

3) $2^{n+1} \rightarrow \Theta(2^n)$

substitute constant a , arbitrarily chosen (but as a positive integer)

O. Yes, by defn. of exponents, a^{n+1} is equal to $a \cdot a^n$, so the inequality $a \cdot a^n \leq c \cdot a^n$ for any $c > a$. ^{So, $\exists c$ s.t. $\forall a (a^{n+1} \geq c \cdot a^n)$}

Ω . Yes, Again by defn. of exponents, a^{n+1} is equal to $a \cdot a^n$, so the inequality $a \cdot a^n \geq c \cdot a^n$ for any $c < a$.

So, $\exists c$ s.t. $\forall a (a^{n+1} \geq c \cdot a^n)$

$\Theta. 2^{n+1} \in O(2^n) \wedge 2^{n+1} \in \Omega(2^n) \rightarrow 2^{n+1} \in \Theta(2^n)$