Homework 2

Note. This homework is due in class on February 20, 2026. Please show detailed steps to explain how you get your solution. Simply providing a final answer may not warrant full credit.

Question 1 [43503/53903] Compute singular value decomposition (SVD) for the following matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}.$$

Question 2 [43503/53903] Compute the SVD of the matrix $\mathbf{A} = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}$ in the form of $\mathbf{A} = \mathbf{U\Sigma V}^*$. Notice, the SVD is not unique, you only need to find one and validate your solution.

(a) What are the 1-norm, 2-norm, ∞-norm, and Frobenius norm of $\mathbf{A}$?

(b) Verify that $\det(\mathbf{A}) = \lambda_1\lambda_2$ and $|\det(\mathbf{A})| = \sigma_1\sigma_2$, where the λ_1, λ_2 are eigenvalues and σ_1, σ_2 are singular values of $\mathbf{A}$.

Question 3 [43503/53903] Show the following statements hold.

(a) Given $\mathbf{A} \in \mathbb{C}^{m\times n}$ with $m \geq n$, show that $\mathbf{A}^*\mathbf{A}$ is nonsingular if and only if $\mathbf{A}$ has full rank.

(b) Let $\mathbf{P} \in \mathbb{C}^{m\times m}$ be a nonzero projector. Show that $\|\mathbf{P}\|_2 \geq 1$. In addition, show that if $\mathbf{P}$ is an orthogonal projector, then $\|\mathbf{P}\|_2 = 1$.

Question 4 [43503/53903] Compute the reduced and full QR factorization of the following matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Question 5 [53903]

(a) Let $\mathbf{A}$ be an $m \times n$ matrix ($m \geq n$), and let $\mathbf{A} = \widehat{\mathbf{Q}}\widehat{\mathbf{R}}$ be a reduced QR factorization. Show that $\mathbf{A}$ has rank n if and only if all the diagonal entries of $\widehat{\mathbf{R}}$ are nonzero.

(b) Implement your own code to compute the QR factorization of the following matrix and compare your result with MATLAB build-in function.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{pmatrix}.$$