

Homework 1

Note. This homework is due in class on January 30, 2026. Please show detailed steps to explain how you get your solution. Simply providing a final answer may not warrant full credit.

Question 1 [43503/53903] The decimal representation of the numbers in a floating point system is specified by

$$(\beta, t, L, U) = (10, 4, -2, 1).$$

- (a) Explain what are β , t , L , and U .
- (b) Find the largest possible number in the system.
- (c) Find the smallest positive number in the system.
- (d) Find the total different fractions and exponents in the system.
- (e) Find the total different numbers in the system.

Question 2 [43503/53903] How would you perform the following calculations to avoid cancellation? Justify your answers.

- (a) Evaluate $\sqrt{x+1} - 1$ for $x \approx 0$.
- (b) Evaluate $\frac{1}{\cos^2 x - \sin^2 x}$ for $x \approx \frac{\pi}{4}$.

Question 3 [43503/53903] Let $\mathbf{S} \in \mathbb{C}^{m \times m}$ denote a skew-hermitian matrix, i.e., $\mathbf{S}^* = -\mathbf{S}$

- (a) Show the eigenvalues of \mathbf{S} are pure imaginary.
- (b) Show that $\mathbf{I} - \mathbf{S}$ is nonsingular. Here \mathbf{I} denote an identity matrix.
- (c) Show the matrix $\mathbf{Q} = (\mathbf{I} - \mathbf{S})^{-1}(\mathbf{I} + \mathbf{S})$ is unitary.

Question 4 [43503/53903] Let $\|\cdot\|$ denote any norm on \mathbb{C}^m and also the induced matrix norm on $\mathbb{C}^{m \times m}$. Show that $\rho(\mathbf{A}) \leq \|\mathbf{A}\|$, where $\rho(\mathbf{A})$ is the spectral radius of \mathbf{A} , i.e., the largest absolute eigenvalue of \mathbf{A} .

Question 5 [53903] In this question, we need to implement our own code. Consider a floating point system defined by the parameters

$$(\beta, t, L, U) = (2, 3, -2, 4).$$

- (a) Plot the decimal representation of all numbers that can be represented in this system, and zoom in to examine numbers close to zero.
- (b) Describe and explain your observations regarding the distribution of the floating point numbers. How is the spacing between consecutive numbers changes near zero, any patterns in the density of representable numbers, and the symmetry of the distribution?