## Mathematics - 3

1 (a) Find the velocity and position vector of an object with the given acceleration, initial velocity and position,

$$a(t) = -32k, \quad v(0) = i + 2j, \quad \gamma(0) = 128 \, K$$

(b) Find the directional derivative of $f(x,y) = \sqrt{xy}$ at $(1,4)$ in the direction of the unit vector that makes an angle $\pi/3$ with the positive x-axis.

(5)

2. (a) Define curvature of a plane curve

(b) Find the curvature of a circle of radius a.

(c) Find the circle of curvature (osculating circle) of the parabola $y = x^2$ at the origin.

(5)

3. (a) Define TNB-frame

(b) A particle moves along a curve described by the vector function $\gamma(t) = ti + t^2 j + t^3 k$. Find the tangential scalar and normal scalar components of acceleration and of the particle at any time $t$. Also find the curvature of the path at any time $t$.

(5)

4 (a) If $z = 3x^3 y + 4xy^2 - 2x + 4y - 5$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$

(b) verify that $\dfrac{\partial^2 w}{\partial y \partial x} = \dfrac{\partial^2 w}{\partial x \partial y}$, $w = x^y + \sin(xy)$

(5)

5. (a) Give the formula for divergence and curl of the vector

   (b) Compute the divergence and curl of the vector,

   $$F = xyz \, i + 3x^2 y \, j + (xz^2 - y^2 z) k \quad \text{at} \quad (1, 2, -1)$$

   (c) Evaluate $\int_C xy \, dx + x^2 \, dy$ where C is given by $y = x^2, \; -1 \le x \le 2$

   (5)