Show directly that $f(n) = n^2 + 3n^3 \in \Theta(n^3)$

i) CLAIM: $n^2 + 3n^3 \in O(n^3)$

Proof: by O definition, $f(n)$ is $O(n^3)$ iff $f(n) \leq c \cdot n^3$ for some real numbers $n \geq k = 1$ and c. Let $c = 4$ and $k = 2$. $(2)^2 + 3(2)^3 \leq 4(2)^3$

$$4 + 24 \leq 32$$

$$28 \leq 32 \checkmark$$

$\exists c$ such that $f(n) \leq c \cdot n^3$, so $f(n) \in O(n^3)$.

ii) CLAIM: $n^2 + 3n^3 \in \Omega(n^3)$

Proof: By Ω definition, $f(n)$ is $\Omega(n^3)$ iff $f(n) \geq c \cdot n^3$ for some real numbers c and k, where $n \geq k$. Let $c = 3$ and $k = 2$. $(2)^2 + 3(2)^3 \geq 3(2)^2$

$$4 + 24 \geq 24$$

$$28 \geq 24 \checkmark$$

$\exists c$ such that $f(n) \geq c \cdot n^3$, so $f(n) \in \Omega(n^3)$.

iii) CLAIM: $f(n) = n^2 + 3n^3 \in \Theta(n^3)$.

Given $f(n) = n^2 + 3n^3 \in O(n^3) \wedge f(n) = n^2 + 3n^3 \in \Omega(n^3)$, it follows that $f(n) = n^2 + 3n^3 \in \Theta(n^3)$ by the definition of Θ.