ILRST/STSCI 2100 Discussion 4: Ch. 6 Probability

Probability: Numerical quantity that expresses the likelihood of an event.
Can be thought of the long-term relative frequency of an event happening.

Sample space: All possible outcomes for an experiment.
Ex. What is the sample space of flipping two coins?

Event: Collection of outcomes with a designated feature. It is a subset of the sample space.
Ex. When flipping two coins, what are the possible events of getting at least one tail?

Can use a tree diagram to keep track of sample space and events.

Subset: Denoted by $\subset$
Ex. If $B = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$ and $C = \{6, 7, 8\}$, is $A \subset B$? Is $C \subset B$?

Complement: Denoted by A^C, any event such that A does not occur
Ex. Suppose A is the event of flipping heads, then what is A^C?

Union: Given two sets A and B, $A \cup B$ are the elements that fall in either A, B or both.
Ex. A = {apple, orange, banana} and B = {orange, watermelon, pear}. What is $A \cup B$?

Intersection: Given two sets A and B, $A \cap B$ are the elements that fall in both A and B.
Ex. With A and B given above, what is $A \cap B$?

With numbers, if $A = \{1, 4, 5, 7, 8, 9\}$ and $B = \{1, 2, 6, 7, 10\}$, then,
$A \cup B = \{1, 2, 4, 5, 6, 7, 8, 9, 10\}$
$A \cap B = \{1, 7\}$
$A \cap B^C = \{4, 5, 8, 9\}$
$A^C \cap B = \{2, 6, 10\}$
Notice that $A \cap B + A \cap B^C = A$ and similarly, $A \cap B + A^C \cap B = B$.

Can also show graphically with Venn Diagrams

Probability Axioms

1. $P(A) > 0$

2. $P(S) = 1$
   Ex. Suppose $P(A) = 0.3$, $P(B) = 0.2$, and $P(C) = 0.4$. Are A, B, and C the only elements in the set?
3. $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_n) = P(A_1) + P(A_2) + ... + P(A_n)$, assuming that no two events can happen at the same time.

Implications

1. Complement Rule: $P(A) = 1 - P(A^C)$
   Ex. When rolling a weighted die, the probability of it landing a "2" is 1/3. What is the probability of not landing a "2"?
2. Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
   Ex. If $P(A^C) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, then what is P(A or B)?

Conditional Probability
"Probability of A given B"
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
Multiplication rule of probability: $P(A \cap B) = P(B) * P(A|B) = P(A) * P(B|A)$

Can be visualized using a Venn Diagram
Applicable for contingency tables