TD4_FFT_Python_Corrige

October 29, 2020

1 TD 4

1.1 Transformée de Fourrier

- Soyez certain d'avoir installé les packages nympy, matplotlib, scipy et skimage (pour la derniere question optionnelle)
- Créer un Jupyter notebook pour le TD d'aujourd'hui
- Téléchargez les fichiers SingingGlass.wav et Rotating_Superfluid.tif de la page Github du cour

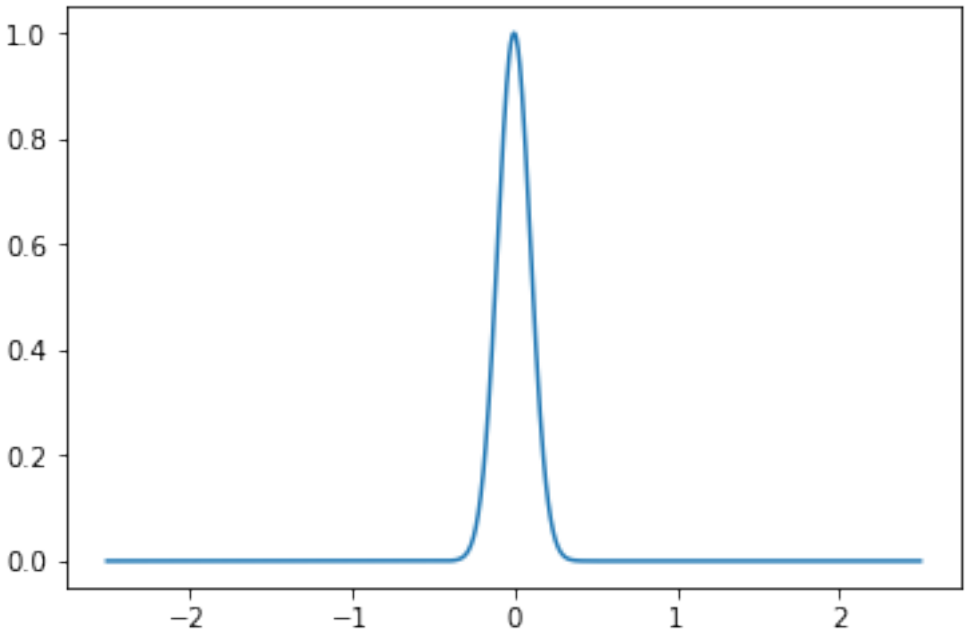
[1]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
from numpy.fft import fft, ifft, rfft, irfft, rfftfreq, fftfreq
```

1.1.1 1 exemple de DFT

[35]:
```
def f(sigma,L,N):
    Delta_x = L/N
    x = Delta_x*np.arange(N)-L/2
    y = np.exp(-x**2/(2*sigma**2))
    return x, y
```

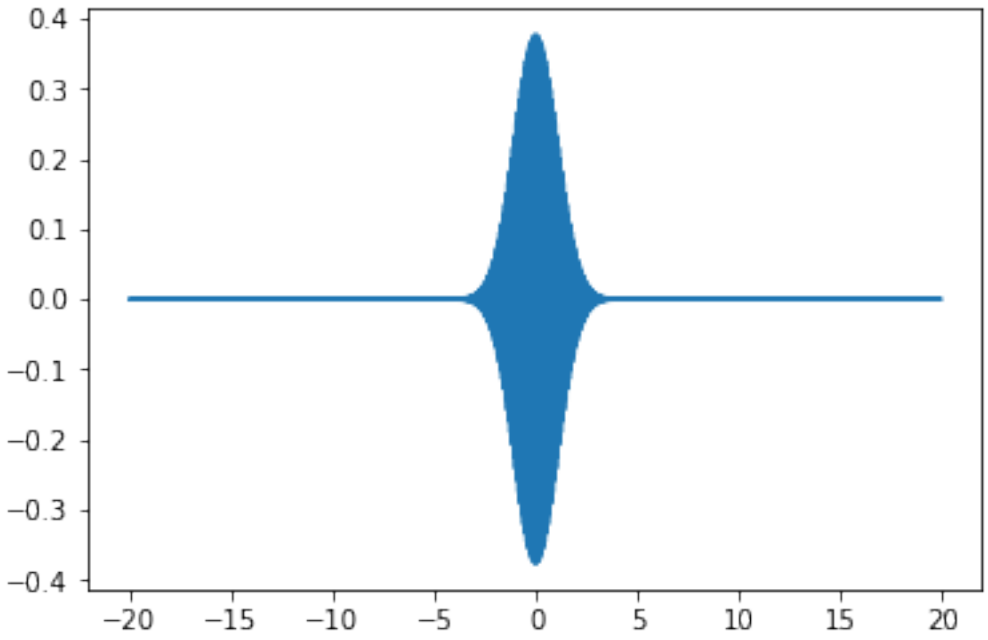
[36]:
```
L=5
N=1000
sigma=0.1
x, y = f(sigma, L, N)
plt.plot(x, y)
```

[36]:
```
[<matplotlib.lines.Line2D at 0x7fe23c7e9350>]
```



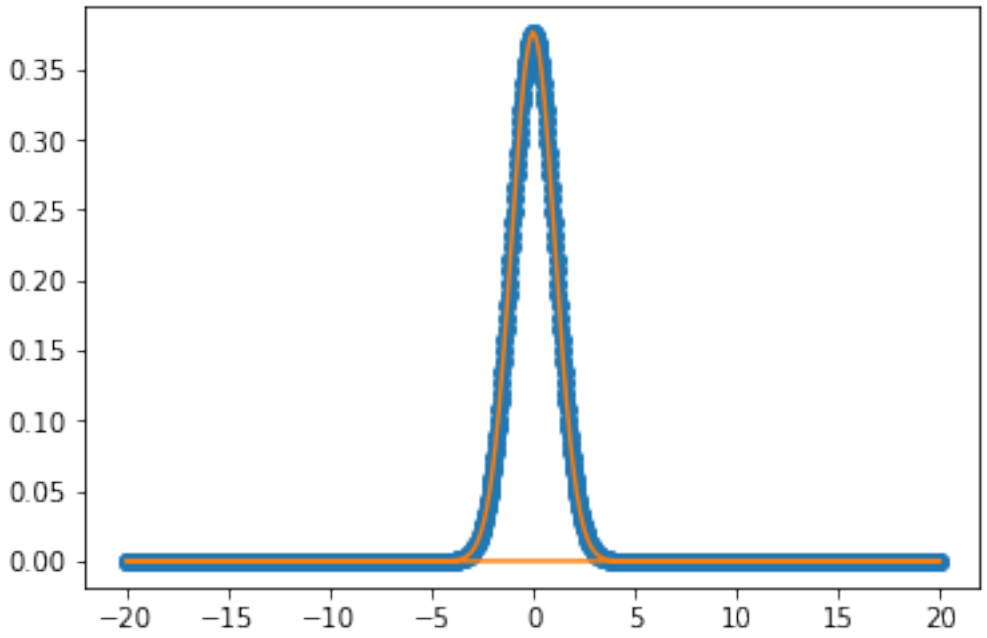
[55]:
```
L = 1/0.04
N=int(2*20*L) # f_max = dx/2
sigma=0.15
x, y = f(sigma,L,N)

f_tilde = fft(y)
f_axis = fftfreq(N, L/N)
plt.plot(f_axis, f_tilde*L/N)

# La TF présente des valeurs négatives car elle n'est pas centrée en j=0 mais en␣
 ↪j=N/2
# La phase vaut exp(2ipi N/2 k/N) donc \pm 1
```

[55]:
```
[<matplotlib.lines.Line2D at 0x7fe238bba750>]
```



[56]:
```
f_tilde = fft(y)
f_axis = fftfreq(N, L/N)
plt.plot(f_axis, np.abs(f_tilde)*L/N, 'o')

plt.plot(f_axis, np.sqrt(2*np.pi)*sigma*np.exp(-(2*np.pi*f_axis)**2*sigma**2/2))
#plt.xlim(-50, 50)
```

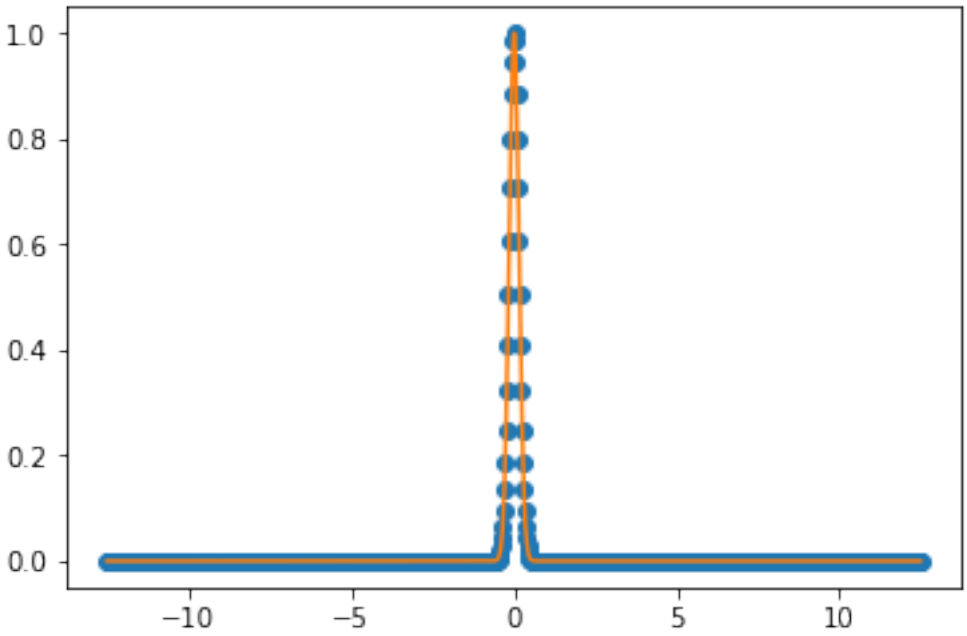
[56]:
```
[<matplotlib.lines.Line2D at 0x7fe238a2ab10>]
```



[57]:
```
signal_initial = ifft(f_tilde)
plt.plot(x, signal_initial, 'o')
plt.plot(x, y)
```

```
/dd_int/anaconda3.7/lib/python3.7/site-packages/numpy/core/_asarray.py:85:
ComplexWarning: Casting complex values to real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
```

[57]:
```
[<matplotlib.lines.Line2D at 0x7fe2390c0490>]
```

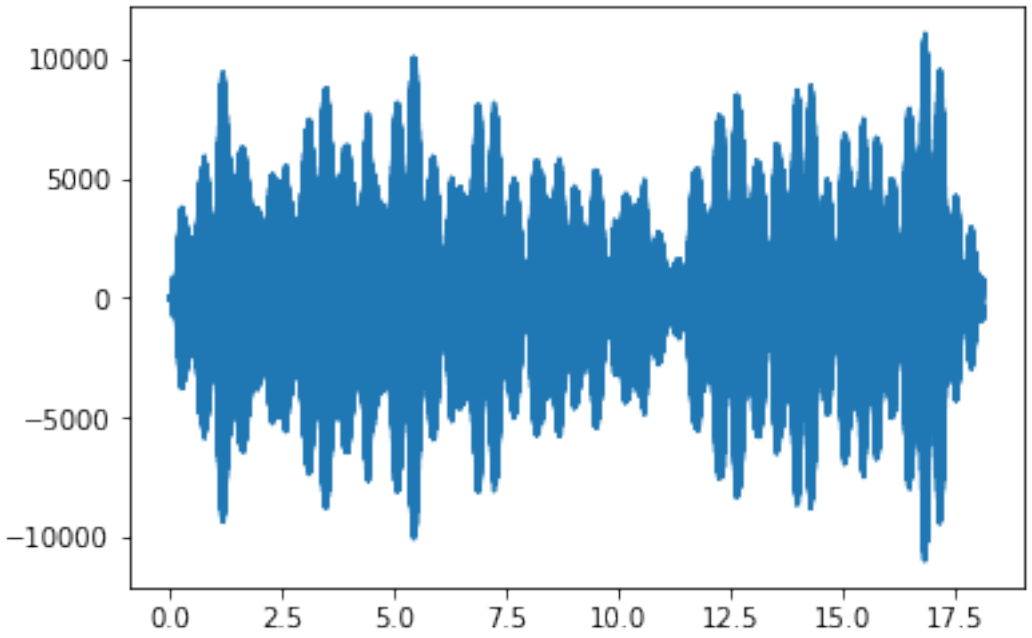


1.1.2 2 Les verres chantants

[60]:
```
from scipy.io.wavfile import read
Fs , amplitude = read('SingingGlass.wav')
N = len(amplitude)
print('N =',N)
dt = 1/Fs
print('dt = ', dt,'s')
T = dt*N
print('T =',T,'s')
time = np.arange(N)*dt
plt.plot(time,amplitude)
```

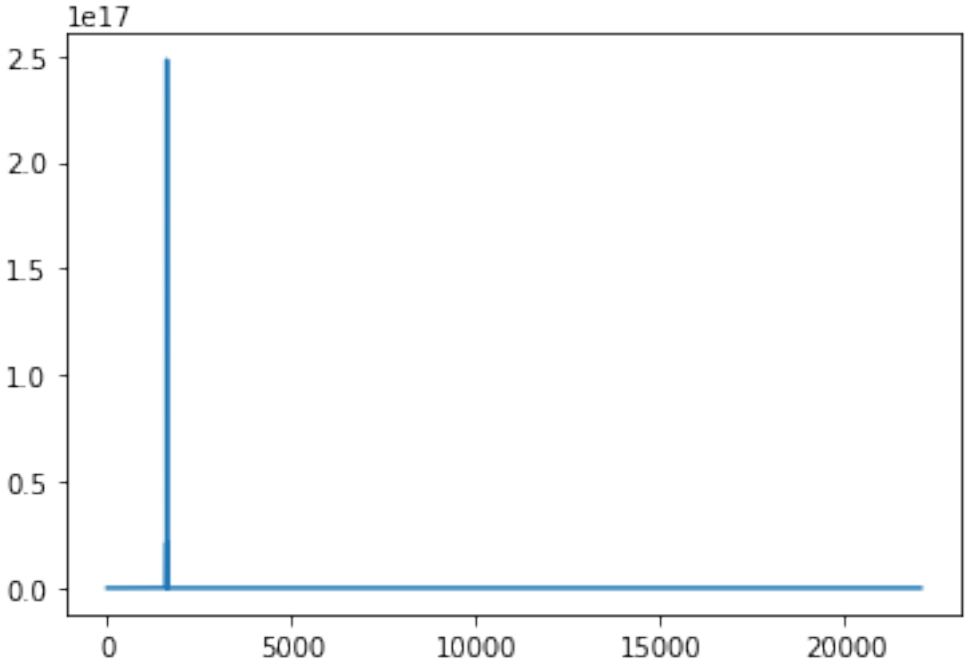
```
N = 800001
dt =  2.2675736961451248e-05 s
T = 18.14061224489796 s
```

[60]: `[<matplotlib.lines.Line2D at 0x7fe238058b50>]`



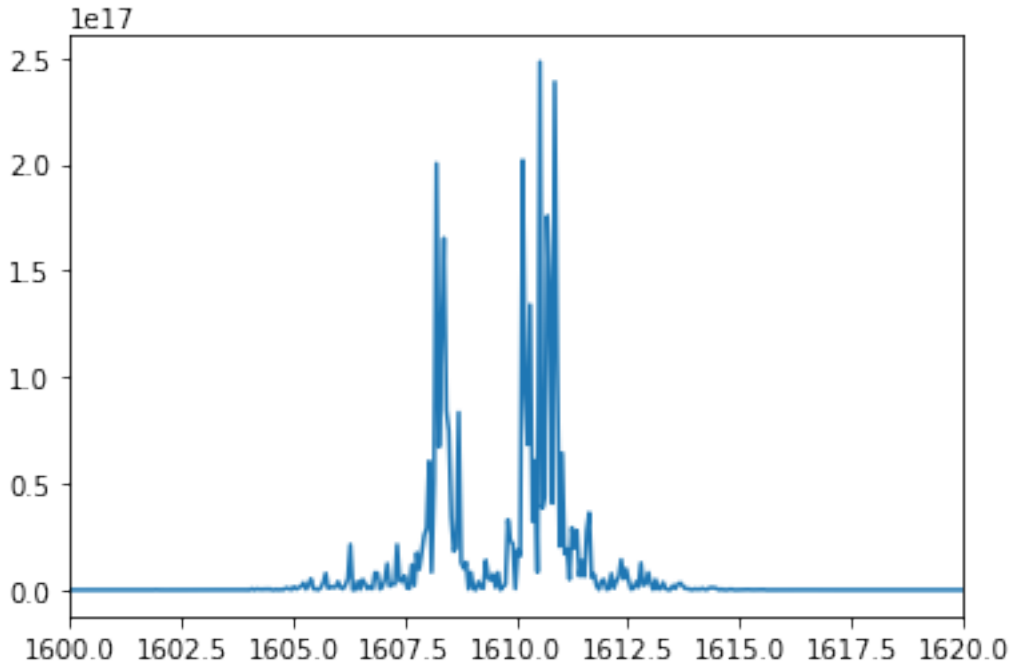
[61]:
```
frequency = rfft(amplitude)
freqs = rfftfreq(N, dt)
plt.plot(freqs, np.abs(frequency)**2, '')
```

[61]: [<matplotlib.lines.Line2D at 0x7fe225a73e50>]



[62]:
```
plt.plot(freqs, np.abs(frequency)**2, '')
plt.xlim(1600, 1620)
```

[62]: (1600, 1620)



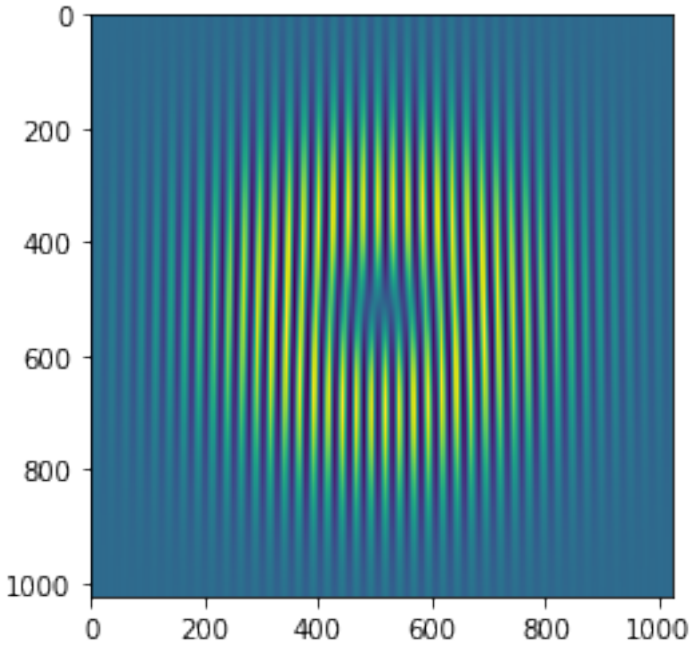
1.2 4 - Application 2D : récupérer la phase d'une fonction d'onde

[66]:
```
from numpy.fft import fft2, ifft2
```

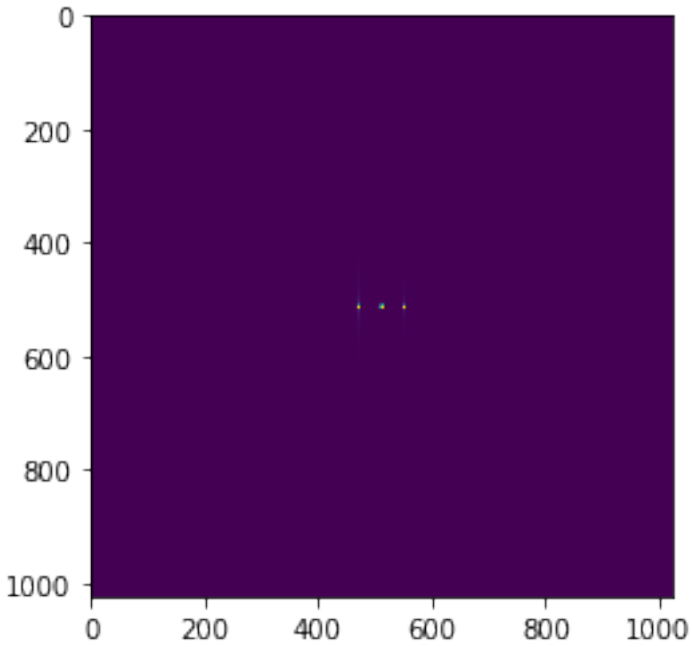
[67]:
```
def Laguerre_Gauss(x, y, l, w):
    r = np.sqrt(x**2+y**2)
    phi = np.arctan2(y,x)
    Laguerre = np.exp(1J*phi*l)*np.exp(-r**2/w**2)*(r*np.sqrt(2)/w)**abs(l)
    return Laguerre

def plane_wave(x,y,kx,ky):
    return 1.0*np.exp(1J*(kx*x + ky*y))

w0 = 1
L=4*w0
N=1024
```

```
x0 = np.arange(N)*L/N - L/2
x,y = np.meshgrid(x0,x0)

kx = 2*np.pi/(0.1)
ky = 0

l = 1

signal = Laguerre_Gauss(x, y, l, w0)
reference = plane_wave(x,y,kx,ky)

interf = np.abs(signal+reference)**2
plt.imshow(interf)
```

[67]: <matplotlib.image.AxesImage at 0x7fe225996450>



[87]:
```
interf_tilde = np.fft.fftshift(fft2(interf))
plt.imshow(np.abs(interf_tilde), vmin=0, vmax=1E4)
plt.show()
```



[ ]:

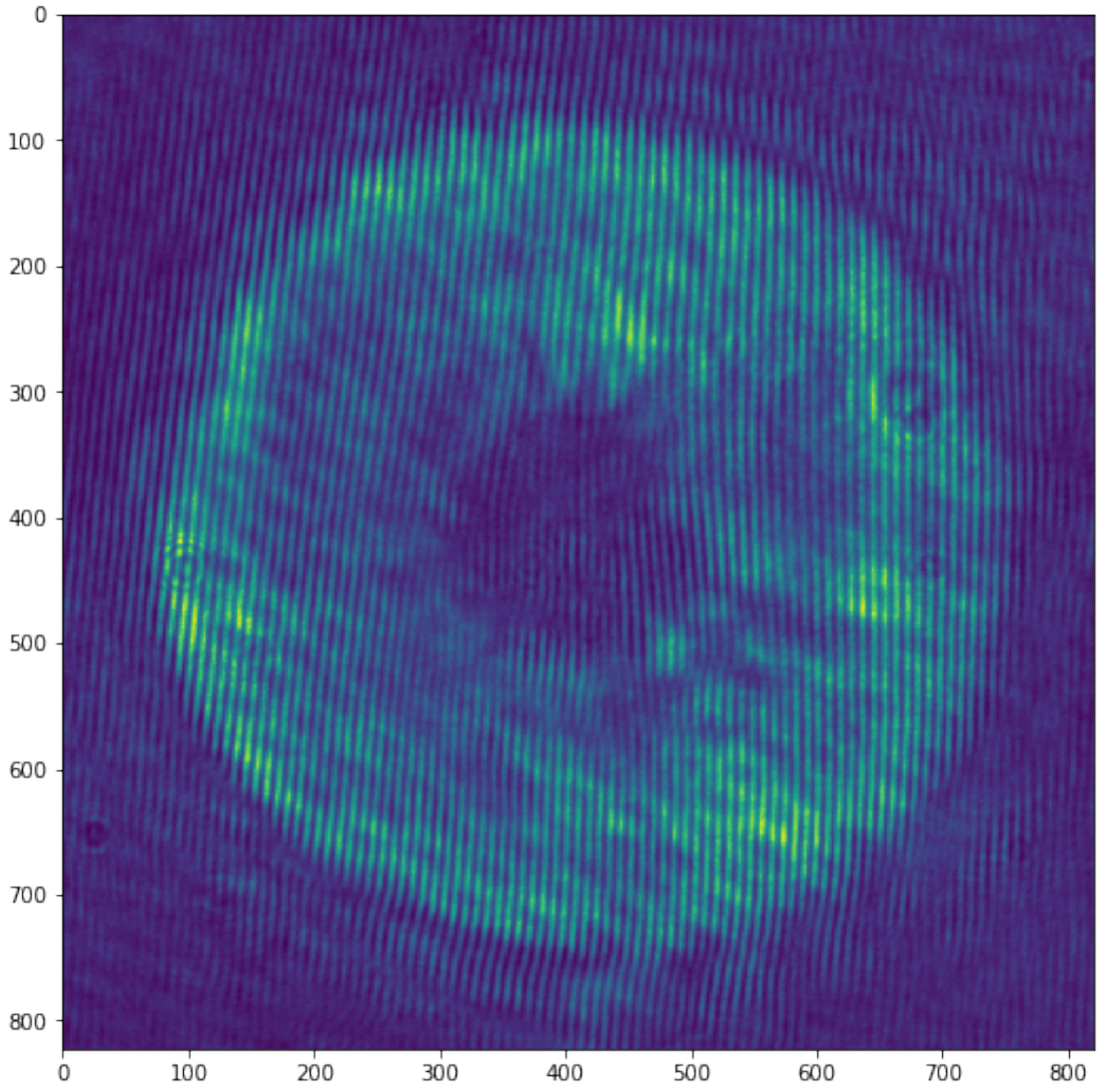
[ ]:

[ ]:

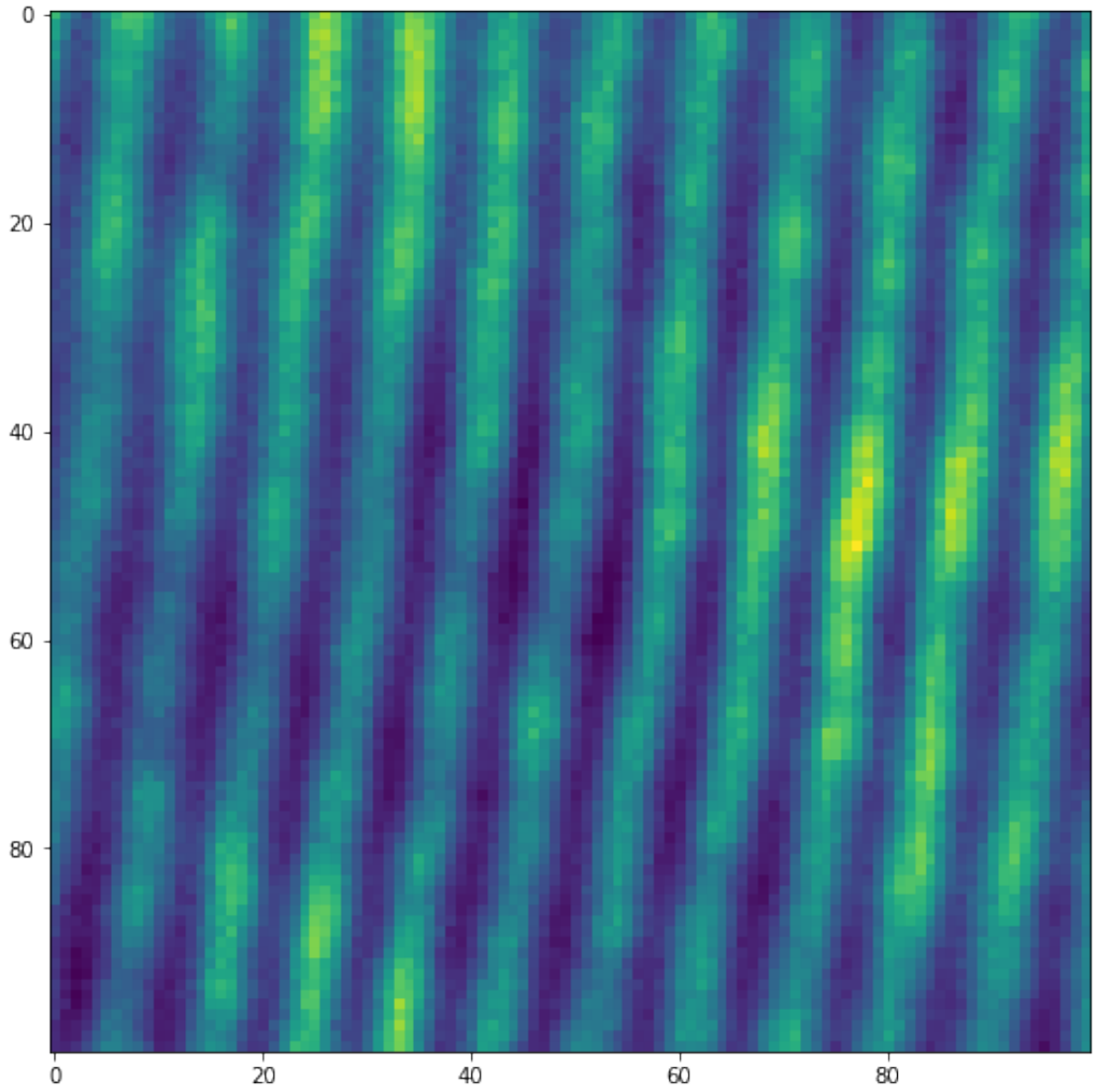
[ ]:

[89]:
```
plt.rcParams["figure.figsize"] = (15,9)
```

[90]:
```
import matplotlib.image
image = matplotlib.image.imread("Rotating_Superfluid.tif")
plt.imshow(image)
plt.show()
```

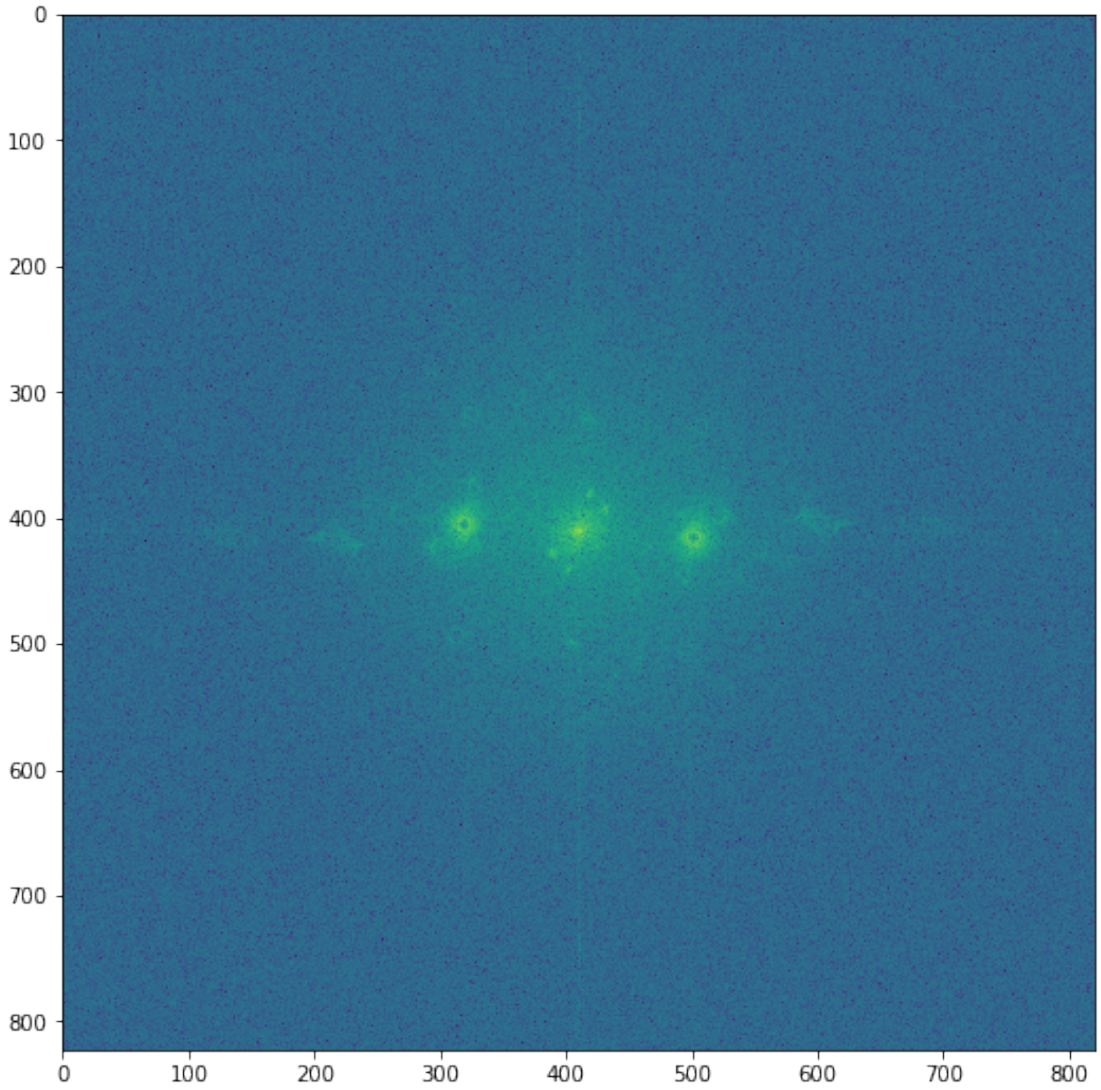


[180]:
```
plt.imshow(image[200:300,200:300])
plt.show()
```



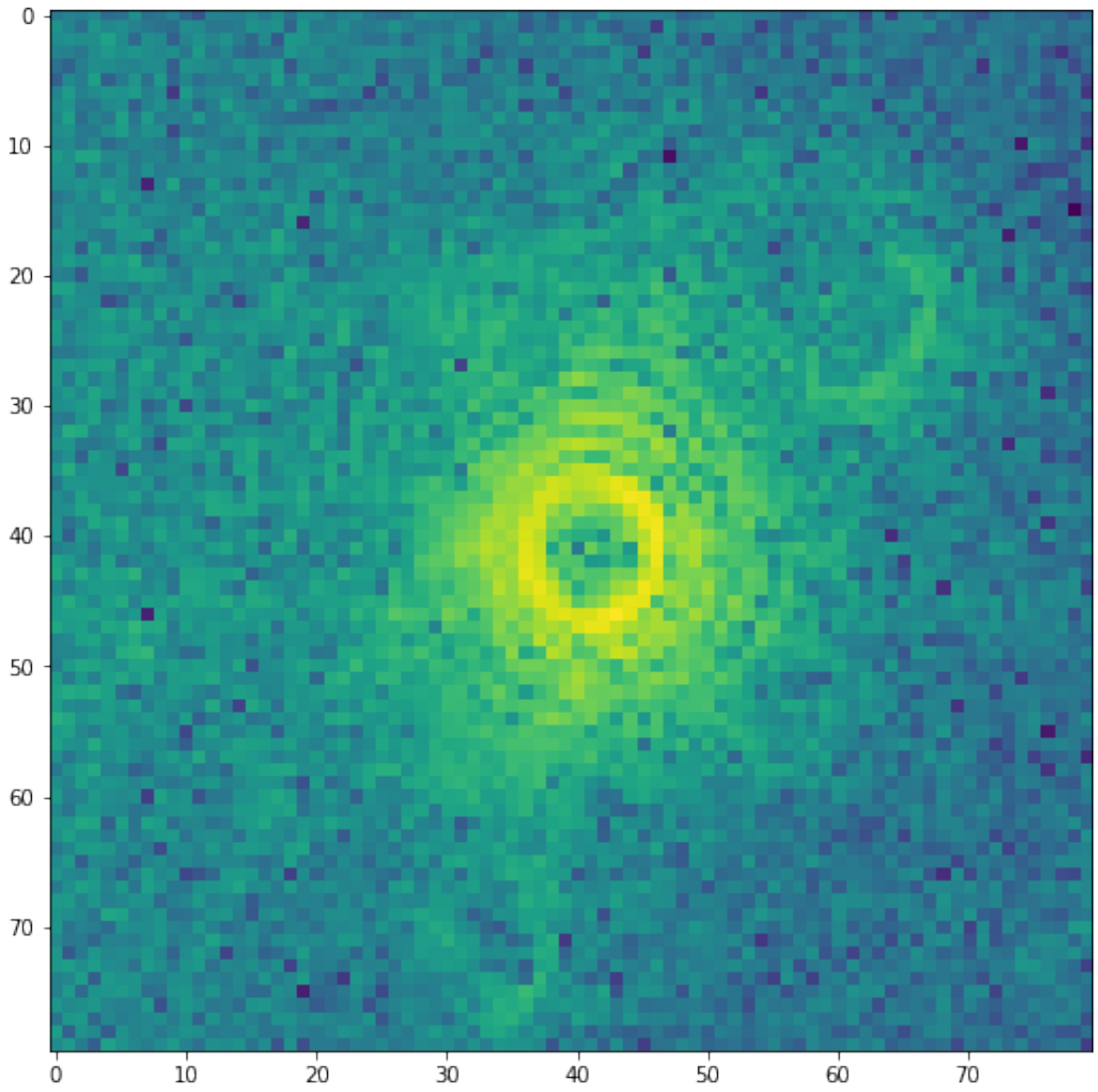
[91]:
```
image_tilde = np.fft.fftshift(fft2(image))
plt.imshow(np.log(np.abs(image_tilde)))
```

[91]: <matplotlib.image.AxesImage at 0x7fe223addfd0>



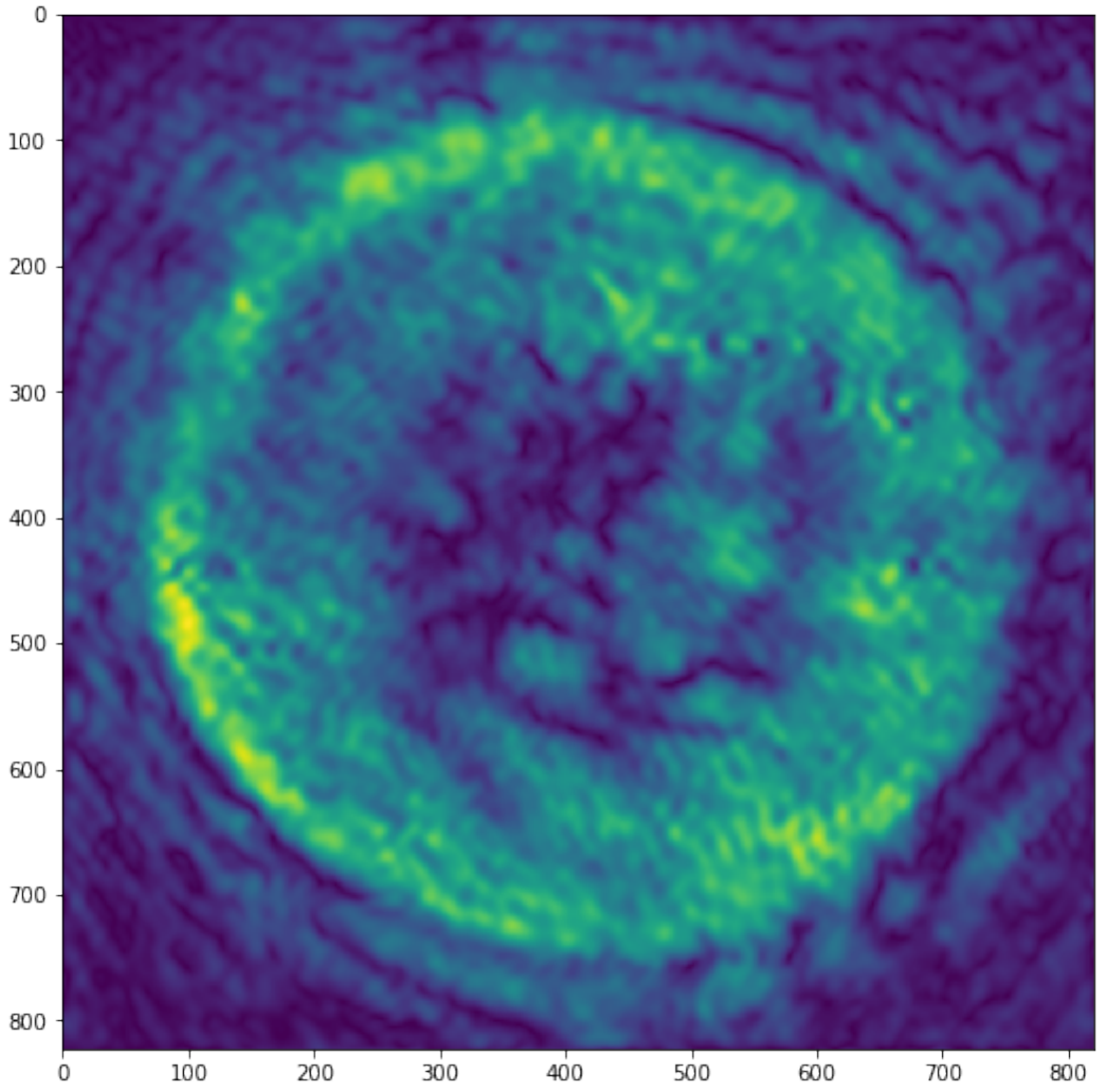
[99]:
```
Lx, Ly = image_tilde.shape

center_x, center_y = (415,500)
dx = dy = 40
signal = image_tilde[center_x-dx : center_x+dx,
                     center_y-dy : center_y+dy]

reconstructed_signal_tilde = np.zeros((Lx, Ly), dtype='complex128')

reconstructed_signal_tilde[Lx//2-dx : Lx//2+dx , Ly//2-dy : Ly//2+dy] = signal
```

[100]:
```
plt.imshow(np.log(np.abs(signal)))
plt.show()
```

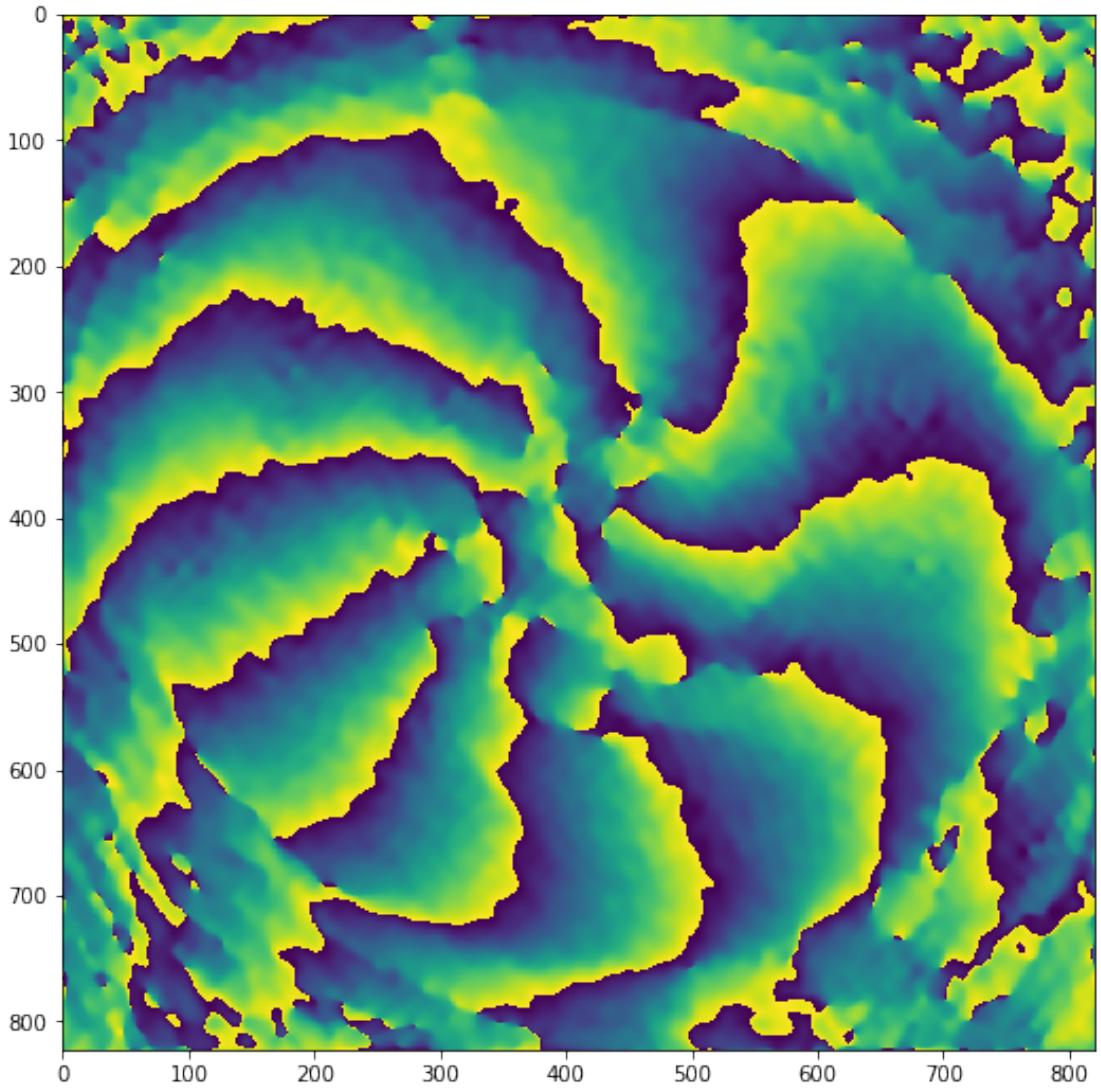


[101]:
```
reconstructed_signal = ifft2(np.fft.ifftshift(reconstructed_signal_tilde))
amplitude = np.abs(reconstructed_signal)
phase = np.angle(reconstructed_signal)
```

[102]:
```
plt.imshow(amplitude)
plt.show()
```

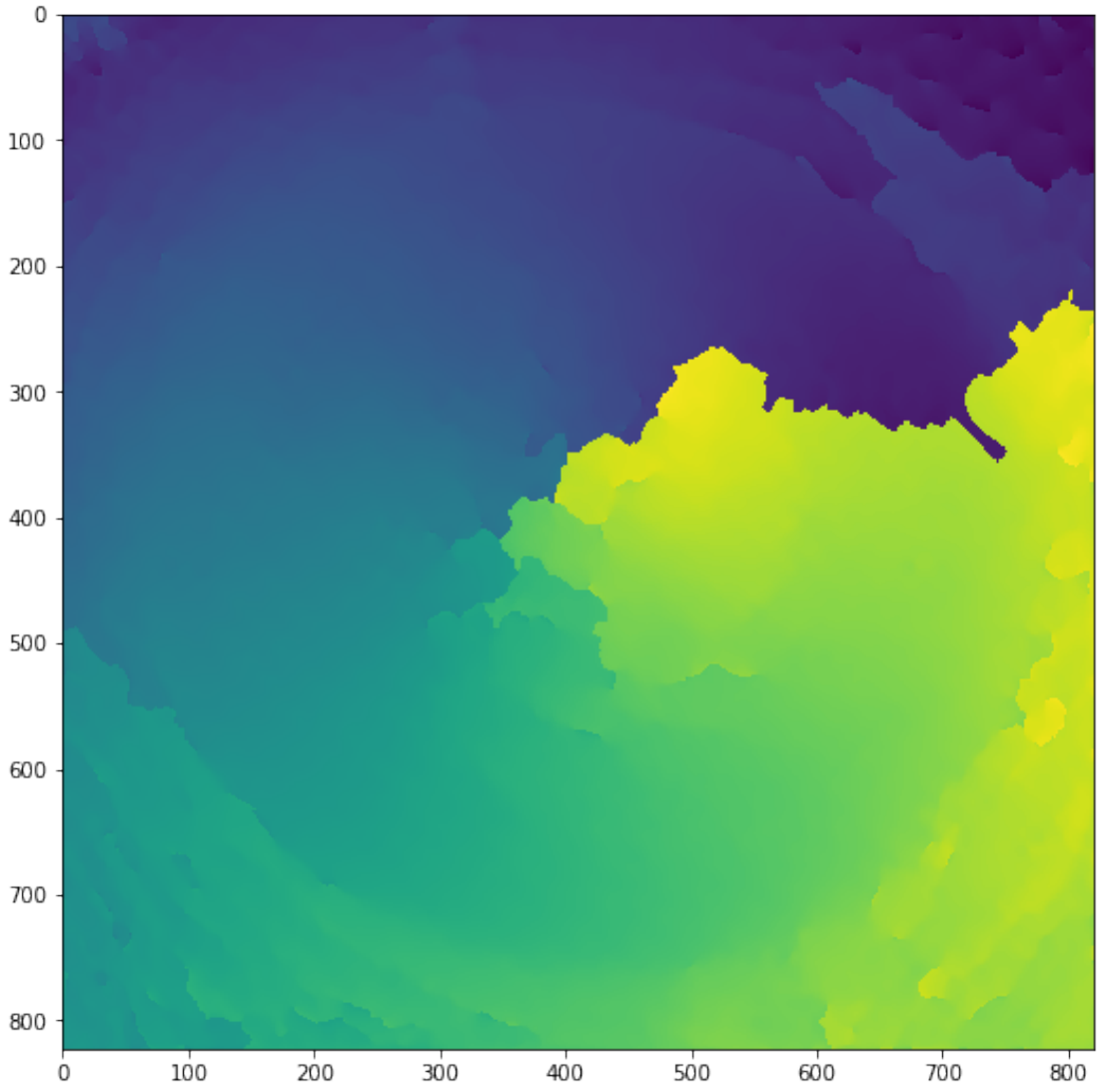


[105]:
```
plt.imshow(phase)
plt.show()
```



1.2.1 Optionnel, requiert le package skimage

[106]:
```
from skimage.restoration import unwrap_phase
phase_unwrapped = unwrap_phase(phase)
plt.imshow(phase_unwrapped)
plt.show()
```



[107]:
```
phase_winding = np.amax(phase_unwrapped)
winding_number = round(phase_winding/(2*np.pi))
print(f"This vortex has a topological charge of {winding_number}")
```

```
This vortex has a topological charge of 10.0
```

[ ]:
```

```