Simulation of $\left| \sum_j V_{jk} e^{i w X_j} \right|^2$

Let $\{Z(t)\}$ be a mean zero, weakly stationary process.

$$\mathrm{Var}\left(\sum_j V_{jk} Z(X_j) \right) = \mathbb{E}\left[\left(\sum_j V_{jk} Z(X_j) \right)^2 \right]$$

$$\sum_j C_j Z(X_j) = \sum_j C_j e^{i w X_j}$$

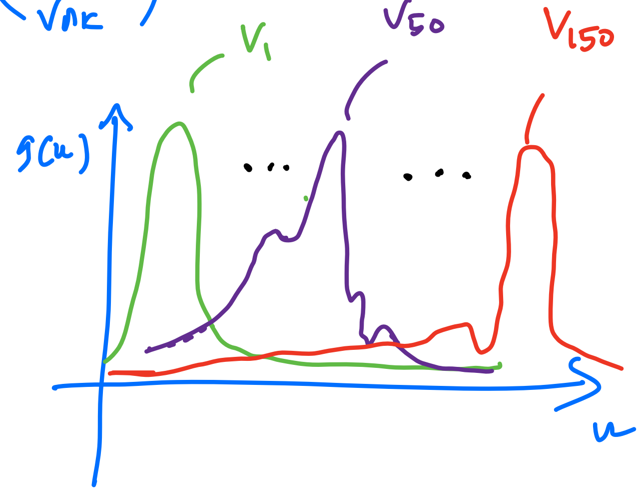
$$\int_{-\infty}^{\infty} \left| \sum_j V_{jk} e^{i w X_j} \right|^2 f(w)\, dw$$

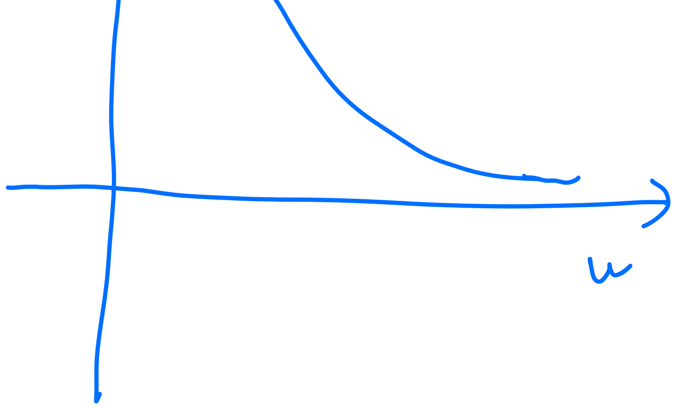
$= \lambda_k$, k-th eigenvalue,

where $v_k = \begin{pmatrix} v_{1k} \\ \vdots \\ v_{nk} \end{pmatrix}$ is a k-th eigenvector.

$g(w) = \left| \sum_j V_{jk} e^{i w X_j} \right|^2$,

for a given v_k





$$\int g(u, v_1) f(u)\, du = \lambda_1$$

$$\geq \int g(w, V_{50}) f(u)\, du = \lambda_{50}$$

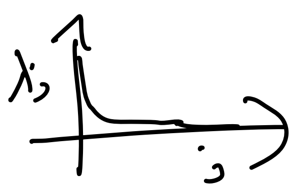
$$\geq \int g(w, V_{150}) f(u)\, du = \lambda_{150}$$

Q_1: how to show $\sum_j c_j z(t_j) = \sum_j c_j e^{iw x_j}$?

should I be able to show this ?

Q_2: numerical issue, singular matrix.

There is no negative eigen values, but

λ decrease fast 

what I do:

1) add $1e^{-8}$ on diagonal

2) add $1e^{-8}$ to space distance

i.e. $|s| \Leftarrow |s| + 1e^{-8}$

to preserve symmetric structure.