Problem 1 ***Exercise 3.1***

Solution: *a)*

$$\begin{aligned}\exists x\exists y(Pxu \wedge Pyv)\frac{u,u,u}{x,y,v} &= \exists x\exists y[(Pxu \wedge Pyv)\frac{u}{v}] \\ &= \exists x\exists y(Pxu \wedge Pyu)\end{aligned}$$

b)

$$\begin{aligned}\exists x\exists y(Pxu \wedge Pyv)\frac{v,fuv}{u,v} &= \exists x\exists y[(Pxu \wedge Pyv)\frac{v,fuv}{u,v}] \\ &= \exists x\exists y(Pxu\frac{v}{u} \wedge Pyv\frac{fuv}{v}) \\ &= \exists x\exists y(Pxv \wedge Pyfuv)\end{aligned}$$

c)

$$\begin{aligned}\exists x\exists y(Pxu \wedge Pyv)\frac{u,x,fuv}{x,u,v} &= \exists w\exists y[(Pwu \wedge Pyv)\frac{x,fuv}{u,v}] \\ &= \exists w\exists y(Pwu\frac{x}{u} \wedge Pyv\frac{fuv}{v}) \\ &= \exists w\exists y(Pwx \wedge Pyfuv)\end{aligned}$$

d)

$$\begin{aligned}&[\forall x\exists y(Pxy \wedge Pxu) \vee \exists ufuu \equiv x]\frac{x,fxy}{x,u} \\ =&[\forall x\exists y(Pxy \wedge Pxu)\frac{x,fxy}{x,u}] \vee [\exists ufuu \equiv x\frac{x,fxy}{x,u}] \\ =&[\forall v\exists w(Pvw \wedge Pvu)\frac{fxy}{u}] \vee \exists ufuu \equiv x \\ =&\forall v\exists w(Pvw \wedge Pvfxy) \vee \exists ufuu \equiv x\end{aligned}$$