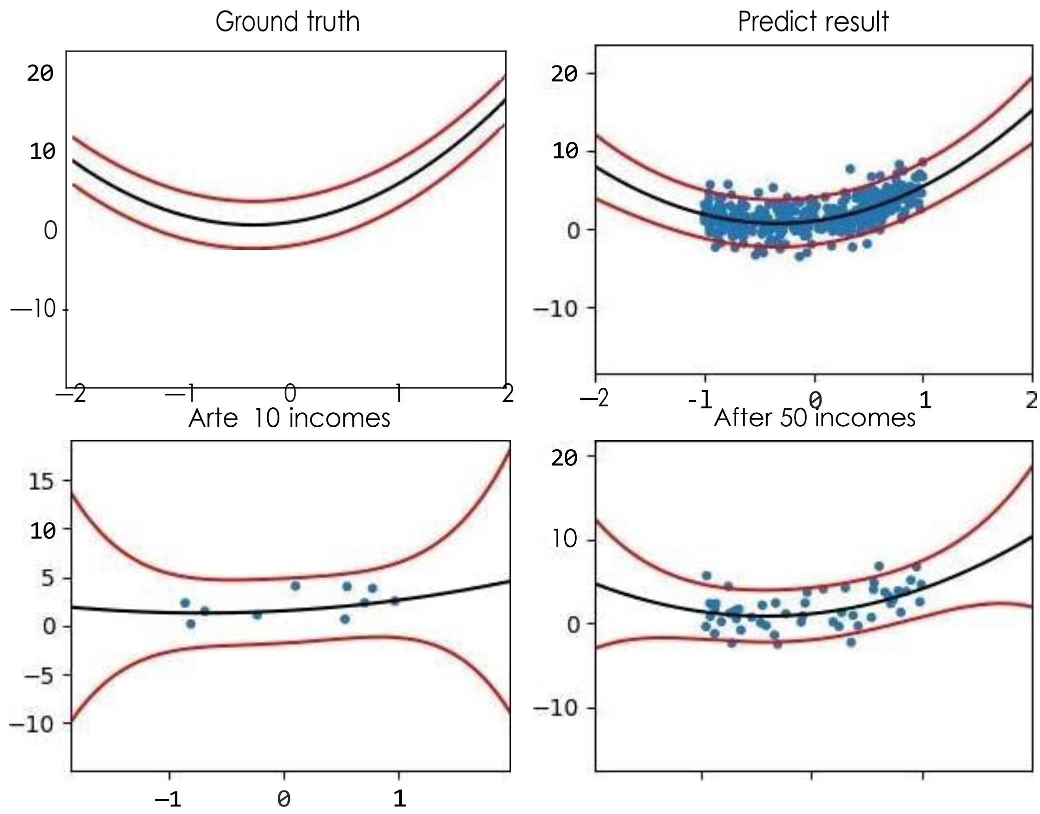

2. b = 100, n = 4, a = 1, w = [1, 2, 3, 4]

```
(Console output omitted)
```




3. b = 1, n = 3, a = 3, w = [1, 2, 3]

```
(Console output omitted)
```



Ground truth
Predict result
Arte 10 incomes
After 50 incomes
20
10
0
−10
−2
−1
0
1
2
15
5
−5