

Outline

- What is part of speech tagging?
- Markov chains
- Hidden Markov models
- Viterbi algorithm
- Example
- Coding assignment!

What is part of speech?

Why not learn something ?

adverb	adverb	verb	noun	punctuation mark, sentence closer
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Part of speech (POS) tagging

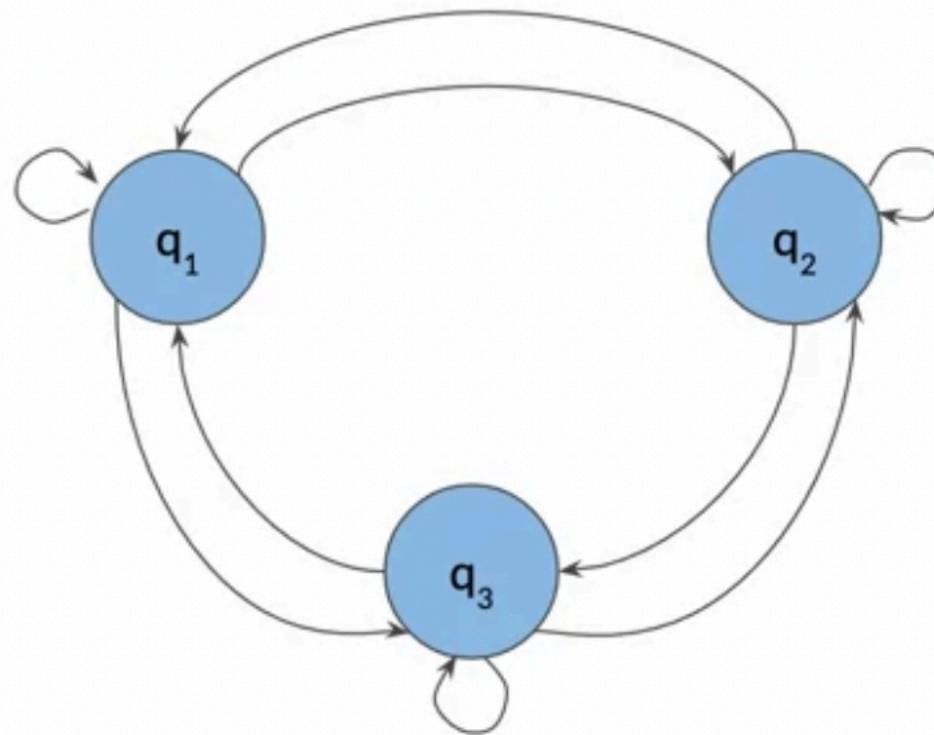
Part of speech tags:

lexical term	tag	example
noun	NN	something, nothing
verb	VB	learn, study
determiner	DT	the, a
w-adverb	WRB	why, where
...	...	

Why not learn something ?

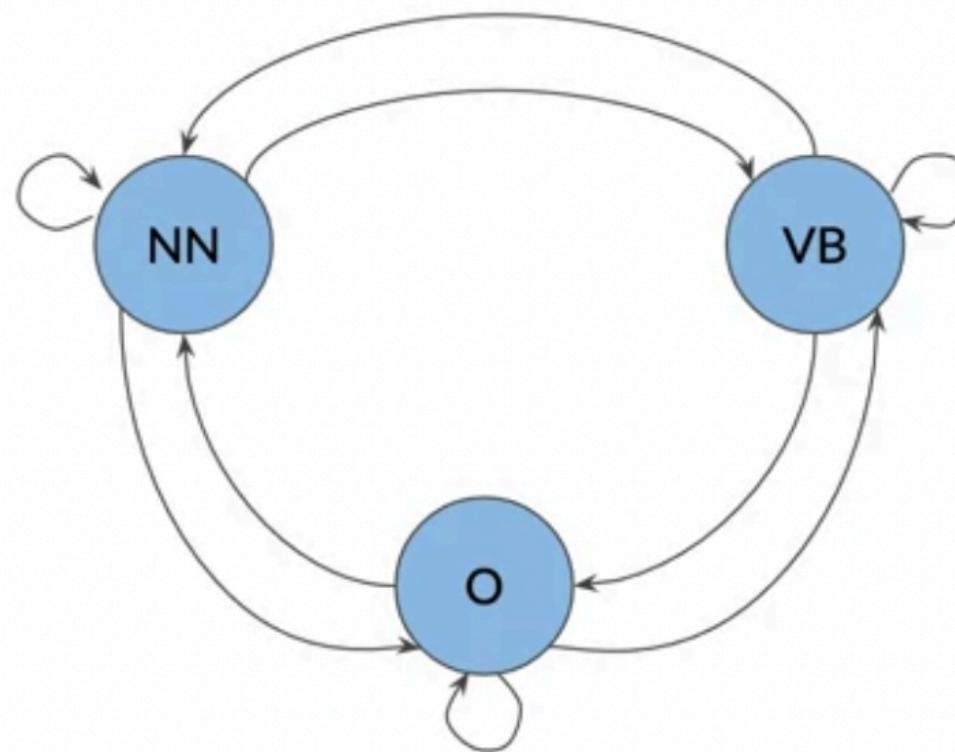
WRB RB VB NN .

States

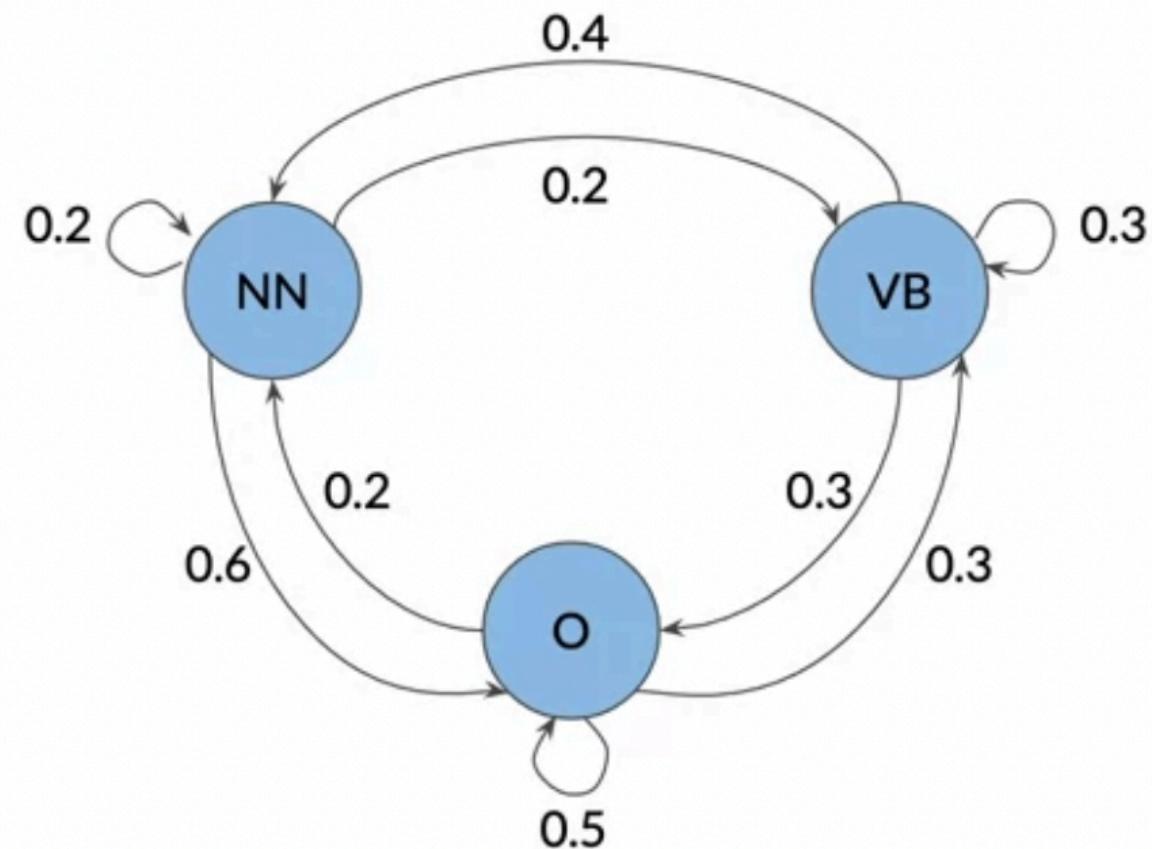


$$Q = \{q_1, q_2, q_3\}$$

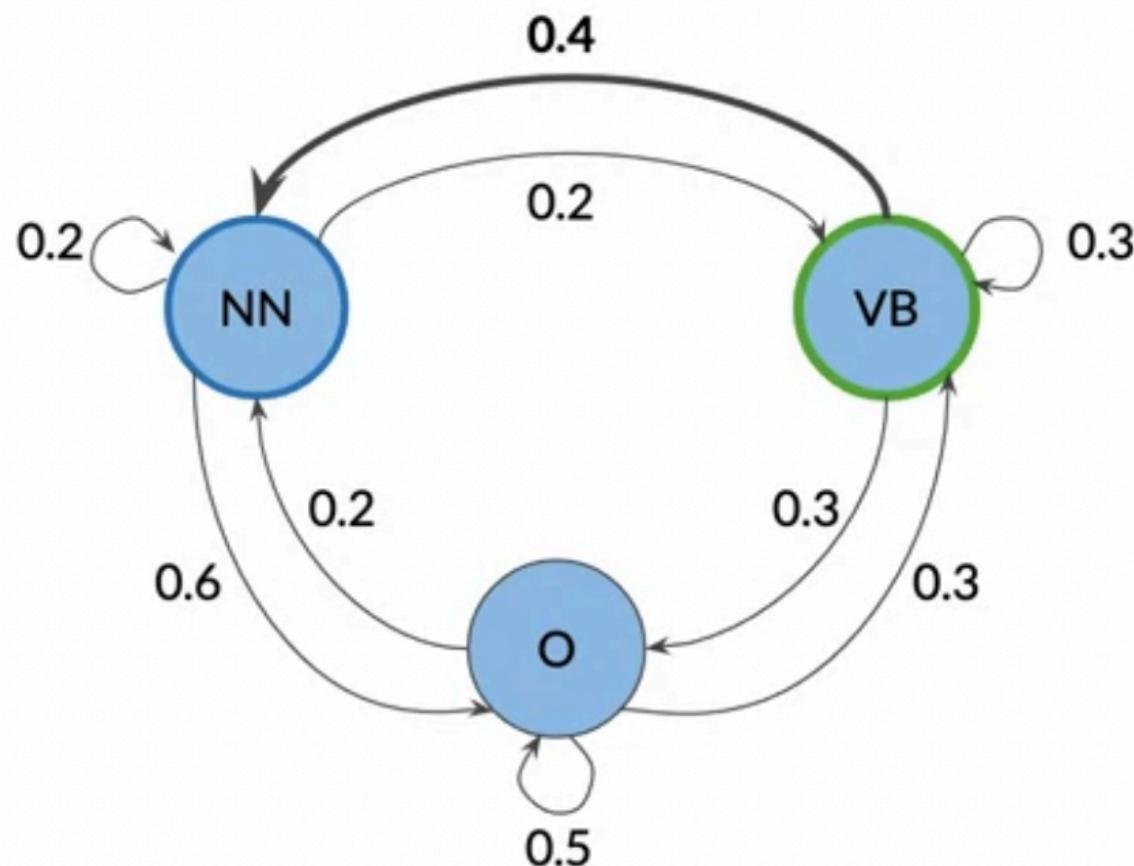
POS tags as States



Transition probabilities

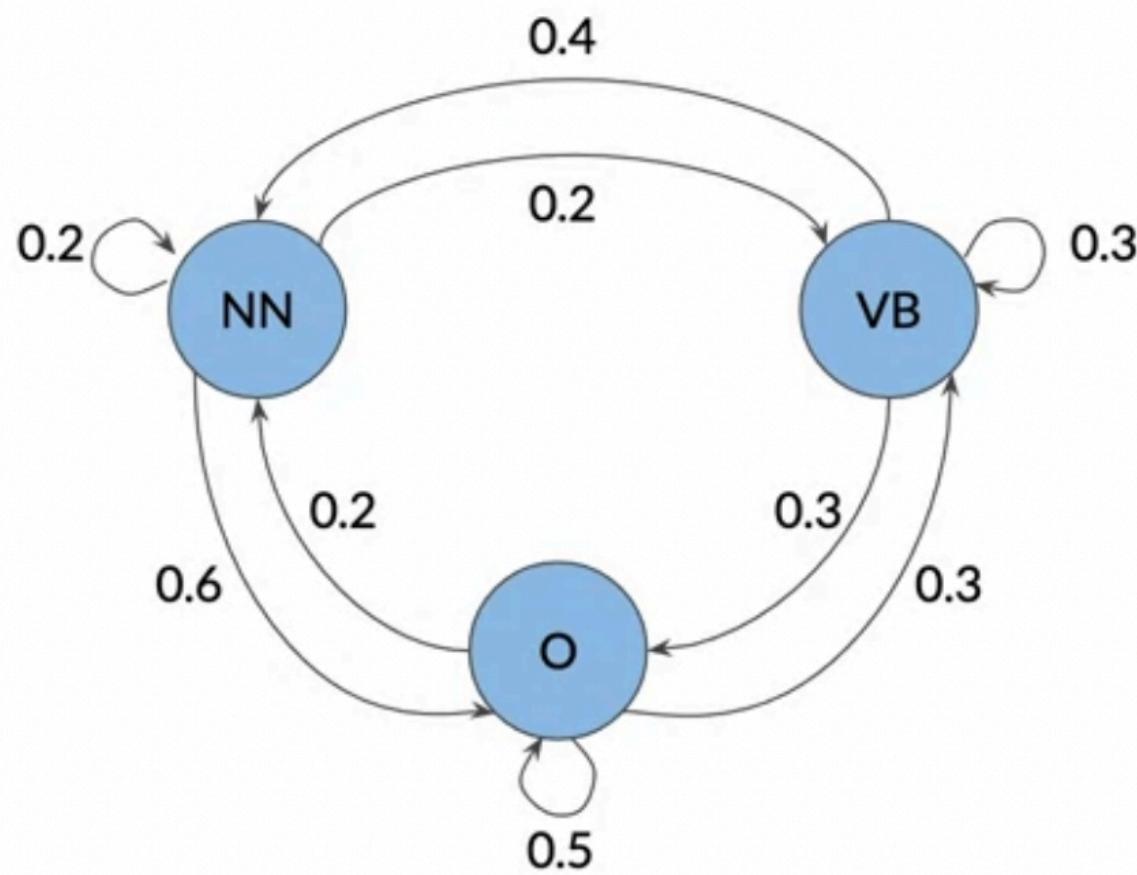


Transition probabilities



Why not **learn something**?

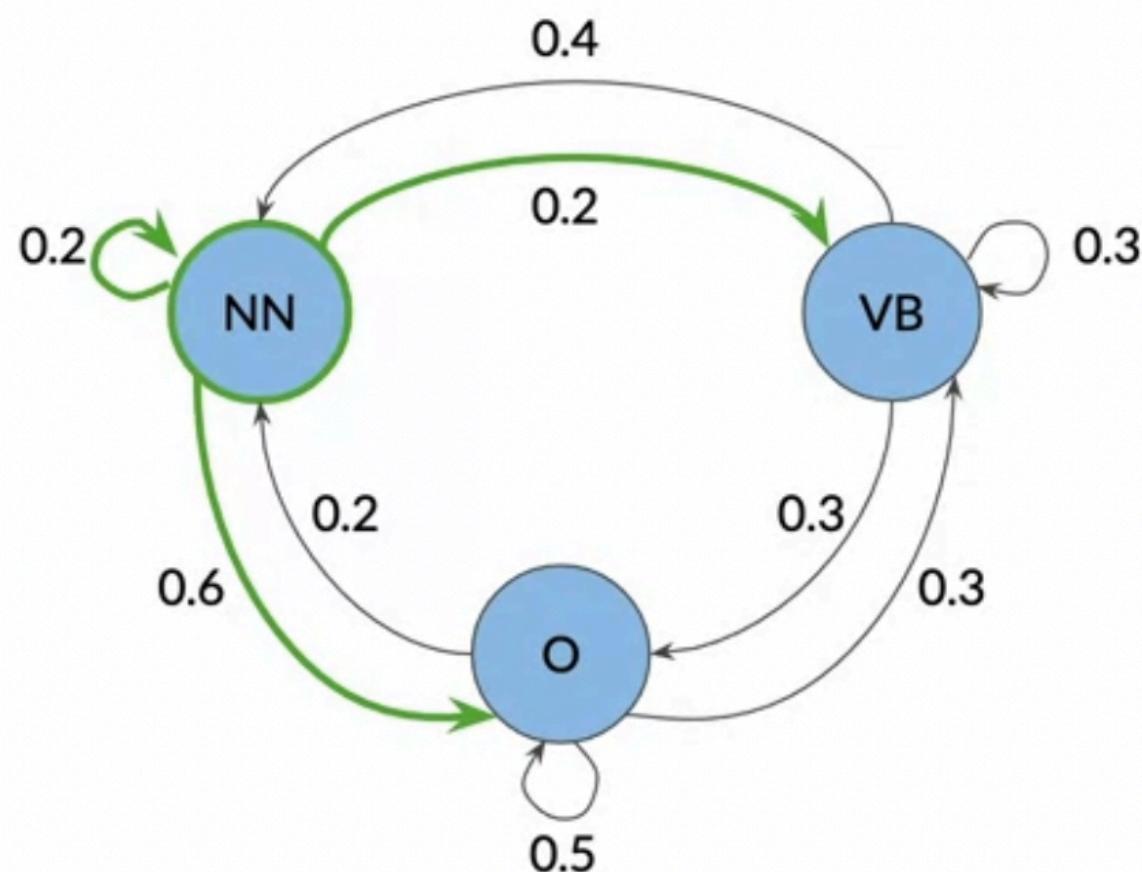
The transition matrix



$A =$

	NN	VB	O
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

The transition matrix

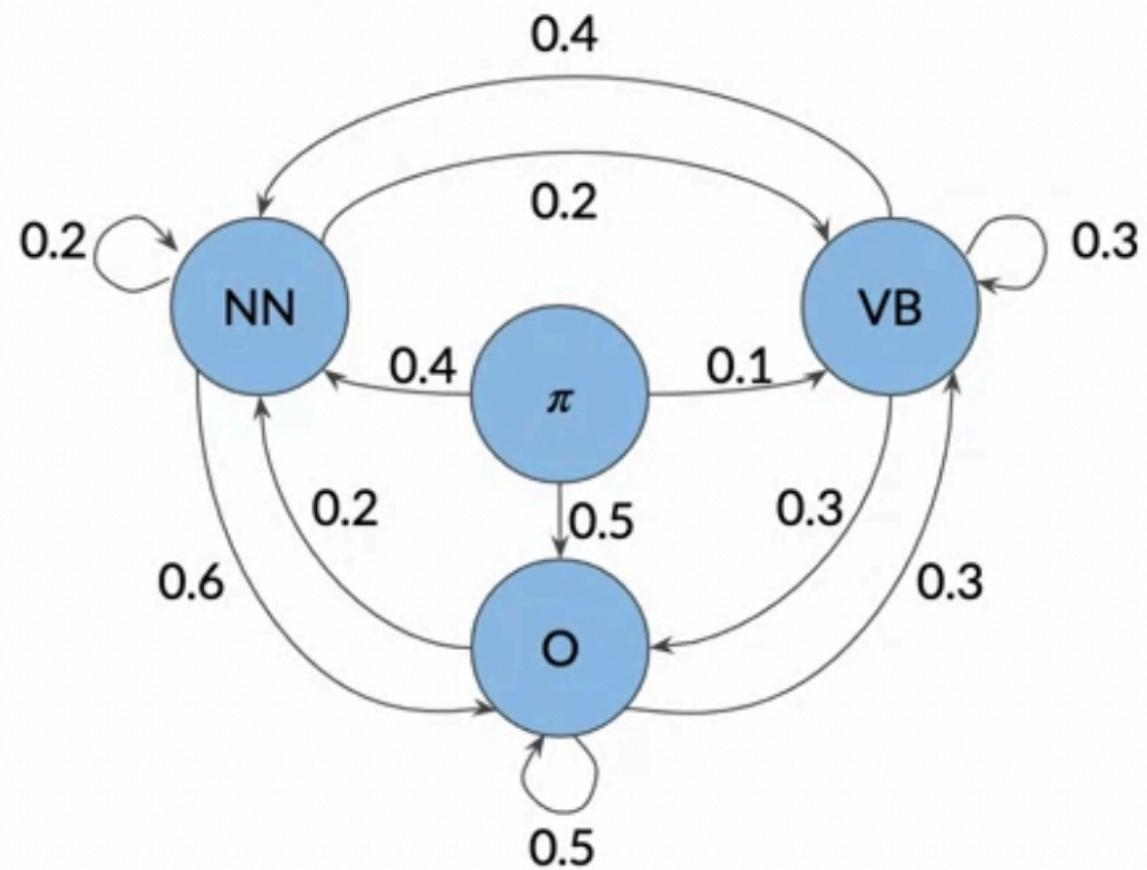


$A =$

	NN	VB	O
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

$$\sum_{j=1}^N a_{ij} = 1$$

Initial probabilities



$A =$

	NN	VB	O
π (initial)	0.4	0.1	0.5
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

Transition table and matrix

	NN	VB	O
π (initial)	0.4	0.1	0.5
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

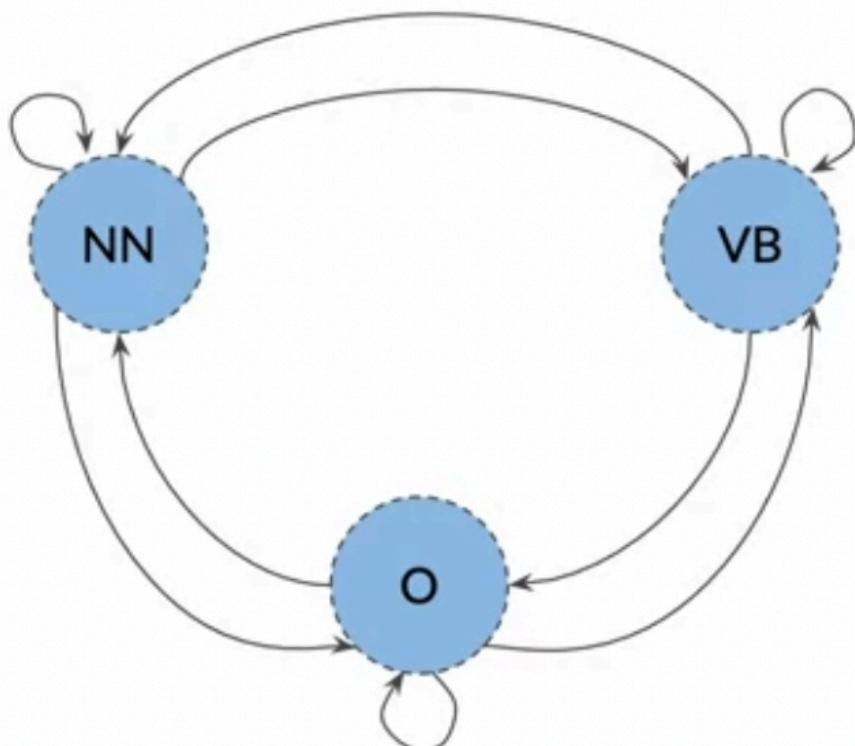
$$A = \begin{pmatrix} 0.4 & 0.1 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

Markov Chains and POS Tags

Summary

States	Transition matrix
$Q = \{q_1, \dots, q_N\}$	$A = \begin{pmatrix} a_{1,1} & \dots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N+1,1} & \dots & a_{N+1,N} \end{pmatrix}$

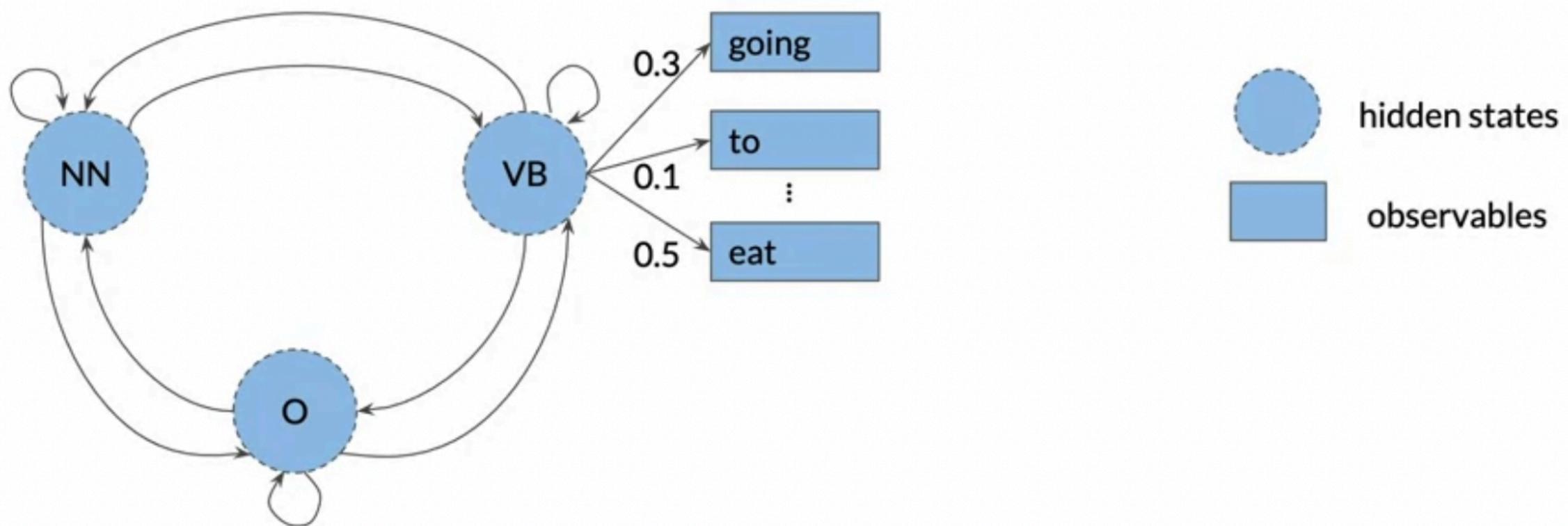
Hidden Markov Model



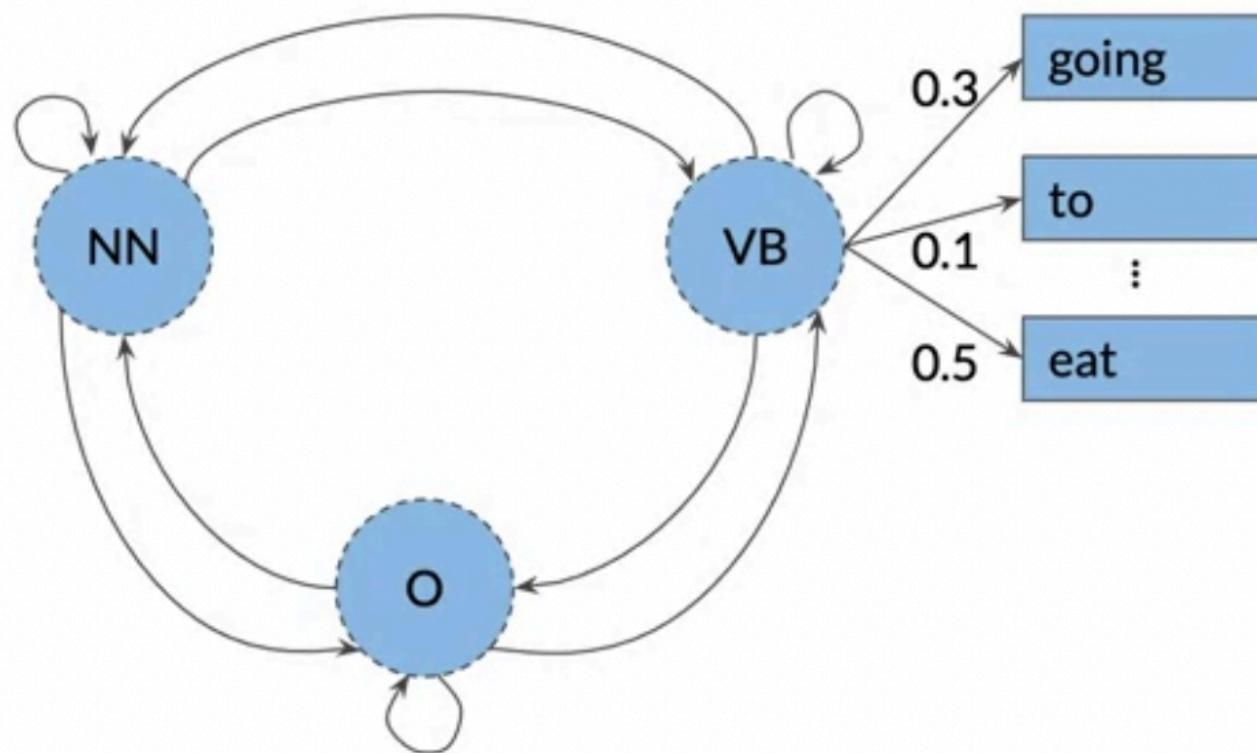
hidden states

Hidden Markov

Emission probabilities



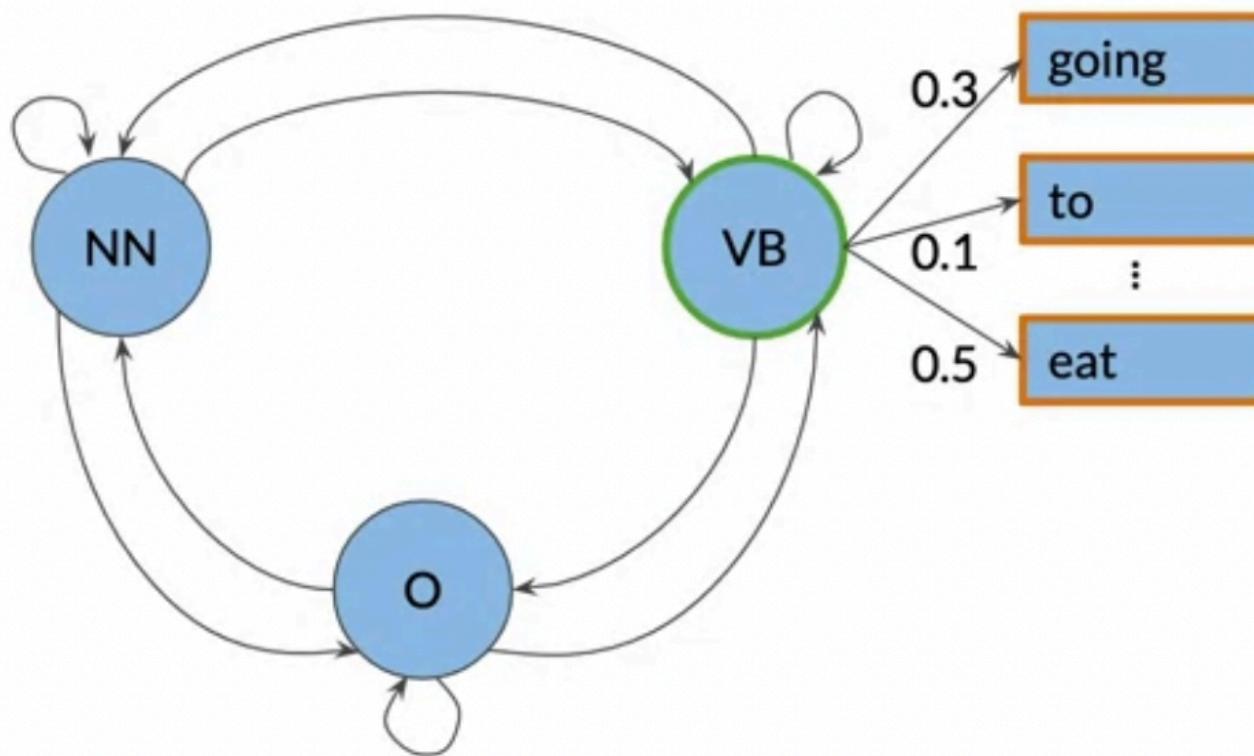
Emission probabilities



$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

Emission probabilities


$$B =$$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

The emission matrix

$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

$$\sum_{j=1}^V b_{ij} = 1$$

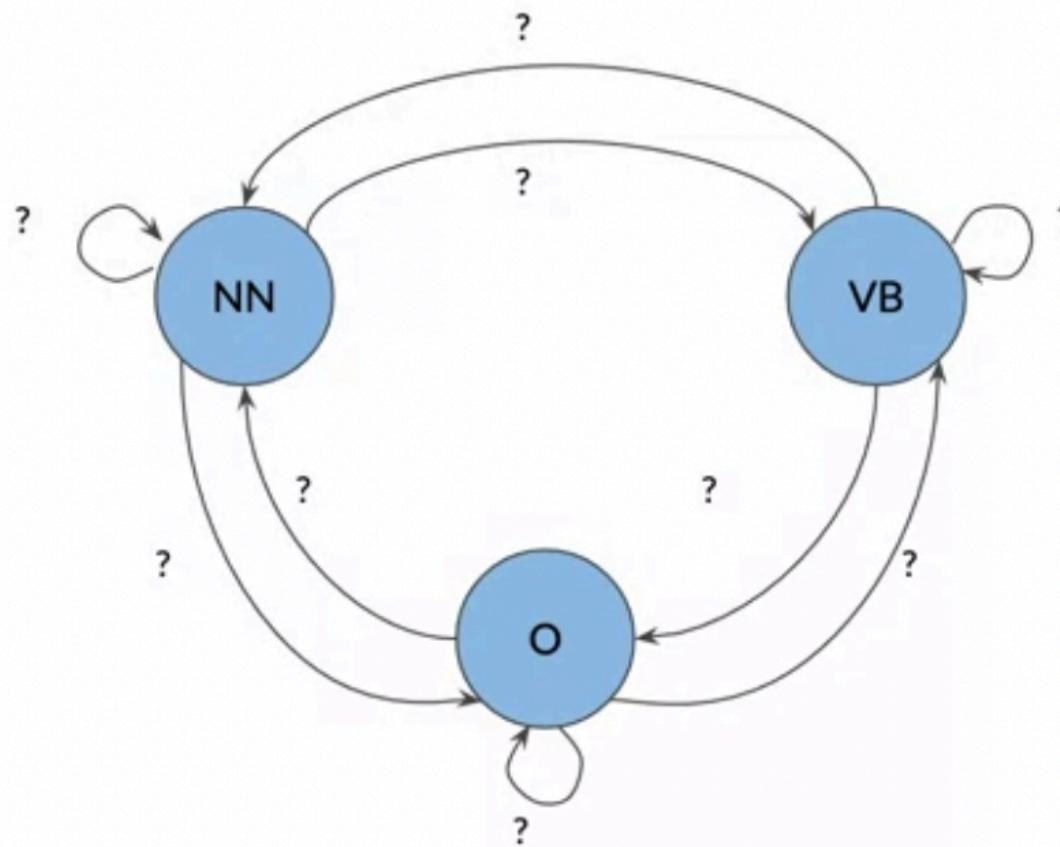
He lay on his back.

I'll be back.

Summary

States	Transition matrix	Emission matrix
$Q = \{q_1, \dots, q_N\}$	$A = \begin{pmatrix} a_{1,1} & \dots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N+1,1} & \dots & a_{N+1,N} \end{pmatrix}$	$B = \begin{pmatrix} b_{11} & \dots & b_{1V} \\ \vdots & \ddots & \vdots \\ b_{N1} & \dots & b_{NV} \end{pmatrix}$

Transition probabilities



1. Count occurrences of tag pairs

$$C(t_{i-1}, t_i)$$

2. Calculate probabilities using the counts

$$P(t_i | t_{i-1}) = \frac{C(t_{i-1}, t_i)}{\sum_{j=1}^N C(t_{i-1}, t_j)}$$

Populating the transition matrix

	NN	VB	O
π	$C(\pi, \text{NN})$		
NN (noun)	$C(\text{NN}, \text{NN})$		
VB (verb)	$C(\text{VB}, \text{NN})$		
O (other)	$C(\text{O}, \text{NN})$		

<S> in a station of the metro
<S> the apparition of these faces in the crowd:
<S> petals on a wet , black bough .

Ezra Pound - 1913

Populating the Transition Matrix

Populating the transition matrix

$$A = \begin{array}{|c|c|c|c|} \hline & \text{NN} & \text{VB} & \text{O} \\ \hline \pi & 1 & & \\ \hline \text{NN (noun)} & 0 & & \\ \hline \text{VB (verb)} & 0 & & \\ \hline \text{O (other)} & 6 & & \\ \hline \end{array}$$

<S> in a station of the metro
<S> the apparition of these faces in the crowd:
<S> petals on a wet, black bough.

Ezra Pound – 1913

Populating the transition matrix

$$A = \begin{array}{|c|c|c|c|} \hline & \text{NN} & \text{VB} & \text{O} \\ \hline \pi & 1 & & \\ \hline \text{NN (noun)} & C(\text{NN,NN}) & & \\ \hline \text{VB (verb)} & C(\text{VB,NN}) & & \\ \hline \text{O (other)} & C(\text{O,NN}) & & \\ \hline \end{array}$$

<S> in a station of the metro
<S> the apparition of these faces in the crowd:
<S> petals on a wet, black bough.

Ezra Pound - 1913

Populating the transition matrix

$$A = \begin{array}{|c|c|c|c|} \hline & \text{NN} & \text{VB} & \text{O} \\ \hline \pi & 1 & 0 & 2 \\ \hline \text{NN (noun)} & 0 & 0 & 6 \\ \hline \text{VB (verb)} & 0 & 0 & 0 \\ \hline \text{O (other)} & 6 & 0 & 8 \\ \hline \end{array}$$

<S> in a station of the metro

<S> the apparition of these faces in the crowd:

<S> petals on a wet, black bough.

Ezra Pound - 1913

Populating the transition matrix

$$A = \begin{array}{|c|c|c|c|c|} \hline & \text{NN} & \text{VB} & \text{o} & \\ \hline \pi & 1 & 0 & 2 & 3 \\ \hline \text{NN} & 0 & 0 & 6 & 6 \\ \hline \text{VB} & 0 & 0 & 0 & 0 \\ \hline \text{o} & 6 & 0 & 8 & 14 \\ \hline \end{array}$$

$$P(\text{NN}|\pi) = \frac{C(\pi, \text{NN})}{\sum_{j=1}^N C(\pi, t_j)} = \frac{1}{3}$$

Populating the transition matrix

$$A = \begin{array}{|c|c|c|c|c|}\hline & \text{NN} & \text{VB} & \text{O} & \\ \hline \pi & 1 & 0 & 2 & 3 \\ \hline \text{NN} & 0 & 0 & 6 & 6 \\ \hline \text{VB} & 0 & 0 & 0 & 0 \\ \hline \text{O} & 6 & 0 & 8 & 14 \\ \hline \end{array}$$

$$P(\text{NN}|O) = \frac{C(O, \text{NN})}{\sum_{j=1}^N C(O, t_j)} = \frac{6}{14}$$

Smoothing

	NN	VB	O	
π	$1+\epsilon$	$0+\epsilon$	$2+\epsilon$	$3+3^*\epsilon$
NN	$0+\epsilon$	$0+\epsilon$	$6+\epsilon$	$6+3^*\epsilon$
VB	$0+\epsilon$	$0+\epsilon$	$0+\epsilon$	$0+3^*\epsilon$
O	$6+\epsilon$	$0+\epsilon$	$8+\epsilon$	$14+3^*\epsilon$

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i) + \epsilon}{\sum_{j=1}^N C(t_{i-1}, t_j) + N * \epsilon}$$

The emission matrix

$$B = \begin{array}{|c|c|c|c|} \hline & \text{in} & \text{a} & \dots \\ \hline \text{NN (noun)} & C(\text{NN}, \text{in}) & & \\ \hline \text{VB (verb)} & C(\text{VB}, \text{in}) & & \\ \hline \text{O (other)} & C(\text{O}, \text{in}) & & \\ \hline \end{array}$$

<S> in a station of the metro

<S> the apparition of these faces in the crowd :

<S> petals on a wet , black bough .

Ezra Pound – 1913

Populating the
Emission Matrix

The emission matrix

$$B = \begin{array}{|c|c|c|c|}\hline & \text{in} & \text{a} & \dots \\ \hline \text{NN (noun)} & 0 & & \\ \hline \text{VB (verb)} & 0 & & \\ \hline \text{O (other)} & 2 & & \\ \hline \end{array}$$

<S> in a station of the metro
<S> the apparition of these faces in the crowd:
<S> petals on a wet, black bough.

Ezra Pound – 1913

The emission matrix

$B =$

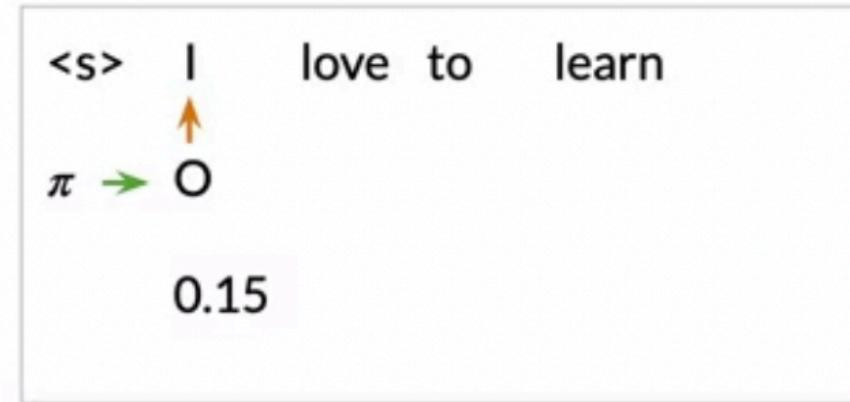
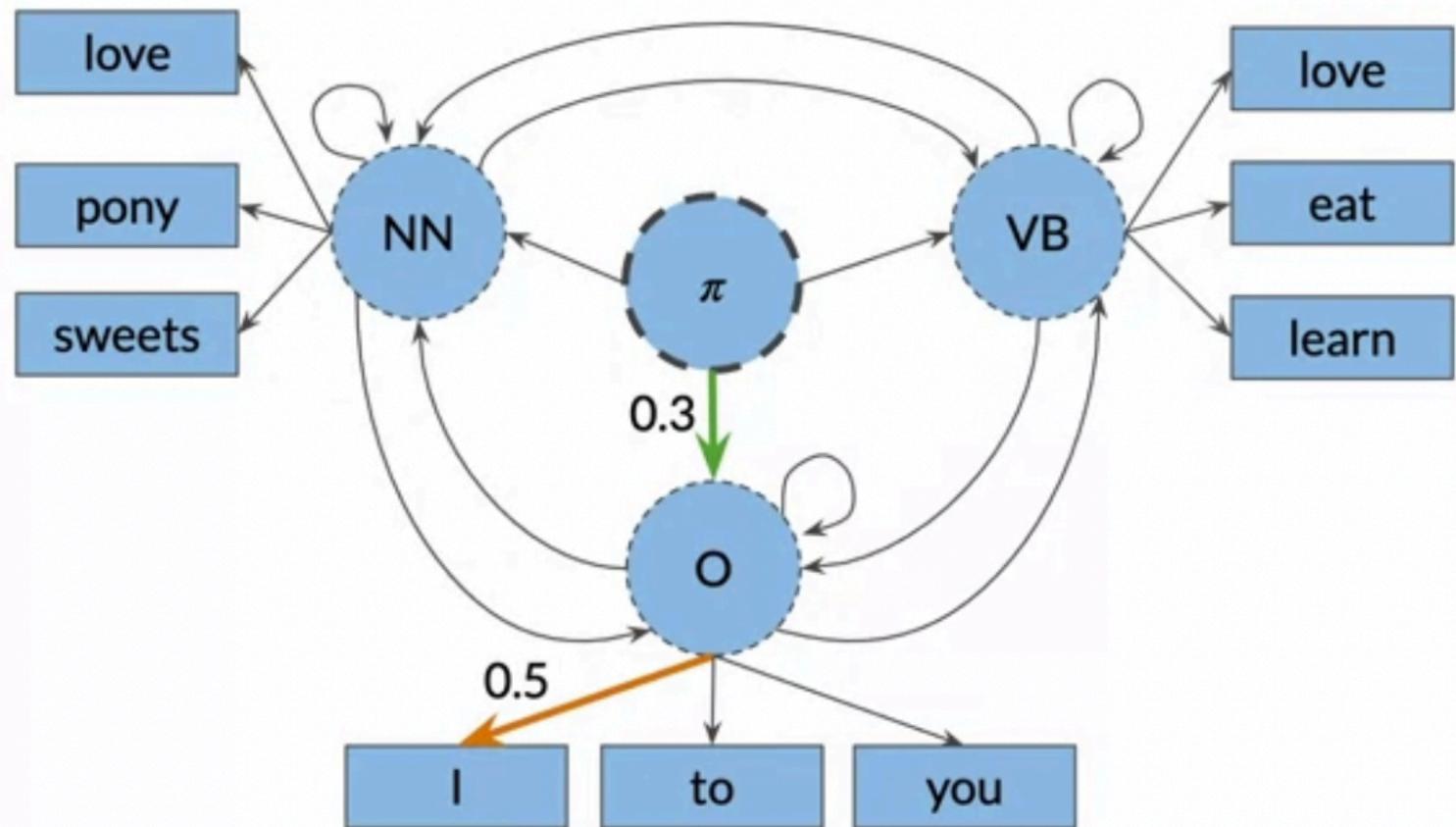
	in	a	...
NN (noun)	0
VB (verb)	0
O (other)	2

$$\begin{aligned} P(w_i|t_i) &= \frac{C(t_i, w_i) + \epsilon}{\sum_{j=1}^V C(t_i, w_j) + N * \epsilon} \\ &= \frac{C(t_i, w_i) + \epsilon}{C(t_i) + N * \epsilon} \end{aligned}$$

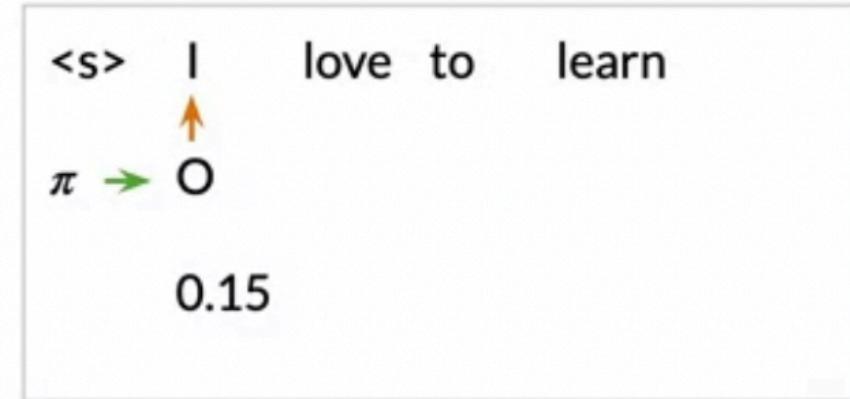
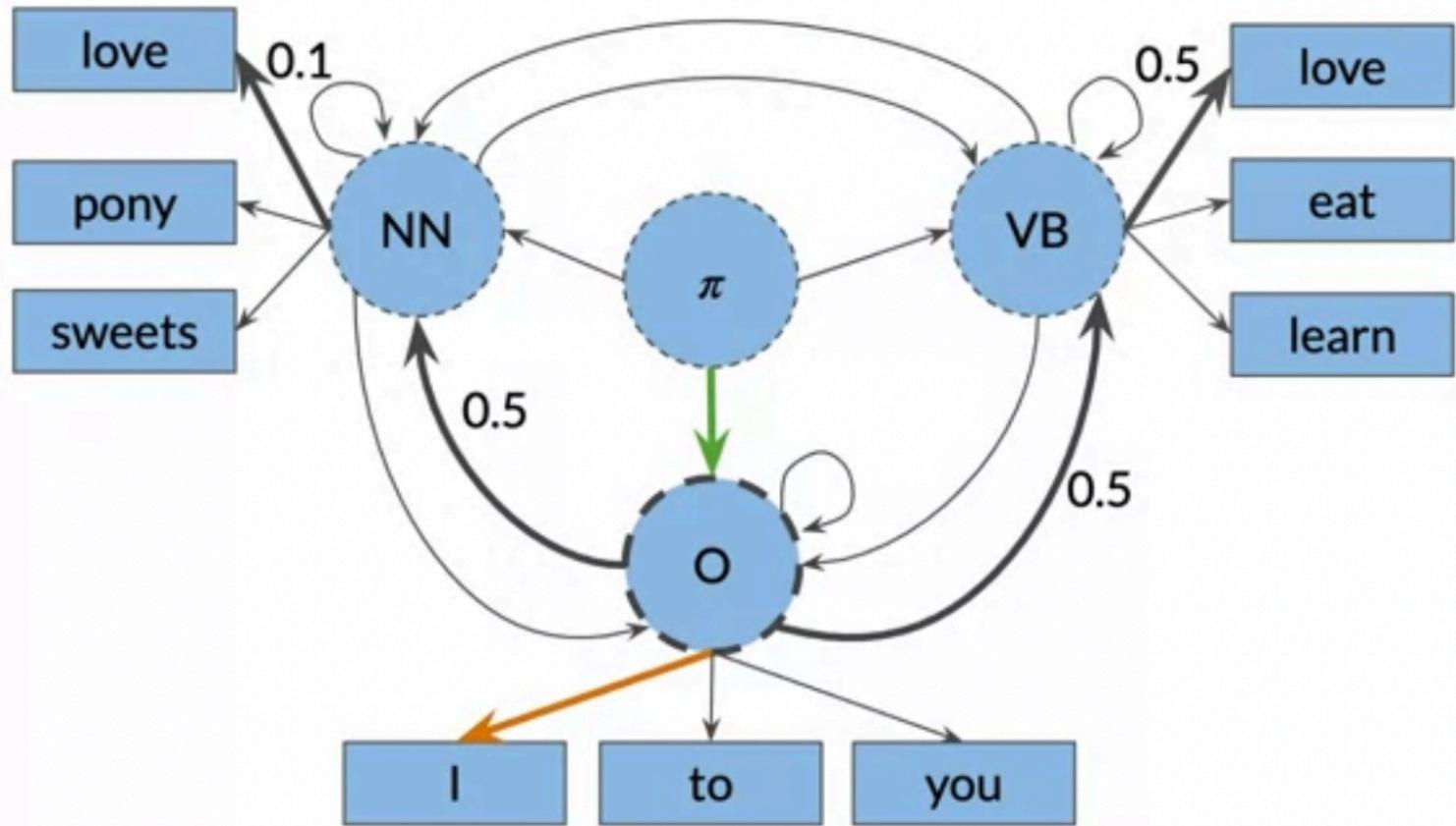
Summary

1. Calculate transition and emission matrix
2. How to apply smoothing

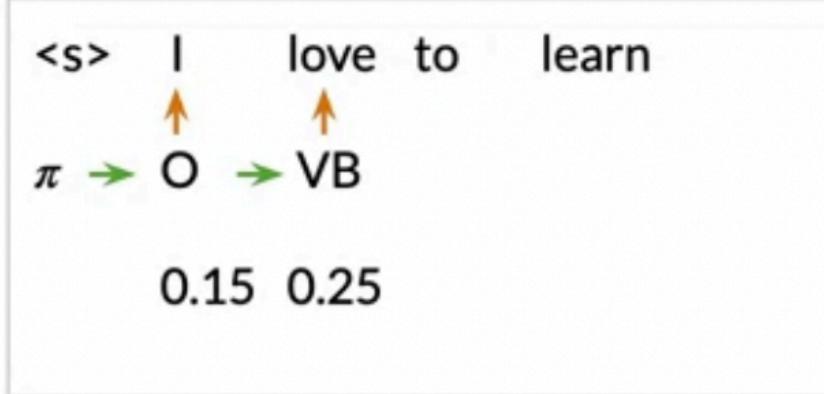
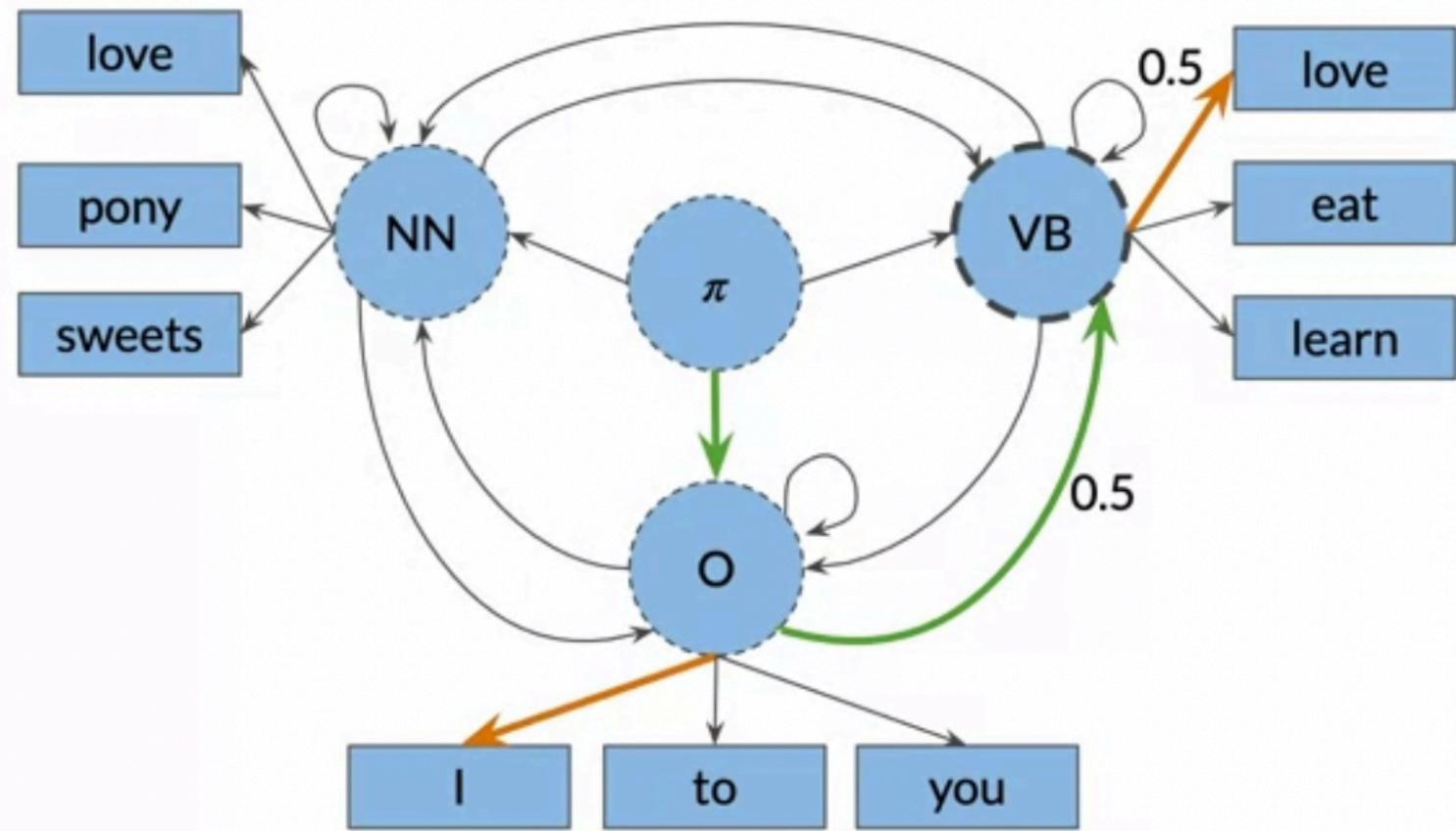
Viterbi algorithm – a graph algorithm



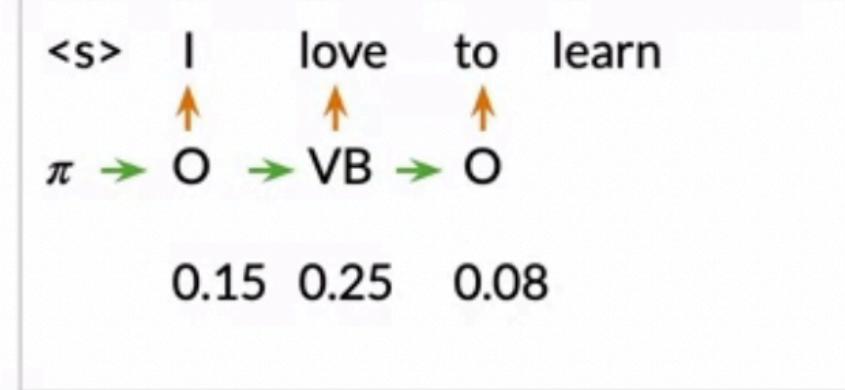
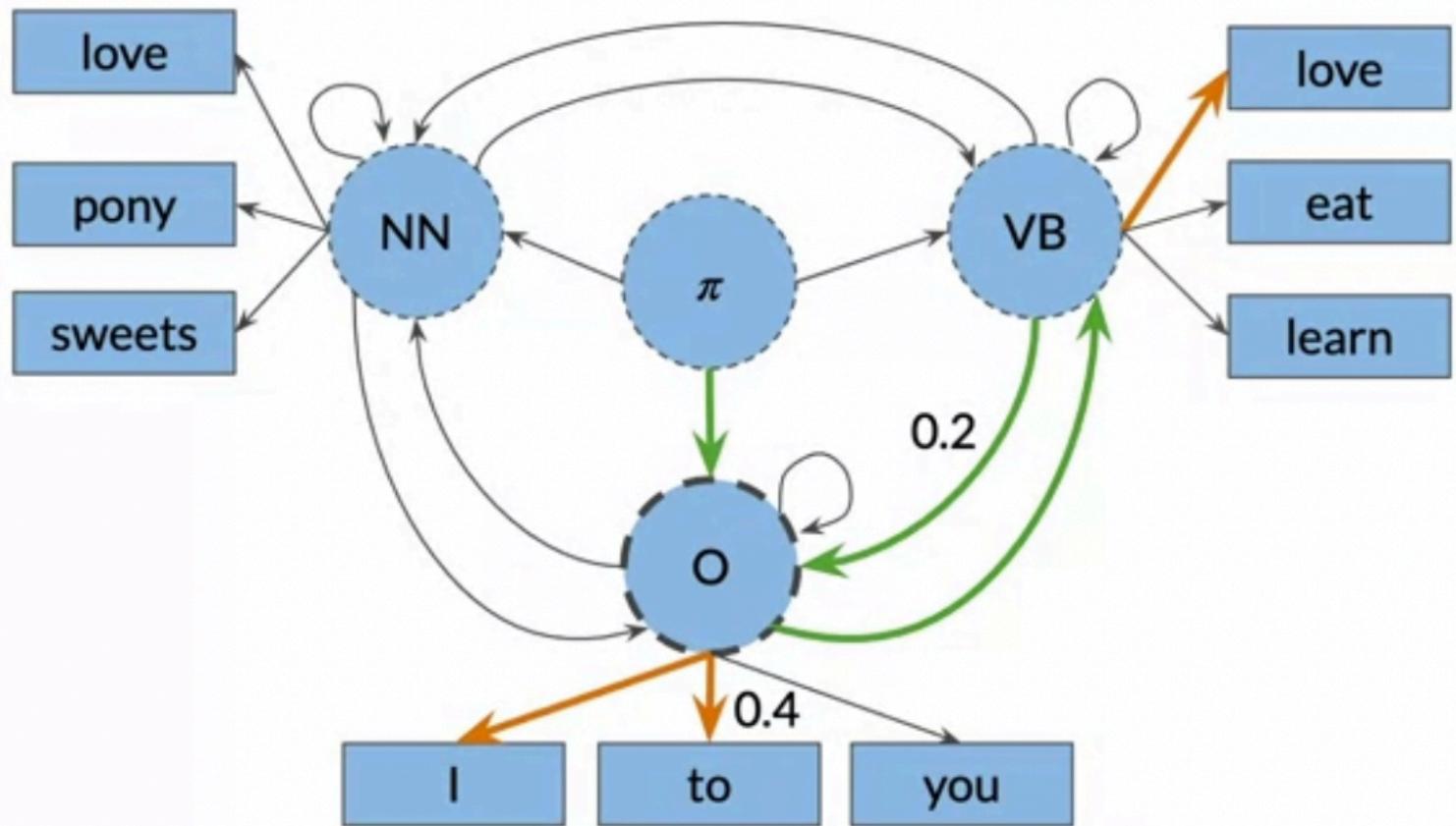
Viterbi algorithm – a graph algorithm



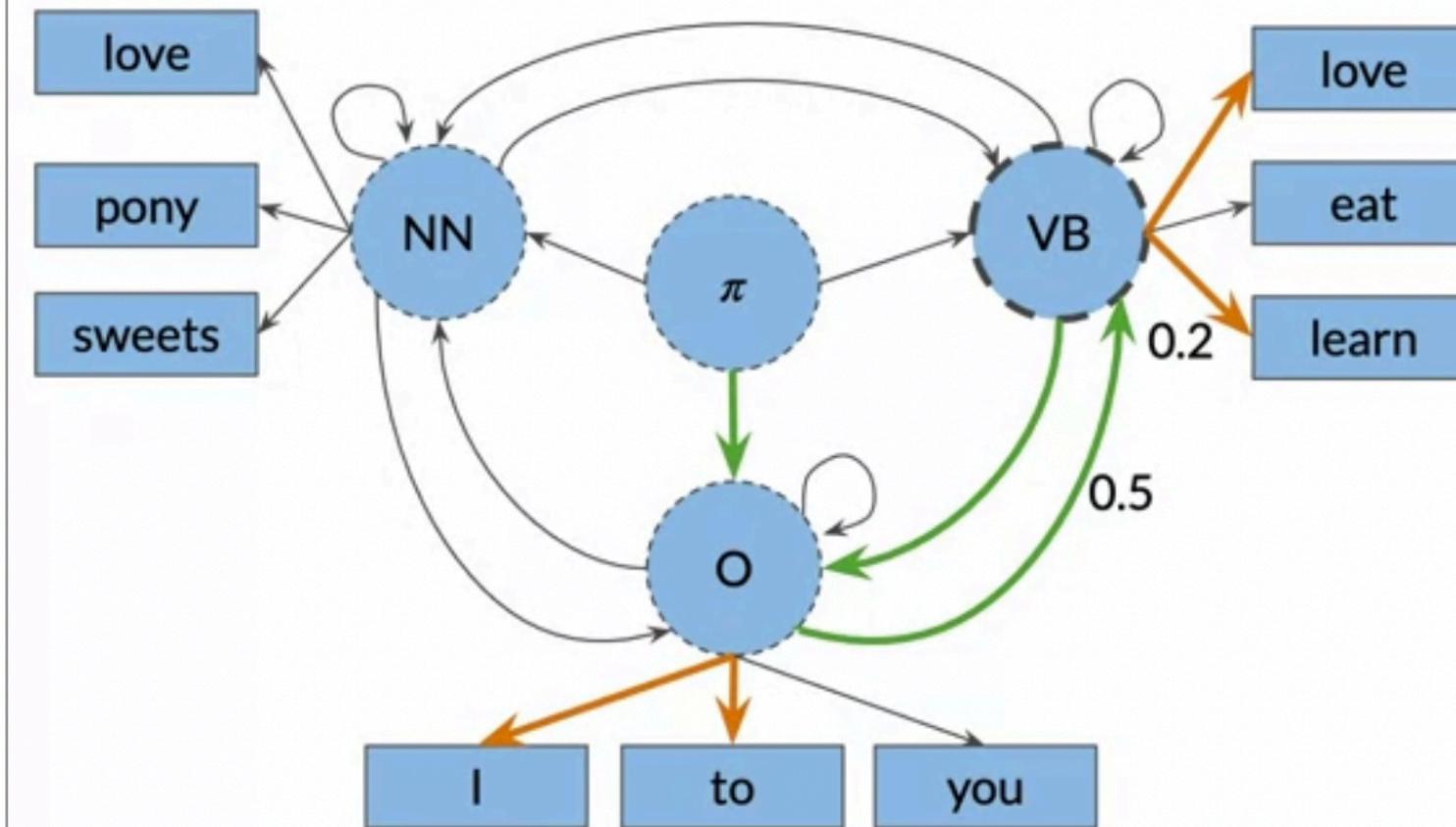
Viterbi algorithm - a graph algorithm



Viterbi algorithm – a graph algorithm

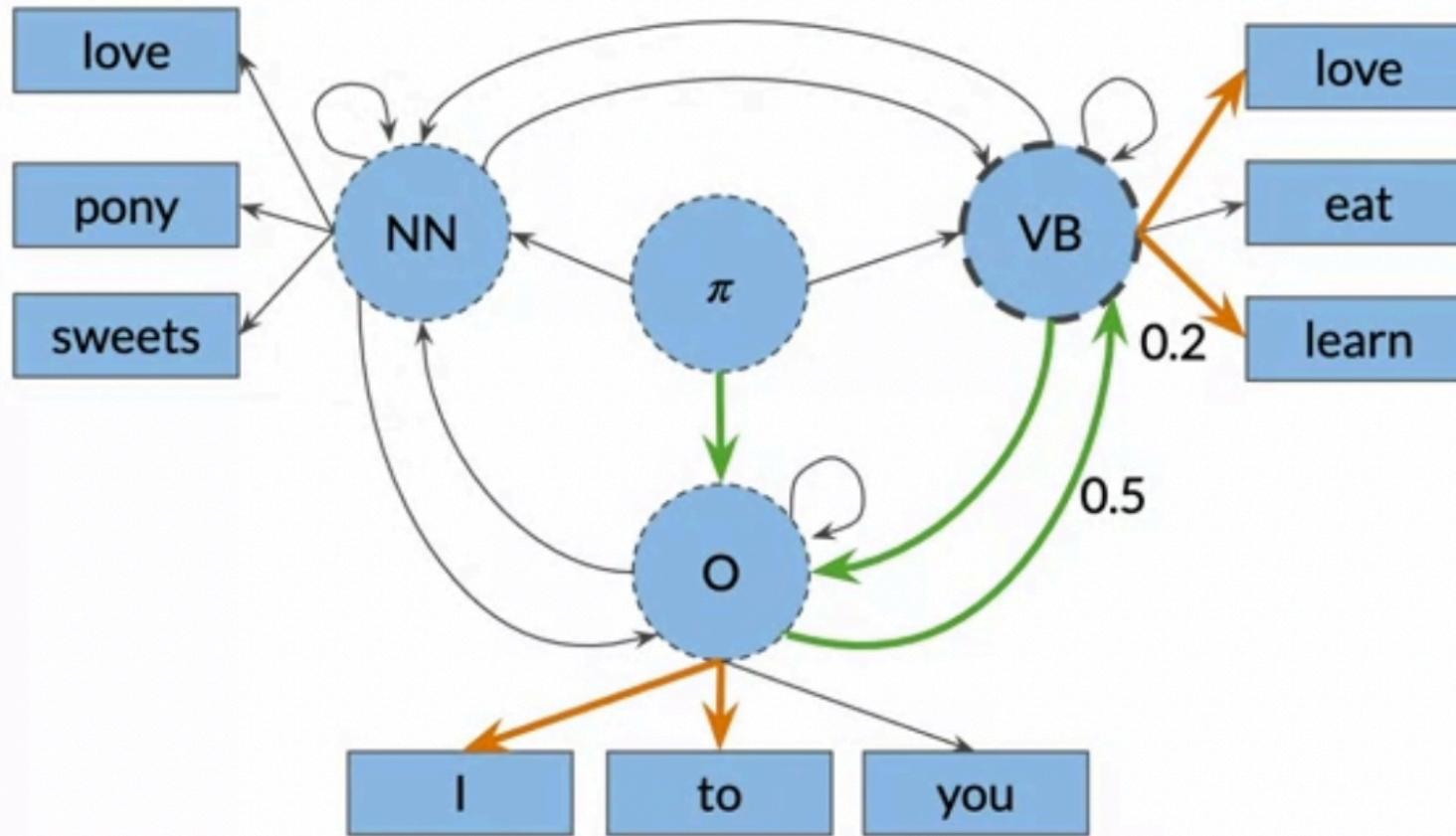


Viterbi algorithm – a graph algorithm



<s>	I	love	to	learn
$\pi \rightarrow$	O	VB	O	VB
	0.15	0.25	0.08	0.1

Viterbi algorithm – a graph algorithm



$$\begin{array}{ccccccc} <s> & I & \text{love} & \text{to} & \text{learn} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \pi \rightarrow O \rightarrow VB \rightarrow O \rightarrow VB \end{array}$$
$$0.15 * 0.25 * 0.08 * 0.1$$

Probability for this sequence of hidden states: 0.0003

Viterbi algorithm – Steps

1. Initialization step
2. Forward pass
3. Backward pass

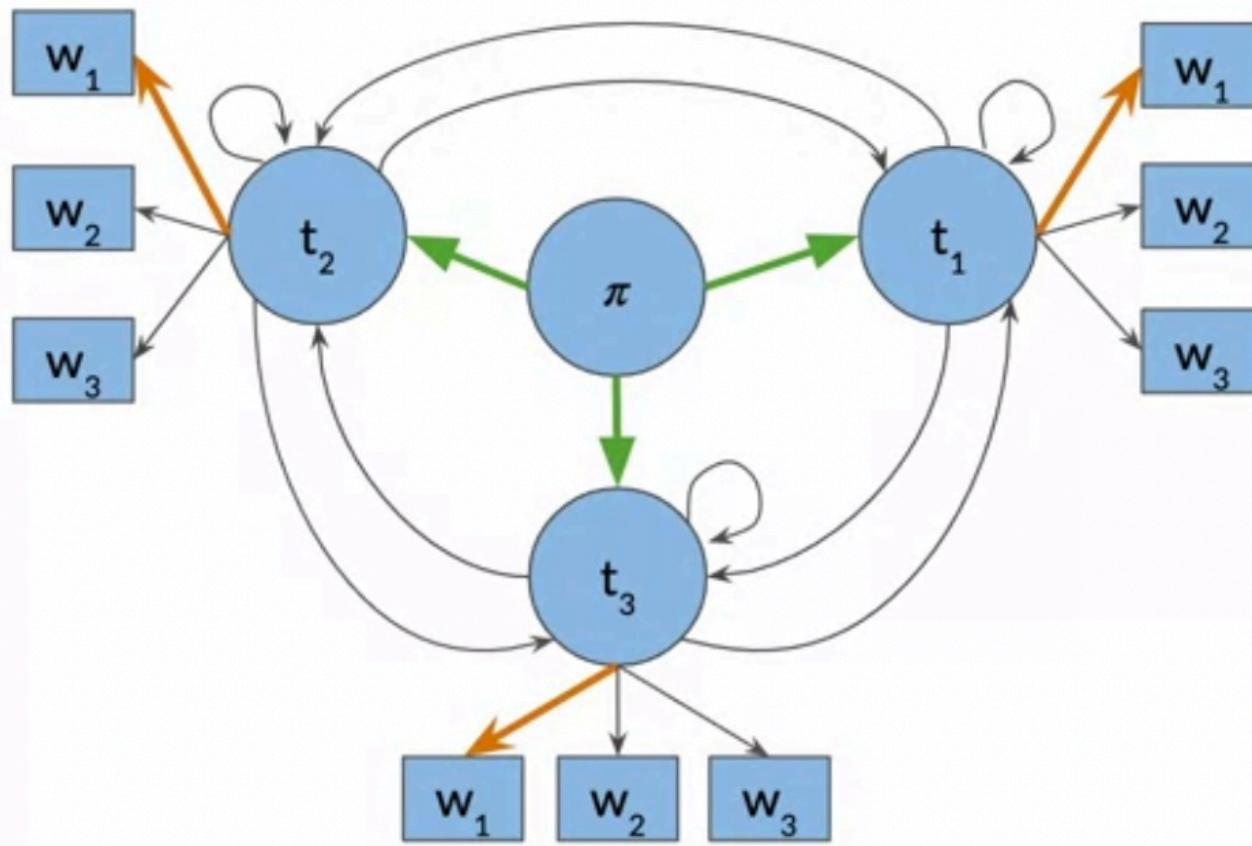
$C =$

	w_1	w_2	\dots	w_K
t_1				
\dots				
t_N				

$D =$

	w_1	w_2	\dots	w_K
t_1				
\dots				
t_N				

Initialization step

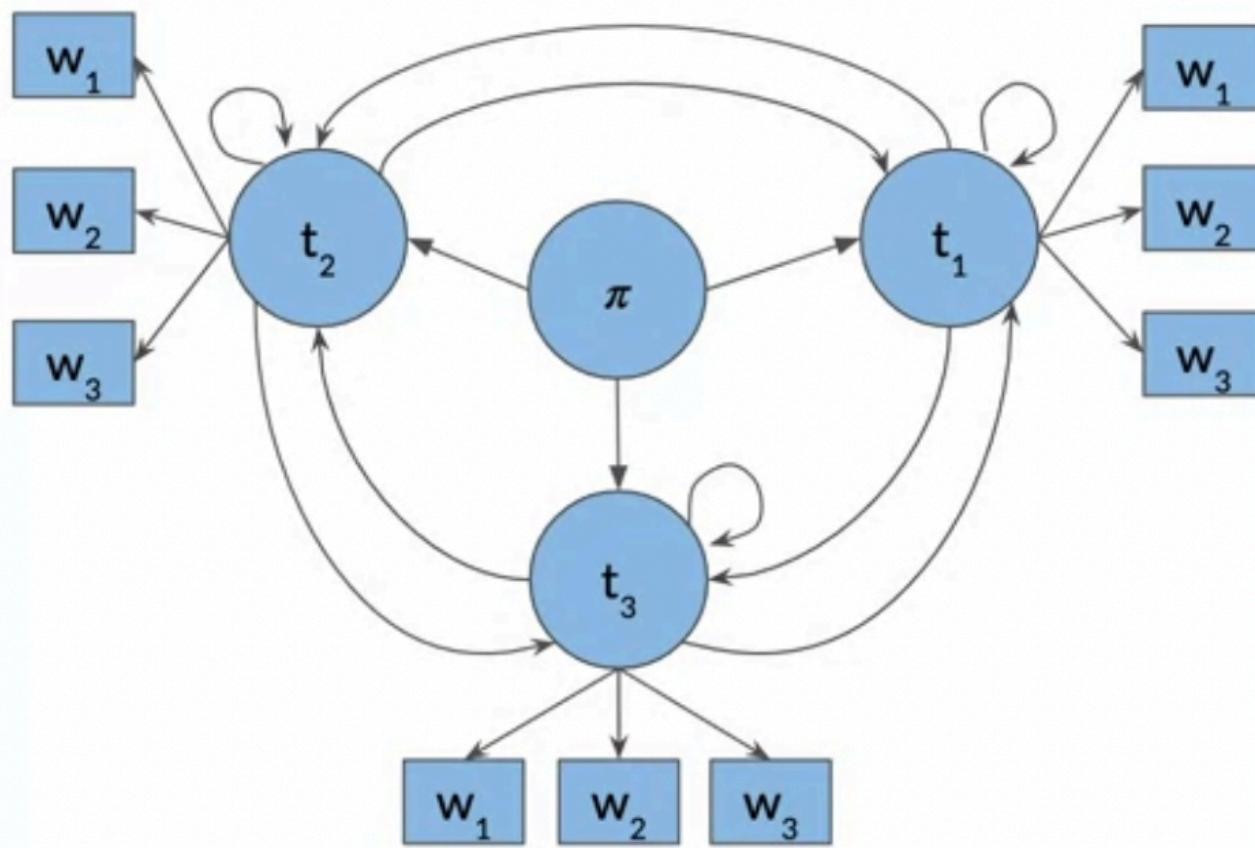


$C =$

	w_1	w_2	\dots	w_K
t_1	$c_{1,1}$			
\dots				
t_N	$c_{N,1}$			

$$c_{i,1} = \boxed{\pi_i} * \boxed{b_{i,cindex(w_1)}} \\ = a_{1,i} * b_{i,cindex(w_1)}$$

Initialization step

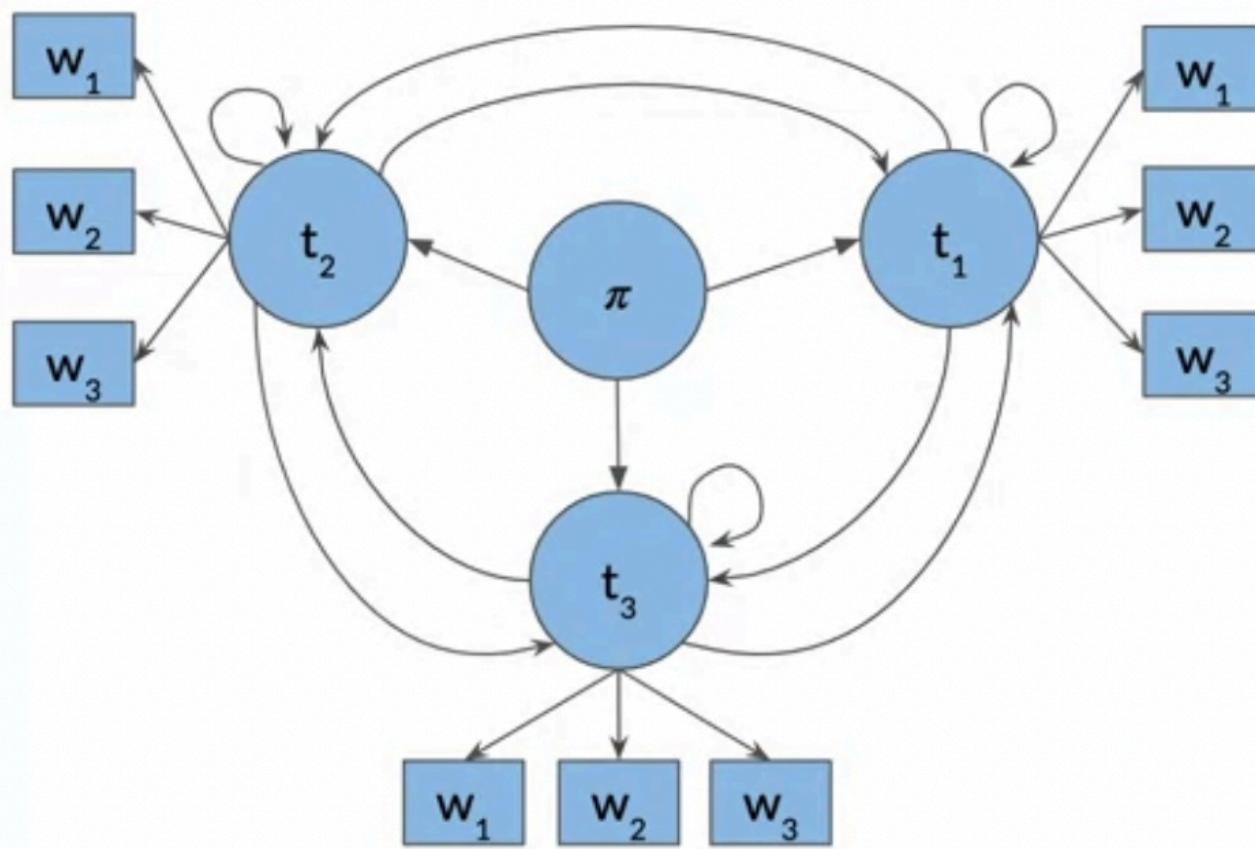


$D =$

	w_1	w_2	\dots	w_K
t_1	$d_{1,1}$			
\dots				
t_N	$d_{N,1}$			

$$d_{i,1} = 0$$

Forward pass

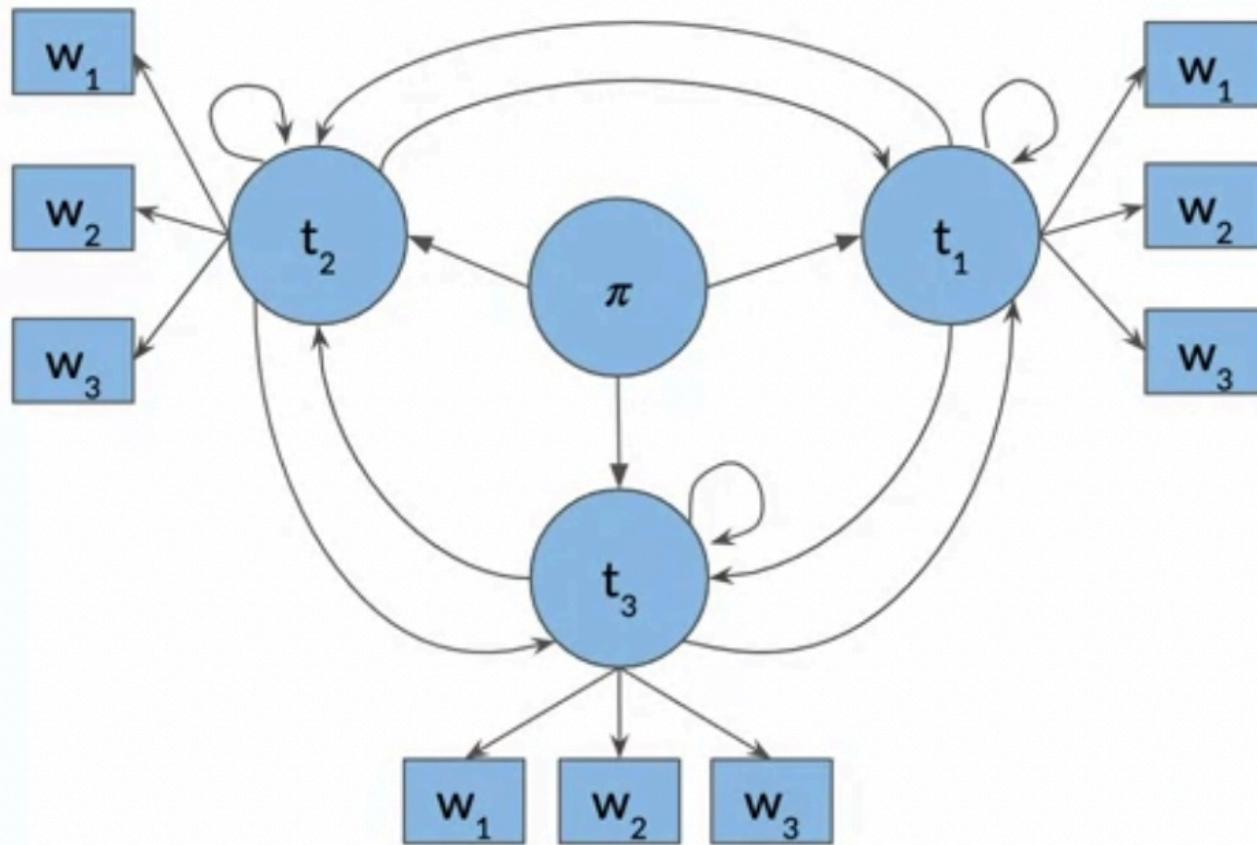


$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,c\text{index}(w_j)}$$

Forward pass

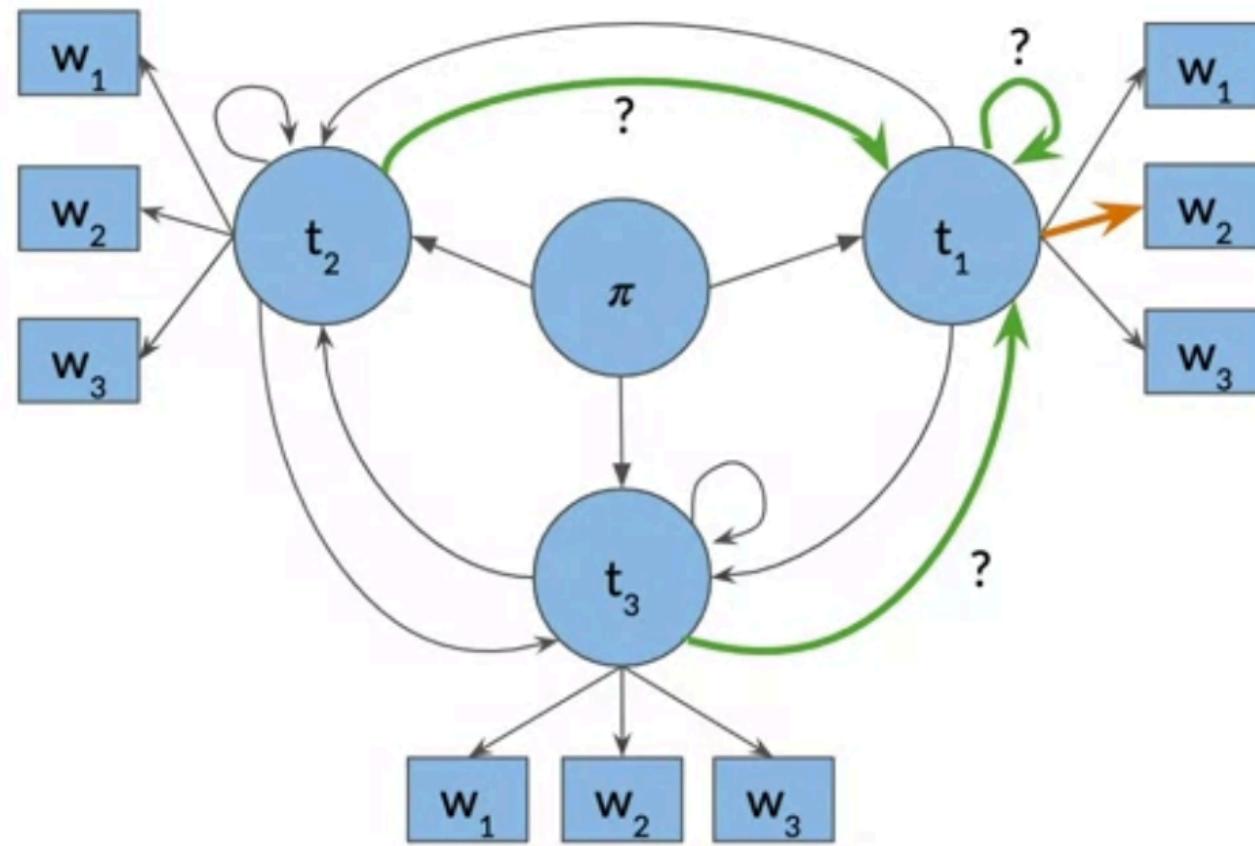


$C =$

	w_1	w_2	\dots	w_K
t_1	$c_{1,1}$	$c_{1,2}$	\dots	$c_{1,K}$
\dots				
t_N	$c_{N,1}$	$c_{N,2}$	\dots	$c_{N,K}$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,c\text{index}(w_2)}$$

Forward pass

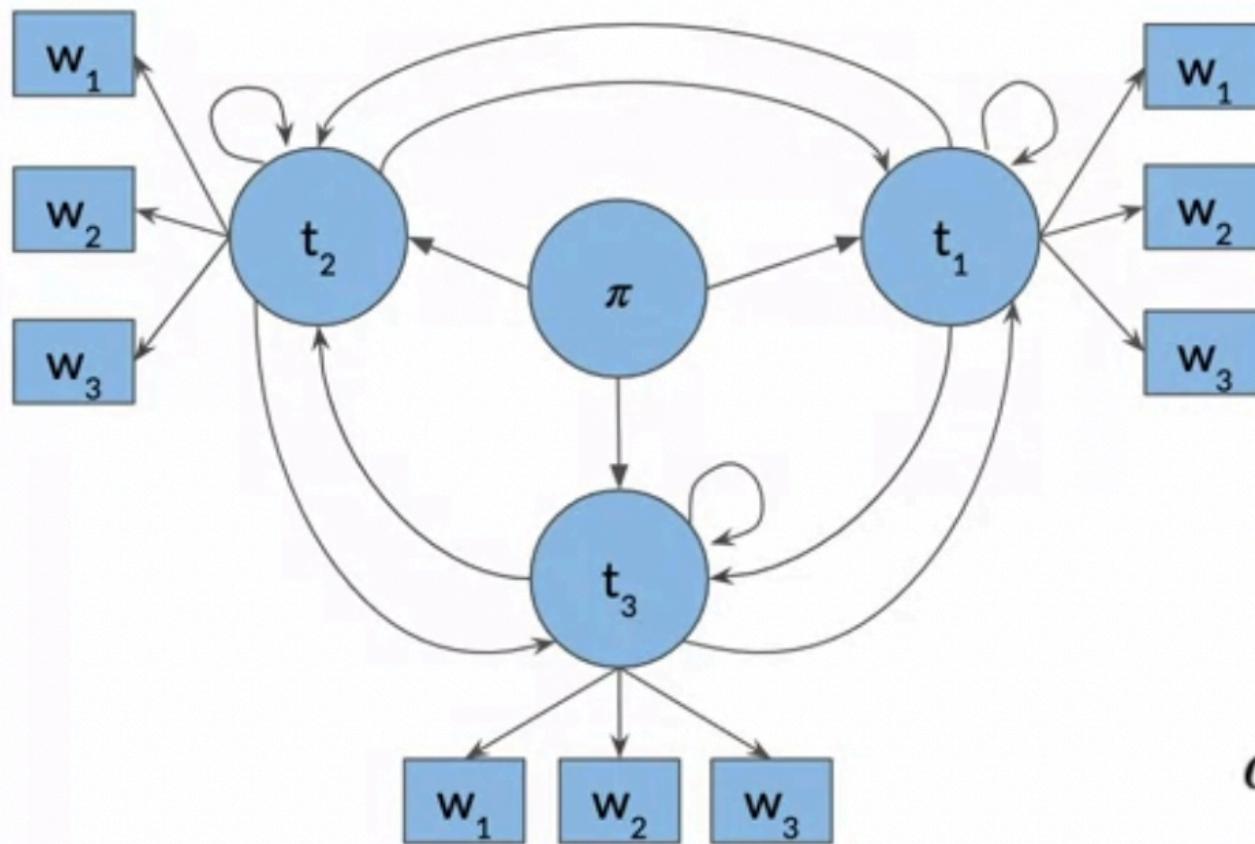


$C =$

	w_1	w_2	\dots	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
\dots				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,c\text{index}(w_2)}$$

Forward pass



$D =$

	w_1	w_2	\dots	w_K
t_1	$d_{1,1}$	$d_{1,2}$		$d_{1,K}$
\dots				
t_N	$d_{N,1}$	$d_{N,2}$		$d_{N,K}$

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,c\text{index}(w_j)}$$

$$d_{i,j} = \operatorname{argmax}_k c_{k,j-1} * a_{k,i} * b_{i,c\text{index}(w_j)}$$

Backward pass

$$C = \begin{array}{|c|c|c|c|c|} \hline & w_1 & w_2 & \dots & w_K \\ \hline t_1 & c_{1,1} & c_{1,2} & & c_{1,K} \\ \hline \dots & & & & \\ \hline t_N & c_{N,1} & c_{N,2} & & c_{N,K} \\ \hline \end{array}$$

$D = \begin{array}{|c|c|c|c|c|} \hline & w_1 & w_2 & \dots & w_K \\ \hline t_1 & d_{1,1} & d_{1,2} & & d_{1,K} \\ \hline \dots & & & & \\ \hline t_N & d_{N,1} & d_{N,2} & & d_{N,K} \\ \hline \end{array}$

$s = \operatorname{argmax}_i c_{i,K}$

Backward pass

$C =$

	w_1	w_2	w_3	w_4	w_5
t_1	0.25	0.125	0.025	0.0125	0.01
t_2	0.1	0.025	0.05	0.01	0.003
t_3	0.3	0.05	0.025	0.02	0.0000
t_4	0.2	0.1	0.000	0.0025	0.0003

$$s = \operatorname{argmax}_i c_{i,K} = 1$$

Backward pass

$$D = \begin{array}{|c|c|c|c|c|c|}\hline & w_1 & w_2 & w_3 & w_4 & w_5 \\ \hline t_1 & 0 & 1 & 3 & 2 & 3 \\ \hline t_2 & 0 & 2 & 4 & 1 & 3 \\ \hline t_3 & 0 & 2 & 4 & 1 & 4 \\ \hline t_4 & 0 & 4 & 4 & 3 & 1 \\ \hline\end{array}$$


<s>	w1	w2	w3	w4	w5

$$s = \operatorname{argmax}_i c_{i,K} = 1$$

Backward pass

$$D = \begin{array}{|c|c|c|c|c|c|}\hline & w_1 & w_2 & w_3 & w_4 & w_5 \\ \hline t_1 & 0 & 1 & 3 & 2 & 3 \\ \hline t_2 & 0 & 2 & 4 & 1 & 3 \\ \hline t_3 & 0 & 2 & 4 & 1 & 4 \\ \hline t_4 & 0 & 4 & 4 & 3 & 1 \\ \hline\end{array}$$

<s>	w1	w2	w3	w4	w5
					t_1

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

```
<s> w1 w2 w3 w4 w5  
 $\pi \leftarrow t_2 \leftarrow t_3 \leftarrow t_1 \leftarrow t_3 \leftarrow t_1$ 
```

Implementation notes

1. In Python index starts with 0!
2. Use log probabilities

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,cindex(w_j)}$$


$$\log(c_{i,j}) = \max_k \log(c_{k,j-1}) + \log(a_{k,i}) + \log(b_{i,cindex(w_j)})$$