ILRST/STSCI 2100: Discussion 5: Ch. 6 Probability (Review) and Ch. 7 Random Variable

Review from last week:

Addition rule: P(A or B) = P(A) + P(B)–P(A and B)

Multiplicative rule: P(A and B) = P(A|B) $\times$ P(B) = P(B|A) $\times$ P(A)

Complement: $P(A^c) = 1 - P(A)$

Conditional probability: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Some helpful "formulas" - can derive with drawing Venn Diagrams:

$P(A) = P(A \text{ and } B^c) + P(A \text{ and } B)$, similarly, $P(B) = P(A^c \text{ and } B) + P(A \text{ and } B)$

P(A or B or C) = P(A) + P(B) + P(C) - P(A and B) - P(A and C) - P(B and C) + P(A and B and C)

Independence : P(A|B) = P(A) and P(B|A) = P(B)

Note: The multiplication rule: $P(A \cap B) = P(A|B) * P(B)$

So $P(A \cap B) = P(A) * P(B)$ when A and B are independent.

Errors typically arise when you assume A and B are independent and do calculations as if they are independent events. Always assume dependence, unless you have shown otherwise.

Note: Independent events vs mutually exclusive events:

- Mutually exclusive events are not independent
  - Flipping a head and a tail are mutually exclusive (they can't both happen at the same time). So, you if flip a head, then you know that you didn't flip a tail. That is, prior knowledge (head), helped you make a statement about probability of flipping tail, meaning these are not independent events.
- Independent events cannot be mutually exclusive
  - If two events are independent, $P(A \cap B) = P(A) * P(B)$ is nonzero, implying both can happen at the same time, meaning not mutually exclusive.
- Exception, independent events appear mutually exclusive when P(A) or P(B) is 0.

Random Variables

Either discrete (countable) or continuous (any number on the number line)

Denoted with a capital letter, ex. "X"

Random variables are associated with a probability function

Discrete: probability mass function (pmf)

f(x) = P(X = x) denotes the probability of observing x

$0 \leq f(x) \leq 1$ and $\sum_x f(x) = 1$

Ex. Let X be a random variable that is the number of tails when you flip 3 coins. What is the p.m.f of X?

Continuous: probability density function (p.d.f)

$f(x) \geq 0$ for all x

$\int_{-\infty}^{+\infty} f(x)dx = 1$: area under the entire curve is 1.

For a continuous (only!) variable, $P(a \leq X \leq b) = P(a < X < b)$, can be thought of as the area under the curve bounded by a range. And this is because $P(X = x) = 0$ for a continuous variable.

Ex. What is the probability that a car on the road is going 65 mph?

Expected value

The center/mean of a random variable X, $E(X) = \mu$, in the discrete case, this is: $E(X) = \sum_x xf(x) = \mu$

Variance of a discrete random variable: $Var(X) = \sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 f(x)$ Given the following p.m.f, find the variance:

Standard Deviation = Square root of variance

Practice Problems: Given $P(A) = 0.5$; $P(B) = 0.2$; $P(C) = 0.2$

1. Are A, B, and C the only elements in the sample space?
2. If A and B are independent, what is $P(A \cap B)$?
3. If A and B are mutually exclusive, what is $P(A \cup B)$?
4. Now suppose $P(A \cap B) = 0.2$. What is $P(A|B)$?

Answers:

1. No, we can find $P(A)+P(B)+P(C) = 0.9$, which isn't equal to one, so there must be more element(s) in the sample space.
2. Since A and B are independent, $P(A \cap B) = P(A) * P(B) = 0.5 * 0.2 = 0.1$
3. Using the general additional rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Since A and B are mutually exclusive, $P(A \cup B) = 0$, so $P(A \cup B) = P(A) + P(B) - 0 = 0.2 + 0.5 - 0 = 0.7$.
4. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.2} = 1$