Introduction to pairwise comparisons (seminar)

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On inconsistency in pairwise comparisons

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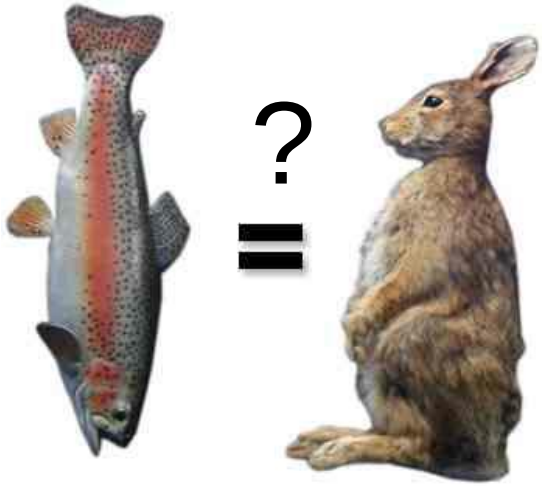
Outline

- Historical perspective on measures
- Pairwise comparisons – motivation
- Inconsistency analysis
- Possible areas of application
- Consistency-driven pairwise comparisons (CDPC) contribution to improving measures of subjective knowledge

Pairwise comparisons motivation

(Four slides donated by http://scholar.google.ca/citations?user=Ipv1KW4AAAAJ&hl=en)

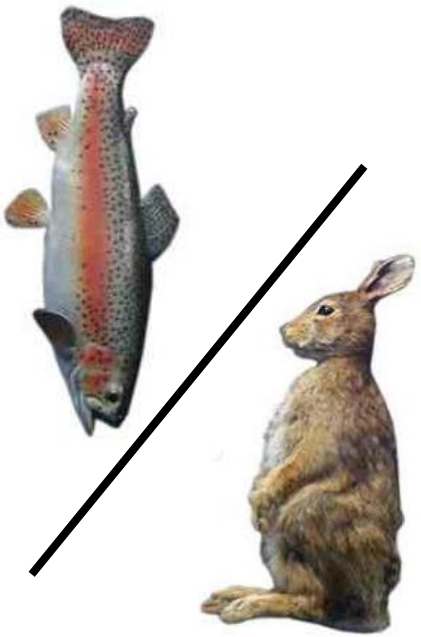
- Before money - barter
  - (see FED Museum: http://www.frbatlanta.org/about/tours/virtual/money/



- History of trade is in fact history of pairwise comparisons

Pairwise comparisons motivation

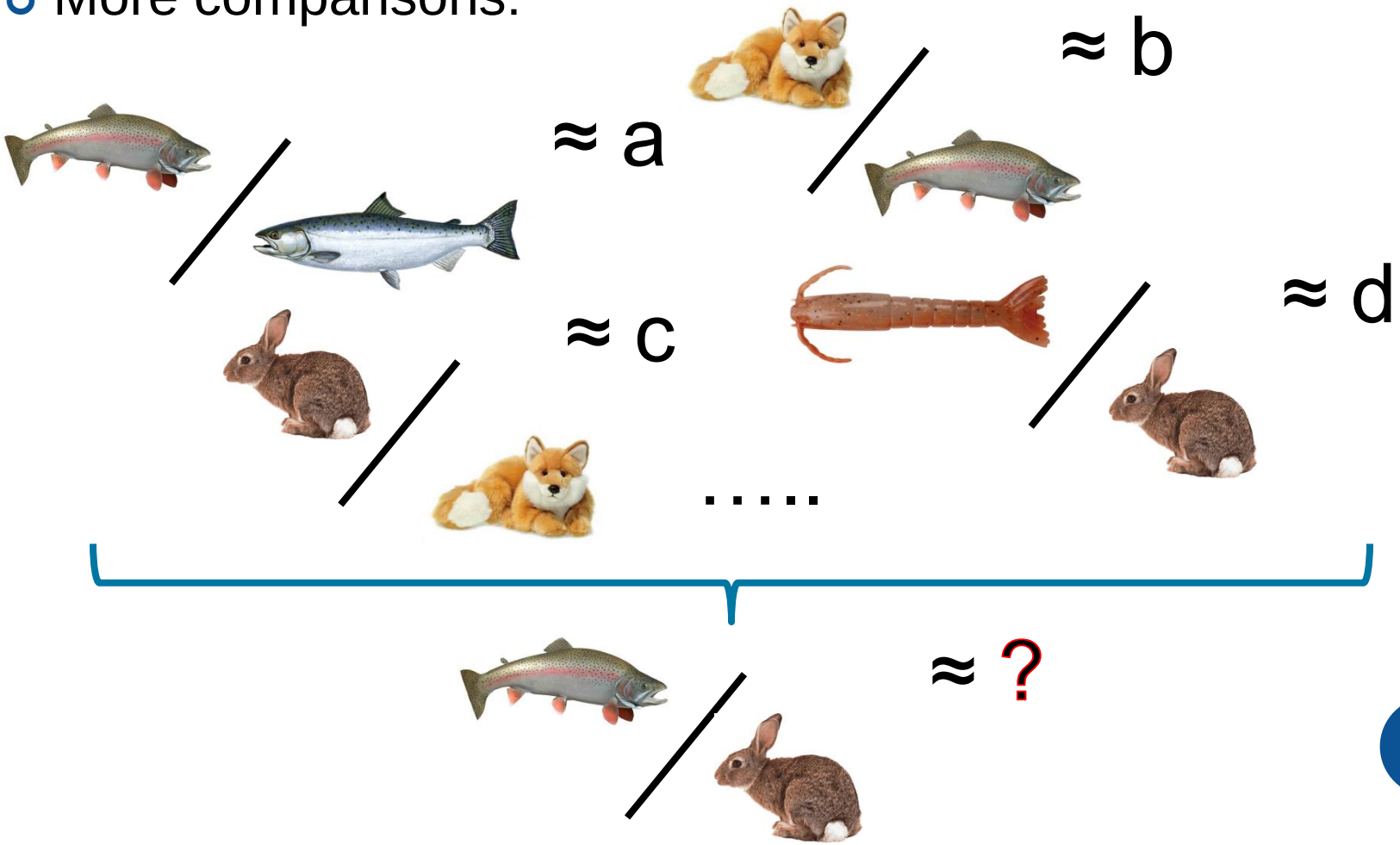
- Experts judgment implies relative value of goods:



$$\approx \frac{1}{2}$$

Pairwise comparisons motivation

- More comparisons:



Result of synthesis

- Pairwise comparisons matrix

$$M = \begin{bmatrix} 1 & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & 1 & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & 1 & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & 1 & m_{45} \\ m_{51} & m_{52} & m_{53} & m_{54} & 1 \end{bmatrix}$$

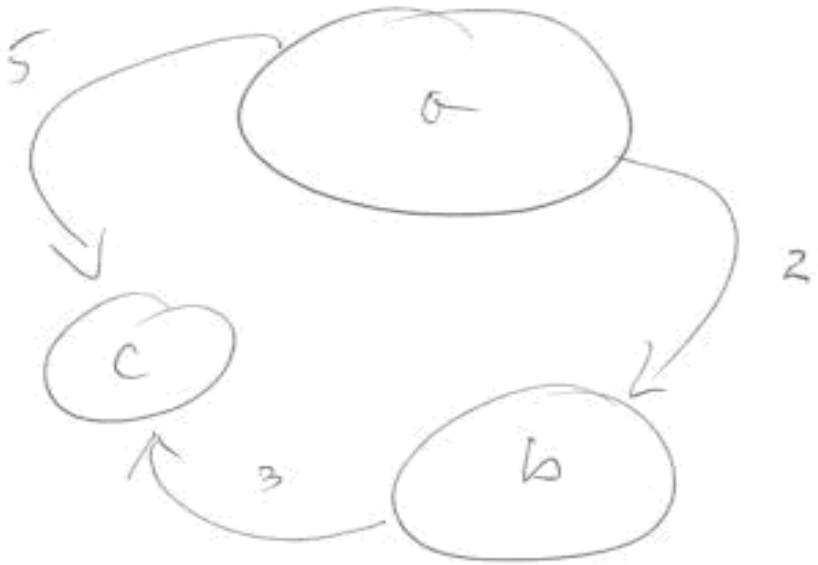
where $m_{ij} \in \mathbb{R}_+$

THE PAIRWISE COMPARISONS PRELIMINARIES

- Pairwise comparisons method creates a matrix (A) of values (a_{ij}) of the i-th object compared with its corresponding (j-th) object (in our case, questionnaire item)
- PC matrix is reciprocal: (a_{ij}) =1/(a_{ji}) since i to j is (or at least, is expected to be) the reciprocal of j to i.
- A small scale [1/c,c] is used for for comparisons from 5 to 9 (in most practical applications)
- inconsistency analysis is essential since it allows to locate the most inconsistent input

WHAT IS INCONSISTENCY?

- It only takes place when we have at least three object to compare.
- For example, taking into consideration their area, we may provide the following estimates:



What is inconsistency?

- The previous picture is fuzzy since our knowledge is nearly always: fuzzy, inexact, incomplete, and/or inaccurate.
- By looking at the above picture, one can even wrongly conclude that if “a” is two times bigger than “b” and “b” is three times bigger than “c” than “a” should be 6, not 5, times bigger than “c”. (By the Internet?)
- Why? Our input cannot be questioned!
- We really do not know if 3 is correct or not (it could be 2.5) or 2 could be 5/3!

INCONSISTENCY ANALYSIS IN BRIEF

- Defined as combinatorial in 1940s
- Saaty defined it as a deviation of an eigenvalue of the pairwise comparing matrix in 1977 but it turned to be incorrect (as evidenced by Koczkodaj and Szwarc in 2014)
- Eigenvalue is a global characteristic of a matrix
- It does not localizes inconsistency
- In 1993, a localizing distance based inconsistency definition was proposed by Koczkodaj; axiomatization for all inconsistency indicators provided by Koczkodaj/Szwarc in 2014

WHAT IS INCONSISTENCY?

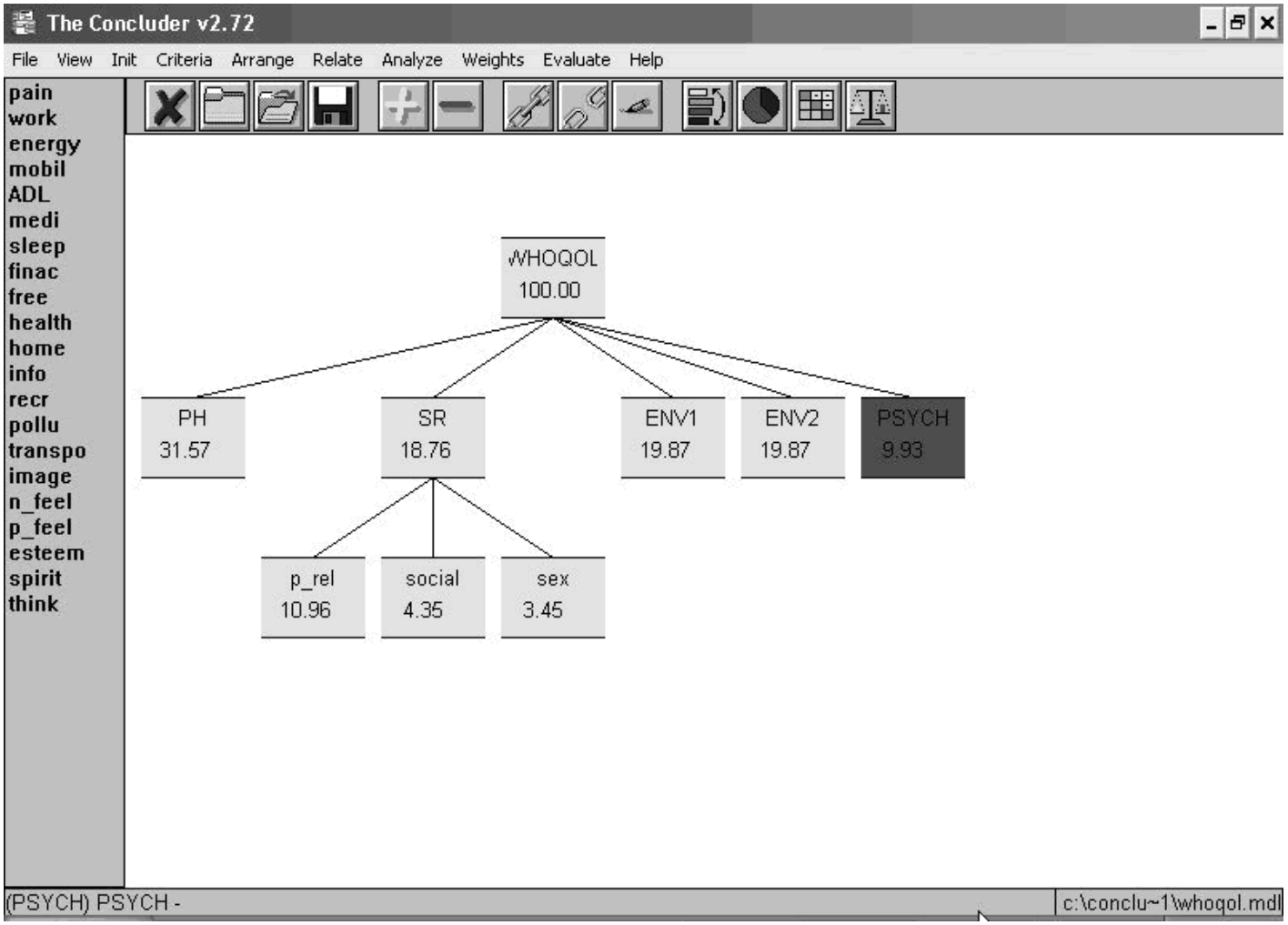
- Our comparisons can be subjective and in the case of QOL, even highly subjective. Often, we cannot be even sure of them. This is a true case of "approximate reasoning" and "incomplete knowledge"

- for a triad, we define inconsistency index as follows:

$$ii = Min\left(\left|1-\frac{a_{ij}}{a_{ik}\cdot a_{kj}}\right|,\left|1-\frac{a_{ik}\cdot a_{kj}}{a_{ij}}\right|\right)$$

and it is the minimum distance from the nearest consistent triad.

LET US BEGIN WITH A BASIC MODEL:



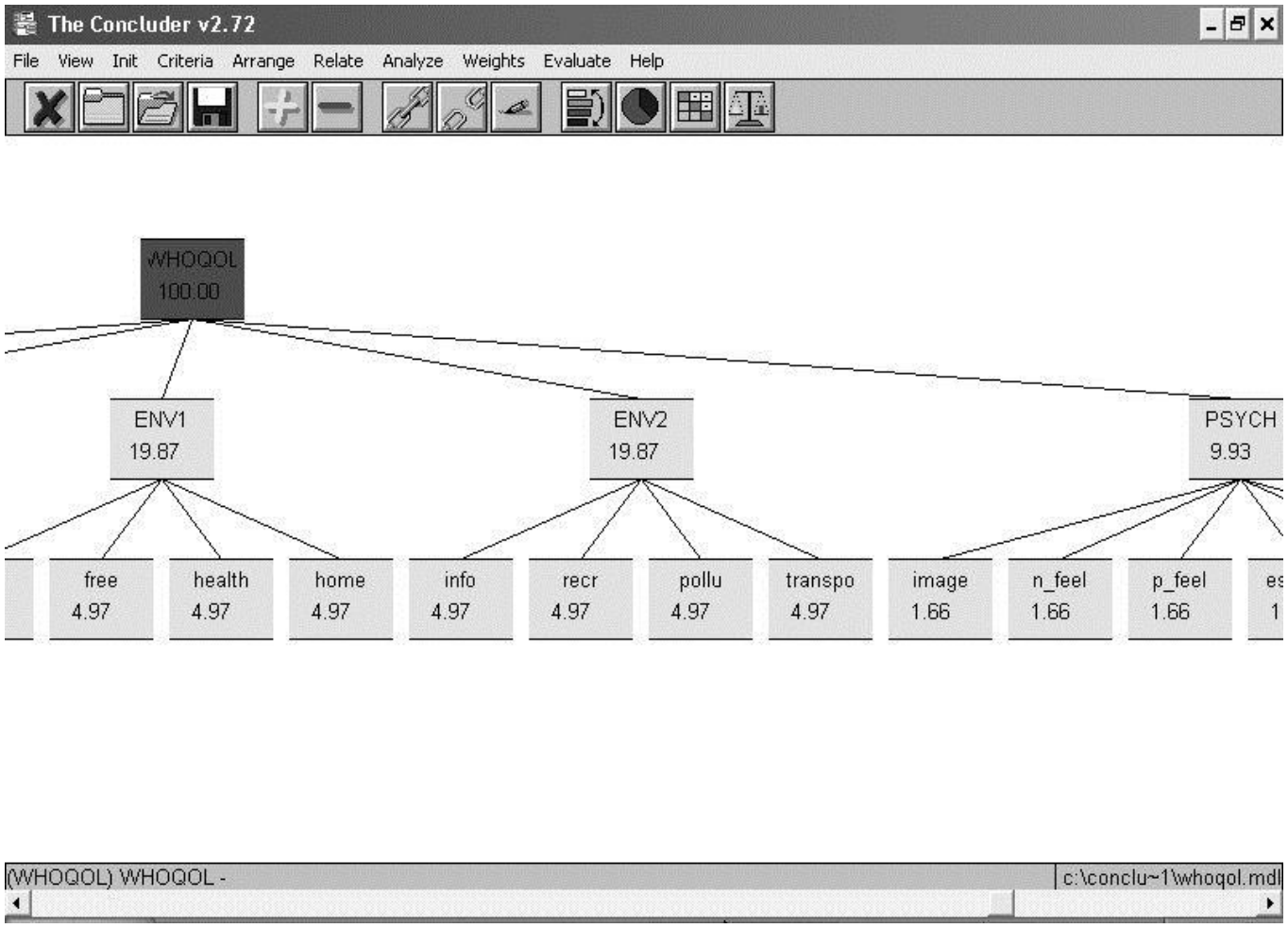
- Explanation: all WHOQOL items have been organized into hierarchical structure

Using inconsistency

- It is not the purpose of this presentation to explain “why” or even “how” the above inconsistency indicator works but how to use it to improve the subjectivity measure by using the inconsistency index.

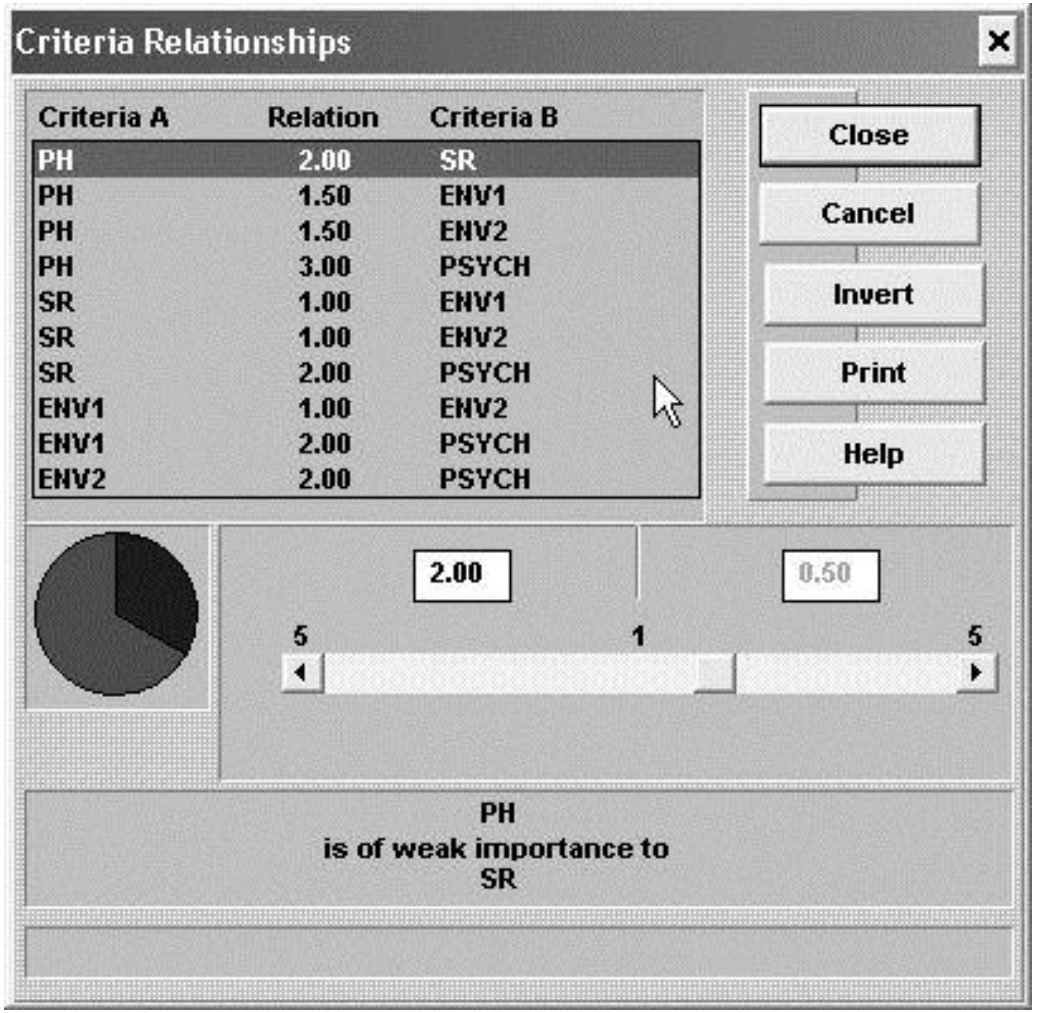
- WHOQOL and APACHE II were improved and published

SO LET US DO IT BY LOOKING AT SEVERAL SCREEN IMAGES:



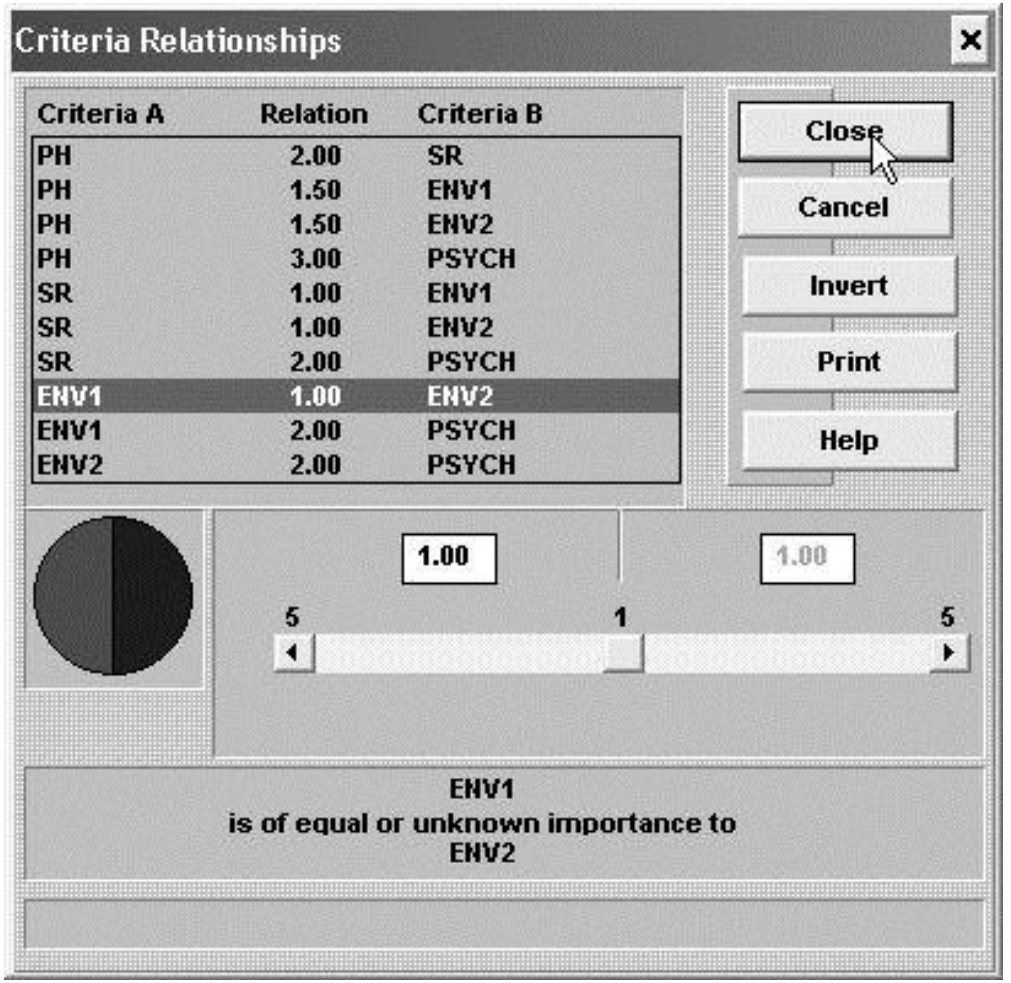
- **This is a part of a model for WHOQOL measure since the full model has not fit into the screen**

SCREEN IMAGES:



- This is how we compare objects against each other starting from the top level

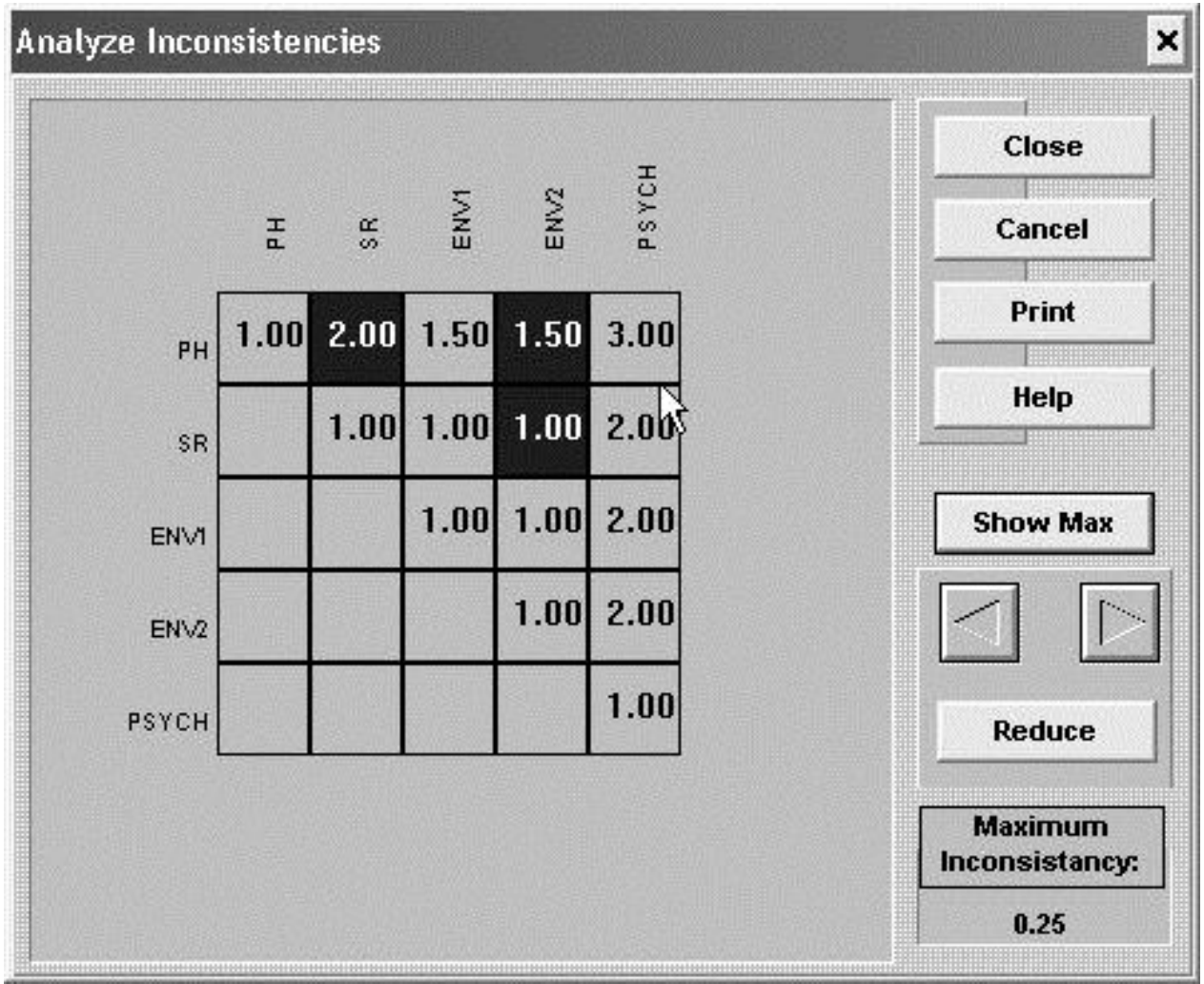
SCREEN IMAGES:



- After a new value (for a pair or more) is entered, we close this window and…

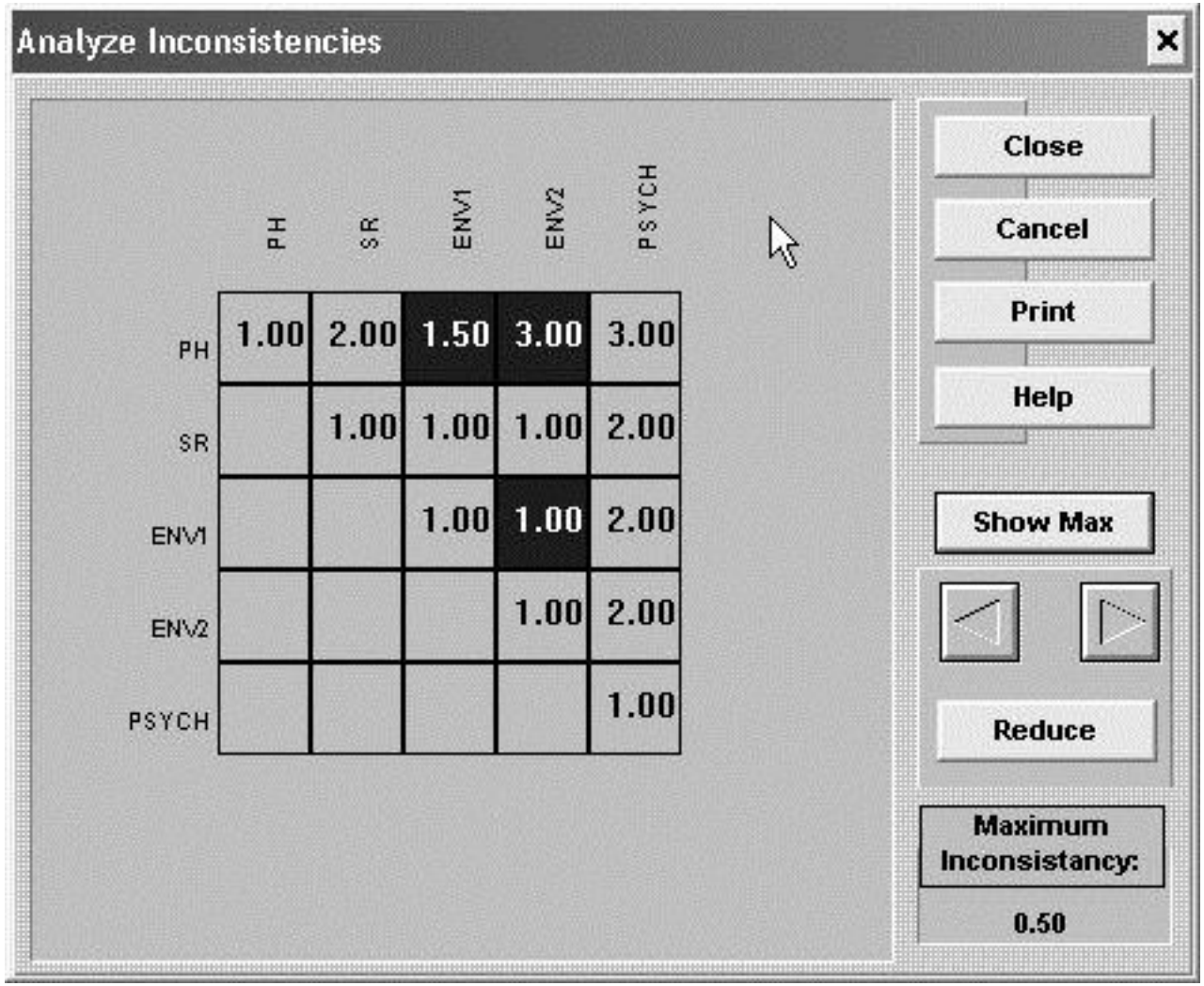
SCREEN IMAGES:

- We can check inconsistency by ANALYZE window selecting "Show Max" below



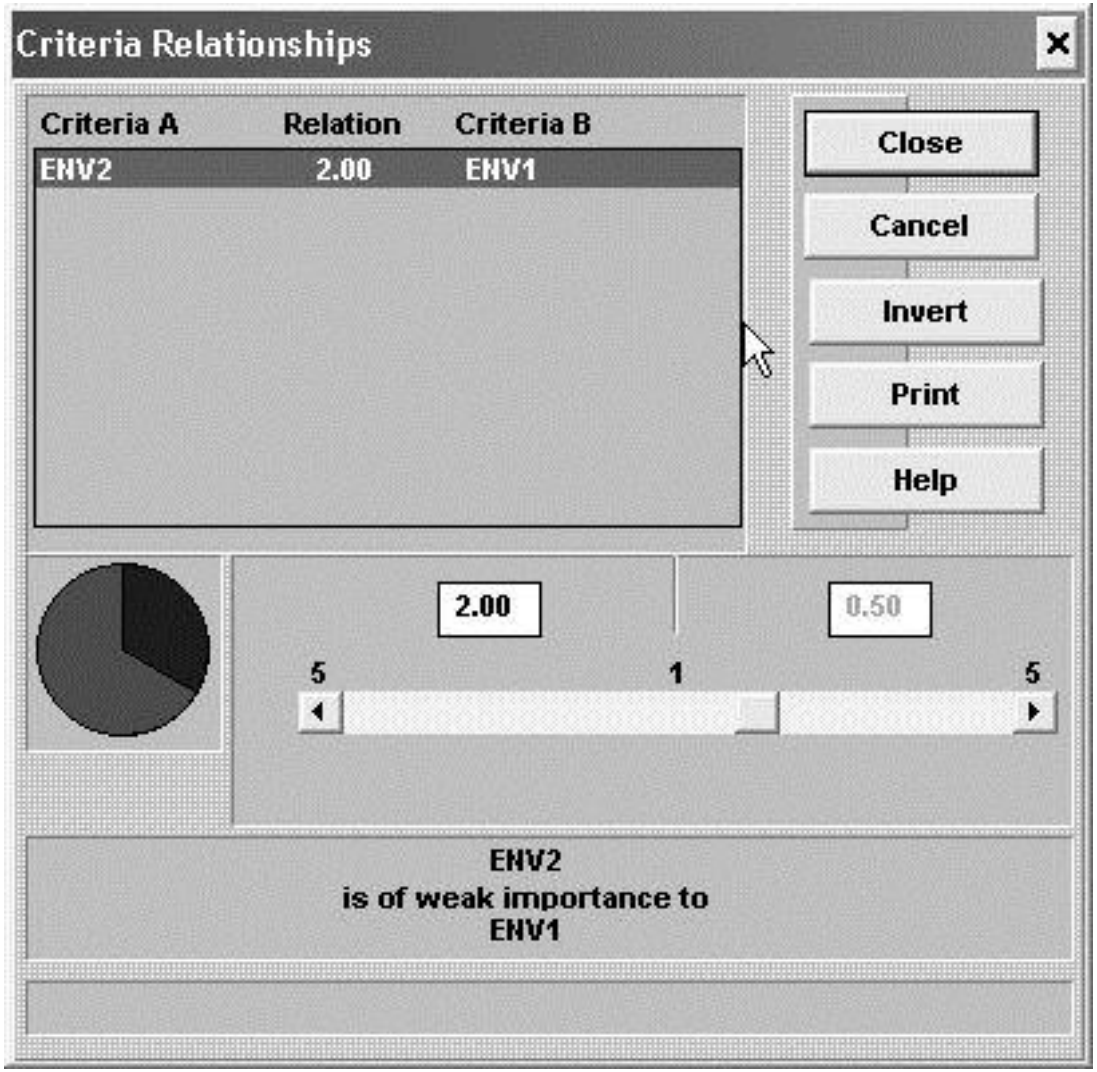
SCREEN IMAGES:

- We can alter values of the most inconsistent triad (the dark highlight)



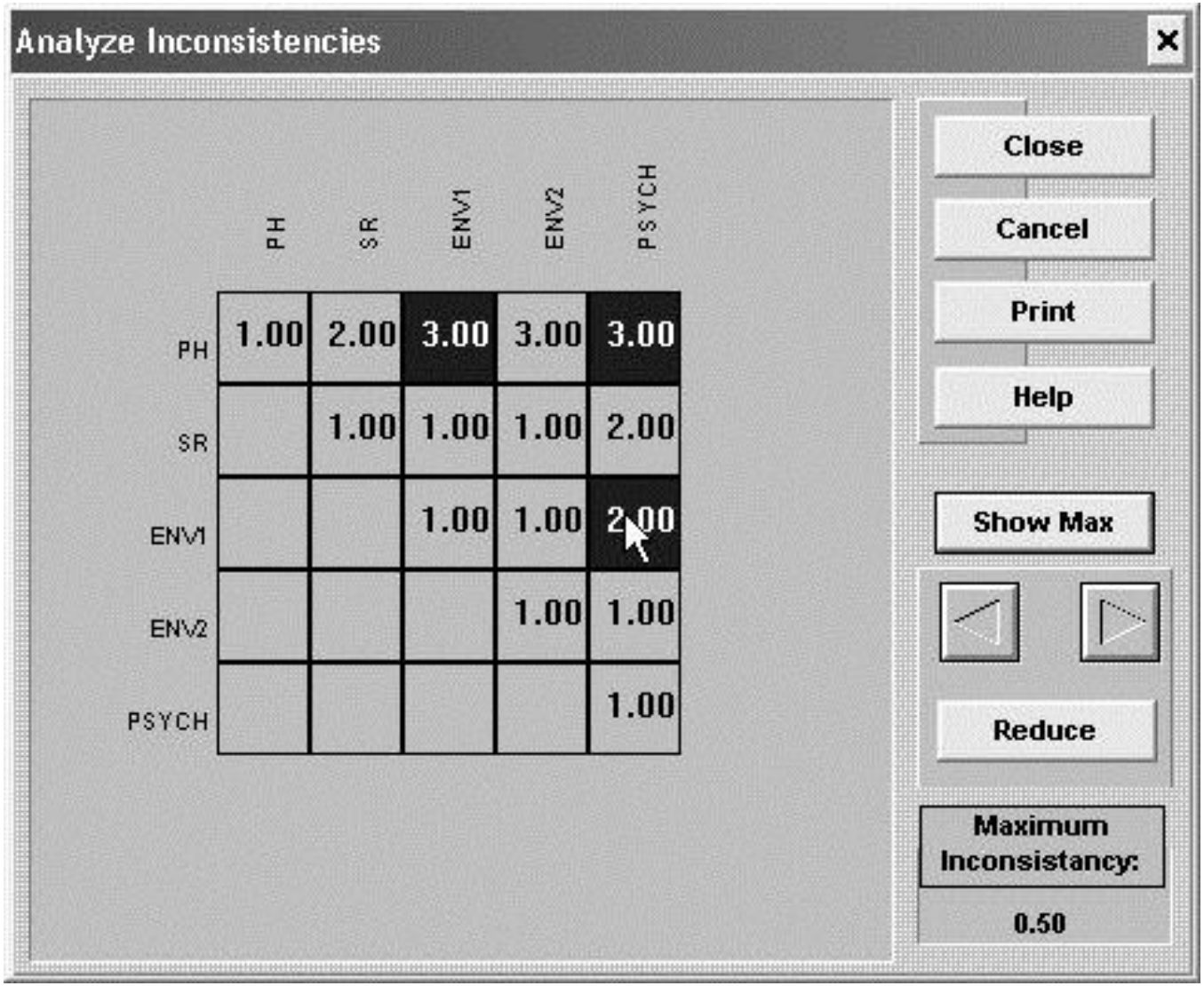
SCREEN IMAGES:

- It brings us to this (previously shown) screen:



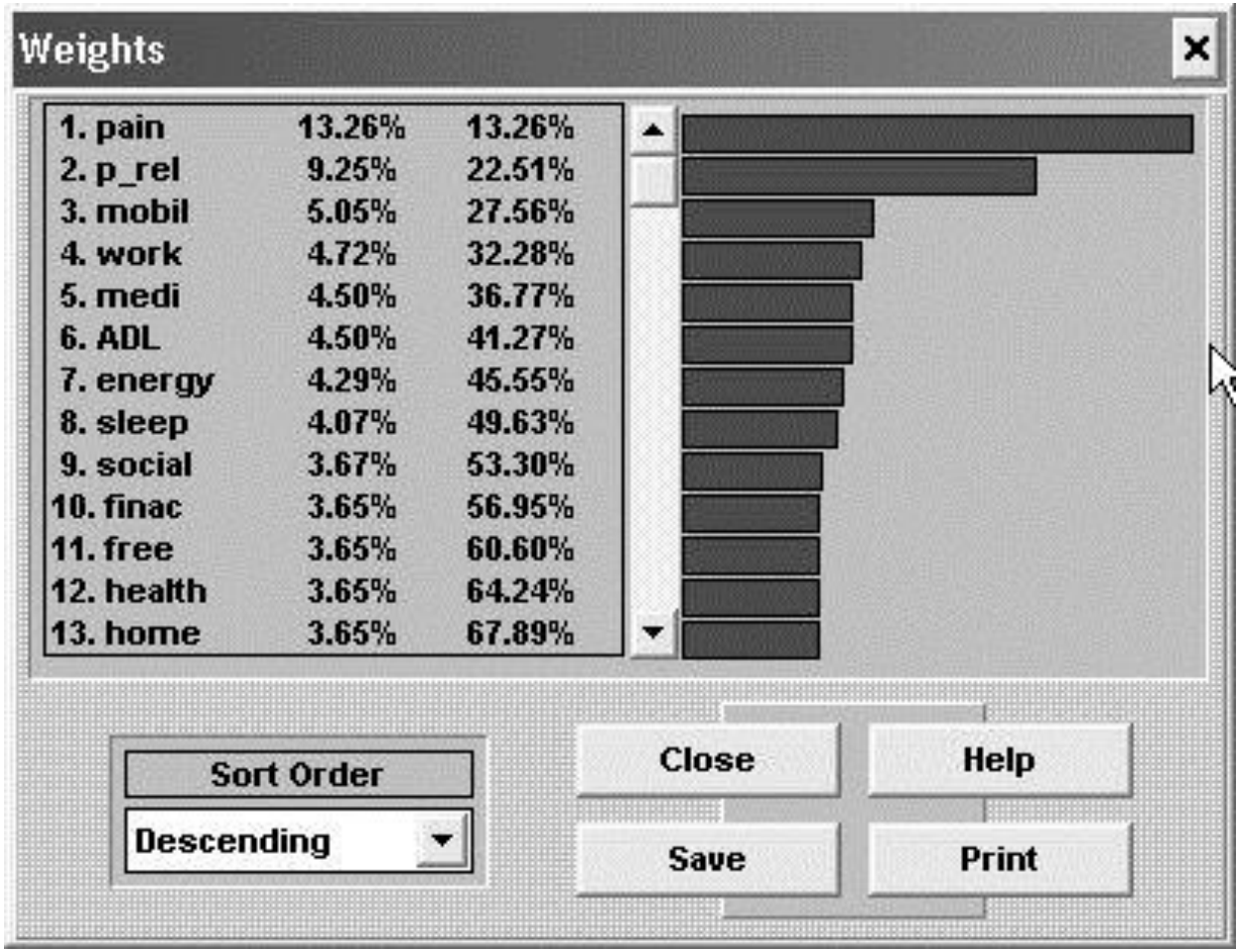
SCREEN IMAGES:

- We can check the inconsistency again:



SCREEN IMAGES:

- When we have inconsistency 1/3 or less (but do not even try to get not zero since it is unrealistic for highly subjective values), we obtain weights:



FUTURE RESEARCH

- Many health measures, including the WHOQOL, do not have weights for items
- Applying the pairwise comparisons method to WHOQOLhelps to improve its precision as the preliminary results show (slide or screen image) shows.
- There is very little dispute that pain is more important for QOL than transportation

Open problems

- Non-reciprocal PC matrix heuristics
  - Lack of reciprocity:

$$m_{ij} \; 1 \; \frac{1}{m_{ji}}$$

  - Let us transform:
    - where:

$$M \rightarrow \hat{M}$$

$$\hat{m}_{ij} = \left(m_{ij} \frac{1}{m_{ji}} \right)^{1/2}$$

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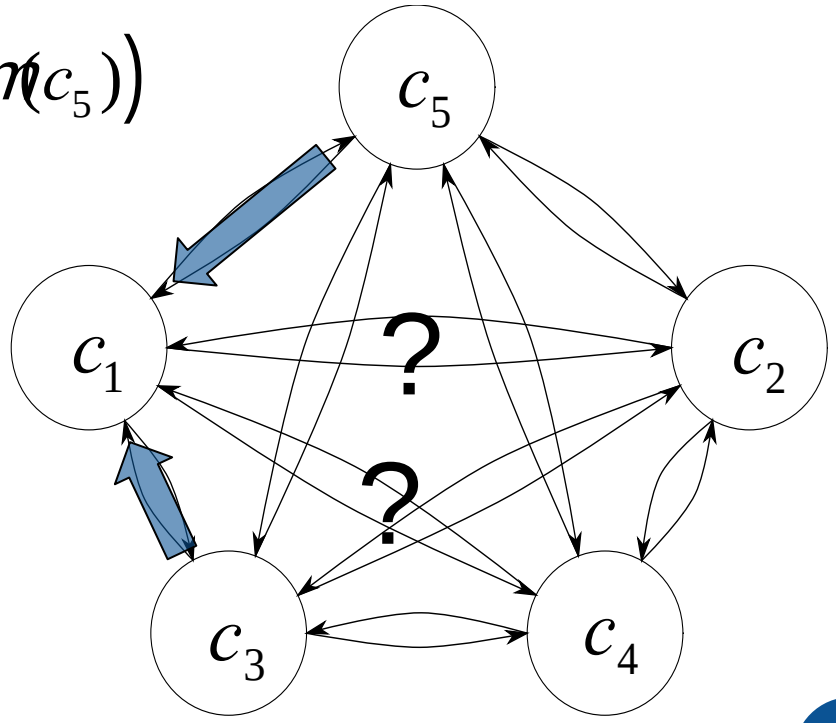
PC method, HRE approach
Supporting heuristics

- Incomplete PC matrix heuristics:

$$m(c_1) = \frac{1}{2}\left(m_{13}m(c_3) + m_{15}m(c_5)\right)$$

- Impacts the lack of reciprocity heuristics



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In conclusion…

- It is a work-in-progress so more graduate students are needed

- An initial undertaking is the examination of the feasibility of CDPC method to improve the precision of PC method

Questions

THANK YOU FOR COMING ☺