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(1) $M = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{bmatrix}$

$\quad\quad\quad v_1 \quad\quad v_2$

a) $\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} = \lambda\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

$\frac{1}{\sqrt{5}} = \lambda \frac{2}{\sqrt{5}} \rightarrow \lambda = \frac{1}{2}$

$\frac{2}{\sqrt{5}} = \lambda\left(\frac{-1}{\sqrt{5}}\right) \rightarrow \frac{2}{\sqrt{5}} = \frac{1}{2}\left(\frac{-1}{\sqrt{5}}\right) \rightarrow 2 \neq \frac{-1}{2} \Rightarrow$ (no solution, there exists no λ which follows this)

So, there is no λ so that $v_1 = \lambda v_2$. That implies that these two vectors are linearly independent. If the two row vectors are linearly independent, the matrix rank is 2, which is equivalent to say that M is invertible

b) $\left(\begin{array}{cc|cc} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 1 & 0 \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & 0 & 1 \end{array}\right) \xrightarrow[R_2 = R_2\sqrt{5}]{R_1 = R_1\sqrt{5}} \left(\begin{array}{cc|cc} 1 & 2 & \sqrt{5} & 0 \\ 2 & -1 & 0 & \sqrt{5} \end{array}\right) \rightarrow$

$\xrightarrow{R_1 = R_1 + 2R_2} \left(\begin{array}{cc|cc} 5 & 0 & \sqrt{5} & 2\sqrt{5} \\ 2 & -1 & 0 & \sqrt{5} \end{array}\right) \xrightarrow{R_1 = R_1 \cdot \frac{1}{5}} \left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \\ 2 & -1 & 0 & \sqrt{5} \end{array}\right) \rightarrow$

$\xrightarrow{R_2 = R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \\ 0 & -1 & \frac{-2\sqrt{5}}{5} & \sqrt{5} - \frac{4\sqrt{5}}{5} \end{array}\right) \xrightarrow{R_2 = R_2(-1)} \cancel{\left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \\ 0 & 1 & \frac{2\sqrt{5}}{5} & 4\sqrt{5} \end{array}\right)}$

$R_2 = R_2(-1)$

$$\longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \\ 0 & 1 & \frac{2\sqrt{5}}{5} & \frac{4\sqrt{5}}{5} - \frac{5\sqrt{5}}{5} \end{array}\right) = \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 0 & 1 & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{array}\right)$$

$\underbrace{\qquad}_{M^{-1}}$

c) $\det(M) = \frac{1}{\sqrt{5}}\left(\frac{-1}{\sqrt{5}}\right) - \left(\frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}\right) = -\frac{1}{5} - \frac{4}{5} = \frac{-5}{5} = -1$

So $\det(M^{-1}) = -1$ because $M^{-1} = M$

2) $\det(AB) = \det(A)\det(B)$

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

a) Recursive definition of a determinant $\begin{cases} a_{11} \text{ if } n = 1 \\ \text{choose } 1 \leq j \leq n \\ \sum (-1)^{i+j} a_{ij} \det(A_{-ij}) \end{cases}$

$D = \mathrm{diag}(\lambda_1, \dots, \lambda_n)$
$\det(D) = \lambda_1 \cdot \dots \cdot \lambda_n$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Without loss of generality $\rightarrow \det(D) = (-1)^{1+1}\lambda_1 \det(D_{-11}) + (-1)^{1+2} \cdot 0 \cdot \det(D_{-12}) + (-1)^{1+3} \cdot 0 \cdot \det(D_{-13}) = \lambda_1 \det(D_{-11}) + (-1)^3 \cdot 0 \cdot \det(D_{-12}) + (-1)^4 \cdot 0 \cdot \det(D_{-13}) = \lambda_1 \det(D_{-11}) + 0 + 0 = \lambda_1 (-1)^{1+1} \lambda_2 \det(D_{-11(-22)}) + 0 + 0 = \lambda_1 \lambda_2 (-1)^{1+1} \lambda_3 = \lambda_1 \lambda_2 \lambda_3$

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(2)

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$$\det(I) = 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 1$$

or $\det(I) = \lambda_1 \lambda_2 \lambda_3 = 1.1.1 = 1$ as I is a diagonal matrix.

c)

$$T = \begin{bmatrix} a_{11} & b_1 & b_2 \\ 0 & a_{22} & b_3 \\ 0 & 0 & a_{33} \end{bmatrix} \rightarrow \det(T) = a_{11}\begin{vmatrix} a_{22} & b_3 \\ 0 & a_{33} \end{vmatrix} +$$

$$0 \cdot \begin{vmatrix} b_1 & b_2 \\ 0 & a_{33} \end{vmatrix} + 0\begin{vmatrix} b_1 & b_2 \\ a_{22} & b_3 \end{vmatrix} = a_{11}\left(a_{22} \cdot a_{33} + 0 \cdot b_3\right) =$$

$$= a_{11}\ a_{22}\ a_{33}$$

Because if in order to compute the matrix of an upper triangular matrix we take the first columns, only the first element would be non-zero, so only the submatrix determinant multiplied by the a_{11} element will be computed. The same will happen consecutively for each a_{ii} element at each submatrix, until we get to ~~...~~ $M_{1\times 1}$, which is the base case and gives ann as a result by definition. Thus, the global result would be:

$$\det(T) = \prod_{i=1}^{n} a_{ii}$$

d) $\det(AB) = \det(A) \cdot \det(B)$

$$A \cdot B = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 7 \\ 0 & 2 & -6 \\ 0 & 0 & -14 \end{bmatrix}$$

$$\det(AB) = 2 \begin{vmatrix} 2 & -6 \\ 0 & -14 \end{vmatrix} \mp 0 \begin{vmatrix} 1 & 7 \\ 0 & -14 \end{vmatrix} + 0 \begin{vmatrix} 1 & 7 \\ 2 & -6 \end{vmatrix} =$$

$$2(-28) = \boxed{-56}$$

$$\det(A) = 1 \begin{vmatrix} 2 & 2 \\ 0 & 7 \end{vmatrix} + 0 \begin{vmatrix} 0 & -3 \\ 0 & 7 \end{vmatrix} + 0 \begin{vmatrix} 0 & -3 \\ 2 & 2 \end{vmatrix} =$$

$$1 \cdot 14 = 14$$

$$\det(B) = 2 \begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix} + 0 \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} + 0 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} =$$

$$2(-2) = -4$$

$$\det(A) = 14 \ ; \ \det(B) = -4 \ ; \ \det(A) \cdot \det(B) = 14(-4) = \boxed{-56}$$

e) $A \cdot A^{-1} = A^{-1} \cdot A = I$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

<u>Proof</u> $\det(A) = \det(A)$; $\det(\overbrace{\underbrace{A^{-1} \cdot A}_{I} \cdot A}^{A}) = \det(A)$; $\det(A^{-1}) \cdot \det(A \cdot A)$ $= \det(A)$; $\det(A^{-1}) = \det(A) / \det(A \cdot A)$; $\det(A^{-1}) = \dfrac{\cancel{\det(A)}}{\cancel{\det(A)} \cdot \det(A)}$

$\therefore \det(A^{-1}) = \dfrac{1}{\det(A)}$

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(3) $A = \begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$f_A = \mathbb{R}^3 \to \mathbb{R}^3$

$f_A(v) = Av$

a) $\det(A) = (-1)^{3+3} \cdot 1 \cdot \begin{vmatrix} -4 & 2 \\ 2 & -1 \end{vmatrix} = (-1)^6 \cdot 1 \cdot (4-4) = 0$

$\det(A_{11}) = -1 \cdot 1 - 0 \cdot 1 = -1 \neq 0$

So rank = n of the highest squared matrix which gives $\det \neq 0 \Rightarrow$ rank = 2 (of A)

b) $\{e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)\} = B \in \mathbb{R}^3$

$$f(e_1) = \underset{3\times 3}{\begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} -4 \\ 2 \\ 0 \end{bmatrix}}$$

$$f(e_2) = \begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$f(e_3) = \begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

c) $S \subset \mathbb{R}^3$

Although $f(e_1)$, $f(e_2)$ and $f(e_3) = \langle f(e_1), f(e_2), f(e_3) \rangle$ they don't form a basis as they are not linearly independent because $f(e_1) = 2f(e_3) - 2f(e_2)$.
So, a basis of the vector subspace generated by $\langle f(e_1), f(e_2), f(e_3) \rangle$ could be $\mathbb{B} = \{f(e_2), f(e_3)\}$

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$$A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} \qquad A^t = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

3×2 $\qquad$ 2×3

a) $A A^t \Rightarrow$ By definition, the multiplication of matrices takes a matrix $M_{n \times K}$ and multiplies it to a $M_{K \times m}$, giving rise to a $M_{n \times m}$ matrix. Because A is a $M_{3\times 2}$ and A^t a $M_{2 \times 3}$, $A \cdot A^t$ would result in a $M_{3 \times 3}$

The same reasoning applies to $A^t \cdot A$, giving rise to a $M_{2\times 2}$ matrix.

b) $$A \cdot A^t = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 \\ -3 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A^t A = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$$

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c) Yes, both AA^t and A^tA are symmetric

as $\begin{array}{c} t\begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix} \\ v_1 \quad v_2 \end{array}$ is $\begin{array}{cc} \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \end{array} \end{array}$ in the case of A^tA

and $\begin{array}{c} t\begin{bmatrix} 2 & -3 & 0 \\ -3 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix} \\ v_1 \quad v_2 \quad v_3 \end{array}$ is $\begin{array}{cc} \begin{bmatrix} 2 & -3 & 0 \\ -3 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \end{array}$ in $A.A^t$

5

$x = (x_1, \dots, x_n)$; $y = (y_1, \dots, y_n)$

$$E(x) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$Var(x) = \frac{1}{n} \sum_{i=1}^{n} \left(x_i - E(x)\right)^2$$

$$Cov\,(x, y) = \frac{1}{n} \sum_{i=1}^{n} \left(x_i - E(x)\right)\left(y_i - E(y)\right)$$

	x	y	z
Sample 1	-1	3	5
Sample 2	0	6	3
sample 3	1	9	1

a) $E(x) = \frac{-1+0+1}{3} = 0$; $E(y) = \frac{3+6+9}{3} = 6$;

$E(z) = \frac{5+3+1}{3} = 3$

b) $Var(X) = \dfrac{(-1-0)^2 + (0-0)^2 + (1-0)^2}{3} = 2/3$

$Var(Y) = \dfrac{(3-6)^2 + (6-6)^2 + (9-6)^2}{3} = 18/3 = 6$

$Var(Z) = \dfrac{(5-3)^2 + (3-3)^2 + (1-3)^2}{3} = 8/3$

c) $Cov(X,Y) = \dfrac{((-1-0)(3-6) + (0-0)(6-6) + (1-0)(9-6))}{3} =$

$= \left(\dfrac{(-1)(-3) + 1 \cdot 3}{3}\right) = 2$

d) Let $1 = \underset{3\times 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$. Compute $1 \cdot 1^t$

$1^t = \underset{1\times 3}{[1, 1, 1]}$; $1 \cdot 1^t = M_{3\times 3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

e) $Y = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ and define $C_Y = Y - E(Y)\,1$

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5) e) Compute $E(C_Y)$; $C_Y = Y - E(Y) \cdot 1$

$$E(Y) = \frac{3+6+9}{3} = 6 \; ; \; 1 = \underset{3\times 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \; ; \; E(Y)\cdot 1 = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$Y = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \; ; \; C_Y = Y - E(Y)\cdot 1 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} - \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}$$

$$E(C_Y) = \frac{-3+0+3}{3} = 0$$

f) $C_3 = I_3 - \frac{1}{3} 1 1^t$; $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $1 1^t = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$;

$$\frac{1}{3} 1 1^t = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \; ; \; C_3 = I_3 - \frac{1}{3} 1 1^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} =$$

$$= \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$$

Check that $C_3 Y = C_Y$

$$C_3 = \underset{3\times 3}{\begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}} \cdot \underset{3\times 1}{\overset{Y}{\overset{\shortparallel}{\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}}}} = \underset{3\times 1}{\begin{bmatrix} \frac{2\cdot 3}{3} - \frac{6}{3} - \frac{9}{3} \\ -\frac{3}{3} + \frac{2\cdot 6}{3} - \frac{9}{3} \\ -3/3 - 6/3 + \frac{2\cdot 9}{3} \end{bmatrix}} =$$

$$\begin{bmatrix} -9/3 \\ 0 \\ 9/3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = C_Y$$

It is known as a centering matrix of $\mathbb{R}_3$ because what it does is make the center of mass of vectors $\in \mathbb{R}_3$ be located at the 0 point.
a

That means that $E\left((_3 \cdot v(\in \mathbb{R}^3)\right) = 0$.

As $C_3 Y = C_Y$ we can see how C_3 achieves that easily.

$$C_3 \cdot Y = C_Y ; \left(I_3 - \frac{1}{3} 11^t\right) Y = Y - E(Y) \cdot 1 ; I_3 Y - \frac{1}{3} 11^t Y = Y - E(Y) \cdot 1$$

$$\frac{1}{3} 11^t Y = E(Y) \cdot 1 \xrightarrow[\frac{1}{3} 11^t Y = 1 \frac{1}{3} 1^t Y]{E(Y) \cdot 1 = 1 \cdot E(Y) \to \lambda} 1 E(Y) = 1 \left(\frac{1}{3} 1^t Y\right)$$

So we can see that when multiplying a $v \in \mathbb{R}_3$ by C_3 we are just substracting the mean $\left(1 \frac{1}{3} 1^t Y\right)$, representing $1^t Y$ the summation, $\frac{1}{3}$ the $\frac{1}{n}$ component and the 1 matrix "to do so for each component of the vector") to the vector $(I_3 Y = Y)$ thus giving rise to a matrix with the same variance but displaced exactly μ units so that $E(Y) = 0$.

g)

$$C_3 = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} ; \quad A = \begin{bmatrix} -1 & 3 \\ 0 & 6 \\ 1 & 9 \end{bmatrix}$$

3x3 3x2

$$B = C_3 A = \begin{bmatrix} -2/3 - 1/3 & 2 - 2 - 3 \\ 1/3 - 1/3 & -1 + 4 - 3 \\ 1/3 + 2/3 & -1 - 2 + 6 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 0 & 0 \\ 1 & 3 \end{bmatrix}$$

(5) s) So, the effect that C_3 has in A is having the dummy data X and Y, centered in 0.
means

h) $w = \frac{1}{3} B^t B$

$$\underset{3\times 2}{\begin{bmatrix} -1 & -3 \\ 0 & 0 \\ 1 & 3 \end{bmatrix}} = B \quad ; \quad \underset{2\times 3}{\begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \end{bmatrix}} = B^t$$

Var(X)

$$B^t B = \underset{2\times 2}{M} = \begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix} \rightarrow B^t \cdot B \cdot \frac{1}{3} = \begin{bmatrix} \frac{2}{3} & 2 \\ 2 & 6 \end{bmatrix}$$

Cov(X,Y) Var(Y)

~~Test~~

Yes, the entries in the main diagonal correspond to the Cov(X,X) and Cov(Y,Y) or in different words the Var(X) and the Var(Y) and the other diagonal corresponds to Cov(X,Y) = Cov(Y,X) = 2.

(6) $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n$

$\|v\| = \sqrt{v^t v}$

a) $v = (1,1,0) \rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $v^t = [1\ 1\ 0]$
$w = (1,2,0) \rightarrow \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$; $w^t = [1\ 2\ 0]$

~~a)~~ $||v|| = \sqrt{v^t v}$; $[1\ 1\ 0] \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = [1+1+0] = 2$

~~$||\alpha v|| = 1$~~ $\quad ||v|| = \sqrt{2}$

$||w|| = \sqrt{w^t w}$; $[1\ 2\ 0] \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = [1+4+0] = 5$

$||w|| = \sqrt{5}$

b) $||\alpha v|| = 1$

$||v|| = \sqrt{2}$

$||\alpha v|| = \alpha\, ||v|| = 1 \rightarrow \alpha = \frac{1}{||v||} = \frac{1}{\sqrt{2}}$

$v_2^t = v^t \cdot \alpha = [1/\sqrt{2}\ \ 1/\sqrt{2}\ \ 0]$

$v_2 = v \cdot \alpha = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

$||v|| = \sqrt{v^t \cdot v}$

$v_2^t \cdot v_2 = \left[\frac{1}{2} + \frac{1}{2} + 0\right] = [1] \rightarrow \sqrt{1} = 1$

$||\alpha w|| = 1$

$||w|| = \sqrt{5}$

$||\alpha w|| = \alpha\, ||w|| = 1 \rightarrow \alpha = \frac{1}{||w||} = \frac{1}{\sqrt{5}}$

$w_2^t = w^t \cdot \alpha = \left[\frac{1}{\sqrt{5}}\ \ \frac{2}{\sqrt{5}}\ \ 0\right]$, $w_2 = w \cdot \alpha = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix}$

$||w|| = \sqrt{w^t \cdot w}$

$w_2^t \cdot w_2 = \left[\frac{1}{5} + \frac{4}{5} + 0\right] = [1] \rightarrow \sqrt{1} = 1$

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c) $v^t w = w^t v$; $v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $w = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$v^t . w$ gives $v_{11} w_{11} + v_{12} w_{21} + \dots + v_{1n} w_{n1}$ and $w^t v$ gives $w_{11} v_{11} + w_{12} v_{21} + \dots + w_{1n} v_{n1}$ which is the same number

d) $1a - 2c = 0$

$1a = 2c$

$\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $\cancel{\begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

are l.i that are orthogonal