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Derivation of Consistent Pairwise Matrices

A Thesis Presented to

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Masters of Science

by

Chris Kuske

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Signature page for the Masters in Computer Science Thesis of Christopher Kuske

APPROVED FOR THE COMPUTER SCIENCE PROGRAM

Dr. Michael Soltys, Thesis Advisor   Date

Committee Member's name, Thesis Committee   Date

APPROVED FOR THE UNIVERSITY

Dr. Gary A. Berg, AVP Extended University   Date

To my family, in gratitude for their encouragement and support.

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Abstract

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This thesis will give an overview of pairwise matrices and their properties. After this introduction, a summary of existing literature on Pairwise Matrices will follow.

A method of generating a consistent Pairwise Matrix from an inconsistent matrix will be presented, along with a method to find a consistent matrix that is as close to the original inconsistent matrix as possible using a calculated distance.

After this methodology has been described, an analysis of the results and further work to be done will follow.

Contents

1. Introduction

Over the past few decades, pairwise comparisons and the Analytical Hierarchical Process (*AHP*) have given decision makers a new set of tools that empower them to make more informed decisions. AHP uses matrices to help rank the evaluation criteria based on importance. This type of problem is also knows as Multiple Attribute Decision Making *(MADM)*.

In pairwise matrices, each criterion has a relative rank. Consider the following matrix:

$$\begin{bmatrix} 1 & x_{12} & x_{13} & x_{1n} \\ x_{21} & 1 & x_{23} & x_{2n} \\ x_{31} & x_{32} & 1 & x_{33} \\ x_{d1} & x_{d2} & x_{d3} & 1 \end{bmatrix}$$

When decision makers are trying to make their evaluation(s), they will often bring in subject matter experts *(SMEs)* to help develop the relative rankings of how one preference should be ranked compared to another. When experts define their preferences in a pairwise matrix, they often generate matrices that do not meet the criteria for consistency.

A pairwise comparison matrix A has the following properties:

- A is square (nxn).
- All elements on the diagonal of A have a value of 1.
- A has the property where each element a_{ij} has an element that is the reciprocal, located at a_{ji}.
- A is consistent. This means that for each element a_{ij}, $a_{ij} = a_{ik} * a_{kj}$. For all i, j, k $\{1, \ldots, n\}$. If $a_{ij}! = a_{ik} * a_{kj}$, the matrix is not consistent.

[1]

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{d1} & x_{d2} & x_{d3} & \dots & x_{dn} \end{bmatrix}$$

1.1. Notation and Terminology.

- **Consistency:** A rectangular array of numbers. The numbers in the matrix are called *entries* or *elements*. The horizontal lines of entries are *rows* while the vertical lines of entries are *columns*.
- **Matrix:** A rectangular array of numbers. The numbers in the matrix are called *entries* or *elements*. The horizontal lines of entries are *rows* while the vertical lines of entries are *columns*.
- **Pairwise Matrix:** A Pairwise Matrix is defined as a square matrix

- **Reciprocal:** The number $1/x$, which multiplied by x gives the product of 1.

2. Literature Review

Saaty

3. Methodology

3.1. Formation of Consistent Matrix.
Dude

3.2. Example.

3.3. Distance Calculation.
If M is consistent, any row or column of M may be selected such that:

$[w_1,w_2,...,w_n] = [a_{11},\ a_{21},\ a_{31},\ ...,\ a_n 1]$

By consistency of M, it is also true that $a_{1n} = a_{1i} * a_{in}$

$a'_{in} := \frac{w_i}{w_n}$

which when further decomposed, the following holds true:

$\frac{w_i}{w_n} = \frac{a_{1i}}{a_{ni}}$ which is in turn is equivalent to $\frac{\frac{a_{1n}}{a_{in}}}{a_{ni}}$

$\frac{a_{1n}}{a_{in} * ani} = 1$

which reduces to simply a_{1n}, since $\frac{a_{1n}}{1} = 1$

$a_{in} * a_{ni}$ is always 1 via the properties of pairwise matrices.

Let M' to be the $M' < W >$ where $\|M - W\|$ is smallest.

4. Results

Consider addressing what the next person to work in this area might tackle.

5. Conclusion

Here is more stuff where I prove that this paper has something worthwhile inside of it.

References

[1] Saaty, T. L. *Fundamentals of decision making and priority theory with the analytic hierarchy process*, vol. 6. RWS Publications, 2000.