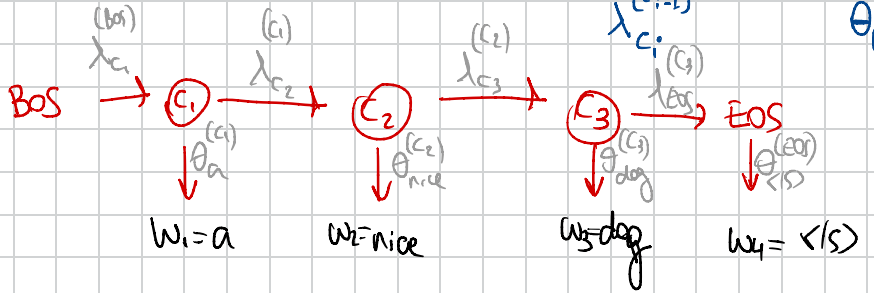
HMM

(POS tagging)

$$P_X(w_{1:\ell}, c_{1:\ell}) = \prod_{i=1}^{\ell} \underbrace{P_{C|C_{prev}}(c_i|c_{i-1})}_{\text{transition}} \times \underbrace{P_{W|C}(w_i|c_i)}_{\text{emission}}$$

$\lambda_{c_i}^{(c_{i-1})}$ $\quad$ $\theta_{w_i}^{(c_i)}$



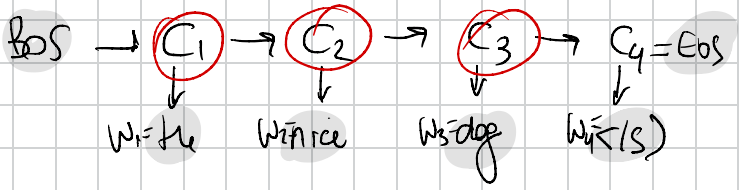
The Value Recursion

[Viterbi] $O(L \times T^2)$

[Forward]

POS tagging : $\underset{c_1 \in C \cdots c_\ell \in C}{\text{Argmax}} P_{Y|X}(c_{1:\ell} | w_{1:\ell}) = \underset{c_1 \in C \cdots c_\ell \in C}{\text{Argmax}} P_{XY}(w_{1:\ell}, c_{1:\ell})$

LM : $P_X(w_{1:\ell}) = \sum_{c_1 \in C} \cdots \sum_{c_\ell \in C} P_{XY}(w_{1:\ell}, c_{1:\ell})$



$G = \{A, B\}$

$w_1 = \text{the} \quad \cdots \quad w_i$

$\uparrow \qquad\qquad \uparrow$

$C_1, \cdots, C_i = j$

$\alpha(i, j)$ ~~marginal prob of~~ probability of the best sequence ending in $(C_i = j, W_i = w_i)$

$$\alpha(i,j) = \begin{cases} \lambda_j^{(\text{BOS})} \theta_{w_i}^{(j)} & \text{if } i = 1 \\ \bigoplus_{r \in C} \alpha(i-1, r) \otimes \lambda_j^{(r)} \theta_{w_i}^{(j)} & \text{if } i > 1 \end{cases}$$

Semirings

	Forward	Forward (log)	Viterbi	Viterbi (log)
$s(c_{i-1}, c_i, w_i)$	$\lambda_{c_i}^{(c_{i-1})} \times \theta_{w_i}^{(c_i)}$	$\log \lambda_{c_i}^{(c_{i-1})} + \log \theta_{w_i}^{(c_i)}$	$\lambda_{c_i}^{(c_{i-1})} \times \theta_{w_i}^{(c_i)}$	$\log \lambda_{c_i}^{(c_{i-1})} + \log \theta_{w_i}^{(c_i)}$
$a \oplus b$	$a + b$	$\log(\exp(a) + \exp(b)) = \text{logsumexp}(a, b)$	$\max(a, b)$	$\max(a, b)$
$a \otimes b$	$a \times b$	$a + b$	$a \times b$	$a + b$

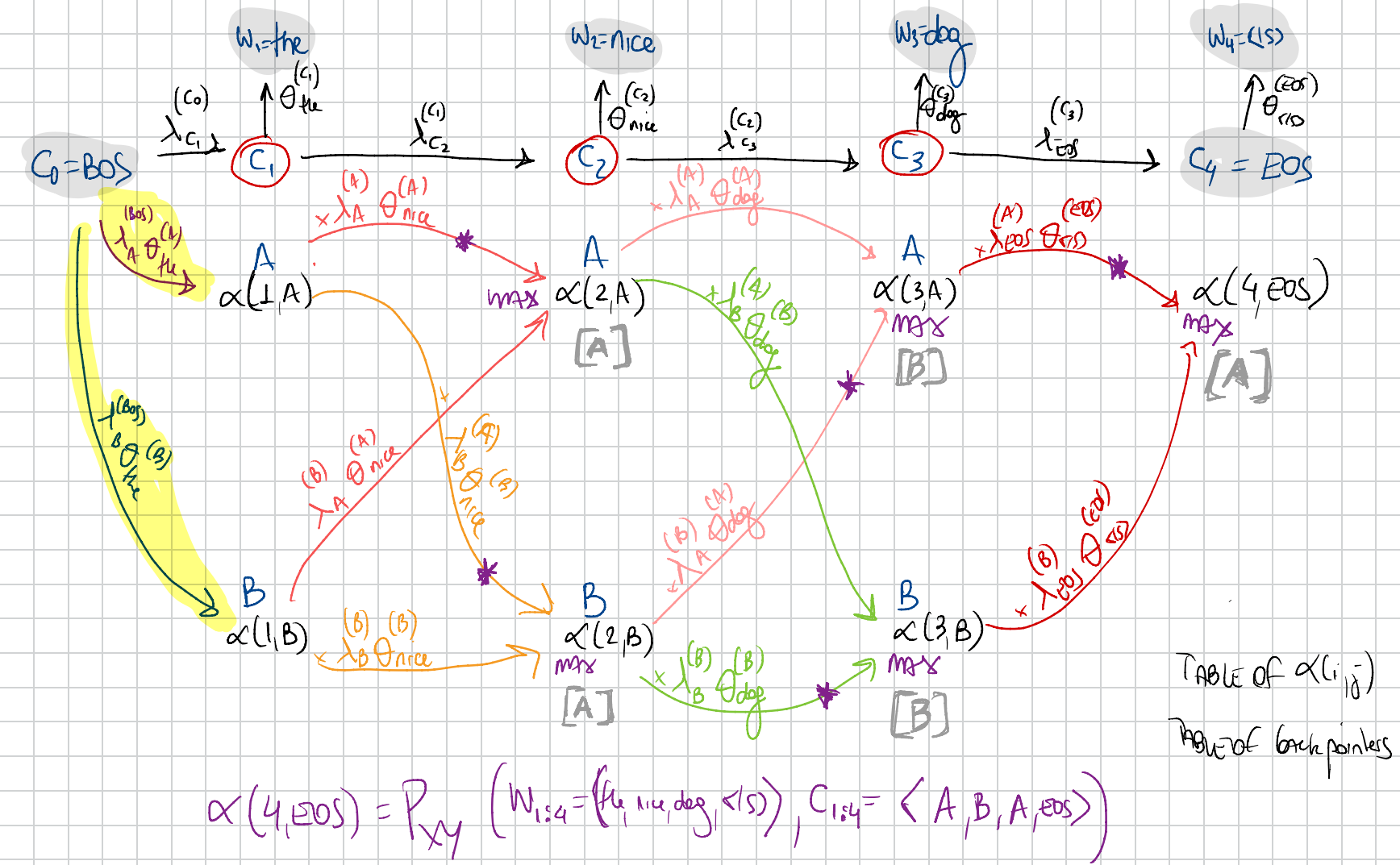


TABLE OF $\alpha(i,j)$

TABLE OF backpointers

$$\alpha(4,\text{EOS}) = P_{XY}\left(W_{1:4}=\langle \text{the, nice, dog, }\langle/s\rangle\rangle,\ C_{1:4}=\langle A,B,A,\text{EOS}\rangle\right)$$